BOSTES
BOARD OF STUDIES TEACHING & EDUCATIONAL STANDARDS NSW

NSW SENIOR SECONDARY REVIEW & EVALUATION: MATHEMATICS

REFERENCE REPORT

1. Historical overview

Courses

The current Mathematics ('2 Unit') and Mathematics Extension 1 courses were revised following a major review in 1982. Since then only minor amendments have been made to the courses. The Mathematics Extension 2 course was last revised in 1980, with some minor amendment in 1989.

In response to the NSW Government's White Paper *Securing their Future*, the Board of Studies, Teaching and Educational Standards NSW (BOSTES) undertook a comprehensive review of the Higher School Certificate (HSC) in the late 1990s. The General Mathematics syllabus was developed as part of this review and was endorsed in June 1999, implemented with the Preliminary cohort in 2000 and first examined for the HSC in 2001.

In recognition of the principle that the post-compulsory years of schooling should cater for all students who choose to participate, eight Stage 6 Life Skills courses were also developed in 1999, including one for Mathematics. These courses extended the curriculum and reporting arrangements that were established in Stage 5 to HSC students with intellectual disabilities. The Mathematics Life Skills course provides a curriculum option for students unable to access the outcomes and content of the regular Mathematics syllabuses. The Mathematics Life Skills course has Board Developed status and can be used to meet the requirements for the award of the HSC without an external examination. Minor amendments were made to the guidelines and assessment advice for the Mathematics Life Skills course in 2007.

The Preliminary Mathematics General course was implemented with Year 11 in 2013, with students studying either the HSC Mathematics General 1 course (CEC) or the HSC Mathematics General 2 course from 2014. The HSC Mathematics General 1 course was developed to better meet the needs of students who wish to study a senior secondary Mathematics course but whose purposes are not accommodated through the study of the HSC Mathematics General 2 course. The Preliminary Mathematics General/HSC Mathematics General 2 pathway replaced the General Mathematics course (1999). The outcomes and content for the pathway were designed to be quite similar in nature to the outcomes and content of the original General Mathematics course in order to accord with Broad Direction 8 of the 2006–2008 Mathematics Stage 6 Review and Development Project that ' the current General Mathematics course material be largely maintained within the structure of Stage 6 non-calculus-based Mathematics courses'.

Assessment and examination

In 2008, the BOSTES undertook a major review of HSC assessment, and changes to examination specifications and school assessment requirements were implemented for HSC courses from 2010. The revised HSC internal assessment requirements and examination specifications set by the BOSTES following this review, were applied to the HSC General Mathematics course in 2012 and 2013, and have been applied to the new HSC Mathematics General 2 course from 2014.

For Mathematics General 2, the HSC assessment components and weightings are *Concepts, skills and techniques* 50% and *Reasoning and communication* 50%, while the HSC examination has been structured as a two-and-a-half hour examination with two sections: Section I, 25 multiple-choice questions, Section II, five 15-mark short-answer questions.

For Mathematics ('2 Unit'), Mathematics Extension 1 and Mathematics Extension 2, the revised HSC assessment components and weightings implemented with the 2012 HSC are *Concepts, skills and techniques* 50% and *Reasoning and communication* 50%.

For Mathematics ('2 Unit'), Mathematics Extension 1 and Mathematics Extension 2 revised HSC examination specifications were also implemented with the 2012 HSC. The examination structures are now as follows:

- Mathematics ('2 Unit') – a three-hour examination with two sections: Section I, 10 multiple-choice questions, Section II, six 15-mark short-answer questions
- Mathematics Extension 1 – a two-hour examination with two sections: Section I, 10 multiple-choice questions, Section II, four 15-mark short-answer questions
- Mathematics Extension 2 – a three-hour examination with two sections: Section I, 10 multiple-choice questions, Section II, six 15-mark short-answer questions.

Further detail about the history of the current NSW Stage 6 Mathematics courses and the associated assessment requirements and examination specifications is provided in Appendix 1.

2. Course requirements

Courses within the Mathematics key learning area offer flexibility for teachers to develop a program of study that is suited to the needs and interests of students, as well as for students to demonstrate their knowledge, skills and understanding in a range of ways.

The following table provides an overview of the structures and assessment experiences provided within the Stage 6 Mathematics courses.

Table 1: Stage 6 Mathematics course requirements

Course	Structure		Assessment		
	Core (topic areas)	Options	School-based	HSC examination	Submitted work
Preliminary Mathematics General	7	–	Yes		
HSC Mathematics General 1	9	–	Yes	No	No
HSC Mathematics General 2	7	–	Yes	Yes	No
Preliminary Mathematics ('2 Unit')	7	–	Yes		
HSC Mathematics ('2 Unit')	9	–	Yes	Yes	No
Preliminary Mathematics Extension	9	–	Yes		
HSC Mathematics Extension 1	12	–	Yes	Yes	No
HSC Mathematics Extension 2	8	–	Yes	Yes	No
Mathematics Life Skills	–	6	Yes	No	No

Mathematics General

The purpose of the Mathematics General courses is to provide an appropriate mathematical background for students who wish to enter occupations that require the use of a variety of mathematical and statistical techniques and tools, including relevant technologies. As well as introducing some new mathematical content, the Focus Studies within the courses give students the opportunity to apply, and develop further, the knowledge, skills and understanding initially developed in the five strands of the courses. Through the Focus Studies, students develop the capacity to integrate their knowledge, skills and understanding across the strands in contemporary

contexts chosen for their ongoing relevance to the students' everyday lives and likely vocational pathways.

The Preliminary Mathematics General course is structured to provide appropriate pathways to both HSC Mathematics General 1 and HSC Mathematics General 2. Preliminary Mathematics General provides an appropriate course of study for students who have demonstrated competence in Mathematics up to and including at least Stage 5.1 by the end of Year 10.

HSC Mathematics General 1 has been written to meet the needs of students who have demonstrated competence in Preliminary Mathematics General. The course provides students with the opportunity to develop an understanding of and competence in further aspects of Mathematics through a large variety of real-world applications for concurrent HSC studies, such as in vocational education and training courses, other practically oriented courses, and some humanities courses, and for vocational pathways, in the workforce or in further training.

HSC Mathematics General 2 has been written on the assumption that students have demonstrated a high level of competence in Preliminary Mathematics General. The course provides students with the opportunity to develop an understanding of and competence in further aspects of Mathematics through a large variety of real-world applications for a range of concurrent HSC studies, such as in the life sciences, the humanities and business studies. The course also provides a strong foundation for vocational pathways, in the workforce and in further training, and for university courses in the humanities, nursing and paramedical sciences.

Mathematics and Mathematics Extensions

The calculus-based Mathematics ('2 Unit'), Mathematics Extension 1 and Mathematics Extension 2 courses are designed to promote, at progressively higher levels, the development of knowledge, skills and understanding in relation to important concepts within areas of Mathematics that have applications in an increasing number of contexts. Across the three courses, this includes the development of deductive and inductive reasoning skills and understanding of proof, the ability to interpret and use mathematical models, and the ability to make more and more advanced generalisations, with increasing levels of communication and literacy. Students learn to use a range of techniques and tools, including relevant technologies, in order to develop solutions to a wide variety of problems relating to their present and future needs and aspirations.

The Mathematics ('2 Unit')/Mathematics Extension 1 syllabus (*Mathematics 2/3 Unit Syllabus – Years 11–12* (1982)) states that the Mathematics ('2 Unit') course is intended for students who have completed Mathematics to Year 10 and demonstrated general competence in all the associated skills, and is useful for concurrent studies in Science and Commerce. The syllabus also states that the course is a sufficient basis for further studies in Mathematics as a minor discipline at tertiary level in support of courses such as the life sciences or Commerce.

The Mathematics ('2 Unit')/Mathematics Extension 1 syllabus states that the Mathematics Extension 1 course is intended for students who have completed Mathematics to Year 10 and demonstrated a mastery of the associated skills, and who are interested in the study of further skills and ideas in Mathematics. The syllabus also states that the course is useful for concurrent studies of Science, Industrial Arts and Commerce and is a recommended minimum basis for further studies in Mathematics as a major discipline at a tertiary level, and for the study of Mathematics in support

of the physical and engineering sciences. It is recommended in the syllabus that, although the Mathematics Extension 1 course is sufficient for these purposes, students of outstanding mathematical ability should consider undertaking the Mathematics Extension 2 course.

Mathematics Life Skills

Students enrolling in a Stage 6 Life Skills course will usually have completed Years 7–10 Life Skills outcomes and content in one or more courses. The Life Skills course provides greater flexibility for teachers to select outcomes and content that meet students' individual learning needs, strengths, goals and interests.

The Mathematics Life Skills course is designed for the small percentage of students, particularly those with an intellectual disability, for whom adjustments to teaching, learning and assessment are not sufficient to access some or all of the regular Mathematics outcomes. The course develops students' knowledge, skills and understanding in numerical concepts and the application of these to everyday situations.

3. Candidature

The following tables summarise candidature in each course within the Mathematics key learning area from 2009–2013.

Table 2: HSC candidature by Mathematics course 2009–2013

Course	2009			2010			2011			2012			2013		
	M	F	T	M	F	T	M	F	T	M	F	T	M	F	T
HSC General Mathematics*	14937	14974	**29911**	15390	15602	**30992**	15610	16023	**31633**	15804	15898	**31702**	15881	16498	**32379**
HSC Mathematics ('2 Unit')	9233	7964	**17197**	9289	7863	**17152**	8854	7710	**16564**	9022	7678	**16700**	8953	7510	**16463**
HSC Mathematics Extension 1	4960	3671	**8631**	5363	3755	**9118**	5154	3670	**8824**	5247	3678	**8925**	5191	3648	**8839**
HSC Mathematics Extension 2	1897	1273	**3170**	2175	1295	**3470**	2198	1243	**3441**	2181	1273	**3454**	2030	1168	**3198**
Mathematics Life Skills (Year 12)	601	456	**1057**	632	464	**1096**	710	417	**1127**	697	447	**1144**	802	506	**1308**

M = Male *F = Female* *T = Total*

*Figures are for the General Mathematics course (study of this course concluded with the 2013 HSC course cohort).

Table 3: Preliminary candidature by Mathematics course 2009–2013

Course	2009			2010			2011			2012			2013		
	M	F	T	M	F	T	M	F	T	M	F	T	M	F	T
Preliminary General Mathematics*	17340	17395	**34735**	17649	18120	**35769**	17372	17972	**35344**	17656	18455	**36111**	20058	19649	**39707**
Preliminary Mathematics ('2 Unit')	14727	12107	**26834**	14255	11877	**26132**	14390	11644	**26034**	13980	11544	**25524**	13654	11096	**24750**
Preliminary Mathematics Extension	7538	5352	**12890**	7334	5279	**12613**	7330	5362	**12692**	7253	5267	**12520**	7306	5123	**12429**
Mathematics Life Skills (Year 11)	728	488	**1216**	822	477	**1299**	887	517	**1404**	984	584	**1568**	1121	514	**1635**

M = Male *F = Female* *T = Total*

*Figures for 2009–2012 are for the Preliminary General Mathematics course (study of this course concluded with the 2012 Preliminary course cohort). The 2013 Preliminary course cohort represented the initial cohort for Preliminary Mathematics General

Table 4: Candidature by system for HSC Mathematics courses 2009–2013

Course	2009			2010			2011			2012			2013		
	Gov	Ind	Syst	Gov	Ind	Syst	Gov	Ind	Syst	Gov	Ind	Syst	Gov	Ind	Syst
HSC General Mathematics*	17255	6717	5546	18041	6672	5826	18456	6771	6028	17994	7226	6139	18331	7268	6493
HSC Mathematics ('2 Unit')	8829	5799	2423	9015	5538	2484	8730	5265	2445	8949	5314	2307	8900	4998	2492
HSC Mathematics Extension 1	4801	2891	896	5123	3082	884	5075	2788	927	5128	2849	914	5090	2720	1018
HSC Mathematics Extension 2	1903	1038	213	2100	1116	242	2131	1052	247	2120	1061	260	2020	930	244
Mathematics Life Skills (Year 12)	909	78	70	991	49	56	994	62	71	985	85	74	1122	100	85

Gov = Government schools *Ind* = Independent schools *Syst* = Systemic schools

*Figures are for the General Mathematics course (study of this course concluded with the 2013 HSC course cohort).

Table 5: Candidature by area for HSC Mathematics courses 2009–2013

Course	2009			2010			2011			2012			2013		
	Gov	Ind	Syst	Gov	Ind	Syst	Gov	Ind	Syst	Gov	Ind	Syst	Gov	Ind	Syst
HSC General Mathematics*	17013	12823	75	17688	13222	82	17695	13859	79	17851	13752	99	18531	13736	112
HSC Mathematics ('2 Unit')	12238	4678	281	12226	4657	269	11946	4373	245	12001	4484	215	12032	4242	189
HSC Mathematics Extension 1	6752	1739	140	7186	1817	115	7091	1640	93	7095	1745	85	7122	1652	65
HSC Mathematics Extension 2	2678	461	31	2936	509	25	2942	474	25	2959	474	21	2759	424	15
Mathematics Life Skills (Year 12)	544	513	–	579	517	–	605	522	–	614	530	–	683	625	–

Met = Schools in the Sydney metropolitan region *Reg* = Schools in country areas of NSW *O'seas* = Schools located outside Australia

*Figures are for the General Mathematics course (study of this course concluded with the 2013 HSC course cohort).

Table 6: Retention of candidates from Preliminary to HSC Mathematics courses 2009–2013

Course	2008–09			2009–10			2010–11			2011–12			2012–13		
	Prelim	HSC	% Ret	Prelim	HSC	% Ret	Prelim	HSC	% Ret	Prelim	HSC	% Ret	Prelim	HSC	% Ret
General Mathematics*	33136	29911	90	34735	30992	89	35769	31633	88	35344	31702	90	36111	32379	90
Mathematics	27110	17197	63	26834	17152	64	26132	16564	63	26034	16700	64	25524	16463	65
Mathematics Extension 1	12691	8631	68	12890	9118	71	12613	8824	70	12692	8925	70	12520	8839	71
Mathematics Extension 2	–	3170	N/A	–	3470	N/A	–	3441	N/A	–	3454	N/A	–	3198	N/A
Mathematics Life Skills	1191	1057	89	1216	1096	90	1299	1127	87	1404	1144	82	1568	1308	83

*Figures are based on the previous General Mathematics course, as 2014 is the first HSC candidature for HSC General Mathematics 1 and HSC General Mathematics 2.

There is continuing strong demand for all five available Mathematics courses for the HSC. However, over the period 2009–2013 there was an overall fall in the proportion of HSC Year Mathematics students studying one or more of the calculus-based Mathematics ('2 Unit'), Mathematics Extension 1 and Mathematics Extension 2 courses for the HSC, while there was a related increase in the proportion studying General Mathematics. This decline in the proportion of students studying the calculus-based courses has been even more significant over the period of the current HSC ie 2001–2013.

The gender balance in the HSC cohorts for the four courses has remained relatively stable over the period 2009–2013. Females continue to be under-represented in the calculus-based courses, especially in the Mathematics Extension 2 course.

Over the period 2009–2013 there was an overall fall in the proportion of Preliminary Mathematics students studying one or both of the Preliminary Mathematics ('2 Unit') and Preliminary Mathematics Extension courses, while there was a related increase in the proportion studying Preliminary General Mathematics (2009–2012) and Preliminary Mathematics General (2013).

The gender balance in the Preliminary cohorts for the four courses has remained relatively stable over the period 2009–2013. Females continue to be under-represented in the calculus-based courses.

Number and percentage of HSC students completing an HSC Mathematics course

Table 7 shows the number and percentage of students awarded the HSC who completed an HSC Mathematics course during the period 2003–2013. The number of students studying a mathematics course in 2013 was very similar to the number of students studying a Mathematics course in 2003, but because of the increase in the overall number of HSC students, the proportion of students completing a Mathematics course has declined from 88% to 80%. The number of students who completed the Preliminary Mathematics ('2 Unit') course and then transferred to General Mathematics for the HSC has been relatively stable over the same period at between 4000 and 4500 students annually, or between 15% and 18% of the Preliminary Mathematics ('2 Unit') enrolment.

It should be noted that there were approximately an additional 4000 students who completed a Preliminary Mathematics course only – this would increase the participation rate from 80% to approximately 86%.

Table 7: Number and percentage of students awarded the HSC who completed an HSC Mathematics course 2003–2013

HSC Year	Total HSC Students	HSC Mathematics	
		No.	%
2003	60 762	53 485	88%
2004	61 652	53 065	86%
2005	61 106	51 311	84%
2006	61 477	50 934	83%
2007	62 277	50 797	82%
2008	63 203	51 021	81%
2009	63 565	50 739	80%
2010	65 214	52 188	80%
2011	66 191	52 962	80%
2012	66 674	53 901	81%
2013	66 841	53 348	80%

4. NSW consultation on the senior secondary Australian curriculum

The BOSTES conducted consultation on the draft Australian senior secondary curriculum for Mathematics during June–July 2012. The NSW consultation consisted of metropolitan and regional face-to-face focus group meetings with teachers and key stakeholders, as well as an online survey. A range of submissions was also received from the NSW education sectors, professional associations and individuals. The *Senior Secondary Mathematics Consultation Report* can be accessed through the BOSTES website at <http://www.boardofstudies.nsw.edu.au/australian-curriculum/11-12-eng-maths-sci-hist.html>.

The 2012 consultation identified similar content between the senior secondary Australian curriculum for Mathematics and the senior secondary NSW Mathematics curriculum.

Essential Mathematics (Australian curriculum) and Preliminary Mathematics General/HSC Mathematics General 1 Pathway (NSW)

The rationales for the two courses imply a similar target audience.

The Essential Mathematics course has a significant remediation element with approximately 30% of the content studied in NSW in Stage 5.1 or earlier, while approximately 50–60% of the content is studied in Preliminary Mathematics General/HSC Mathematics General 1.

General Mathematics (Australian curriculum) and Preliminary Mathematics General/HSC Mathematics General 2 Pathway (NSW)

The rationales for the two courses state that they focus on mathematical skills and techniques that have direct application to everyday activity, in contrast with the more abstract approach taken by calculus-based senior Mathematics courses.

The General Mathematics (Australian curriculum) content has a high emphasis on graph theory, networks and statistics without any clear justification for their inclusion and/or extent. Approximately 50% of the content is studied in Preliminary Mathematics General/HSC Mathematics General 2.

Mathematical Methods (Australian curriculum) and Mathematics ('2 Unit') (NSW)

The rationales for the two courses state that they are designed for students whose future pathways may involve Mathematics and Statistics and their applications in a range of disciplines at the tertiary level. This implies that the target audience for the Mathematical Methods course is equivalent to the target audience for the NSW Mathematics ('2 Unit') course.

Mathematical Methods is too difficult for many students wishing to study an introductory calculus-based course only, but its target audience does not appear to include students who require substantial Mathematics at a tertiary level supporting the physical sciences, Computer Science or Engineering, as is the case for the Mathematics Extension 1 course. The course content has a high emphasis on statistics without any clear justification for its inclusion and/or extent. Approximately 50–60% of the content is studied in the Mathematics ('2 Unit') course, while approximately 5–10% is studied in the Mathematics Extension 1 course.

Specialist Mathematics (Australian curriculum) and Mathematics Extension 1, Mathematics Extension 2 (NSW)

The rationale for the Specialist Mathematics course is in common with that of the Mathematics Extension 1 course in its statement that it lays the foundations for future studies in quantitative subjects in a coherent and structured fashion. Its target audience includes students intending to study Mathematics, Statistics, all sciences and associated fields, Economics or Engineering at university. This implies that the target audience for the Specialist Mathematics course is at least equivalent to the target audience for the NSW Mathematics Extension 1 course. It falls short of stating (as is stated in the Introduction to the NSW Mathematics Extension 2 course) that 'it is designed for students with a special interest in mathematics who have shown that they possess special aptitude for the subject'.

The rationale relates to that of the Mathematics Extension 2 course, but to a lesser extent, in its statement that it is designed for students with a strong interest in Mathematics, and that students of Specialist Mathematics will be able to appreciate the true nature of Mathematics, its beauty and its functionality. The course, which is required to be taken in conjunction with Mathematical Methods, does not provide the appropriate degree of difficulty and challenge for very able students.

Specialist Mathematics is a two-year course beginning in Year 11 and is a succession of largely disconnected topics. The course is too difficult for many able students, while being of insufficient challenge for many of the most able students. Approximately 5–10% of the Specialist Mathematics content is studied in the Mathematics ('2 Unit') course, approximately 15–20% is studied in the Mathematics Extension 1 course, while approximately 20% is studied in the Mathematics Extension 2 course.

Mathematical Methods and Specialist Mathematics are not sufficient in themselves to meet the needs of the range of students wishing to study a calculus-based course. Many students are likely to see the two courses as being of different nature, rather than as an integrated whole.

Clear rationales for the selection or omission of different content within the Australian curriculum courses have not been provided. Without these rationales it is not clear that the content of the courses represents appropriate scopes of learning for the targeted students.

There is a lack of flexibility in the Australian curriculum to enable students to move between courses. This is of greatest concern in relation to General Mathematics and Mathematical Methods. These two courses are at the middle of the suite and have minimal content overlap. This will make it very difficult for students to change courses from Mathematical Methods to General Mathematics.

Following consultation, the BOSTES provided advice to the Australian Curriculum, Assessment and Reporting Authority (ACARA) about the senior secondary Australian curriculum for Mathematics, including:

- Further consideration, particularly in the cases of General Mathematics, Mathematical Methods and Specialist Mathematics, needs to be given to the alignment between the course rationales and the respective sets of content prescribed for the courses.
- Flexibility is needed to arrange and deliver the content as appropriate to New South Wales. While the totality of the content and the appropriateness of structures for its hierarchy need to be reconsidered, the similarity of difficulty level and the minimal content overlap of the proposed General Mathematics and Mathematical Methods courses need particular reconsideration.
- In order to develop focused and coherent senior Mathematics courses, a rationale for the selection of specific content and the exclusion of other content is required.
- In aiming to achieve appropriate balance, sequencing and level of difficulty in the further development of the proposed curriculum, the appropriateness of a unitised structure needs careful investigation.

- In order to cater for the full range of students wishing to undertake the study of calculus-based Mathematics in the senior secondary years and the broad range of their aspirations, an appropriately coherent set of content needs to be developed that allows the construction of a rigorous and unified 'middle' calculus-based course of study.
- The expected role of ICT needs to be identified and applications of ICT need to be explicitly included in the content.

In revising the senior secondary NSW curriculum for Mathematics, the advice to ACARA will be considered.

5. Literature review

5.1 ACARA literature review

The National Curriculum Board (now ACARA) began work on framing the Australian curriculum for Mathematics in 2008 by recruiting a writer who worked with a small advisory group to draft an initial advice paper that provided a broad scope and sequence from Foundation to Year 12.

The initial advice paper was discussed at a national forum in October 2008. A small group of nominees from the Australian Association of Mathematics Teachers (AAMT) met with the writers to discuss the feedback from the forum and its implications for developing the curriculum.

The *National Mathematics Curriculum: Framing paper* built on this initial advice, as well as advisory group feedback; submissions through the National Curriculum Board's website; individual responses by academics and teachers; responses from national, state and territory forums; and responses received through email and letters.

In developing the four Australian curriculum senior secondary Mathematics courses, ACARA indicated in its *Senior Secondary Mathematics – Information Sheet* that it reviewed state and territory Mathematics curriculums, that its work was further guided by some key international and national references, and that the senior secondary courses were reviewed by overseas experts and international curriculum authorities.

ACARA's consultation of key international and national references included consideration of the structures and content of the senior Mathematics curriculums of Finland, Singapore, Hong Kong and the United Kingdom, the 2008 *Maths? Why Not?* report prepared for the Department of Education, Employment and Workplace Relations (DEEWR), and international research on the inclusion of statistics in the Mathematics curriculum.

The *Maths? Why Not?* report conveys concerns regarding Australia's capacity to ensure young people have the requisite mathematics background and skills to pursue careers in Science, Technology, Engineering and Mathematics (STEM). This would in turn affect Australia's competitiveness in a global market. Findings of the report also indicate that nationally levels of mathematical literacy in the early years of secondary schooling are leading to increased enrolments in senior mathematics courses, although enrolments in higher-level mathematics courses are declining in comparison with enrolments in lower-level mathematics courses (McPhan et al 2008).

The research question identified for the project, *Why is it that capable students are not choosing to take higher-level mathematics in the senior years of schooling?*, was therefore also an important consideration for ACARA in developing its senior secondary Mathematics courses.

5.2 NSW literature review

HSC Subject Evaluation 1998

As part of the 1998 HSC subject evaluation process for Mathematics following the release of the Government's White Paper, *Securing Their Future* (1997), the BOSTES commissioned Professor Kaye Stacey et al, of the University of Melbourne, to compile a curriculum and literature review, *Review of Senior Secondary Mathematics Curriculum*. The review compared the NSW curriculum model with interstate and overseas Mathematics curriculums and listed issues that needed to be further considered in any future syllabus review.

Mathematics Stage 6 Review and Development Project (2006–2008)

In 2006, the BOSTES commissioned Dr Mary Coupland of the Department of Mathematical Sciences, University of Technology, Sydney to write a curriculum review and literature review (available on the BOSTES website) of the 2006–2008 Mathematics Stage 6 Review and Development Project. Further information in relation to the reviews can be found at Appendix 2.

In the curriculum review, *A critical analysis of selected Australian and international mathematics syllabuses for the post-compulsory years of secondary schooling*, the Executive Summary included that:

- the current NSW Mathematics courses provide an appropriate level of challenge but need to reflect current uses of technology and contemporary applications of Mathematics
- the NSW Mathematics courses do not provide the same opportunities offered in other states for students needing more time to reach the numeracy levels expected by the end of Year 10
- the use of technology for the teaching and learning of Mathematics requires consideration, including the ways in which technology can be used to engage students in real-world applications and the ways that technology is changing the way Mathematics is used in the workplace
- a shift in focus is required that places tasks, rather than tools, at the centre of learning.

The literature review, *Literature Review and Annotated Bibliography to Inform the Stage 6 Mathematics Review*, included sections 'Relevant Theories of Learning', 'Research Concerning Teaching and Learning with New Technologies, including Computer Algebra Systems (CAS) and Graphing Display Calculators (GDCs)', 'Concluding comments on the place of new technologies in the post-compulsory mathematics curriculum', and 'The Place Of Statistics in Post-Compulsory Mathematics Curricula' (see Appendix 2).

The curriculum review and literature review were important strategies among a range of data-gathering strategies used in the first phase of the Mathematics Stage 6 Review and Development Project, *Syllabus Review*.

The information obtained through the data-gathering strategies used in the *Syllabus Review* phase was analysed to identify issues that needed to be considered in the revision and development of NSW Stage 6 Mathematics courses. Key findings were synthesised from the data and a set of Broad Directions (listed below) for the revision and development compiled.

Broad Directions for the revision and development of Stage 6 Mathematics (Mathematics Stage 6 Review and Development Project 2006–2008)

- That the set of Stage 6 Mathematics courses include an additional offering to accommodate the purposes of students who wish to study a Board-developed Mathematics course in Stage 6 but who are currently choosing not to, as well as those whose purposes are not accommodated through the study of General Mathematics.
- That in the revision, due attention be given to clarifying the purpose of each course and identifying future learning or vocational pathways of the intended candidatures.
- That the nested structure of the current Mathematics ('2 Unit'), Mathematics Extension 1 and Mathematics Extension 2 courses, and the calculus basis of these courses, be retained.
- That any revision or development of the calculus-based courses maintains the current rigour and level of challenge of the courses.
- That the amount of content prescribed for Stage 6 Mathematics courses reflects the amount that can be taught, and learnt by the typical student, in the indicative time.
- That in reviewing the content of Stage 6 mathematics courses, particular attention be given to the purpose of the Mathematics ('2 Unit') course for Mathematics ('2 Unit')-only students, and the appropriateness and relevance of the course content for those students.
- That the inclusion of additional study of Statistics be considered, while addressing implications in relation to the extent of relevant teacher expertise, professional development, future pathways of students, school Mathematics staffing, and school timetabling.
- That the current General Mathematics course material be largely maintained within the structure of Stage 6 non-calculus-based Mathematics courses.
- That, in reviewing the content of the calculus-based courses, the appropriateness and relevance of the applications within the courses be explored, with a view to ensuring that they are contemporary and that they meet the needs of students.
- That, in the consideration of the use of technology in Stage 6 Mathematics courses, due regard be given to the related access and equity issues.
- That the use of technology with capabilities beyond the level of scientific calculators be encouraged in the learning and teaching, and school-based assessment, of all Stage 6 Board-developed Mathematics courses.
- That the non-calculus-based Stage 6 Mathematics courses be developed with the view that technology with capabilities beyond the level of scientific calculators will need to be used for aspects of the associated HSC examinations.
- That the use of technology in HSC examinations for the calculus-based courses be further investigated and clarified in the writing brief phase.
- That the appropriateness of the current processes for the examination of Stage 6 Mathematics courses be reviewed, with particular emphasis on the examination of Mathematics ('2 Unit')-only candidates.
- That the syllabus documents within the Stage 6 Mathematics syllabus package incorporate applications, implications and considerations for the teaching of the syllabus content, including in relation to depth of coverage.
- That each Stage 6 Mathematics course be named so as to avoid confusion with the discipline itself.

5.3 Recent significant developments and practices in Mathematics

There is increasing focus on the relevance of Science, Technology, Engineering and Mathematics (STEM) education as a means to lift productivity and economic growth and secure Australia's future in a global context of continual change. Curriculum should provide opportunities for students to become STEM literate and to obtain the knowledge, understanding and skills to participate in a STEM workforce as well as become proficient STEM practitioners. Developments in senior secondary Mathematics syllabuses will consider the incorporation of practical skills, the inclusion of learning that is inquiry-based, fostering creativity and reflection, gender equity in the acquisition of STEM concepts and skills, and the knowledge and skills required to participate in a STEM workforce.

In relation to 21st century learning, including the use of digital technologies and ICT, and developments in pedagogy regarding the teaching of Mathematics, the literature review commissioned by the BOSTES for the 2006–2008 Mathematics Stage 6 Review and Development Project, contains considerable detail under 'Research Concerning Teaching and Learning with New Technologies, including Computer Algebra Systems (CAS) and Graphing Display Calculators (GDCs)' (see Appendix 2).

This includes that:

> Mathematics education in the post-compulsory years has more than one goal. First, students need to be equipped for participation in the world of work and to participate in society as responsible citizens. … Second, students intending to study mathematics at university or to use mathematics to support other university studies need to learn the language of advanced mathematics, based as it is on algebra and functions. While these topics have traditionally been taught with pen and paper methods, the availability of graphics calculators and CAS supports a widening of approaches to include more exploration and reduce time spent perfecting routines.

The review indicates, in summary, that for both of the goals of post-compulsory Mathematics, the new technologies and internet access support a much-enriched approach.

6. Discussion on the proposed revisions to NSW senior secondary Mathematics courses

The NSW Stage 6 Mathematics courses most appropriate for comparison with the respective Australian curriculum senior secondary Mathematics courses are listed in the table below.

Table 8: NSW Stage 6 Mathematics courses most appropriate for comparison with the Australian curriculum senior secondary Mathematics courses

Australian curriculum senior secondary Mathematics courses	NSW Stage 6 Mathematics courses most appropriate for comparison
Essential Mathematics	Preliminary Mathematics General, HSC Mathematics General 1, HSC Mathematics General 2
General Mathematics	Preliminary Mathematics General, HSC Mathematics General 1, HSC Mathematics General 2
Mathematical Methods	Mathematics ('2 Unit'), Mathematics Extension 1
Specialist Mathematics	Mathematics Extension 1, Mathematics Extension 2

An analysis providing a comparison between each Australian curriculum senior secondary Mathematics course and its most closely aligned NSW Stage 6 Mathematics course(s) was undertaken by BOSTES officers and presented to the BOSTES at its May 2013 meeting. The analysis identified similarities and differences and illustrated these with examples. It provided, in broad terms, an indication of the degree of change represented in the Australian curriculum for NSW and summarised conceptual issues that would need to be addressed in considering the incorporation of the Australian curriculum into new NSW senior secondary courses.

It is proposed that the following Stage 6 'non-calculus-based' courses and 'calculus-based' courses make up the suite of NSW Stage 6 Mathematics courses from 2016.

Non-calculus-based courses

Preliminary Mathematics General, HSC Mathematics General 1 and HSC Mathematics General 2

The non-calculus-based courses Preliminary Mathematics General, HSC Mathematics General 1 and HSC Mathematics General 2 are new senior secondary Mathematics courses that were implemented with Year 11 (Preliminary course) in 2013 and with Year 12 (HSC courses) in 2014. Teachers and schools have been familiarising themselves with the new courses since early 2012, undertaking professional development in relation to new content areas within the courses.

For these reasons, it is proposed that the Preliminary Mathematics General, HSC Mathematics General 1 and HSC Mathematics General 2 courses be retained. However, minor revisions of the courses need to be considered and undertaken, as appropriate. This would include careful consideration of the Australian curriculum Mathematics courses and incorporation, where appropriate, of relevant content.

Calculus-based courses

Mathematics ('2 Unit'), Mathematics Extension 1 and Mathematics Extension 2

In the 'HSC Subject Evaluation' of the Stage 6 calculus-based Mathematics courses in 1998, there was an 'overwhelming view identified through the consultative process that the present structure of 2/3/4 Unit Mathematics should be continued for the immediate future, with the same indicative time allocations' (240 hours for 2 Unit, 120 hours 3 Unit, 60 hours for 4 Unit).

In the 2006–2008 Mathematics Stage 6 Review and Development Project, the Broad Directions for the revision and development of Stage 6 Mathematics included:

> That the nested structure of the current Mathematics ('2 Unit'), Mathematics Extension 1 and Mathematics Extension 2 courses, and the calculus basis of these courses, be retained.

and

> That any revision or development of the calculus-based courses maintains the current rigour and level of challenge of the courses.

The 'Mathematics 2 Unit' (suggested title), Mathematics Extension 1 and Mathematics Extension 2 courses will form a continuum to provide opportunities at progressively higher levels for students to acquire knowledge, skills and understanding in relation to important concepts within areas of Mathematics that have applications in an increasing number of contexts. These concepts and

applications are appropriate to the students' continued experience of Mathematics as a coherent, interrelated, interesting and intrinsically valuable study that forms a basis for future learning.

The concept of a function of a real variable, the algebraic and geometrical representations of a number of important functions, and the introductory concepts and techniques of differential and integral calculus, will together form a strong basis of the courses. These concepts, representations and techniques will be developed and utilised across the courses.

Calculus is concerned with how quantities change and is of fundamental importance in Mathematics. It builds on knowledge, skills and understanding developed earlier in Algebra, Geometry and Trigonometry. The concepts and techniques of calculus provide a means of modelling and developing increased understanding of many real-world situations, and of solving a variety of related problems. These situations and problems include many of those arising in the sciences, including in relation to the natural environment and medicine, and in statistics, business, finance and economics. A number of related applications will be studied in the Mathematics 2 Unit, Mathematics Extension 1 and Mathematics Extension 2 courses.

The Mathematics 2 Unit course will be written on the assumption that students have demonstrated competence in Mathematics up to and including the Stage 5.2 level by the end of Year 10. The course will provide such students with the opportunity to develop an understanding of and competence in further aspects of Mathematics through real-world applications for concurrent HSC studies, such as in Science, Business studies and Economics, and for further studies at tertiary level in such areas as the life sciences, Business, Finance, Technology and Education. At the same time, the Mathematics2 Unit course will provide an appropriate basis for the study of the Stage 6 Mathematics Extension courses.

The Mathematics Extension 1 course will be written to meet the needs of students who have demonstrated a high level of competence in Mathematics up to and including the Stage 5.3 level by the end of Year 10. The course will provide such students with the opportunity to develop a thorough understanding of and competence in further aspects of Mathematics through real-world applications for concurrent HSC studies, such as in Science, Engineering Studies and Economics, and for further studies at tertiary level in Mathematics and in such areas as the physical sciences and Engineering.

At the same time, the Mathematics Extension 1 course, together with the Mathematics 2 Unit course, will provide an appropriate basis for the study of the Mathematics Extension 2 course, which will be written to meet the needs of students who have demonstrated outstanding ability in Mathematics. The course will represent a distinctly high level in school Mathematics and provide such students with the opportunity to develop considerable manipulative skills and a high degree of understanding of the fundamental ideas of algebra and calculus. The course, therefore, provides a sufficient basis for a wide range of useful applications of Mathematics as well as a strong foundation for the further study of the subject.

The review and development of the Stage 6 calculus-based courses will include careful consideration of the Australian curriculum Mathematics courses, and incorporation of as broad a range of aspects of the courses as is appropriate.

Possible structures for the new calculus-based courses, Mathematics 1 Unit, Mathematics Extension 1, and Mathematics Extension 2 are provided in the paper *NSW senior secondary review and evaluation: English, Mathematics, Science and History.*

The current suite of NSW senior secondary Mathematics courses, the likely extent of revision of each of these courses, the level of consideration of the content of the senior secondary Australian curriculum Mathematics courses for incorporation in each revised course, and the resultant new course in each case, are listed in the table below.

Table 9: Proposed revisions of NSW Stage 6 Mathematics courses

Current NSW Stage 6 Mathematics course	Likely extent of revision	Consideration of Australian curriculum content for incorporation	New NSW Stage 6 Mathematics course
Preliminary Mathematics General	minor*	some aspects	Preliminary Mathematics General
HSC Mathematics General 1	minor	some aspects	HSC Mathematics General 1
HSC Mathematics General 2	minor*	some aspects	HSC Mathematics General 2
Preliminary Mathematics ('2 Unit')	moderate–major	full consideration	Preliminary Mathematics 2 Unit
HSC Mathematics ('2 Unit')	moderate–major	full consideration	HSC Mathematics 2 Unit
Preliminary Mathematics Extension	moderate–major	full consideration	Preliminary Mathematics Extension
HSC Mathematics Extension 1	moderate–major	full consideration	HSC Mathematics Extension 1
HSC Mathematics Extension 2	moderate–major	full consideration	HSC Mathematics Extension 2
Mathematics Life Skills	moderate–major	n/a	Mathematics Life Skills

* A greater level of revision would be considered if required to accommodate/accord with agreed changes to the Mathematics ('2 Unit') course.

6.1 Assessment and examination specifications

The following table shows the current and proposed assessment requirements and examination specifications for the new senior secondary Mathematics courses.

Table 10: Proposed assessment requirements and examination specifications for new NSW Stage 6 Mathematics courses

Current NSW Stage 6 Mathematics course	Current examination specifications	New NSW Stage 6 Mathematics course	Proposed assessment requirements (all as currently)	Proposed examination specifications
Preliminary Mathematics General		Preliminary Mathematics General	Suggested weightings: concepts, skills and techniques 50%, reasoning and communication 50%	
HSC Mathematics General 1 (CEC)		HSC Mathematics General 1 (CEC)		
HSC Mathematics General 2	Two-and-a-half-hour written exam: objective-response 25, short answer 75	HSC Mathematics General 2	Component weightings: concepts, skills and techniques 50%, reasoning and communication 50%	Two-and-a-half hour written exam: objective-response 25, short answer 75 **(status quo)**
Preliminary Mathematics ('2 Unit')		Preliminary Mathematics 2 Unit	Suggested weightings: concepts, skills and techniques 50%, reasoning and communication 50%	
HSC Mathematics ('2 Unit')	Three-hour written exam: objective-response 10, short answer 90	HSC Mathematics 2 Unit	Component weightings: concepts, skills and techniques 50%, reasoning and communication 50%	Two-and-a-half-hour written exam (83 marks) with two options (30 marks each) for last two short-answer questions (15 marks each). Each version of the exam will therefore have 53 marks in common. For '2 Unit only' students (with 'Option 1' for last two questions): objective-response 8, short answer 75 For Extension 1 students (with 'Option 2' for last two questions): objective-response 8, short answer 75 **(new*)**
Preliminary Mathematics Extension		Preliminary Mathematics Extension	Suggested weightings: concepts, skills and techniques 50%, reasoning and communication 50%	
HSC Mathematics Extension 1	Two-hour written exam: objective-response 10, short answer 60	HSC Mathematics Extension 1	Component weightings: concepts, skills and techniques 50%, reasoning and communication 50%	Two-and-a-half-hour written exam: objective-response 8, short answer 75 **(new ł)**
HSC Mathematics Extension 2	Three-hour written exam: objective-response 10, short answer 90	HSC Mathematics Extension 2	Component weightings: concepts, skills and techniques 50%, reasoning and communication 50%	Three-hour written exam: objective-response 10, short answer 90 **(status quo)**
Mathematics Life Skills		Mathematics Life Skills	Outcomes achieved by student recorded for HSC Profile of Student Achievement	

* This approach is designed as a stronger incentive for students to study an introductory calculus-based course (only) and thereby obtain a more appropriate background for particular fields of tertiary study. The two questions for Extension 1 students at the end of the paper ('Option 2') would be similar in nature and difficulty to questions at the end of the

current Mathematics ('2 Unit') paper, while the two questions for '2 Unit only' students at the end of the paper ('Option 1') would be more routine in nature and therefore less difficult than those for 'Option 2'. Marks obtained by candidates on the different versions of the examination paper would be kept separate for BOSTES reporting purposes. The two sets of marks would then be able to be integrated to give a final mark distribution for scaling purposes, so that, as currently, all students on the same total mark for would receive the same scaled mark.

The reduced examination time of two-and-a-half hours for the Mathematics 2 Unit paper is considered more appropriate in the examination of '2 Unit only' candidates, equating it to the examination time for the Mathematics General 2 examination. It would also mean that the total examination time (five hours) for Mathematics Extension 1 candidates is maintained (see ł below).

ł The proposed extension of the examination time for the Mathematics Extension 1 examination paper is designed to reduce what many perceive as an overemphasis on 'speededn**ess' in the current examination and thereby increase the opportunity for the broad range of Mathematics Extension 1 candidates** to demonstrate more fully the level of their knowledge, skills and understanding.

7. K–12 learning continuum

The NSW *Mathematics K–10 Syllabus* describes a continuum of Mathematics learning from Kindergarten to Year 10. Stage 6 Mathematics syllabuses describe Preliminary and HSC courses in Years 11 and 12 and represent the Mathematics learning for all students who study Mathematics in those years.

In Early Stage 1 to Stage 3, students follow a common course of Mathematics study. For Stage 4 to Stage 5, there is a range of available pathways of learning in Mathematics.

While most students in Stage 4 follow a common course of Mathematics study, Life Skills outcomes and content are provided for a small percentage of students for whom the regular course outcomes and content are not appropriate. By Stage 5, students exhibit a wide range of mathematical skills, levels of competence, and aspirations. Some students may be aiming to develop the mathematical skills necessary to function in daily life and various work contexts. Other students may seek to address more challenging Mathematics to prepare them for the highest level courses in Years 11 and 12. For this reason, Stage 5 of the learning continuum is expressed in terms of the three substages: Stage 5.1, Stage 5.2 and Stage 5.3. These substages are not designed as prescribed courses, and many different 'endpoints' are possible. As well as studying the Stage 5.1 content, the majority of students will study some or all of the Stage 5.2 content. Similarly, as well as studying the Stage 5.2 content, many students will study some or all of the Stage 5.3 content. As for Stage 4, Life Skills outcomes and content are provided for a small percentage of students.

The following assumptions and recommendations regarding Stage 5 Mathematics, typically undertaken by students in Years 9 and 10, are provided in relation to the proposed suite of new senior secondary Mathematics courses.

The Preliminary Mathematics General course will undergo minor revision on the assumption that students have studied the content and achieved the outcomes of the *Mathematics K–10 Syllabus* up to, and including, the content and outcomes of Stage 5.1. For students who intend to study the revised (minor revision of current course) HSC Mathematics General 2 course, it will be recommended that they experience at least some of the Stage 5.2 content, particularly the topics on algebra and trigonometry, if not all of the content.

The new Mathematics 2 Unit course (moderate– major revision of current course) will be constructed on the assumption that students have studied the content and achieved the outcomes of the *Mathematics K–10 Syllabus* up to, and including, the content and outcomes of Stage 5.2. It will be recommended that, where possible, they also experience the topics on real numbers, algebraic techniques and coordinate geometry, as well as at least some of the trigonometry in Stage 5.3, if not all of the content.

The new Preliminary Mathematics Extension and HSC Mathematics Extension 1 courses (moderate–major revisions of the current courses) will be constructed on the assumption that students have studied the content and achieved the outcomes of the *Mathematics K–10 Syllabus* up to, and including, the content and outcomes of Stage 5.3. It will be recommended that, where possible, they also experience the optional topics on polynomials, logarithms, functions and other graphs, and circle geometry.

The new Mathematics Extension 2 course (moderate– major revision of current course) will be constructed on the assumption that students have completed the new Preliminary Mathematics 2 Unit and Preliminary Mathematics Extension courses to very high levels of competence.

The new Mathematics Life Skills course (moderate– major revision of current course) will be constructed on the assumption that the majority of students will have studied Life Skills outcomes and content in Stage 4 and Stage 5.

8. Teaching standards and teacher education

The BOSTES supports quality teaching in all NSW schools. A minimum standard of teacher quality applies uniformly throughout the country in order to ensure rural and socially disadvantaged communities a level of teacher quality comparable to that available to students in advantaged metropolitan locations.

The BOSTES oversees a system of accreditation and recognition of a teacher's professional capacity against professional standards. Part of these responsibilities is to approve initial and continuing teacher education courses and programs.

Currently, there are 76 accredited secondary initial teacher education programs in New South Wales, delivered through 17 institutions. Of these, 36 initial teacher education courses allow graduates to qualify to teach within the Mathematics key learning area in a secondary context. Institutions offering these courses include Australian Catholic University, Charles Sturt University, Macquarie University, University of New England, University of New South Wales, University of Newcastle, University of Notre Dame Australia, University of Sydney, University of Western Sydney, and University of Wollongong. Alternative pathways also exist to post-graduate teacher education courses.

Significant professional learning and support will be required in order for teachers to adequately teach 'new' content that may be introduced into the new calculus-based courses. Consultation on the draft senior secondary Australian curriculum Mathematics courses in New South Wales in 2012 indicated that stakeholders were of the view that significant changes to the NSW senior secondary

Mathematics curriculum would place considerable pressure on teachers, schools and schooling systems. Relevant teacher support will need to be provided to assist teachers in implementing any new syllabuses.

APPENDIX 1

History of current NSW Stage 6 Mathematics courses and examinations

The Preliminary Mathematics General course was implemented with Year 11 in 2013, with students then studying either the HSC Mathematics General 2 course or the HSC Mathematics General 1 course from Year 12, 2014. The HSC Mathematics General 1 CEC was provided to better meet the needs of students who wish to study Mathematics in Stage 6 but whose purposes are not accommodated through the study of the HSC Mathematics General 2 course.

The Preliminary Mathematics General/HSC Mathematics General 2 pathway replaced General Mathematics (implemented with Year 11, 2000 and Year 12 2001). The outcomes and content for the pathway were designed to be quite similar in nature to the outcomes and content of the General Mathematics course to accord with Broad Direction 8 of the 2006–2008 Mathematics Stage 6 Review and Development Project that 'the current General Mathematics course material be largely maintained within the structure of Stage 6 non-calculus-based Mathematics courses'.

The current Mathematics ('2 unit') and Mathematics Extension 1 courses were revised following a major review in 1982 and since then only minor amendments have been made. The Mathematics Extension 2 course was last revised in 1980 and was amended slightly in 1989.

In 1998, the BOSTES evaluated all Stage 6 courses, including the full suite of Mathematics courses against the criteria of the Government's White Paper *Securing Their Future* (1997): the 'non-calculus-based' *Mathematics in Practice* (1989) and *Mathematics in Society* (1981) courses, and the 'calculus-based' *2 Unit Mathematics* (1982)*, 3 Unit Mathematics* (1982*)* and *4 Unit Mathematics* (1989) courses*.* At that time, over 95% of the HSC candidature presented each year for at least one of the five Mathematics courses provided to meet the diverse needs of virtually the entire HSC candidature.

Following this evaluation, the BOSTES developed a single non-calculus-based course, *General Mathematics* (1999), to replace Mathematics in Society and Mathematics in Practice, which were designed for 'students who wish to study mathematics from a practical perspective in relation to a range of contexts drawn from everyday experiences and without formal treatment of the calculus'. At the same time, the BOSTES agreed to a longer-term review of the '2, 3 and 4 Unit' courses, which were renamed Mathematics, Mathematics Extension 1 and Mathematics Extension 2 respectively. The BOSTES announced that in the introductory years of the new HSC, the content and internal assessment arrangements of these courses would be maintained.

In the development of General Mathematics, the stated important considerations included:

- It is essential that the suite of courses meets the needs of the full range of students. It was also specifically stated that the issue of catering for the large group of students who have had difficulty with Mathematics, and have not yet achieved an appropriate standard of mathematical competence, must be considered.
- The inclusion of significantly more statistics and options which are more in line with students' current Mathematics needs and priorities. The lack of background in statistics that affects

students entering many fields of tertiary study such as Psychology and the biomedical sciences needs to be addressed.
- The appropriate use of technology in teaching, learning and assessment needs to be investigated.
- The need for a course such as Mathematics in Practice, as prior to 1990, schools had introduced across the state over 300 Other Approved Studies courses in Mathematics. The students targeted by these courses had historically either performed very poorly in Mathematics in Society, where the demands of the course, particularly in Algebra, were too great for them, or had omitted Mathematics from their course of study altogether, which limited their future options.

The development of new K–6 and Years 7–10 syllabuses then became the focus of Mathematics curriculum development, with the completion of these syllabuses in 2000–2002 a necessary prerequisite to the broader review of Stage 6 Mathematics courses. Following their completion, a plan was established for the review of the Stage 6 Mathematics courses. However, the announcement of a timeline was deferred in recognition of the importance of teachers being given appropriate time to focus on the implementation of the new Years 7–10 syllabus.

A timeline, 2006–2008, was subsequently announced in 2005, with five new Board-developed courses, Mathematics General 1, Mathematics General 2, Mathematics Advanced, Mathematics Extension 1, and Mathematics Extension 2, endorsed by the BOSTES in March 2009. The Mathematics General 1 course was developed to meet Broad Direction 1 for the project that 'the set of Stage 6 Mathematics courses include an additional offering to accommodate the purposes of students who wish to study a Board-developed Mathematics course in Stage 6 but who are currently choosing not to, as well as those whose purposes are not accommodated through the study of General Mathematics'.

At the March 2009 meeting, the BOSTES also agreed to advise the Minister not to approve the courses for implementation pending the outcomes of the Australian curriculum development.

In May 2009, the BOSTES approved the release of the outcomes and content of the newly endorsed Mathematics General 1 course for the use of schools in developing or maintaining Stage 6 school-designed Board-endorsed courses (SDBECs) for the HSC. The BOSTES wrote to schools already offering school-developed courses, suggesting that the new materials could be used to amend their existing courses, and also made the outcomes and content available to schools considering a practical or applied Mathematics course for 2010 and beyond.

In March 2012, the BOSTES advised that General Mathematics would be replaced by two new pathways to be introduced in 2013: the Preliminary Mathematics General/HSC Mathematics General 2 Pathway, and the Preliminary Mathematics General/HSC Mathematics General 1 Pathway. The three component courses were made available on the Board's website in May 2012.

The Preliminary Mathematics General course was provided as a Board Developed Course for implementation with Year 11 from 2013. The HSC Mathematics General 2 course was provided as an (examinable) Board Developed Course, allowing students to count their Mathematics study towards the calculation of an Australian Tertiary Admission Rank (ATAR), for implementation with Year 12 from 2014. The HSC Mathematics General 1 course was provided as a (non-examinable) Content

Endorsed Course (CEC), for implementation with Year 12 from 2014. The availability of the Preliminary Mathematics General/HSC Mathematics General 1 Pathway for all schools meant that the BOSTES would no longer consider SDBEC applications for Stage 6 Mathematics, Applied.

Assessment and examination

The HSC examination for the single non-calculus-based course, General Mathematics, which replaced Mathematics in Society and Mathematics in Practice of the previous HSC, was structured from the 2001 HSC until the 2011 HSC as follows: Two-and-a-half hour examination with two sections: Section I, 22 multiple-choice questions; Section II, six 13-mark questions. The number of question parts and sub-parts in the questions in Section II could vary, both from question to question and from year to year. Candidates could use a calculator, including a graphics calculator, provided it satisfied the published criteria. It was also stated that the General Mathematics specimen examination was constructed so that students with scientific or graphics calculators were equally able to access all questions

Following the evaluation of the five Stage 6 Mathematics courses of the previous HSC and the decision to maintain the content and internal assessment arrangements of the 'calculus-based' courses, the BOSTES also announced that in the introductory years of the new HSC, the examination specifications of these courses would be maintained.

Since 1983, the examination structure for these three courses has utilised a common three-hour paper taken by all Mathematics ('2 unit') and Mathematics Extension 1 candidates, a common two-hour paper taken by all Mathematics Extension 1 and Mathematics Extension 2 candidates, and a single additional three-hour paper taken by Mathematics Extension 2 candidates. For ATAR-scaling purposes, Mathematics ('2 unit') candidates could count both units and Mathematics Extension 1 candidates could count both Mathematics ('2 unit') units and also one additional unit derived from the Mathematics Extension 1/Mathematics Extension 2 common paper. Mathematics Extension 2 candidates could count two units from the Mathematics Extension 1/Mathematics Extension 2 common paper and two units from the additional paper. Since its inception, school-based assessment in these courses has been reported on the same basis as that used for the HSC examination. Equipercentile mapping procedures are used by the BOSTES for comparing final performances of candidates in all three courses.

Revised HSC examination specifications and internal assessment requirements for the Stage 6 Mathematics courses were advised in May 2011 and commenced from Term 4, 2011 for Year 12 students sitting the 2012 HSC examinations. They included:

- a reduction in total mark value for the Mathematics, Mathematics Extension 1 and Mathematics Extension 2 examinations while retaining the current time allocation for each examination, and the inclusion of objective-response sections in each paper
- a minor amendment to the mark values of the two sections of the General Mathematics examination paper
- revised assessment components and weightings:
  - Concepts, skills and techniques 50%
  - Reasoning and communication 50%.

The revised examination specifications and internal assessment requirements applied to the General Mathematics HSC course in 2012 and 2013 have been applied to the new HSC Mathematics General 2 course from 2014.

APPENDIX 2

Research undertaken by the BOSTES for the Mathematics Stage 6 Review and Development Project (2006–2008)

In 2006, the BOSTES commissioned Dr Mary Coupland of the Department of Mathematical Sciences, University of Technology, Sydney to write a curriculum and literature review for the 2006–2008 Mathematics Stage 6 Review and Development Project.

In the curriculum review, *A critical analysis of selected Australian and international mathematics syllabuses for the post-compulsory years of secondary schooling*, the Executive Summary includes that:

> This review analyses syllabus documents and assessment arrangements for post-compulsory mathematics courses in a selection of Australian states and other countries.
>
> Compared to the mathematics courses offered by other Australian States for university-bound students studying calculus, the current NSW mathematics courses provide an appropriate level of challenge but have not changed in content to reflect current uses of technology, and contemporary applications of mathematics.
>
> Compared to the mathematics courses offered by other Australian States for students not intending to proceed to university, the NSW mathematics courses do not provide the same opportunities that are offered in other states for students needing more time to reach the numeracy levels that most of the cohort has achieved by the end of Year 10.
>
> Awarding and assessing bodies at state and national levels necessarily operate within constraints and policies that are peculiar to their own geographical location and social conditions. For this reason, educational innovations that are successful elsewhere need to be evaluated in the context of local traditions, capabilities, and goals. That being said, studying the structure, content and assessment arrangements of the mathematics courses of other countries, and those of Australian states and Territories, can yield useful ideas for innovations in the NSW situation. For example, the Victorian syllabus structure provides students with much choice and flexibility, and we have an opportunity to observe how teachers and students adapt to a course taught with computer algebra systems. The Queensland courses (among others) offer school students a chance to experience aspects of challenging non-calculus mathematics of both a pure and applied nature. One of the reasons for the success of students from Finland in recent international comparative studies is the commitment to achieving educational goals that reflect equitable standards for as many students as possible.
>
> …
>
> While considering new ideas from other systems, NSW can also benefit by being aware of less successful innovations such as the AS (Advanced Supplementary) exams in England. The centrally imposed additional set of external examinations to be held one year before the A Level examinations, and courses overloaded with content, turned many students away from mathematics.
>
> The issue of technology for the teaching and learning of mathematics at this level is still a matter for debate. This is considered more deeply in a separate review, where it is shown how enthusiasts are happy to demonstrate that lessons can be enlivened, real world applications can be discussed, and weaker students can be supported through calculations to a stage where they can grasp concepts and solve interesting problems. With changes occurring rapidly in ICT, questions asked must be broadened beyond the issue of "are graphing calculators going to be in the exams?" New technology is changing the way mathematicians work, and is changing the way mathematics is used in the workplace. If mathematics is regarded as something people *do* rather than a disembodied collection of rules and results, the focus changes from the tools to the tasks. In schools the focus could be nudged from the tests towards the solving of interesting problems.
>
> Practical measures to ensure that valuable by-hand skills are still learned have been found in many of the courses considered in this review. Examples have also been found of ways to encourage teachers

to embrace the positive features of modern technology - its capacity for engaging and exciting students.

This is a time for evaluating, preserving the best, and incorporating the better of the new.

In the literature review, *Literature Review and Annotated Bibliography to Inform the Stage 6 Mathematics Review*, for the 2006–2008 Mathematics Stage 6 Review and Development Project, Dr Mary Coupland includes the following summary of the sections 'Relevant Theories Of Learning' and 'Research Concerning Teaching And Learning With New Technologies, Including Computer Algebra Systems (CAS) And Graphing Display Calculators (GDCs)':

> This literature review began with a description of a dichotomy in the theoretical background of mathematics education research. This dichotomy was described as a map of, on the one hand, *acquisitive* models of learning based on constructivism and on the other hand *participatory* models of learning incorporating a more expansive and situated view of cognition. With this background, research into the introduction of CAS to mathematics teaching was considered. Most of the reviewed research was based in the constructivist paradigm, concerned with the acquiring of concepts and skills by individual students, however there are strong indications that the role of the teacher in encouraging discussion and reflection about techniques is essential to ensure that learning (as in the *acquiring* of concepts) occurred in technology-rich classroom environments.
>
> Early research within the acquisitive model was concerned with demonstrating that there were advantages to learning mathematics with the new tools (GDCs, CAS). Due to the complex nature and many variables in classroom settings, this was (and always will be) difficult to establish conclusively and research next focused on describing the ways that students worked with the new tools. Unforeseen difficulties with the technology were identified. Much effort has been expended in producing innovative teaching activities and assessment tasks to take advantage of the new ways of working with mathematical ideas that are afforded by GDCs and CAS. More recently there has been recognition of the essential role played by the teacher in guiding students' attention when there is much to be considered in a technology rich environment.
>
> From the socio-cultural view, it is precisely that classroom discussion and reflection, motivated by a genuine social need to engage in sense-making, that constitutes participation in authentic mathematical activities and this *participation* is the key to learning. As described by Goos, Galbraith, Renshaw and Geiger (2000, p. 306), a sociocultural perspective on learning is manifested in classrooms where students "are expected to defend and critique ideas by proposing justifications, explanations, and alternatives". In these classrooms students learn to use the tools of GDCs and computers as resources to support the mathematical discussion and problem solving that are the real focus of the lessons.
>
> The role of tools is a complex one in educational theory and this is appropriate in considering the role of computers in mathematics education. As Eric Love (1995) has described the situation:
>
> > There is a dynamic interaction between the work that people do, computer software to carry out operations hitherto taught as part of a mathematics curriculum, and of what 'mathematics' is thought to consist. Trying to pin down parts of this dynamic by speaking of "the mathematics embedded in software", of "producing software for teaching mathematics" or "tools for doing mathematics" will inevitably fail adequately to characterise this continually changing interaction. (p. 117)
>
> The purposes for the CAS tool in "doing mathematics" in a professional context are quite different from the purposes of using a CAS when "doing mathematics" in a mathematics classroom. Mathematics can be regarded as *a social practice* as well as *a collection of knowledge and disembodied skills.* CAS are one of many mathematical artefacts that change the nature of mathematics and mathematics education, not just because they allow mathematics to be done in new ways, but because they change the nature of mathematical practice – what mathematicians *do*.

The literature review also included the following 'Concluding comments on the place of new technologies in the post-compulsory mathematics curriculum':

> Mathematics education in the post-compulsory years has more than one goal. First, students need to be equipped for participation in the world of work and to participate in society as responsible citizens. As Professor Kaye Stacey points out, improved methods of mathematical computation are unproblematic in the world of work, where tools that make computation easier, more accurate and faster have always been important to progress. (Stacey, 2005, p. 8.). One kind of technology readily used in the workplace and also in the home, and perhaps too often overlooked in school mathematics is spreadsheet software.
>
> Second, students intending to study mathematics at university or to use mathematics to support other university studies need to learn the language of advanced mathematics, based as it is on algebra and functions. While these topics have traditionally been taught with pen and paper methods, the availability of graphics calculators and CAS supports a widening of approaches to include more exploration and reduce time spent perfecting routines. As outlined in this review, this does not happen automatically and requires expert teaching and carefully prepared materials.
>
> Barry Kissane (2000) raises the possibility of broadening the curriculum to include topics that were not possible without graphics calculators (or CAS), for example simulation as a means of tackling some situations involving uncertainty, iterative techniques to study some of the mathematics of chaos, and dealing with real world data collected electronically and transmitted to a calculator.
>
> For both of the goals of post-compulsory mathematics, participation in work and society *and* further learning, the new technologies and internet access support a much enriched approach to the study of statistics.

A further section of the literature review 'The Place Of Statistics in Post-Compulsory Mathematics Curricula' includes, under 'The current situation in New South Wales compared to other school systems':

> Compared to other Australian states and territories, NSW has much less content described as statistics and probability in its mathematics course for non-specialists, Mathematics 2 Unit …
>
> …
>
> On the international scene, new developments in Singapore and Hong Kong include radical rearrangements of courses, but are not yet fully implemented. It is interesting to note the inclusion of Statistics and Probability as major components of the new courses, and reduced emphasis on mechanics and applied mathematics. Students in England and in the USA Advanced Placement program are offered separate courses in Statistics, and it is a component of every mathematics syllabus in the International Baccalaureate Diploma Programme. This goes hand in hand with making more use of graphing calculators for teaching, learning, and assessment; and other kinds of technology for graphical display in the classroom.

Arguments for the inclusion of statistics and/or data handling refer to the notion of statistical literacy. Among different definitions of 'statistical literacy', the literature review includes the following 2005 statement of the Royal Statistical Society:

> Our view is that 'statistical literacy' means the ability to understand how quantitative data are generated, and how they can be summarized, modelled and interpreted in ways that allow substantively useful conclusions to be drawn about the functioning of the world from which they are derived. It is also, and most importantly, an understanding of uncertainty and how the measurement of uncertainty can be put to constructive uses, for example in decision making and in handling risk, and especially in the formulation of evidence based policy. It involves the ability critically to evaluate the use of statistical data by others, in the media and elsewhere. This refers especially to the use of official statistics, both in providing 'snapshots' of current situations and in showing important changes over time. It is also our view that statistical literacy has to be acquired through the experience of handling data themselves, but within the context of an appropriate mathematical framework. The process of learning statistics is iterative, moving continually between theory and practice.

The literature review also includes the following 'Arguments for the inclusion of statistics and/or data handling in the Mathematics curriculum USA (adapted from Franklin, Kader, Mewborn, Moreno, Peck, Perry and Schaeffer, 2005)':

- Statistical literacy is essential for citizenship. Students who are reaching the age of voting need to be able to understand the principles behind opinion polling and the data gathering conducted for and by government agencies.
- Personal choices concerning health, safety, investments, and purchases are all informed by statistical information. Interpreting this requires statistical literacy.
- Statistical literacy is required for productive work in many jobs and careers, contributing to individual advancement and the good of a nation's economy.
- Quality control and accountability in the workplace rely on statistical procedures.
- Science contributes to the health of modern societies and the communication of scientific findings relies heavily on statistics.

The following 'Additional arguments especially relevant to the NSW situation' are also included:

- Statistics can be used to motivate topics in the mathematics curriculum. For example, the equation of a straight line is used to describe a line of best fit when finding a trend in data; basic equation solving is required to convert to and from standardized scores.
- For students needing further development of numeracy, statistics provides realistic examples where the calculation of percentages, ratios, and rates is required, along with units of measurement. Newspaper stories illustrated with data and charts provide numbers over a large range, reinforcing number concepts.
- Considering the choice of broad field of study, for many students proceeding to university in NSW, statistics is a more useful application of mathematics than some of the current topics in the calculus courses.

References

Australian Curriculum, Assessment and Reporting Authority 2012, *Senior Secondary Mathematics – Information Sheet*, ACARA, Sydney, available at <www.acara.edu.au/verve/_resources/Senior_Secondary_Mathematics.pdf>.

Coupland, M 2006a, *A critical analysis of selected Australian and international mathematics syllabuses for the post-compulsory years of secondary schooling,* Board of Studies NSW, Sydney, available at <www.boardofstudies.nsw.edu.au/manuals/pdf_doc/maths_st6_lit_curr_rev_pt1.pdf>.

Coupland, M 2006b, *Literature review and annotated bibliography to inform the Stage 6 Mathematics review,* Board of Studies NSW, Sydney, available at <www.boardofstudies.nsw.edu.au/manuals/pdf_doc/maths_st6_lit_curr_rev_pt2.pdf>.

McPhan, G, Morony, W, Pegg, J, Cooksey, R & Lynch, T 2008, *Maths? Why Not?* , report prepared for the Department of Education, Employment and Workplace Relations, Canberra, available at <www.gsu.uts.edu.au/academicboard/cabs/082/papers/082-item6-1.pdf>.

National Curriculum Board 2008a, *National Mathematics Curriculum: Framing paper*, Commonwealth of Australia.

—— 2008b, *National Mathematics Curriculum: Initial advice*, Commonwealth of Australia.

—— 2009, *Shape of the Australian Curriculum: Mathematics*, Commonwealth of Australia.

Stacey, K, Dowsey, J, McCrae, B & Stephens, M 1998, *Review of Senior Secondary Mathematics Curriculum,* Board of Studies NSW, Sydney.