Mathematics

Stage 6

Calculus-based Courses

Draft Writing Brief

2007

Responses to the Mathematics Stage 6 Calculus-based Courses Draft Writing Brief can be made on the associated survey form as follows:

a) written response to:
   Mathematics Curriculum Support Officer
   Office of the Board of Studies NSW
   GPO Box 5300
   SYDNEY NSW 2001

b) email response to david.howe@bos.nsw.edu.au

c) online response through the Board’s website at:
   www.boardofstudies.nsw.edu.au

Consultation period: 1 May 2007 to 1 June 2007

Note: A copy of this document can also be found on the Board’s website at
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1 Draft Writing Brief
1 Draft Writing Brief

Preparation of a writing brief takes place in Phase 2 of the Board of Studies’ syllabus development process (see page 8).

1.1 Purpose

This draft writing brief provides the proposed ‘blueprint’ for the new Mathematics Stage 6 calculus-based syllabuses: Mathematics Advanced/Mathematics Extension 1 Syllabus, and Mathematics Extension 2 Syllabus. (A further draft writing brief has been developed for the new Mathematics Stage 6 non-calculus-based syllabus and is also available for consultation.)

Following consultation, the writing brief will be finalised to provide the detailed blueprint for the development of the calculus-based syllabuses, against which the final syllabuses will be judged.

The final syllabus documents will be developed during Phase 3 of the syllabus development process.

1.2 Structure

The draft writing brief is structured according to the elements of the courses (see Section 5).

The draft writing brief covers the Mathematics Stage 6 calculus-based courses: Mathematics Advanced, Mathematics Extension 1, and Mathematics Extension 2. Responses to the draft writing brief could focus on one or two of these courses, or all three courses.

1.3 For your information

Sections 1 to 3 of this document provide background reading on the development of the draft writing brief. Sections 4, 5 and 6 provide details of the structure of the Mathematics Stage 6 courses, the place of the courses in the K–12 curriculum, and proposed elements of the Mathematics Stage 6 calculus-based courses. In Sections 5, 6, 7 and 8 the following icons are used to assist your reading and response:

<table>
<thead>
<tr>
<th>i</th>
<th>for your information</th>
<th>The information following this icon is general information, in the form of definitions or other items, that assists in reading or understanding the draft writing brief.</th>
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</thead>
<tbody>
<tr>
<td>☑</td>
<td>consult</td>
<td>The material following this icon is material on which responses and views are sought.</td>
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</table>

1.4 Audience

Teachers and the wider community are invited to read the draft writing brief and to comment on the directions it provides to the syllabus writers.
1.5 Consultation

There will be several methods of consultation in relation to the writing brief:
• teacher focus groups will be conducted
• meetings and discussions will be held with key groups
• a survey will be published with the draft writing brief (in hard copy and on the Board’s website) to enable responses from schools and interest groups.

1.6 Matters for consideration

‘Matters for Consideration’ contains questions on a number of issues. Please use your responses to these questions to assist you when completing the survey that accompanies the draft writing brief.

1.7 How to respond to the draft writing brief

Survey for the Mathematics Stage 6 Calculus-based Courses Draft Writing Brief

The purpose of the survey is to obtain detailed comments from individuals and systems/organisations on the Mathematics Stage 6 Calculus-based Courses Draft Writing Brief.

Please comment on both the strengths and the weaknesses of the draft writing brief.

Comments made by you and others will be taken into account when the draft writing brief is amended.

The final version of the writing brief will provide directions to writers of the syllabuses for the Mathematics Stage 6 calculus-based courses.

Schools, teachers and other interested individuals and organisations can respond to the consultation document on the associated survey form as follows:

(1) written response to:

Mathematics Curriculum Support Officer
Office of the Board of Studies NSW
GPO Box 5300
SYDNEY 2001

OR

(2) email response to david.howe@bos.nsw.edu.au

OR

(3) online response through the Board’s website at:

www.boardofstudies.nsw.edu.au

The consultation period is 1 May 2007 to 1 June 2007.
2 Syllabus Development
2 Syllabus Development

2.1 Mathematics Stage 6 syllabus development process

Syllabuses in NSW are developed in accordance with the Board of Studies’ syllabus development process. This process is detailed in the Board’s Syllabus Development Handbook (July 2006), which is available on the Board’s website at www.boardofstudies.nsw.edu.au manuals/#syl_develop_process.

The first phase, Syllabus Review, in the development of new Mathematics Stage 6 syllabuses, is now complete. This phase involved several key data-collection strategies, including oral and written submissions, a symposium, a survey of a sample of schools, and a literature and curriculum review.

**SYLLABUS REVIEW**

**Phase 1**

March 2006–December 2006

This involved:

- consultation with teachers and key groups regarding the existing syllabuses and the general directions for syllabus development
- research, including a review of literature and practice in Australia and overseas
- development of the project plan
- information to schools about the syllabus development
- endorsement by the Board of the broad directions for syllabus development.

**WRITING BRIEF DEVELOPMENT**

**Phase 2**


This involves:

- writing teams developing the writing briefs from analysis of research and analysis of consultation input
- distribution of the draft writing briefs (in hard copy and on the Board’s website) to schools and interest groups for comment
- revision of the draft writing briefs in response to consultation input
- checking out the modifications with key interest groups
- endorsement of the writing briefs by the Board.

**SYLLABUS DEVELOPMENT**

**Phase 3**

August 2007–November 2008

This will involve:

- preparation of the draft syllabus packages developed from the writing briefs
- the distribution of the draft packages (in hard copy and on the Board’s website) to schools and interest groups for comment
- revision of the draft syllabus packages in response to consultation input
- endorsement of the syllabus packages by the Board and approval of the syllabuses by the Minister
- handover of the syllabus packages to school systems and distribution to schools.

*Note: The Board’s Syllabus Development Process provides for the new syllabuses to be in schools for at least one year prior to implementation for familiarisation and programming.*

**IMPLEMENTATION**

**Phase 4**

2010

The new syllabuses will be implemented in Year 11 in 2010 and Year 12 in 2011.

During implementation of the syllabuses the Board will:

- collect, collate and analyse data on the use of the syllabuses
- identify and record issues that need to be taken into account in subsequent syllabus revision.
2.2 Timeline for the development of the syllabus packages for Mathematics Stage 6 courses

The syllabus packages for the Mathematics Stage 6 courses will be developed in accordance with:

- the Board’s *Syllabus Development Handbook*, a copy of which is available on the Board’s website
- the syllabus development plan for the Mathematics Stage 6 syllabuses approved by the Board of Studies on 14 December 2004, details of which are set out below. Details of the timeline were published in *Board Bulletin* Vol. 15, No. 2, 2006, and in subsequent bulletins.

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<thead>
<tr>
<th>Steps in the syllabus development process</th>
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<td><strong>Syllabus Review</strong></td>
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<td>• Oral submissions</td>
<td>24–25 May 2006</td>
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<td>• Written submissions</td>
<td>closed 30 June 2006</td>
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<td>• Symposium</td>
<td>19 August 2006</td>
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<td>• Survey of a sample of schools</td>
<td>closed 18 September 2006</td>
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<td>• Literature and curriculum review</td>
<td>August–September 2006</td>
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<td><strong>Writing Brief Development</strong></td>
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<td>• Preparation of draft writing briefs and surveys</td>
<td>January 2007 – April 2007</td>
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<td>• Consultation (5 weeks)</td>
<td>1 May 2007 – 1 June 2007</td>
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<td>• Development of consultation reports and revision of draft writing briefs</td>
<td>June 2007 – July 2007</td>
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<td><strong>Syllabus Development</strong></td>
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<td>• Development of draft syllabuses and surveys</td>
<td>August 2007 – April 2008</td>
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<td>• Consultation (8 weeks)</td>
<td>5 May 2008 – 27 June 2008</td>
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<td>• Development of consultation reports and revision of draft syllabuses</td>
<td>July 2008 – October 2008</td>
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<td>• Distribution of syllabuses</td>
<td>November 2008</td>
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<td>• Development and distribution of specimen examination papers and marking guidelines</td>
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**Note: Support Materials**

The nature of proposed support materials is presented in section 7.
3 Background Information
3. Background Information

3.1 Evaluation and review of the current Stage 6 Mathematics syllabuses

The current Mathematics 2/3 Unit Syllabus – Years 11–12 was introduced in 1983, while the current 4 Unit Mathematics Syllabus was introduced in 1980, with some amendment in 1989. The current General Mathematics course was introduced in 2000 as part of the New Higher School Certificate.

The Board of Studies initiated the revision of the suite of Stage 6 Mathematics courses, following the completion of new K–6 and Years 7–10 Mathematics syllabuses in 2002, to ensure an up-to-date and cohesive K–12 continuum of mathematics teaching and learning in New South Wales that meets the needs of the full range of students.

In 1998, the Board undertook the evaluation of all existing Higher School Certificate (HSC) courses against the specific criteria of the Government’s White Paper Securing Their Future (August 1997). For Stage 6 Mathematics, two evaluation reports were developed: Mathematics 2/3/4 Unit and Non-Calculus-Based Mathematics (incorporates Mathematics in Society and Mathematics in Practice).

Following the release of these reports, the Board endorsed the development of a single, non-calculus-based course (which became General Mathematics) to replace Mathematics in Society and Mathematics in Practice and the maintenance of the 2, 3 and 4 Unit courses for the immediate future, with ‘minimal change to existing content’. The Board also agreed to ‘a longer term, comprehensive review of the present 2/3/4 Unit courses by an appropriately constituted expert committee’.

The General Mathematics syllabus was released to schools in June 1999, with the study of the Preliminary course undertaken for the first time in 2000, and the first HSC examination undertaken in 2001. As an interim arrangement for the introductory years of the new HSC, the Board of Studies maintained the course content, internal assessment arrangements and examination specifications of the 2, 3 and 4 Unit Mathematics courses from the previous HSC. The new HSC standards-referencing procedures that were introduced for all new HSC Board-developed courses were also put in place for these Mathematics courses, which were renamed Mathematics, Mathematics Extension 1 and Mathematics Extension 2 respectively.

In July 2000, the NSW Government announced the review of Years 7–10 Mathematics following its 1999 commitment in Literacy and Numeracy Plan: Focusing on the basics that ‘A new Years 7–10 Mathematics syllabus will be developed to ensure it provides students with the skills, knowledge and understanding required and appropriately prepares students for the New School Certificate and the New Higher School Certificate’. At about this time, the Board of Studies announced the review and development of the K–6 Mathematics syllabus.

The review and development of K–10 Mathematics consequently became the focus of Mathematics curriculum development in 2000 – 2002, with its completion a necessary prerequisite to the ‘longer-term, comprehensive review of the present 2/3/4 Unit courses’ agreed to by the Board in 1998.

With the completion of the new K–6 and Years 7–10 syllabuses at the end of 2002, a plan was established for the review and development of the Stage 6 calculus-based Mathematics
courses. In considering the plan, the Board felt that it would be necessary to review the full Stage 6 Mathematics provision, taking account of the needs of less able students as well as those who would undertake the higher-level calculus-based courses. The Board endorsed the plan for the review and development of the full Stage 6 Mathematics course provision in December 2004.

3.2 Scope of Mathematics Stage 6 review and development

The implementation of the plan in 2006, with the commencement of the Mathematics Stage 6 Review and Development Project, recognised the importance of teachers having appropriate time to focus on the implementation of the new Mathematics Years 7–10 Syllabus, and the value of obtaining feedback following 2004 and 2005, the initial years of implementation.

The first phase of the project, Syllabus Review, commenced in March 2006. The main purpose in undertaking the Syllabus Review phase was to review the existing Mathematics course provision and to establish Broad Directions for revision and development.

A range of strategies was used to gather data in the Syllabus Review phase. The most significant of these were:
- oral submissions
- written submissions
- a survey of a sample of schools
- a symposium
- a literature and curriculum review.

The information obtained through the data-gathering strategies used in the Syllabus Review phase was analysed to identify issues that need to be considered in the revision and development of Stage 6 Mathematics courses. Key findings were synthesised from the data and a set of draft Broad Directions for the revision and development compiled.

The Board’s endorsement of the Broad Directions at its December 2006 meeting represented the conclusion of the Syllabus Review phase of the project. The Broad Directions have guided the development of the draft Writing Briefs for the calculus-based, and non-calculus-based, Stage 6 Mathematics courses.

(The Broad Directions are appended.)
4 Mathematics in Stage 6
4 Mathematics in Stage 6

4.1 Structure of the proposed Mathematics Stage 6 courses

In the review and development of the Mathematics Stage 6 courses, it is proposed that there be five Board-developed Mathematics courses of study for the Higher School Certificate (in increasing order of difficulty): Mathematics General 1, Mathematics General 2 (together referred to as the ‘non-calculus-based courses’), and Mathematics Advanced, Mathematics Extension 1, and Mathematics Extension 2 (together referred to as the ‘calculus-based courses’).

Mathematics General 1 represents an additional non-calculus-based course of study in the suite of Mathematics Stage 6 courses (currently, there is a single, non-calculus-based Stage 6 course, General Mathematics), while Mathematics General 2 will replace the current General Mathematics course (on which it is largely modelled). The Mathematics General 1 course has been included in accordance with the Broad Direction (see section 8) ‘That the set of Stage 6 Mathematics courses include an additional offering to accommodate the purposes of students who wish to study a Board-developed Mathematics course in Stage 6 but who are currently choosing not to, as well as those whose purposes are not accommodated through the study of General Mathematics.’

It is proposed that there be a largely common Preliminary course for the non-calculus-based courses, Preliminary Mathematics General, leading to the HSC Mathematics General 1 and HSC Mathematics General 2 courses.

Mathematics Advanced, which consists of Preliminary Mathematics Advanced and HSC Mathematics Advanced courses, replaces the current Mathematics (‘2 Unit’) course. Mathematics Extension 1 and Mathematics Extension 2 replace the current Mathematics Extension 1 and Mathematics Extension 2 courses. As in the current situation, students studying the HSC Mathematics Extension 1 course, or the HSC Mathematics Extension 1 and (HSC) Mathematics Extension 2 courses, will be required to have studied the Preliminary Mathematics Extension course.

The structure of the proposed Mathematics Stage 6 courses is illustrated in the diagram on the next page.
Proposed non-calculus-based courses

- Preliminary Mathematics General
  - HSC Mathematics General 1
  - HSC Mathematics General 2

Proposed calculus-based courses

- Preliminary Mathematics Advanced
  - HSC Mathematics Advanced
- Preliminary Mathematics Extension
  - HSC Mathematics Extension 1
  - (HSC) Mathematics Extension 2
4.2 Place of the Mathematics Stage 6 courses in the K–12 curriculum and pathways of learning
5 Mathematics Stage 6 Calculus-based Courses

– course elements
5 Mathematics Stage 6 Calculus-based Courses
– course elements

for your information

This section of the draft writing brief is structured to address the course elements for the Mathematics Stage 6 calculus-based courses, Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2:

• Rationale
• Aim
• Objectives
• Content organisation
• Outcomes
• Content
• Use of technology
• Assessment and HSC examination.

5.1 Rationale

for your information

The rationale describes the nature of the subject and relevant courses in broad terms. It explains the place and purpose of the subject and relevant courses in the curriculum.

consult

Mathematics is deeply embedded in modern society. From the numeracy skills required to manage personal finances, to making sense of data in various forms, to leading-edge technologies in the Sciences and Engineering, Mathematics provides the framework for interpreting, analysing and predicting, and the tools for effective participation in an increasingly complex society.

The need to interpret the large volumes of data made available through technology draws on skills in logical thought and the ability to check claims and assumptions in a systematic way. Mathematics is the appropriate training ground for the development of these skills and abilities. The thinking required to enhance further the power and usefulness of technology in real world applications requires advanced mathematical training. The rapid advances in technology experienced in recent years have driven, and been driven by, advances in the discipline of Mathematics.

The development of Mathematics throughout history has been catalysed by its utility in explaining real-world phenomena and its inherent beauty. In this way, the discipline has continued to evolve through a process of observation, conjecture, proof and application. Similarly, effective Mathematics teaching and learning often sees teachers and students involved in a collaborative and interactive process as ‘creators’ of Mathematics, rather than as ‘consumers’ of a static and irrefutable body of knowledge.
The calculus-based Mathematics courses provide the opportunity for students to acquire knowledge, skills and understanding in relation to important mathematical concepts, and applications in a range of contexts, that are appropriate to their continued experience of Mathematics as a coherent, interrelated, interesting and intrinsically valuable study that forms a basis for future learning. Students develop an appreciation of Mathematics as a study with high levels of internal structure that provide opportunities for the development of logical and disciplined thought.

Through the learning experiences within the courses, students are able to progress from a knowledge and understanding of facts, procedures and applications in idealised contexts to facility in the use of mathematical models that situate the Mathematics in context and provide information on the behaviour of real-world systems, and to more advanced generalisations based on deductive and inductive reasoning processes. This involves the development and use of an increasingly sophisticated level of communication and literacy.

The courses provide students with the opportunity to study applications of Mathematics in a range of contexts relevant to contemporary professional practice, including examples from the Sciences, Engineering, Business and Finance.

Matters for Consideration
- Does the rationale adequately describe the nature of Mathematics in broad terms?
- Does the rationale reflect a contemporary view of Mathematics?
- How well does the rationale reflect the purpose of the calculus-based courses?

### 5.2 Aim

For your information

*The aim states the overall purpose of the courses. It indicates the educational benefits that are intended to accrue for students who satisfactorily complete programs of study based on the courses.*

Consult

The calculus-based Mathematics courses are designed to promote the development of knowledge, skills and understanding in relation to important concepts within areas of Mathematics that have applications in an increasing number of contexts. This includes the development of deductive and inductive reasoning and the ability to construct, solve and interpret mathematical models.

Students will learn to use a range of techniques and tools, including relevant technologies, in order to develop solutions to a wide variety of problems relating to their present and future needs and aspirations.

Matter for Consideration

Does the aim adequately describe the overall purpose of the calculus-based courses?
5.3 Objectives

Objectives provide more specific statements of the intent of the courses. They amplify the aim and provide direction to teachers on the teaching and learning process emerging from the course/s. They define in broad terms the knowledge, skills and understanding and values and attitudes fundamental to the key learning area/subject. They act as organisers of the intended outcomes. The same objectives may apply across Stages in a key learning area/subject.

Knowledge, skills and understanding

Students will develop:
- the ability to apply deductive and inductive reasoning, and the use of appropriate language, in the construction of proofs and mathematical arguments
- the ability to use concepts and apply techniques to the solution of problems in series, differential and integral calculus, probability and variation
- the ability to construct, solve and interpret mathematical models in a range of contemporary contexts
- the ability to interpret and communicate Mathematics in a variety of forms.

Values and attitudes

Students will develop:
appreciation of the scope, usefulness, beauty and elegance of Mathematics.

Matters for Consideration

- Do the objectives demonstrate the intention of the calculus-based courses?
- Do the objectives adequately define knowledge, skills and understanding and values and attitudes essential for the calculus-based courses?
5.4 Content organisation

The Content organisation section describes how the course content is to be organised.

The course content is organised into topics in each of the Preliminary and HSC courses for the Mathematics Advanced and Mathematics Extension 1 courses, and in the Mathematics Extension 2 (HSC) course.

Mathematics Extension 1 students (including those students who also study Mathematics Extension 2) study all of the Mathematics Advanced and Mathematics Extension 1 course material. For Mathematics Extension 1 students this means studying a number of separate, additional topics, as well as additional material within topics studied for the Mathematics Advanced course that is beyond the scope of the Mathematics Advanced course. For this reason the Mathematics Advanced and Mathematics Extension 1 courses are presented within a single syllabus, Mathematics Advanced/Mathematics Extension 1 Syllabus.

The separate, additional topics within the Mathematics Advanced/Mathematics Extension 1 Syllabus studied by Mathematics Extension 1 students, as well as the additional material studied by these students within topics studied for the Mathematics Advanced course, will be marked with the symbol §. This will signify that this material is not part of the Mathematics Advanced course and is to be studied by Mathematics Extension 1 (and Mathematics Extension 2) students only. (See section 5.6, pp 29–41.)
Presentation of Content

Course topics are presented in the following format:

**Name of topic**
A brief summary of the content/purpose of the topic.

**Outcomes addressed**
A list of the course outcomes that will be addressed in the study of the topic.

**Students learn and acquire the following knowledge, skills and understanding**
The mathematical content to be addressed in the topic.

**Terminology introduced in this topic**
A list of words and/or phrases that may be new to students, and which may be used in relevant assessment tasks.

**Technology that may be used in support of this topic**
Advice about the nature and use of technology that is appropriate to the topic.

**Applications, considerations, examples**
The provision of examples defining the range and style of applications used to introduce and illustrate the mathematical content of the topic, as well as important considerations for the teaching and learning of the topic.

**Matters for Consideration**
- Does the proposed content organisation assist your understanding of the way the study of the calculus-based courses is to be structured?
- Is the proposed presentation of content appropriate for the calculus-based courses?
## 5.5 Outcomes

Syllabus outcomes express the specific intended student learning that results from the teaching of the course/s. They are derived from the objectives and content. Outcomes provide clear statements of the knowledge, skills and understanding expected to be gained by most students as a result of effective teaching and learning by the end of a Stage.

A proposed set of outcomes for the Mathematics Stage 6 calculus-based courses is presented in the table below. The outcomes are derived from the content of the courses, and together with the content, determine the breadth and depth of study to be undertaken by students.

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<td>Students will develop:</td>
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<td>the ability to apply deductive and inductive reasoning, and the use of appropriate language, in the construction of proofs and mathematical arguments</td>
<td>P1 provides reasoning to support conclusions which are appropriate to the context</td>
<td>H1 constructs arguments to prove and justify results in a variety of contexts</td>
<td>PE1 uses multi-step deductive reasoning to solve problems/prove results in circle geometry</td>
<td>HE1 applies the binomial theorem to probability and the proof of identities</td>
<td>E1 chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings</td>
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<td>PE2 uses ideas in algebra and quadratic functions to prove results and identities</td>
<td>E2 applies algebraic, graphical and calculus techniques to establish proofs of inequalities</td>
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<td>Students will develop:</td>
<td>Mathematics Advanced</td>
<td>Mathematics Extension 1</td>
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<td><strong>Objectives</strong></td>
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<td>HSC Outcomes</td>
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<td><strong>Preliminary Outcomes</strong></td>
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<td><strong>HSC Outcomes</strong></td>
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<td>PE3 uses inductive reasoning in the construction of proofs</td>
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<td><strong>HSC Outcomes</strong></td>
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<td>PE4 uses the geometric representation of functions in the transformation of graphs</td>
<td>HE2 applies differential calculus to problems involving polynomials</td>
<td>E3 uses the relationship between algebraic and geometric representations of complex numbers</td>
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<td>the ability to use concepts and apply techniques to the solution of problems in series, differential and integral calculus, probability and variation</td>
<td>P2 applies the concept of a function and the relationship between its algebraic, numerical and graphical representations</td>
<td>H2 uses the derivative to determine the features of the graph of a function and applies in practical situations</td>
<td>PE5 uses algebraic techniques to solve inequalities involving variable denominators and absolute value functions</td>
<td>HE3 uses the properties of trigonometric functions to derive sum and difference formulae and to solve trigonometric equations</td>
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<td><strong>P3</strong></td>
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<td>HE3 uses the fundamental theorem of algebra and the conjugate root theorem in problems involving complex numbers and polynomials</td>
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<td>uses circular angle measure to solve problems involving area and arc length in circles</td>
<td>H3 uses the features of a graph to deduce information about the derivative and applies in practical situations</td>
<td>PE5 uses algebraic techniques to solve inequalities involving variable denominators and absolute value functions</td>
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<td>Objectives</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>P5</td>
<td>solves problems</td>
<td>H5</td>
<td>PE7</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>involving graphs</td>
<td></td>
<td>HE5</td>
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<tr>
<td></td>
<td>of trigonometric</td>
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<tr>
<td></td>
<td>functions</td>
<td></td>
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</tr>
<tr>
<td>P6</td>
<td>develops understanding</td>
<td></td>
<td>HE6</td>
<td></td>
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<tr>
<td></td>
<td>of the tangent</td>
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<td>as the limiting</td>
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<td>position of the</td>
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<td>secant and</td>
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<td>differentiates</td>
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<td></td>
<td>from first principles</td>
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<tr>
<td>P7</td>
<td>represents</td>
<td>H6</td>
<td>HE7</td>
<td></td>
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<td></td>
<td>inverse functions</td>
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<td>algebraically and</td>
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<tr>
<td></td>
<td>geometrically</td>
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</tr>
<tr>
<td>P8</td>
<td>combines the ideas</td>
<td>H7</td>
<td>HE8</td>
<td></td>
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<tr>
<td></td>
<td>of algebra and</td>
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<td></td>
<td>calculus to</td>
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<td>determine the</td>
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<td></td>
<td>important features</td>
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<td>of the graphs of</td>
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<td></td>
<td>a wide variety of</td>
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<td></td>
<td>functions</td>
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</tbody>
</table>
## Objectives

<table>
<thead>
<tr>
<th>Mathematics Advanced</th>
<th>Mathematics Extension 1</th>
<th>Mathematics Extension 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preliminary Outcomes</td>
<td>HSC Outcomes</td>
<td>Preliminary Outcomes</td>
</tr>
<tr>
<td>Students will develop:</td>
<td>A student:</td>
<td>A student:</td>
</tr>
<tr>
<td><strong>P7</strong> determines the derivative of a function through routine application of the rules of differentiation</td>
<td><strong>H7</strong> standardises normal random variables and solves related probability problems</td>
<td><strong>E8</strong> uses standard techniques to solve first-order and second-order ordinary linear differential equations</td>
</tr>
<tr>
<td><strong>P8</strong> solves problems involving sequences and series</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>P9</strong> uses techniques of integration to calculate areas and volumes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>the ability to construct, solve and interpret mathematical models in a range of contemporary contexts</td>
<td><strong>P10</strong> applies numerical techniques to the counting of ordered and unordered selections</td>
<td><strong>H8</strong> uses the concepts of point and interval estimation and constructs and interprets confidence intervals for the population mean</td>
</tr>
<tr>
<td>Objectives</td>
<td>Mathematics Advanced</td>
<td>Mathematics Extension 1</td>
</tr>
<tr>
<td>------------------------------------------------</td>
<td>----------------------</td>
<td>-------------------------</td>
</tr>
<tr>
<td>Students will develop:</td>
<td>A student:</td>
<td>A student:</td>
</tr>
<tr>
<td>P11 calculates probabilities involving mutually exclusive and independent events and counting methods, in practical situations</td>
<td>H9 applies differential and integral calculus to mathematical modelling situations involving linear motion and exponential growth and decay</td>
<td></td>
</tr>
<tr>
<td>P12 uses linear and quadratic functions in mathematical modelling of practical situations</td>
<td>H10 applies series to the solution of financial problems</td>
<td></td>
</tr>
<tr>
<td>the ability to interpret and communicate Mathematics in a variety of forms</td>
<td>P13 interprets and uses the language of Mathematics in a variety of contexts</td>
<td>H11 communicates using mathematical language, notation, diagrams and graphs</td>
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<td>--------------------------------</td>
<td>-------------------------------------------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>Students will develop:</td>
<td>A student:</td>
<td>A student:</td>
</tr>
<tr>
<td>appreciation of the scope,</td>
<td>P/VA demonstrates confidence in using</td>
<td>H/VA seeks to apply mathematical techniques to problems in a wide range of practical contexts</td>
</tr>
<tr>
<td>usefulness, beauty and</td>
<td>Mathematics to obtain realistic solutions to problems</td>
<td></td>
</tr>
<tr>
<td>elegance of Mathematics</td>
<td>(Values and attitudes)</td>
<td></td>
</tr>
</tbody>
</table>

**Matter for Consideration**
Are the outcomes for the Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 courses, appropriate?
5.6 Content

for your information

**Content** describes the knowledge, skills, understanding and values to be studied and developed by students over a Stage or Stages in a course and the development of processes of learning so that students are encouraged to be effective learners.

* consult

**Note:** In the information presented below, topics shown in *italics* are new topics in the calculus-based courses or are new in the particular course described.

Further information in relation to proposed changes to the content of the current calculus-based courses, in the development of the new Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 courses, is provided at the end of section 5.6, pp 38–41.

**Mathematics Advanced**

Course topics:

- Counting techniques
- Probability
- Real functions and their graphs
- The trigonometric functions
- Differential calculus
- Data analysis
- Sequences and series
- Integral calculus
- Logarithmic and exponential functions
- Mathematical modelling

In accordance with the Board’s ‘Pathways’ provisions, the course topics are designated as Preliminary or HSC topics, or are divided into Preliminary and HSC course components, as follows:

- Counting techniques  (Preliminary)
- Probability (simple and counting methods)  (Preliminary)
- Real functions and their graphs (including modelling with linear and quadratic functions)  (Preliminary)
- The trigonometric functions  (Preliminary)
  - Introduction to trigonometric functions (using circular angle measure)
  - Calculus of trigonometric functions (including applications)  (HSC)
- Differential calculus  (Preliminary)
  - Introduction to differential calculus
  - Geometrical applications of differentiation  (HSC)
• **Data analysis**
  – *Descriptive statistics, variation, the normal distribution* (Preliminary)
  – *Sampling distributions, the Central Limit Theorem, confidence intervals* (HSC)
• **Sequences and series** (Preliminary)
• **Integral calculus**
  – *Introduction to integral calculus* (Preliminary)
  – *Integral calculus – areas and volumes* (HSC)
• **Logarithmic and exponential functions** (HSC)
  (including applications of calculus)
• **Mathematical modelling**
  – *Applications of calculus to motion* (HSC)
  – *Applications of calculus to finance* (HSC)
  (involves applications of series)

The table below illustrates the division of the Mathematics Advanced course into a Preliminary course and an HSC course, each of 120 (indicative) hours of school study, and estimated hours of study for the designated topics. The sequence of topics within the Preliminary and HSC courses has been constructed not only to meet appropriately the needs of students studying the Mathematics Advanced course, but also to meet the needs of students studying the Mathematics Extension 1 course.

### Mathematics Advanced

<table>
<thead>
<tr>
<th>Preliminary Course</th>
<th>Estimated Hours</th>
<th>HSC Course</th>
<th>Estimated Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Counting techniques</em></td>
<td>12</td>
<td><em>Integral calculus – areas and volumes</em></td>
<td>12</td>
</tr>
<tr>
<td><em>Probability (simple and counting methods)</em></td>
<td>12</td>
<td><em>Geometrical applications of differentiation</em></td>
<td>24</td>
</tr>
<tr>
<td><em>Real functions and their graphs</em></td>
<td>22</td>
<td><em>Logarithmic and exponential functions</em></td>
<td>22</td>
</tr>
<tr>
<td>(including modelling with linear and quadratic functions)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Introduction to trigonometric functions (using circular angle measure)</em></td>
<td>18</td>
<td><em>Calculus of trigonometric functions</em></td>
<td>14</td>
</tr>
<tr>
<td><em>Introduction to differential calculus</em></td>
<td>20</td>
<td><em>Sampling distributions, the Central Limit Theorem, confidence intervals</em></td>
<td>12</td>
</tr>
<tr>
<td><em>Descriptive statistics, variation, the normal distribution</em></td>
<td>12</td>
<td><em>Mathematical modelling (applications of calculus to motion)</em></td>
<td>24</td>
</tr>
<tr>
<td><em>Sequences and series</em></td>
<td>16</td>
<td><em>Mathematical modelling</em></td>
<td>12</td>
</tr>
<tr>
<td><em>Introduction to integral calculus</em></td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total indicative hours</strong></td>
<td><strong>120</strong></td>
<td><strong>Total indicative hours</strong></td>
<td><strong>120</strong></td>
</tr>
</tbody>
</table>
Mathematics Extension 1

Course topics:

- Circle geometry
- Further algebra
- Transformations of graphs
- Other inequalities
- Polynomials
- *Elementary difference equations and the discrete logistic growth model*
- Mathematical induction
- The binomial theorem, binomial identities and the binomial probability distribution
- Further trigonometry
- Methods of integration
- Inverse functions
- Further applications of calculus involving *mathematical modelling*
- Normal approximation to the binomial distribution

In accordance with the Board’s ‘Pathways’ provisions, the course topics are designated as Preliminary or HSC topics, or are divided into Preliminary and HSC course components, as follows:

- Circle geometry (Preliminary)
- Further algebra (including sum and product of roots of quadratic equations, quadratic identities) (Preliminary)
- Transformations of graphs (Preliminary)
- Other inequalities (Preliminary)
- Polynomials
  - Polynomial equations, graphs (Preliminary)
  - *Multiple roots of polynomials* (HSC)
- *Elementary difference equations and the discrete logistic growth model* (Preliminary)
- Mathematical induction (series and divisibility only) (Preliminary)
- The binomial theorem, binomial identities and the binomial probability distribution (HSC)
- Further trigonometry (sums and differences, general solutions, auxiliary angle, and angle between two lines) (HSC)
- Methods of integration (including substitution, the primitive of \( \sin^2 x \) and \( \cos^2 x \)) (HSC)
- Inverse functions (including inverse trigonometric functions) (HSC)
- Further applications of calculus involving *mathematical modelling* (including motion, modified growth and decay, and Newton’s method) (HSC)
- *Normal approximation to the binomial distribution* (HSC)
The table below illustrates the division of the \textit{Mathematics Extension 1} course into a Preliminary course and an HSC course, each of 60 (indicative) hours of school study, and estimated hours of study for the designated topics. The sequence of topics within the Preliminary and HSC courses has been constructed not only to meet appropriately the needs of students studying the \textit{Mathematics Extension 1} course, but also to meet the needs of students studying the \textit{Mathematics Extension 2} course.

\begin{center}
\textbf{Mathematics Extension 1}
\end{center}

\begin{table}[h]
\centering
\begin{tabular}{|l|c|l|c|}
\hline
\textbf{Preliminary Course} & \textbf{Estimated Hours} & \textbf{HSC Course} & \textbf{Estimated Hours} \\
\hline
Circle geometry & 12 & The binomial theorem, binomial identities and the binomial probability distribution & 10 \\
\hline
Further algebra (including sum and product of roots of quadratic equations, quadratic identities) & 6 & \textit{Multiple roots of polynomials} & 2 \\
\hline
Transformations of graphs & 4 & Further trigonometry (sums and differences, general solutions, auxiliary angle, and angle between two lines) & 8 \\
\hline
Other inequalities & 6 & Methods of integration (including substitution, the primitive of \( \sin^2 x \) and \( \cos^2 x \)) & 8 \\
\hline
Polynomial equations, graphs & 14 & Inverse functions (including inverse trigonometric functions) & 12 \\
\hline
\textit{Elementary difference equations and the discrete logistic growth model} & 12 & Further applications of calculus involving \textit{mathematical modelling} (including motion, modified growth and decay, and Newton’s method) & 12 \\
\hline
Mathematical induction (series and divisibility only) & 6 & \textit{Normal approximation to the binomial distribution} & 8 \\
\hline
\textbf{Total indicative hours} & \textbf{60} & \textbf{Total indicative hours} & \textbf{60} \\
\hline
\end{tabular}
\end{table}
Mathematics Extension 2

Course topics:

- Further inequalities (including induction with inequalities)
- Complex numbers and polynomials over the complex field
- Graphs
- Integration techniques (including $t$-formulae and partial fractions)
- Volumes
- *First and second order ordinary differential equations and modelling* (including aspects of mechanics, simple harmonic motion)

The Mathematics Extension 2 course consists of an HSC course only, of 60 (indicative) hours of school study.

The table below illustrates a possible sequence of topics, with estimated hours of study, for the course.

### Mathematics Extension 2

<table>
<thead>
<tr>
<th>HSC Course</th>
<th>Hrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Further inequalities (including induction with inequalities)</td>
<td>6</td>
</tr>
<tr>
<td>Complex numbers and polynomials over the complex field</td>
<td>16</td>
</tr>
<tr>
<td>Graphs</td>
<td>8</td>
</tr>
<tr>
<td>Integration techniques (including $t$-formulae and partial fractions)</td>
<td>10</td>
</tr>
<tr>
<td>Volumes</td>
<td>6</td>
</tr>
<tr>
<td><em>First and second order ordinary differential equations and modelling</em> (including aspects of mechanics, simple harmonic motion)</td>
<td>14</td>
</tr>
<tr>
<td><strong>Total indicative hours</strong></td>
<td><strong>60</strong></td>
</tr>
</tbody>
</table>
Sample Topic  (HSC course Mathematics Advanced)

The following topic from the HSC course for Mathematics Advanced is provided to demonstrate:

- the representation of the content that needs to be experienced by:
  - Mathematics Advanced (only) students (all of the content other than that indicated by §)
  - Mathematics Extension 1 students (all of the content including the content indicated by §)
- the intended format of the topics
- the nature of new course material.

Data Analysis: Sampling distributions, the Central Limit Theorem and confidence intervals

The main focus of this unit is the relationship between the sampling distribution for a statistic and the parent population.

Outcomes Addressed

A student:

H7  standardises normal random variables and solves related probability problems

H8  uses the concepts of point and interval estimation and constructs and interprets confidence intervals for the population mean

Students learn and acquire the following knowledge, skills and understanding:

- definition of a sampling distribution (see next page for explanation of underlined terms)
- the Central Limit Theorem and implications
- identifying situations that satisfy ‘large sample’ assumptions
- the relationship between the point estimate and interval estimate for a population parameter
- calculating and interpreting the standard error for the population mean
- § calculating and interpreting the standard error for a population proportion
- calculating and interpreting the margin of error for a confidence interval
- calculating probabilities for the sample mean using the standard error
- § calculating probabilities for the sample proportion using the standard error
- generating large sample confidence intervals for the population mean
- § generating large sample confidence intervals for the population proportion
- calculating the required sample size given the tolerable margin of error
sampling distribution
A sampling distribution for a statistic (population mean or proportion) is the set of all possible values that may result when random samples of size $n$ are drawn repeatedly from the parent population.

Central Limit Theorem
The Central Limit Theorem states that the sums and means of random samples of measurements drawn repeatedly from a population have an approximately normal (bell-shaped) distribution.

point estimate
A single number, calculated from sample data, to estimate an unknown population parameter.

interval estimate
Two numbers, calculated from sample data, between which an unknown population parameter is expected to lie.

population parameter
A numerical, descriptive measure of a population characteristic. (Typical examples are the mean, standard deviation and binomial proportion. Population parameters need to be calculated using data from all members of a population and are usually unknown.)

standard error for the population mean
The standard error for the mean gives a measure of the ‘uncertainty’ associated with using the sample mean as an estimate for the unknown population mean.

standard error for a population proportion
The standard error for the population proportion gives a measure of the ‘uncertainty’ associated with using the sample proportion as an estimate for the unknown binomial population proportion.

margin of error for a confidence interval
A practical upper bound on the error of estimation when using a confidence interval to estimate a population parameter. (In the unlikely event that the confidence interval does not capture the actual population parameter, the error of estimation will be larger than the margin of error.) In practice the margin of error is equal to half the width of the confidence interval.

tolerable margin of error
The margin of error associated with an interval estimate that is within tolerable bounds (in terms of the cost of gathering the information etc). (It is sensible then to be able to determine the minimum sample size required. This reduces in practice to solving an inequation of the form: Margin of error $\leq B$ for the sample size $n$ where $B$ is the maximum allowable error for a given confidence level.)
Sample Topic  (Preliminary course Mathematics Extension 1)

The following information is provided to indicate the nature of new material within the Mathematics Extension 1 course.

**Elementary difference equations and the discrete logistic growth model**

- the contrast between discrete and continuous variables
- situations which require modelling using discrete variables eg financial applications, population growth
- recognising and classifying difference equations (see below for explanation of underlined terms)
- methods of solution for simple first-order and second-order difference equations
- modelling with difference equations
- the discrete logistic growth model
- logistic growth sequences and chaos

**difference equation**

An expression which gives a particular term in a sequence in terms of previous terms is called a difference equation, or recurrence relation.

**discrete logistic growth model**

A model of population growth that takes into account the fact that the growth rate depends on the size of the population at any time. Populations growing according to the logistic model have a maximum possible population size known as the ‘carrying capacity’. A discrete logistic growth model predicts the population size at discrete times, eg at yearly intervals.

**logistic growth sequence**

A sequence of population sizes at discrete times generated by a logistic growth model. Small changes in the model parameters can produce large changes in the behaviour of these sequences and the populations they represent.

Sample Topic  (HSC course Mathematics Extension 1)

The following information is provided to indicate the nature of new material within the Mathematics Extension 1 course.

**Normal approximation to the binomial distribution**

- expected value of a binomial random variable
- rationale for using the normal distribution
- the normal approximation to the binomial distribution
- necessary assumptions for the use of the normal distribution
- confidence intervals for the population proportion
- estimating the required sample size and applications to polling
Sample Topic (Mathematics Extension 2)

The following information is provided to indicate the nature of new material within the Mathematics Extension 2 course.

First-order and Second-order Ordinary Differential Equations and Modelling

Direction Fields
- sketch direction fields for differential equations

First-order Linear Differential Equations
- recognise linear differential equations
- derive the formula for the integrating factor for first-order linear differential equations
- solve first-order linear differential equations using an integrating factor

Modelling with First-order Linear Differential Equations
- use the techniques of solving first-order differential equations to solve problems that occur in the real world

Second-order Linear Differential Equations
- recognise second-order linear differential equations
- find the characteristic equation for homogeneous second-order linear differential equations
- solve homogeneous second-order linear differential equations using the roots of the characteristic equation and given initial conditions
- solve non-homogeneous linear second-order differential equations (in the context of projectile motion with resistance)

Modelling with Second-order Differential Equations
- use the techniques of solving second-order differential equations to solve problems that occur in the real world

Matters for Consideration
- Is the proposed content for the Mathematics Advanced course appropriate?
- Is the proposed content for the Mathematics Extension 1 course appropriate?
- Is the proposed content for the Mathematics Extension 2 course appropriate?
Proposed changes to the content of the current calculus-based courses

Proposed changes to the content of the current Mathematics (‘2 Unit’) course, for the new Mathematics Advanced course, aim to enhance its appeal and effectiveness as a stand-alone course, and thereby make it a more meaningful endpoint to secondary studies in Mathematics. Additions to the content have been accommodated by removing content that does not build on Stage 5 syllabus outcomes, or that is of a level of abstraction unlikely to result in the development of conceptual understanding.

The mathematical rigour of the course has been maintained. Mathematics Extension 1 students who undertake the Mathematics Advanced course as a component of their Mathematics study will be challenged and prepared sufficiently for the more difficult and theoretical concepts within the Mathematics Extension 1 course material. Data Analysis, an important contemporary application of mathematical thought, has been included in the Mathematics Advanced course. The mathematical modelling process will also be given explicit attention and could be thought of as a logical adjunct to the ‘Working Mathematically’ strand of the Mathematics Years 7–10 Syllabus.

The aim of the focus on modelling is to further develop students’ understanding of the Mathematics involved, and to improve their ability to interpret and critically analyse results in context. These skills improve students’ mathematical literacy and are invaluable in workplace contexts and in the tertiary education environment, where high emphasis is placed on understanding, applying and interpreting Mathematics.

The increase in the use and importance of Statistics and modelling can be linked to the rapid technological advances of the last two decades. Statistical techniques allow professionals in almost all fields to analyse, interpret and make predictions in relation to the large volumes of data that can be readily obtained and stored via technological means. Statistical literacy was identified as an important area of study in Year 12 Curriculum Content and Achievement Standards¹ (Matters and Masters, January 2007) and has become increasingly important in a wide range of professions. An introduction to aspects of ‘non-deterministic’ thinking will assist students’ transition to undergraduate courses requiring Statistics and/or Data Analysis.

Most of the content in the current Mathematics (‘2 Unit’) course topic Basic arithmetic and algebra is taught within the new Mathematics Years 7–10 Syllabus. This content is included in the Mathematics Advanced course as a review section to be flexibly programmed by schools to meet the needs of their students. The current material on absolute value has been moved to the topic Real functions and their graphs.

The Mathematics (‘2 Unit’) course topic Plane geometry has not been included in the Mathematics Advanced course because the content is largely covered within the Mathematics Years 7–10 Syllabus. The notion of geometric proof was rated by academics in a recent survey² as low in importance for students entering university from ‘2 unit Mathematics’ level.

¹ In May 2006 the Australian Government commissioned the Australian Council for Educational Research (ACER) to analyse and report on the content, curriculum and standards in Year 12 English (including Literature), Mathematics, Physics, Chemistry and Australian History in Australian States and Territories.

² Survey of academics conducted by Ayres, Brown, Pahor and Wong, of the University of New South Wales, and described in their written submission to the Syllabus Review phase of the Mathematics Stage 6 Review and Development Project.
The Preliminary course material in Trigonometric ratios (in degrees) is covered in the Mathematics Years 7–10 Syllabus and has been replaced by Introduction to trigonometric functions (using circular angle measure), so that exposure to the key ideas of trigonometric functions can begin earlier in the course.

Much of the current Mathematics (‘2 Unit’) topic Linear functions and lines and aspects of The quadratic polynomial are covered in the Mathematics Years 7–10 Syllabus and are now taught as a review section. Identity of two quadratic expressions is felt to be too abstract for the majority of ‘2 unit-only’ students and has been moved to the Mathematics Extension 1 course.

The (perpendicular) distance of a point from a line and The equation of a line through the point of intersection of two given lines in the current course have not been included in the Mathematics Advanced course as there are no obvious applications (with Locus problems also removed) anywhere in the course. The parabola defined as a locus was also rated by academics in the survey conducted by Ayres et al as low in importance for students entering university from ‘2 unit Mathematics’ level. Feedback from teachers has indicated that the topic is of little interest or value to ‘2 unit-only’ students. The general concept of transformation of graphs will be covered in Real functions and their graphs.

Repayment problems (within Series and applications in the Mathematics (‘2 Unit’) course) is a difficult and time-consuming application for most ‘2 unit-only’ students. As the topic does not add value to the Mathematics Extension 1 course, it has not been included in the Mathematics Advanced course. Applications to recurring decimals has also been removed as the expression of recurring decimals in the form \( \frac{P}{q} \) is better done by algebraic means.

It is felt that Trapezoidal rule (with its proof accessible to ‘2 unit-only’ students) is sufficient at this level as an example of approximate methods within Integration. Hence, Simpson’s rule (with proof) has been moved to the Mathematics Extension 1 course.

Submissions to the first phase of the Mathematics Stage 6 Review and Development Project indicated strong overall support for the level and content of the current Mathematics Extension 1 and Mathematics Extension 2 courses. For this reason, only minor changes have been made to the content of the courses, while ensuring that the rigour of the courses is maintained.

The modelling theme developed in the Mathematics Advanced course is further developed in the Mathematics Extension 1 course. Elementary difference equations and the logistic growth model has been added to expose students to contemporary ideas in Discrete Mathematics, which have a range of important applications. This, together with the application of calculus in physical, biological and business scenarios will provide opportunities for students to develop further their understanding of algebra, function and calculus concepts in context and through the analysis of real-world problems.

The revised Mathematics Extension 1 course improves the balance of topics between Mathematics Extension 1 and Mathematics Extension 2 by moving some material to the Mathematics Extension 2 course for a more ‘natural’ development of the mathematical ideas, by removing small amounts of content that have limited conceptual benefit or are not likely to be developed further at tertiary level, and by adding applications of Discrete Mathematics.
Content involving $t$-formulae (within *Trigonometric ratios*) has been moved to Mathematics Extension 2 for a more natural development as a method of integration by substitution. *Internal and external division of an interval in a given ratio* has not been included since it does not have links to other major course themes; if required can be performed more simply by similar-triangle methods.

*Mathematical induction* to prove results involving inequalities has been moved to ‘Further inequalities’ in Mathematics Extension 2 and is treated in depth within an inequalities theme. *Parametric representation of the parabola* has not been included in the revised Mathematics Extension 1 course. This aspect of the current course was identified in submissions and feedback during the *Syllabus Review* phase as a possible area for removal and was also rated by academics in the survey conducted by Ayres et al as low in importance for students entering university from Mathematics Extension 1 or Mathematics Extension 2 level. This material can be easily introduced, if required at tertiary level, in relation to problems in Mechanics, dynamical systems, etc.

*Multiple roots* within *Polynomials* in the current Mathematics Extension 2 course has been moved to the Mathematics Extension 1 course to provide a more appropriate endpoint to the treatment of Polynomials in that course. Simple harmonic motion has not been included in the revised Mathematics Extension 1 course, but is treated in its entirety as an application of ordinary differential equations in Mathematics Extension 2. The topic can be developed more naturally when viewed as an application of Newton’s laws and framed as a differential equation.

Feedback during the first phase of the Mathematics Stage 6 Review and Development Project in relation to the Mathematics Extension 2 course questioned the value of retaining the *Conics* topic and the value of the *Mechanics* topic in its current form. *Conics* was rated by academics in the survey conducted by Ayres et al as low in importance for students entering university from Mathematics Extension 2 level. It was decided, therefore, not to include *Conics* in the revised Mathematics Extension 2 course, and to make other changes to ensure that applications are more contemporary and more likely to stimulate the interest of students. A new focus on ordinary differential equations will also better prepare students for tertiary studies requiring Mathematics as a major discipline.

The balance of topics between Mathematics Extension 1 and Mathematics Extension 2 is further improved by combining some concepts that arise more naturally in tandem in the Mathematics Extension 2 course, eg Polynomials and Complex Numbers. *Harder 3 Unit topics* in the current Mathematics Extension 2 course has been revised to a focus on inequalities.

Most of the content of the *Polynomials* topic in the current Mathematics Extension 2 course has been retained, with the Fundamental Theorem of Algebra and factorisation combined with the topic on Complex Numbers in the revised course. (*Roots and coefficients of a polynomial equation* is covered in Mathematics Extension 1, while *Multiple roots* has been moved to the Mathematics Extension 1 course.) *Euler’s formula* \( e^{i\theta} = \cos \theta + i \sin \theta \) has been included in the revised Mathematics Extension 2 course as it provides a more efficient method for performing complex number operations in ‘mod/arg’ form and develops further students’ knowledge and skills in relation to exponential functions. The formula is also useful in the solution of second-order ordinary differential equations.
Circular motion within Mechanics in the current Mathematics Extension 2 course has not been included in the revised course. This provides for a reduced focus on Mechanics in favour of more contemporary applications. The rigorous treatment of Circle geometry in Mathematics Extension 1 frees time in the Mathematics Extension 2 course to focus on substantially new concepts.

The current Mathematics Extension 2 course touches on ordinary differential equations but avoids the language and techniques commonly associated with this branch of Mathematics. In the revised Mathematics Extension 2 course, the topic includes the basic theory of the solution methods and applications of simple first and second-order equations, with a focus on mathematical modelling and updated applications including simple harmonic motion, resisted motion in one and two dimensions, and electric circuits.
5.7 Use of technology

for your information

(a) in learning and teaching, and school-based assessment
There are no restrictions on the use of different types of technology in the learning and teaching of courses within the Mathematics Key Learning Area. The appropriateness and viability of particular types of technology in the development of students’ knowledge, skills and understanding, in relation to the courses within the suite of Mathematics Stage 6 courses, are decisions for students, teachers and schools.

The final syllabuses will provide a range of opportunities for the use of calculators and computer software packages in learning and teaching. This will include opportunities to utilise the graphing functions and financial and statistical capabilities of calculators, and dynamic geometry and statistics software packages.

The Broad Directions developed during the first phase of the Mathematics Stage 6 Review and Development Project included ‘That the use of technology with capabilities beyond the level of scientific calculators be encouraged in the learning and teaching, and school-based assessment, of all Stage 6 Board-developed Mathematics courses.’

(b) in the Board’s HSC examinations

In accordance with the Broad Direction ‘That the use of technology in HSC examinations for the calculus-based courses be further investigated and clarified in the Writing Brief phase’, it is proposed that the Board prescribe for use in HSC examinations for the calculus-based courses, a clear set of calculator functions and capabilities. These functions and capabilities will be consistent with and support the knowledge and skills students should be able to demonstrate after completing the Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 courses. For example, if students are expected to use the binomial theorem in probability, functions such as $n!$, $_nP_r$, and $_nC_r$ would be appropriate to prescribe. The functions and capabilities would be carefully chosen so that basic algebraic, procedural, graphical and conceptual skills are not replaced by technology. For example, where the syllabus indicates that students should be able to differentiate and integrate basic functions, then these capabilities would not be included in the set of functions and capabilities for these courses.

These functions and capabilities may be consistent across both the calculus-based and non-calculus-based Stage 6 Mathematics courses. However, as the detailed syllabuses are written, consulted upon and finalised, it may be necessary for different sets of functions and capabilities to be developed for these two Mathematics course groupings.

Matter for Consideration
Is the proposed approach to the use of technology in the HSC examinations appropriate?
5.8 Assessment and HSC examination

for your information

The general assessment and reporting advice in the Mathematics Stage 6 calculus-based syllabuses will focus on the role of assessment in improving teaching and learning.

The following requirements for internal assessment and external examinations will be contained in the syllabus:
• Preliminary course components and weightings (advisory)
• HSC course components and weightings (mandatory)
• HSC specifications.

The following materials will be developed for each course:
• performance band descriptions
• a specimen HSC examination, including a mapping grid linking specimen examination questions to syllabus outcomes, content and targeted performance bands, and sample marking guidelines.

It is proposed that the current situation be maintained in terms of the examination papers to be sat by candidates for the calculus-based courses. This means that there would be a single HSC examination paper for the Mathematics Advanced course. Mathematics Extension 1 candidates would need to sit for two examination papers, one for the Mathematics Advanced course and one for the Mathematics Extension 1 course. Mathematics Extension 2 candidates would also need to sit for two examination papers, one for the Mathematics Extension 1 course and one for the Mathematics Extension 2 course.

Consideration needs to be given to:
• what the components should be for internal assessment, and their weightings
• the length of the examinations
• the types of items to be included in the examinations
• the balance of Preliminary and HSC content to be included in the examinations
• any tables, formulae or other information to be included with the examinations.

Matter for Consideration
What suggestions would you like to make regarding assessment components and weightings, examination specifications and the proposed arrangements for examination papers to be sat by candidates for the calculus-based courses?
6 Glossary
6 Glossary

The Glossary explains terms that will assist teachers in the interpretation of the Mathematics Stage 6 calculus-based syllabuses.

Syllabus terminology

Content

Content describes the knowledge, skills, understanding and values and attitudes to be studied and developed by students over a Stage or Stages in a syllabus and the development of processes of learning so that students are encouraged to be effective learners.

Outcomes

Syllabus outcomes express the specifically intended student learning that will result from the teaching of the syllabus. They are derived from the objectives and content of the syllabus. They provide clear statements of the knowledge, skills and understanding expected to be gained by most students as a result of effective teaching and learning by the end of a Stage. They also describe the values and attitudes expected to be developed by students.

Standards

The term standards refers to the knowledge, skills and understanding expected to be learned by:

- students as a result of studying a subject – the content standards
- the levels of achievement of the knowledge, skills and understanding – the performance standards.

Both content standards and performance standards are based on the aims, objectives, outcomes and content of a course. Together they specify what is to be learned and how well it is to be achieved.

Content standards specify what students are expected to know, understand and be able to do as a result of studying a course. Teacher understanding of content standards comes from their consideration of the aims, objectives, outcomes and content of the syllabus.

Performance standards are the different levels of achievement demonstrated by students.
Subject

A subject is a name given to a defined area of knowledge. There may be several courses offered in a subject.

Syllabus

A document that describes for a key learning area or a course of study what students are expected to learn in terms of aims, objectives, outcomes, content and assessment requirements.

A syllabus package includes a syllabus document with additional information on assessment and examination, and support materials.
7 Support Materials
7 Support Materials

It is proposed that a range of support materials be produced to assist teachers with the implementation of the new Mathematics Stage 6 calculus-based courses.

In accordance with the Broad Direction ‘That the syllabus documents within the Stage 6 Mathematics syllabus package incorporate applications, implications and considerations for the teaching of the syllabus content, including in relation to depth of coverage’, it is proposed that each of the calculus-based courses include an ‘applications, considerations and examples’ section within the syllabus document for each of the Preliminary and HSC topic areas.

The support materials could include:
• sample teaching and learning units
• advice on programming
• program overviews
• advice in relation to teaching ‘new’ areas of course content
• teaching suggestions to assist the development of meaningful and engaging units of work
• sample HSC assessment programs
• advice on the utilisation of different types of assessment tasks
• sample assessment tasks
• suggestions for the utilisation of technology in teaching and learning
• suggested applications that relate to real-world problems.

Matter for Consideration
Would this range of support materials provide teachers with sufficient support for the implementation of the calculus-based courses?
8 Appendix
Broad Directions for the Mathematics Stage 6 Writing Briefs – endorsed by the Board of Studies on 12 December 2006.

Note: The Board of Studies endorsed these Broad Directions on 12 December 2006. They reflect the consultation undertaken in Phase 1 of the syllabus development process and inform the development of Section 4 of this draft writing brief, which is subject to consultation. The broad directions are not subject to consultation.

Broad Directions for the Mathematics Stage 6 Writing Briefs:

• That the set of Stage 6 Mathematics courses include an additional offering to accommodate the purposes of students who wish to study a Board-developed Mathematics course in Stage 6 but who are currently choosing not to, as well as those whose purposes are not accommodated through the study of General Mathematics.

• That in the revision, due attention be given to clarifying the purpose of each course and identifying future learning or vocational pathways of the intended candidatures.

• That the nested structure of the current Mathematics (‘2 Unit’), Mathematics Extension 1 and Mathematics Extension 2 courses, and the calculus-basis to these courses, be retained.

• That any revision or development of the calculus-based courses maintain the current rigour and level of challenge of the courses.

• That the amount of content prescribed for Stage 6 Mathematics courses reflect the amount that can be taught, and learnt by the typical student, in the indicative time.

• That in reviewing the content of Stage 6 Mathematics courses, particular attention be given to the purpose of the Mathematics (‘2 Unit’) course for Mathematics (‘2 Unit’) only students, and the appropriateness and relevance of the course content for those students.

• That the inclusion of additional study of statistics be considered, while addressing implications in relation to the extent of relevant teacher expertise, professional development, future pathways of students, school Mathematics staffing, and school timetabling.

• That the current General Mathematics course material be largely maintained within the structure of Stage 6 non-calculus-based Mathematics courses.

• That, in reviewing the content of the calculus-based-courses, the appropriateness and relevance of the applications within the courses be explored, with a view to ensuring that they are contemporary and that they meet the needs of students.

• That, in the consideration of the use of technology in Stage 6 Mathematics courses, due regard must be given to the related access and equity issues.
• That the use of technology with capabilities beyond the level of scientific calculators be encouraged in the learning and teaching, and school-based assessment, of all Stage 6 Board-developed Mathematics courses.

• That the non-calculus-based Stage 6 Mathematics courses be developed with the view that technology with capabilities beyond the level of scientific calculators will need to be utilised for aspects of the associated HSC examinations.

• That the use of technology in HSC examinations for the calculus-based courses be further investigated and clarified in the Writing Brief phase.

• That the appropriateness of the current processes for the examination of Stage 6 Mathematics courses be reviewed, with particular emphasis on the examination of Mathematics (‘2 Unit’)–only candidates.

• That the syllabus documents within the Stage 6 Mathematics syllabus package incorporate applications, implications and considerations for the teaching of the syllabus content, including in relation to depth of coverage.

• That each Stage 6 Mathematics course be named so as to avoid confusion with the discipline itself.