

**Mathematics,  
Mathematics Extension 1 and  
Mathematics Extension 2  
Senior Years**

**Writing Brief**

**February 2016**

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## **1. Introduction**

In 2014, the Board of Studies, Teaching and Educational Standards NSW (BOSTES) commenced a review of NSW senior secondary syllabuses for English, Mathematics, Science and History learning areas to determine directions for the incorporation of the senior secondary Australian curriculum. BOSTES conducted consultation in August and September 2014 on proposed directions outlined in *NSW Senior Secondary Review & Evaluation: English, Mathematics, Science and History*.

The broad directions for each learning area, developed following consultation, were endorsed by BOSTES in December 2014 and are available in Appendix I of this writing brief.

The development of the *Mathematics, Mathematics Extension 1 and Mathematics Extension 2 Senior Years Writing Brief* takes account of the broad directions and feedback gathered through consultation conducted in October and November 2015.

The purpose of the writing brief is to inform the directions for draft syllabus development. The writing brief is structured according to the elements of a Senior Years syllabus. Each element includes proposed actions and key considerations for writers in the writing of the draft syllabus. These elements are:

- Rationale
- The place of the Mathematics, Mathematics Extension 1, Mathematics Extension 2 Senior Years syllabuses in the K–12 curriculum
- Aim
- Objectives
- Outcomes
- Course structure
- Content, including how Australian curriculum content may be incorporated
- Glossary.

The draft syllabus package will include the elements of a syllabus and Australian curriculum content identified with codes, learning across the curriculum content identified by icons, further information about meeting the diversity of learners, and internal and external assessment.

The draft syllabuses for Mathematics, Mathematics Extension 1 and Mathematics Extension 2 will be developed and available for consultation during 2016.

A summary of the BOSTES syllabus development process is available at <http://www.boardofstudies.nsw.edu.au/syllabuses/syllabus-development/>.

### **Diversity of learners**

NSW senior secondary syllabuses will be inclusive of the learning needs of all students. The draft syllabuses will be designed to accommodate teaching approaches that support student diversity under the sections 'Students with special education needs', 'Gifted and talented students' and 'Students learning English as an additional language or dialect (EAL/D)'.

For example:

## **Special education needs**

All students with special education needs are entitled to participate in and progress through the curriculum. Some students may require additional support or adjustments to teaching, learning and assessment activities. Adjustments are measures or actions taken in relation to teaching, learning and assessment that enable a student to access syllabus outcomes and content and demonstrate achievement of outcomes.

Most students with special education needs will undertake regular Board Developed courses and/or Board Endorsed courses. Students with special education needs can access Years 11 and 12 outcomes and content in a range of ways. They should choose the most appropriate courses for the HSC in keeping with their goals, interests and learning needs.

Students may engage with:

- syllabus outcomes and content with adjustments to teaching, learning and/or assessment activities
- selected outcomes and content appropriate to their learning needs.

For some students with special education needs, the Years 11–12 Life Skills outcomes and content provided in the Mathematics General Senior Years draft syllabus may provide learning more appropriate to their individual needs.

## **Australian curriculum**

BOSTES began its syllabus development process for Stage 6 English, Mathematics, Science and History in 2014. This follows state and territory Education Ministers' endorsement of senior secondary Australian curriculum in these learning areas as the agreed and common base for development of state and territory senior secondary courses. It was also agreed that states and territories would have the flexibility to integrate the approved senior secondary Australian curriculum as appropriate. The writing brief determines how Australian curriculum content can be modified, reordered and supplemented in each learning area, while remaining compatible with the NSW Senior Years assessment and examinations structures.

## 2. Mathematics, Mathematics Extension 1 and Mathematics Extension 2 key

**i** for your information

The following codes will be used in the Mathematics, Mathematics Extension 1 and Mathematics Extension 2 Senior Years draft syllabuses.

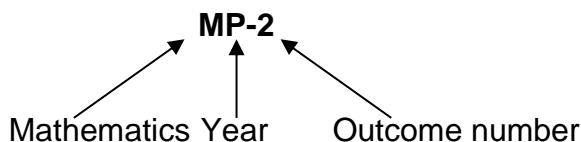
### Outcome coding

Syllabus outcomes will be coded in a consistent way. The code identifies the subject, Year and outcome number.

Years of learning will be represented by the following codes:

Year	Code
Year 11	P
Year 12	H

In the Mathematics, Mathematics Extension 1 and Mathematics Extension 2 syllabuses, outcome codes indicate the subject, Year and outcome number. For example:



### Coding of Australian curriculum content

Australian curriculum content descriptions included in the syllabus will be identified.

### Actions for writers and key considerations

- Identify Australian curriculum content descriptions by using Australian curriculum codes.
- The code should appear in brackets at the end of each content description.

For example:

- sketch curves associated with simple polynomials; find stationary points, and local and global maxima and minima; and examine behaviour as  $x \rightarrow \infty$  and  $x \rightarrow -\infty$  (ACMMM095)
- solve optimisation problems arising in a variety of contexts involving simple polynomials on finite interval domains. (ACMMM096).

### **3. Rationale**

**i** for your information

The rationale describes the distinctive nature of the subject and outlines its relationship to the contemporary world and current practice. It explains the place and purpose of the subject in the curriculum, including:

- why the subject exists
- the theoretical underpinnings
- what makes the subject distinctive
- why students would study the subject
- how it contributes to the purpose of the Senior Years curriculum
- how it prepares students for post-school pathways.

#### **Proposed rationale for Mathematics, Mathematics Extension 1 and Mathematics Extension 2 Senior Years**

Mathematics is deeply embedded in modern society. From the numeracy skills required to manage personal finances, to devices and scales for measuring something of interest, to leading-edge technologies in the sciences and engineering, mathematics provides the framework for interpreting, analysing and predicting, and the tools for effective participation in an increasingly complex society.

The need to interpret the large volumes of data made available through technology draws on skills in logical thought and skills in checking claims and assumptions in a systematic way. Mathematics is the appropriate training ground for the development of these skills. The thinking required to further enhance the power and usefulness of technology in real-world applications requires advanced mathematical training. The rapid advances in technology experienced in recent years have driven, and have been driven by, advances in the discipline of Mathematics.

The development of mathematics throughout history has been catalysed by its utility in explaining real-world phenomena and its inherent beauty. In this way, the discipline has continued to evolve through a process of observation, conjecture, proof and application.

The Mathematics, Mathematics Extension 1 and Mathematics Extension 2 courses form a continuum to provide opportunities at progressively higher levels for students to acquire knowledge, skills and understanding in relation to important concepts within areas of mathematics that have applications in an increasing number of contexts. These concepts and applications are appropriate to the students' continued experience of Mathematics as a coherent, interrelated, interesting and intrinsically valuable study that forms a basis for future learning.

Students develop an appreciation of Mathematics as a study with high levels of internal structure that provide opportunities for the development of logical and disciplined thought. Through the learning experiences within the courses, students are able to progress from a knowledge and understanding of facts, procedures and applications in idealised contexts to facility in the use of mathematical models that

situate the mathematics in context, and to more advanced generalisations based on deductive and inductive reasoning processes.

This involves the development and use of an increasingly sophisticated level of communication and literacy.

The concept of a function of a real variable, the algebraic and geometrical representations of a number of important functions, and the introductory concepts and techniques of differential and integral calculus, together form a strong basis of the courses. These concepts, representations and techniques are developed and utilised across the courses.

Differential calculus is concerned with how quantities change and is of fundamental importance in Mathematics. It builds on knowledge, skills and understanding developed earlier in algebra, geometry and trigonometry. The concepts and techniques of calculus provide a means of modelling and developing increased understanding of many real-world situations and of solving a variety of related problems. These situations and problems include many of those arising in the sciences, including in relation to the natural environment and medicine, and in statistics, business, finance and economics. A number of related applications are studied in the Mathematics, Mathematics Extension 1 and Mathematics Extension 2 courses.

The Mathematics course has been written on the assumption that students have demonstrated competence in Mathematics up to and including the Stage 5.2 level by the end of Year 10. The course provides such students with the opportunity to develop an understanding of and competence in further aspects of Mathematics through real-world applications for concurrent HSC studies, such as in science, business studies and economics, and for further studies at tertiary level in such areas as the life sciences, business, finance, technology and education. At the same time, the Mathematics course provides an appropriate basis for the study of the Mathematics Extension Senior Years courses.

The Mathematics Extension 1 course has been written to meet the needs of students who have demonstrated a high level of competence in Mathematics up to and including the Stage 5.3 level by the end of Year 10. The course provides such students with the opportunity to develop a thorough understanding of and competence in further aspects of Mathematics through real-world applications for concurrent HSC studies, such as in science, engineering studies and economics, and for further studies at tertiary level in Mathematics, and in such areas as the physical sciences and engineering.


At the same time, the Mathematics Extension 1 course, together with the Mathematics course, provides an appropriate basis for the study of the Mathematics Extension 2 course, which has been written to meet the needs of students who have demonstrated outstanding ability in Mathematics. The course represents a distinctly high level in school mathematics and provides such students with the opportunity to develop considerable manipulative skills and a high degree of understanding of the fundamental ideas of algebra and calculus. The course, therefore, provides a sufficient basis for a wide range of useful applications of mathematics as well as a strong foundation for the further study of the subject.



### **Actions for writers and key considerations**

- Write separate rationales for the Mathematics, Mathematics Extension 1 and Mathematics Extension 2 courses to clarify the similarities and differences between the courses.
- Each rationale should be concisely stated and a net reduction in length should be achieved.
- Clearly identify the possible pathways from Stage 5 to Senior Years. For example, the intention that Stage 5.2 is not a sufficient preparation for successful study of Mathematics, without the inclusion of suggested topics from Stage 5.3.
- Review any reference to geometry in the rationales considering that there is no geometry topic in the proposed content for Mathematics.
- Include reference in the rationales to mathematical understanding, specifically fluency, communication, problem-solving, reasoning and understanding (as a reference to building on the Working Mathematically skills from Stage 5).
- Include a reference to the use of appropriate technology and preparing students for a technological society.
- Add detail to the rationales to indicate possible post-school pathways for all courses.
- Ensure that the rationale aligns with the direction and nature of the course.

#### **4. The place of the Mathematics, Mathematics Extension 1 and Mathematics Extension 2 Senior Years syllabuses in the K–12 curriculum**

 for your information

NSW syllabuses will include a diagram that illustrates how the syllabus relates to the learning pathways K–12. This section places the Senior Years syllabuses in the K–12 curriculum as a whole.

This diagram will be included in the draft syllabuses.

## **5. Aim**

**i** for your information

In NSW syllabuses, the aim provides a statement(s) of the overall purpose of the syllabus. It indicates the general educational benefits for students from programs based on the syllabus.

The aim, objectives, outcomes and content of a syllabus are clearly linked and sequentially amplify details of the intention of the syllabus.

### **Proposed aim for Mathematics, Mathematics Extension 1 and Mathematics Extension 2 Senior Years**

The aim of the Mathematics course is to encourage students to:

- develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment
- recognise how a situation may be represented mathematically and understand the relationship between 'real world' problems and mathematical models and how these can be refined and improved
- acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations
- use mathematics as an effective means of communication
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

The aim of the Mathematics Extension 1 course is to encourage students to:

- develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment
- develop abilities to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs
- develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected
- acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations
- use mathematics as an effective means of communication.

The aim of the Mathematics Extension 2 course is to encourage students to:

- develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment
- extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems
- read and comprehend mathematical arguments and articles concerning the applications of mathematics
- develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected

- acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations.

**Actions for writers and key considerations**

- Review the aims to ensure they present the overall purpose of the syllabus and are appropriate for each course.
- Review the aims to ensure consistency of length, detail and complexity with other senior syllabuses.
- Include reference in the aims to mathematical understanding, specifically fluency, communication, problem-solving, reasoning and understanding (as a reference to building on the Working Mathematically skills from Stage 5).

## **6. Objectives**

**i** for your information

In NSW syllabuses, objectives provide specific statements of the intention of a syllabus. They amplify the aim and provide direction to teachers on the teaching and learning process emerging from the syllabus. They define, in broad terms, the knowledge, understanding, skills, values and attitudes to be developed through study in the subject. They act as organisers for the intended outcomes.

### **Proposed objectives for Mathematics, Mathematics Extension 1 and Mathematics Extension 2 Senior Years**

#### **Knowledge, understanding and skills**

Students:

- apply deductive reasoning, and use appropriate language, in the construction of mathematical arguments and proofs
- use concepts and techniques, including technology, in the solution of problems
- interpret and use mathematical models in a range of contexts
- analyse solutions to problems and conclusions, and communicate in appropriate mathematical forms.

#### **Values and attitudes**

Students:

- appreciate the scope, usefulness, power and elegance of mathematics.

#### **Actions for writers and key considerations**

- Review and amend the objectives to include aspects of technology and define the knowledge, understanding, skills, values and attitudes to be developed through study in each course.
- The objectives should include that students propose models and draw conclusions.

## 7. Outcomes

**i** for your information

In NSW syllabuses, outcomes provide detail about what students are expected to achieve at the end of each Stage in relation to the objectives. They indicate the knowledge, understanding and skills expected to be gained by most students as a result of effective teaching and learning. They are derived from the objectives of the syllabus.

### Proposed outcomes for Mathematics, Mathematics Extension 1 and Mathematics Extension 2 Senior Years

Up to 10–12 outcomes for each course will be developed during draft syllabus writing. The following table presents a sample of some of the proposed outcomes.

Objectives	Year 11 Mathematics outcomes	Year 12 Mathematics outcomes	Year 11 Mathematics Extension outcomes	Year 12 Mathematics Extension 1 outcomes	Year 12 Mathematics Extension 2 outcomes
Students:	A student:	A student:	A student:	A student:	A student:
apply deductive reasoning, and use appropriate language, in the construction of mathematical arguments and proofs	MP–1 provides reasoning to support conclusions appropriate to the context	MH–1 constructs mathematical arguments to prove and justify results	MX1P–1 uses deductive reasoning to solve problems and prove results in circle geometry  MX1P–2 uses algebraic techniques to solve inequalities and prove results and identities	MX1H–1 uses mathematical induction in the construction of proofs  MX1H–2 uses the binomial theorem and algebraic and calculus techniques to prove identities	MX2H–1 constructs mathematical arguments and proofs in concrete and abstract settings  MX2H–2 applies algebraic, graphical and calculus techniques in the construction of proofs, including those involving inequalities
use concepts and techniques, including technology, in the solution of problems	MP–2 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions, absolute values and logarithms	MH–2 manipulates algebraic expressions and solves problems involving exponential and logarithmic functions	MX1P–3 uses the relationship between the algebraic and geometrical representations of a function in the solutions of problems	MX1H–3 uses the concept of inverse functions in the solution of problems	MX2H–3 combines the ideas of algebra and calculus to determine features of graphs

**Actions for writers and key considerations**

- Review the course outcomes to include aspects of technology and define the knowledge, skills, understanding, values and attitudes to be developed by the students in each course.
- Investigate the idea of a separate set of outcomes of Working Mathematically skills, as in the K–10 syllabus.

## 8. Course structure

**i** for your information

The following provides an outline of the Year 11 and Year 12 course structure for the Mathematics, Mathematics Extension 1 and Mathematics Extension 2 Senior Years syllabuses with indicative course hours and the arrangement of course content, along with outlining relationships between specific components and between core and options.

### Proposed course structure for Mathematics, Mathematics Extension 1 and Mathematics Extension 2 Senior Years

	Mathematics ('2 Unit')	Indicative hours
Year 11 course (120 hours)	<p>Six strands. For example:</p> <ul style="list-style-type: none"> <li>• Real Numbers and Algebraic Techniques</li> <li>• Functions and Graphs</li> <li>• Trigonometric Functions</li> <li>• Arithmetic and Geometric Sequences and Series</li> <li>• Introduction to Differential Calculus</li> <li>• Data and Statistics</li> </ul> <p>Modelling and applications should be an integral part of each topic and should also merge topics together, enabling candidates to make connections and appreciate the use of mathematics and appropriate technology.</p>	120
Year 12 course (120 hours)	<p>Six strands. For example:</p> <ul style="list-style-type: none"> <li>• Integral Calculus</li> <li>• Exponential and Logarithmic Functions</li> <li>• Further Differentiation and Applications</li> <li>• Further Trigonometric Functions</li> <li>• Counting and Probability</li> <li>• Discrete and Continuous Random Variables and the Normal Distribution</li> </ul> <p>Modelling and applications should be an integral part of each topic and should also merge topics together, enabling candidates to make connections and appreciate the use of mathematics and appropriate technology.</p>	120



	<b>Mathematics Extension 1</b>	<b>Indicative hours</b>
<b>Year 11 course (60 hours)</b>	Five strands. For example: <ul style="list-style-type: none"> <li>• Functions and Graph Sketching</li> <li>• Further Trigonometry</li> <li>• Geometry</li> <li>• Descriptive Statistics</li> <li>• Series and Elementary Difference Equations</li> </ul>	60
<b>Year 12 course (60 hours)</b>	Five strands. For example: <ul style="list-style-type: none"> <li>• Binomial Theorem</li> <li>• Methods and Applications of Integration</li> <li>• Further Applications of Calculus involving Mathematical Modelling</li> <li>• Inverse Functions</li> <li>• Mathematical Induction</li> </ul>	60


	<b>Mathematics Extension 2</b>	<b>Indicative hours</b>
<b>Year 12 course (60 hours)</b>	Eight strands. For example: <ul style="list-style-type: none"> <li>• The Nature of Proof</li> <li>• Complex Numbers</li> <li>• Graphs</li> <li>• Integration Techniques</li> <li>• Rates of Change and Differential Equations</li> <li>• Mechanics</li> <li>• Statistical Inference</li> <li>• Vectors</li> </ul>	60

### **Actions for writers and key considerations**

- The above list of topics is not comprehensive and in developing content, some topics could be reconsidered for inclusion, but the overall structure of the course should be maintained.
- Investigate the inclusion of some additional content such as networks and matrices, while maintaining the course structure. For example, 'Vectors' could be considered for inclusion in Mathematics Extension 1.
- Overall a net reduction in content when compared with the current Mathematics courses should be achieved to provide opportunities for depth of learning.
- A significant amount of the common content between the Mathematics course and the Mathematics General 2 course should be built into the respective Year 11 courses. This could include topics from the 'Data and Statistics' strand.
- Include appropriate common content between the Year 11 Mathematics General course and the Year 11 Mathematics course.

- Include appropriate common content between the Year 12 Mathematics General 2 course and the Year 12 Mathematics course.
- Include appropriate common content between the Mathematics course and the Mathematics Extension 1 course.
- Include appropriate common content between the Mathematics Extension 1 course and the Mathematics Extension 2 course.
- The Mathematics course should reflect the academic requirements of a two-unit course, providing appropriate opportunities for students to be successful in developing the knowledge, skills, understanding, values and attitudes with similar time requirements and expectations of other two-unit courses.

## **9. Content**

 for your information

In NSW syllabuses for Senior Years, courses of study and educational programs are based on the outcomes of syllabuses. The content describes in more detail how the outcomes are to be interpreted and used, and the intended learning appropriate for each Year. In considering the intended learning, teachers will make decisions about the emphasis to be given to particular areas of content, and any adjustments required based on the needs, interests and abilities of their students.

### **Organisation of the content**

The Mathematics, Mathematics Extension 1 and Mathematics Extension 2 Senior Years syllabuses will be organised as shown in the following pages.

## Mathematics

The following overview illustrates a sample structure for the Mathematics course.

Year 11 Mathematics Course	Year 12 Mathematics Course
<p><b>Real Numbers and Algebraic Techniques</b></p> <ul style="list-style-type: none"> <li>Algebraic expressions, equations and inequalities</li> <li>Surds and indices</li> </ul> <p><b>Functions and Graphs</b></p> <ul style="list-style-type: none"> <li>Function notation and properties of functions and graphs</li> <li>Applications of functions</li> <li>Direct and inverse variation</li> </ul> <p><b>Trigonometric Functions</b></p> <ul style="list-style-type: none"> <li>Sine, cosine and area rules</li> <li>Angles of any magnitude</li> <li>Trigonometric functions and graphs</li> <li>Trigonometric identities and equations</li> </ul> <p><b>Applications of Real Functions (modelling topic)</b></p> <p><b>Arithmetic and Geometric Sequences and Series</b></p> <ul style="list-style-type: none"> <li>Arithmetic sequences and series</li> <li>Geometric sequences and series</li> <li>Applications of series to finance</li> </ul> <p><b>Introduction to Differential Calculus</b></p> <ul style="list-style-type: none"> <li>Rates of change</li> <li>Derivative function</li> <li>Computation of derivatives</li> </ul> <p><b>Data and Statistics</b></p> <ul style="list-style-type: none"> <li>Statistics and society</li> <li>Univariate data and summary statistics</li> <li>Describing bivariate data</li> </ul> <p><b>Applications of Rates of Change (modelling topic)</b></p>	<p><b>Integral Calculus</b></p> <ul style="list-style-type: none"> <li>The primitive function</li> <li>Indefinite and definite integrals</li> <li>The Fundamental theorem</li> <li>Applications of integration</li> </ul> <p><b>Exponential &amp; Logarithmic Functions</b></p> <ul style="list-style-type: none"> <li>Exponential functions and equations</li> <li>Differentiation of exponential functions and applications</li> <li>Logarithmic functions and equations</li> <li>Differentiation of logarithmic functions and applications</li> <li>Integration with logarithms</li> </ul> <p><b>Further Differentiation &amp; Applications</b></p> <ul style="list-style-type: none"> <li>Differentiation rules</li> <li>Applications of derivatives including stationary points, the second derivative, curve sketching, maximums and minimums</li> <li>Real life applications of derivatives such as motion and optimisation</li> </ul> <p><b>Applications to the Natural Environment (modelling topic)</b></p> <p><b>Further Trigonometric Functions</b></p> <ul style="list-style-type: none"> <li>Circular and radian measure</li> <li>Differentiation of trigonometric functions</li> <li>Integration of trigonometric functions</li> </ul> <p><b>Counting and Probability</b></p> <ul style="list-style-type: none"> <li>Permutations and Combinations</li> <li>Language of events and sets</li> <li>Conditional probability and independence</li> </ul> <p><b>Discrete and Continuous Random Variables and the Normal Distribution</b></p> <ul style="list-style-type: none"> <li>General discrete random variables</li> <li>Bernoulli and binomial distributions</li> <li>General continuous random variables</li> <li>Normal distributions</li> </ul> <p><b>Applications of Probability and Finance (modelling topic)</b></p>

## Mathematics Extension 1

The following overview illustrates a sample structure for the Mathematics Extension 1 course.

Year 11 Mathematics Extension 1 Course	Year 12 Mathematics Extension 1 Course
<p><b>Functions and Graph Sketching</b></p> <ul style="list-style-type: none"> <li>• Transformations of graphs</li> <li>• Remainder and factor theorems</li> <li>• Polynomial functions and their graphs</li> </ul> <p><b>Further Trigonometry</b></p> <ul style="list-style-type: none"> <li>• Reciprocal functions</li> <li>• Rewriting an expression as <math>R\cos(x+a)</math></li> <li>• Applications of trigonometric functions to model periodic phenomena</li> </ul> <p><b>Geometry</b></p> <ul style="list-style-type: none"> <li>• The nature of proof</li> <li>• Circle geometry</li> </ul> <p><b>Descriptive Statistics</b></p> <ul style="list-style-type: none"> <li>• Sample means</li> <li>• Confidence intervals for means</li> </ul> <p><b>Series and Elementary Differential Equations</b></p> <ul style="list-style-type: none"> <li>• Limiting sum of a geometric series</li> <li>• Methods of solution of first-order linear differential equations</li> </ul>	<p><b>Binomial Theorem</b></p> <ul style="list-style-type: none"> <li>• Binomial expansions and identities</li> <li>• Binomial probabilities</li> </ul> <p><b>Methods and Applications of Integration</b></p> <ul style="list-style-type: none"> <li>• Integration by substitution</li> <li>• Solids of revolution</li> </ul> <p><b>Further Applications of Calculus involving Mathematical Modelling</b></p> <ul style="list-style-type: none"> <li>• Further exponential growth and decay</li> <li>• Projectile motion</li> <li>• Iterative methods for estimating roots of equations</li> </ul> <p><b>Inverse Functions</b></p> <ul style="list-style-type: none"> <li>• Rules for inverse functions</li> <li>• Inverse trigonometric functions</li> </ul> <p><b>Mathematical Induction</b></p> <ul style="list-style-type: none"> <li>• The nature of proof</li> <li>• Mathematical induction</li> </ul>

## Mathematics Extension 2

The following overview illustrates a sample structure for the Mathematics Extension 2 course.

Mathematics Extension 2 Course
<p><b>The Nature of Proof</b></p> <ul style="list-style-type: none"><li>• Proofs involving inequalities</li></ul> <p><b>Complex Numbers</b></p> <ul style="list-style-type: none"><li>• Arithmetic of complex numbers and geometrical representation</li><li>• Powers and roots, curves and regions</li><li>• Factorising polynomials</li></ul> <p><b>Graphs</b></p> <ul style="list-style-type: none"><li>• Graphing techniques</li><li>• General approach to curve sketching</li></ul> <p><b>Integration Techniques</b></p> <ul style="list-style-type: none"><li>• Integration by parts</li><li>• Recurrence relations</li></ul> <p><b>Rates of Change and Differential Equations</b></p> <ul style="list-style-type: none"><li>• Solve differential equations by integration</li><li>• First-order differential equations</li><li>• Second-order differential equations</li></ul> <p><b>Mechanics</b></p> <ul style="list-style-type: none"><li>• Simple harmonic motion</li><li>• Newton's laws</li></ul> <p><b>Statistical Inference</b></p> <ul style="list-style-type: none"><li>• Sample means</li><li>• Confidence intervals for means</li></ul> <p><b>Vectors</b></p> <ul style="list-style-type: none"><li>• Representing vectors in the plane</li><li>• Algebra of vectors in the plane</li><li>• Algebra of vectors in three dimensions</li></ul>

## **Sample content pages for the Mathematics course**

(The Mathematics Extension 1 and Mathematics Extension 2 courses will be organised using the same approach.)

### **Topic: Data and Statistics**

Data and Statistics relates to the collection, summarisation, display, analysis and interpretation of data from the real world. In recent times, rapid increases in the use of technology for commercial activity and for communication has enabled the collection of very large amounts of data, leading to the increased importance of statistical understanding in order to make best use of this information. Statistical results form the basis of many decisions affecting society, and also inform individual decision-making.

In this topic, students describe and summarise univariate and bivariate data, building on knowledge, skills and understanding developed in Stage 5. Related sets of data are compared visually. The analysis of data in order to make conclusions about trends is emphasised. This topic provides important groundwork for later courses in inferential statistics at a post-secondary level.

### **Outcomes addressed**

A student:

- MP1 provides reasoning to support conclusions appropriate to the context
- MP7 uses concepts and techniques from descriptive statistics to present and interpret data
- MP11 interprets solutions to problems using mathematical language.

### **Content summary**

- DS1 Statistics and society
- DS2 Univariate data and summary statistics
- DS3 Describing bivariate data

### **Prior learning**

- Plotting points and constructing a straight line graph on a number plane.
- Substitution in an algebraic formula.

### **Use of technology**

Students should learn to use appropriate technology as an effective support for mathematical activity. For example:

- to use real-life data and organise it in a spreadsheet
- to produce a variety of graphs and tables
- to calculate measures of location, centre and spread
- to produce lines of best fit.

### DS3 Describing bivariate data









In this subtopic, students use visual displays and calculations to find and investigate relationships in bivariate data.

#### Outcomes addressed





MP1, MP7, MP11

#### Content

Students:

- construct a scatterplot (by hand and with suitable technology) to identify patterns in the data suggesting the presence of an association (ACMGM052) 
- describe an association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak) (ACMGM053) 
- calculate (using suitable technology) and interpret the correlation coefficient ( $r$ ) to quantify the strength of a linear association. (ACMGM054) 
- recognise that the range of possible values for the correlation coefficient,  $r$ , is given by  $-1 \leq r \leq 1$  
- describe the difference between correlation and causality (ACMGM064) 
- estimate by eye and drawing a line of fit on a scatterplot (ACMGM07) 
- define the least-squares line of best fit (ACMGM07) 
- use suitable technology to construct a least-squares line of best fit and obtain its equation, and verify this equation by consideration of the relevant graph (ACMGM057) 

$$m = r \frac{s_y}{s_x}$$

- use the relationship  $m = r \frac{s_y}{s_x}$ , where  $m$  is the gradient of the least-squares line of best fit, and the standard deviations of the  $x$  values and  $y$  values are  $s_x$  and  $s_y$  respectively 
- use the fact that the least-squares line of best fit passes through the point  $(\bar{x}, \bar{y})$ , whose coordinates  $\bar{x}$  and  $\bar{y}$  are the means of the  $x$  and  $y$  distributions respectively, to find the  $y$ -intercept of the least-squares line of best fit 
- use a least-squares line of best fit to interpolate (ACMGM062) 
- identify outliers on a scatterplot as points that do not follow the general pattern, noting that the  $x$  and  $y$  values of such points are not necessarily outliers in the separate  $x$  and  $y$  distributions. 



## Actions for writers and key considerations

- Content is to be based on the structure and model of the sample content pages provided.
- Consider the topics and develop draft content for each. The provided list is not comprehensive and in the course of writing content, some topics could be reconsidered for inclusion.
- Analyse and select Australian curriculum content, and modify, reorder and supplement to align with and complement draft syllabus content as appropriate.
- Identify the Australian curriculum content descriptions by the Australian curriculum code. The code should appear in brackets at the end of each content description.
- Identify, by underlining, specific terms for inclusion in and links to a glossary.
- Appropriate and authentic opportunities to develop knowledge, understanding, skills, values and attitudes specific to learning across the curriculum areas should be identified by icons.
- Overall a net reduction in content when compared with the current Mathematics courses should be achieved to provide opportunities for depth of learning.
- Investigate the inclusion of some additional content such as networks and matrices. For example, 'Vectors' could be considered for inclusion in Mathematics Extension 1.
- A 'Prior Learning' section should be included to identify the knowledge, skills and understanding that are assumed within the topic.
- Investigate the inclusion of other possible sections, such as 'Working Mathematically Skills' which identify aspects of communication, fluency, problem-solving, reasoning and understanding. These skills would build on the Stage 5 outcomes.
- In order to cater for the diversity of learners, practical, real-world applications need to be used and this should be reflected in the choice of content.
- Identify any considerations or guidance that might be needed to interpret the syllabus, and list them for inclusion in a future support materials document. For example: *Small data sets can be constructed to investigate lines of fit and correlation coefficients. It is intended that students then apply these concepts to larger data sets from the real world.*
- Identify any suggested applications that might be used to deliver the syllabus, and list them for inclusion in a future support materials document. For example: *Students could measure body dimensions such as arm-span, height and hip-height, as well as length of stride. It is recommended that students have access to published data to provide suitable and realistic learning contexts. Comparisons could be made using parameters such as age or gender.*

## 10. Learning across the curriculum

 for your information

NSW syllabuses provide a context within which to develop core skills, knowledge and understanding considered essential for the acquisition of effective, higher-order thinking skills that underpin successful participation in further education, work and everyday life including problem-solving, collaboration, self-management, communication and information technology skills.

BOSTES has described learning across the curriculum areas that are to be included in syllabuses. In Senior Years syllabuses, the identified areas will be embedded in the descriptions of content and identified by icons. Learning across the curriculum content, including the cross-curriculum priorities and general capabilities, assists students to achieve the broad learning outcomes defined in the BOSTES *Statement of Equity Principles*, the *Melbourne Declaration on Educational Goals for Young Australians (December 2008)* and in the Australian Government's *Core Skills for Work Developmental Framework (2013)*.

Knowledge, understanding, skills, values and attitudes derived from the learning across the curriculum areas will be included in BOSTES syllabuses, while ensuring that subject integrity is maintained.

Cross-curriculum priorities enable students to develop understanding about and address the contemporary issues they face.

The cross-curriculum priorities are:

- Aboriginal and Torres Strait Islander histories and cultures 🖐️
- Asia and Australia's engagement with Asia 🇦🇺
- Sustainability 🌱

General capabilities encompass the knowledge, skills, attitudes and behaviours to assist students to live and work successfully in the 21st century.

The general capabilities are:

- Critical and creative thinking ⚙️
- Ethical understanding ⚖️
- Information and communication technology capability 💻
- Intercultural understanding 🌐
- Literacy 📖
- Numeracy 📊
- Personal and social capability 👥

BOSTES syllabuses include other areas identified as important learning for all students:

- Civics and citizenship 🗳️
- Difference and diversity 🌈
- Work and enterprise ⭐

## **Sample learning across the curriculum area for Mathematics, Mathematics Extension 1 and Mathematics Extension 2 Senior Years**

### **Critical and creative thinking**

Critical and creative thinking are key to the development of mathematical understanding. Students use critical and creative thinking as they learn to generate and evaluate knowledge, ideas and possibilities, and when seeking new pathways or solutions. Mathematical reasoning and logical thought are fundamental elements of critical and creative thinking. They are integral to mathematical problem-solving as students identify similarities and differences in mathematical situations and engage in reasoning and thinking about solutions to problems, and the strategies needed to find those solutions.

Critical and creative thinking are of fundamental importance in such aspects of the Mathematics curriculum as posing problems, modelling situations, justifying choices and strategies used, and giving reasons to explain mathematical ideas.


### **Actions for writers and key considerations**

For each learning across the curriculum area develop a succinct statement that describes how the subject provides opportunities to develop knowledge, understanding, skills, values and attitudes related to the area and its relevance. The statements below should be considered when writing these statements.

- *Aboriginal and Torres Strait Islander histories and cultures*: opportunities should be identified to use mathematics to consider cultures with relevant historical and contemporary data.
- *Asia and Australia's engagement with Asia*: opportunities should be identified to use mathematics to consider cultures with relevant historical and contemporary data.
- *Sustainability*: opportunities should be identified to consider where the mathematics of sustainability methods and data can be addressed.
- *Critical and Creative Thinking*: the current proposed statement needs to be shortened and made more concise.
- *Ethical Understanding*: opportunities should be identified to raise awareness of ethical issues, particularly in the topics relating to statistics, probability and rates of change.
- *Information and communication technology capability*: students should learn to use appropriate technology as an effective support for mathematical activity throughout the syllabus. Opportunities should therefore be identified where a variety of appropriate technology can be used.
- *Intercultural understanding*: opportunities should be identified to raise awareness of intercultural issues, particularly in the topics relating to statistics and rates of change.
- *Literacy*: the communication, problem-solving, reasoning and understanding aspects of mathematics are strongly linked with literacy and therefore areas where developing these skills is a priority should be identified.

- *Numeracy*: a statement should be written to clearly state that numeracy is embedded throughout the syllabus and therefore will not be tagged individually in the syllabus, as in the K–10 syllabus.
- *Personal and social capability*: opportunities should be identified to develop student awareness of personal and social skills either through consideration of a particular topic or through opportunities for organised group work.
- *Civics and citizenship*: opportunities should be identified to raise awareness of civics and citizenship issues, particularly in the topics relating to rates of change and statistics.
- *Difference and diversity*: opportunities should be identified to raise awareness of difference and diversity issues, particularly in the topics relating to rates of change and statistics.
- *Work and enterprise*: opportunities should be identified to raise awareness of work and enterprise issues, particularly in the topics relating to rates of change and statistics.

## **11. Glossary**

 for your information

One glossary will be developed for each Senior Years learning area. The glossary to be developed for the Mathematics Senior Years draft syllabuses will explain terms that will assist teachers in the interpretation of the subject. The glossary will be based on the NSW K–10 Mathematics glossary and Australian curriculum Senior Years Mathematics glossaries.

### **Actions for writers and key considerations**

- Identify and underline words and/or terms additional to those in the K–10 Mathematics glossary in the content for inclusion in the Senior Years glossary.

## **12. Assessment and reporting**

 for your information

BOSTES continues to promote a standards-referenced approach to assessing and reporting student achievement in NSW, and the importance of assessment for, of and as learning as essential components of quality teaching and learning.

Information on assessment and reporting for the Year 11 and Year 12 courses will be reviewed and developed for draft syllabus consultation in 2016.

The information will include:

- mandatory components and weightings for school-based assessment of the Year 12 course
- HSC examination specifications which describe the format of the HSC examination program for Mathematics, Mathematics Extension 1 and Mathematics Extension 2.

## **13. Appendix I**

### **Broad directions from consultation**

The following broad directions for syllabus development have been informed through consultation with stakeholders. These broad directions will guide the development of the NSW Mathematics Stage 6 syllabuses.

1. In the revision of the courses, consideration be given to how the courses provide flexibility to meet the needs of all students.
2. The content prescribed for each of the revised courses:
  - a. be carefully monitored to reflect the indicative time of the course. In the case of the revised Mathematics General courses and the revised Mathematics ('2 Unit') course, there should be a net reduction in content when compared to the respective current courses
  - b. be reviewed for relevance and opportunities for depth of learning.
3. The status of the revised Mathematics General 1 course be reviewed and consideration be given to:
  - a. making the course a Board Developed course with an optional HSC examination
  - b. developing a separate Year 11 course for the revised Mathematics General 1 course.
4. The nested structure of the current Mathematics ('2 Unit'), Mathematics Extension 1 and Mathematics Extension 2 courses be retained for the respective revised courses.
5. The inclusion of additional study of statistics be considered, while addressing implications in relation to the extent of relevant teacher expertise, professional development, future pathways of students, school mathematics staffing, and school timetabling.
6. In reviewing the content of the calculus-based courses, the appropriateness and relevance of the applications within the courses be explored, with a view to ensuring that they are contemporary and meet the needs of students.
7. The technology available for use by candidates in the mathematics HSC examinations be clarified in the development of the draft writing briefs.
8. The appropriateness of the current structures and durations of the HSC examinations for the senior mathematics courses be reviewed, with emphasis on the examination of '2 Unit'-only candidates.
9. An appropriate formula sheet be provided for all mathematics examinations.
10. In the development of course structures and HSC examinations for the revised Mathematics ('2 Unit') and Mathematics General 2 courses, consideration be given to student movement between the courses and the need to make meaningful comparisons of student performance.
11. The rationale, outcomes and content of the Mathematics Life Skills course be reviewed to better meet the needs of the students for whom the course is intended, as well as to provide an appropriate progression from Mathematics Life Skills Stage 5 outcomes and content and alignment with the regular mathematics Stage 6 courses where appropriate.
12. In the naming of the revised mathematics Stage 6 courses, consideration be given to the nomenclature used for English Stage 6 courses.

## 14. Appendix II

### Key matters raised during draft writing brief consultation and actions

Key matters	Actions
<p>The proposed significant overlap in the content of the Mathematics and Mathematics General 2 courses, and consequently in the respective HSC examinations, is well supported.</p>	<p>The content of the Mathematics and Mathematics General 2 courses will incorporate significant overlap which will provide for the inclusion of common questions in the respective HSC examinations.</p>
<p>The need for the determination of appropriate course relativity in ATAR scaling for Mathematics and Mathematics General 2.</p>	<p>The inclusion of common questions, on the common content in the Mathematics and Mathematics General 2 courses, will mean that data can be provided to the Universities Admissions Centre (UAC) for obtaining appropriate course relativity in ATAR scaling.</p>
<p>The Mathematics course content needs to be reduced to have a similar workload to that required for other 2-unit courses.</p>	<p>The content prescribed for each of the revised courses will be carefully monitored in terms of the indicative time available. For each of the courses, there will be a net reduction in content when compared to the current course.</p>
<p>The development of a strategy to retain students capable of studying a calculus-based course in the Mathematics course and review the current common HSC examination for Mathematics and Mathematics Extension 1.</p>	<p>Examination and assessment requirements will be considered during syllabus development.</p>
<p>The inclusion of statistics in the Mathematics course is well supported, but to a lesser extent for the Mathematics Extension courses.</p>	<p>Statistics will be incorporated in the Mathematics course and will be considered for inclusion in the Mathematics Extension courses.</p>
<p>The proposal that candidates sit one HSC examination only for each course, all at the same time on the same day, with each examination containing questions in common with the HSC examination for the next level.</p>	<p>The proposal of one HSC examination will be considered during syllabus development.</p>