2013 Notes from the Marking Centre – Mathematics Extension 2

Introduction

This document has been produced for the teachers and candidates of the Stage 6 Mathematics Extension 2 course. It contains comments on candidate responses to the 2013 Higher School Certificate examination, highlighting their strengths in particular parts of the examination and indicating where candidates need to improve.

This document should be read along with:

- the *Mathematics Extension 2 Stage 6 Syllabus*
- the 2013 Higher School Certificate Mathematics Extension 2 examination
- the marking guidelines
- *Advice for students attempting HSC mathematics examinations*
- *Advice for HSC students about examinations*
- other support documents developed by the Board of Studies, Teaching and Educational Standards NSW to assist in the teaching and learning of Mathematics in Stage 6.

Question 11

(a) (i) Most candidates could find the conjugate.

Common problems were:

- making errors in transcription
- making errors in elementary algebra.

(a) (ii) This part was also done very well.

A common error was $\arg(w) = \frac{\pi}{6}$ instead of $\frac{\pi}{3}$.

(a) (iii) Many candidates showed $w^{24} = 2^{24}\, cis\ 8\pi$.

Common problems were:

- ignoring the directive (in simplest form) and not converting $cis\ 8\pi$ to 1, leading to the final answer of 2^{24}
- using a calculator when it was not required, making the answer appear more complicated and possibly inaccurate.

(b) Most candidates developed the correct polynomial and equated it with the equivalent form. A number of candidates struggled to solve for *A*, *B* and *C*.

Common problems were:

- not distinguishing between factors and the solution to a quadratic equation – using the quadratic formula correctly and getting $z = i$ or $z = -5i$, but not stating that the factors were $(z - i)(z + 5i)$

- not stating the quadratic formula before using it, eg a number of candidates simply wrote $\frac{-4i \pm \sqrt{-16+20}}{2}$. This incorrect '+20' did not allow the markers to know whether this was due to an incorrect formula being used or to a simple arithmetic error
- not realising that the co-efficient of z was $4i$ and not 4
- many candidates wrote $z^2 + 4iz + 5 = (z + 2i)^2 + 9$, but had difficulty in converting the + 9 to – $9i^2$ to achieve the next step equal to $(z + 2i + 3i)(z + 2i - 3i)$.

(c) Candidates were most successful when they used the substitutions $u^2 = 1 - x^2$ or $u = 1 - x^2$. Candidates who used $u = x^2$ and $\frac{dv}{dx} = x\sqrt{1 - x^2}$ could get the correct result, but found that it was a lengthy procedure.

Common problems were:

- making common arithmetic errors
- using integration by parts by writing $\frac{du}{dx} = x^3$, leading to $\frac{x^4}{4}$ and $v = \sqrt{1 - x^2}$, which leads to a more complicated integral than the original
- where candidates used the substitution $x = \sin\theta$ or $x = \cos\theta$ (which leads to the correct answer), they had to find $\int \sin^5 \theta \, d\theta$ or $\int \cos^5 \theta \, d\theta$, which caused some difficulty
- using de Moivre's theorem or the reduction formula was very involved and was not a recommended method for this style of question.

(d) Most candidates commenced this question by correctly writing $z^2 + \bar{z}^2 = (x + iy)^2 + (x - iy)^2 \leq 8$. Most candidates achieved the correct equation $x^2 - y^2 \leq 4$ and sketched the correct hyperbola.

Common problems were:

- algebraic errors leading to $x^2 + y^2 \leq 4$ or even $x^2 \leq 4$
- not indicating where the vertices were (or any other data)
- not shading the region.

Question 12

(a) In most responses, candidates quoted the relevant expressions of $\cos\theta = \frac{1-t^2}{1+t^2}$ and $\frac{d\theta}{dt} = \frac{2}{1+t^2}$.

Common problems were:

- making errors in substitution
- misquoting the expressions involving t.

(b) While most candidates recognised that implicit differentiation was required, errors occurred when differentiating $-\log_e(1000 - y)$. An alternative method was to make x the subject and then find $\frac{dx}{dy}$.

(c) Candidates who successfully completed this part used the method of cylindrical shells.

Common problems were:

- using $3-x$, $x-4$, $x+1$ instead of $4-x$ as the radius
- having incorrect limits of integration
- losing negative signs and integrating x rather than e^x(where candidates used integration by parts)
- working with logs and difficulty completing the integration (where candidates used the method of slicing perpendicular to the axis of rotation).

(d) (i) Most candidates found the gradient of the tangent and therefore obtained the correct equation.

A common problem was:

- omitting the negative sign when finding $\frac{dy}{dx}$ or $\frac{dy}{dp}$.

(d) (ii) Most candidates found the coordinates of A and B. Some showed that the midpoint of A and B was P and stated $\angle AOB = 90^\circ$.

A common problem was:

- not showing that P was the centre of the circle.

(d) (iii) Candidates attempting this part found the gradients of BC and PQ to prove parallel lines. Ratio of intercepts of parallel lines was another method used by some candidates.

A common problem was:

- not recognising the coordinates of C as $(2cq, 0)$.

Question 13

(a) (i) Most candidates made an attempt to try integration by parts. Some made a trigonometric substitution, such as $x = \sin\theta$, and then attempted integration by parts, which very often led to the desired result.

Common problems were:

- making errors when differentiating $(1-x^2)^{\frac{n}{2}}$ or when simplifying algebraic terms
- not many candidates showed that $x^2(1-x^2)^{\frac{n-2}{2}} = (1-x^2)^{\frac{n-2}{2}} - (1-x^2)^{\frac{n}{2}}$, which was required to relate back to the original integral
- trying to do this part by mathematical induction, without success.

(a) (ii) This part was done successfully by many candidates. Many evaluated I_1 by a trigonometric substitution, instead of the simpler method of realising that the integral was equivalent in value to the area of a quarter of a unit circle. Quite a few candidates thought that $\int \sqrt{1-x^2}\,dx$ was $\sin^{-1} x$.

(b) A large number of candidates did well on this question.

Common problems were:

- failing to label the axes or to make it clear which were preliminary sketches and which were final sketches
- when many different graphs were shown on the same sketch, often it was not clear which sketch was which, since they were not labelled.

(b) (i) Common problems were:

- failing to label the x-axis or the lines $y = \pm 1$
- showing the x-intercepts of the sketch as 'sharp' points.

(b) (ii) Candidates whose final sketch was incorrect but who had clearly labelled a correct working sketch, such as $y = -f(x)$ or $y = 1 - f(x)$, were often able to obtain some marks.

A common problem was:

- not showing lead-up work and only presenting a final sketch – this was fine if the final sketch was correct, but incorrect sketches made it difficult for the examiners to award marks.

(c) (i) This part was well done by a large number of candidates with a variety of methods. Most realised that $\angle ACB = \pi/2$ and that $\angle ABC = \alpha + \beta$.

Common problems were:

- no reasoning or inadequate reasoning
- referring to angles such as x or θ but not making it clear to the examiners which angles they were
- not stating or proving that $\angle ACB = \pi/2$.

(c) (ii) Many candidates realised that $\angle ADB = \pi/2$ and that $\angle DBA = \beta$, but often the reasoning was inadequate or omitted.

(c) (iii) This part was either not done by many or, if attempted, caused some difficulty. A substantial number of candidates realised that CE must equal $2r \sin \alpha \cos \beta$, but failed to give reasons.

Question 14

(a) This part was very well done, with most candidates awarded 2 or 3 marks. Candidates who found the area under the curve by integration by parts and made it greater than the area of the triangle tended to find the answer most easily. Candidates who expanded terms, rather than factorised terms, were much more successful in showing the desired result.

(b) Most candidates were able to demonstrate some level of understanding of mathematical induction. This question had a stated recurrence relationship that needed to be used in the induction step.

Common problems were:

- proving the recurrence relationship rather than proving what they were asked to prove
- using the base case of $n = 3$, possibly distracted by the $n > 2$ in the statement of the recurrence
- not recognising the need to use $|ab| = |a|\,|b|$ in the proof.

(c) (i) Many candidates struggled with this part.

Common problems were:

- not seeing the connection to the binomial theorem
- stating that $\sec^{2n}\theta = 1 + \tan^{2n}\theta$
- trying to show the result by working right to left, usually unsuccessfully
- trying to sum a geometric progression.

(c) (ii) Many candidates correctly found the solution to the integration, even if they had left out part (i). Most attempted to use the information from part (i).

(d) (i) This part was generally well done.

Common problems were:

- making errors in naming correct sides and/or correct proportions
- missing the included angle
- using the wrong side, *DE*.

(d) (ii) Most candidates had the correct idea of proving a characteristic property of a cyclic quadrilateral. Candidates identified the similar triangles, the equal angles and the appropriate theorem (the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle).

Common problems were:

- incorrectly naming angles
- using incorrect or no geometrical reasoning.

(d) (iii) Candidates who attempted this question generally used two applications of the cosine rule. Occasionally, and less efficiently, a sine rule was used in the first step to get cosine of some angle. The other common method was to drop a perpendicular bisector in either the small triangle *ADE* (from *D* to *AE*) or in the big triangle *ABC* (from *C* to *AB*) and use Pythagoras' theorem twice.

(d) (iv) Not many candidates attempted this part. There were several correct solutions that used the theorem: the angle at the centre of the circle is twice the angle at the circumference, subtended by the same arc.

Question 15

(a) The majority of candidates used the modulus–argument approach, initially showing that $z\overline{\omega} - \omega\overline{z} = |z||\omega|\{cis(\theta - \varphi) - cis(\varphi - \theta)\}$. Candidates who arrived at this point usually went on to successfully show the required result.

Common problems were:

- incorrect notation for the modules of z
- difficulty progressing when using z and w in the form $a + ib$.

(b) (i) Most candidates displayed a sound knowledge of the remainder theorem, as well as the theory on repeated roots, allowing them to successfully show the required result.

Common problems were:

- not realising that $P(-1) = P'(-1) = 0$
- difficulty solving simultaneously $P(-1) = 0$ and $P(1) = -3$.

(b) (ii) Candidates who were successful in (i) were usually successful in (ii).

(c) (i) The vast majority of candidates achieved the required result.

(c) (ii) Candidates who did well in this part displayed a sound knowledge of binomial probability and complementary events, $\{P(E) = 1 - P(\tilde{E})\}$.

A common problem was:

- failing to see the connection between (i) and (ii).

(d) (i) Almost all candidates successfully obtained the result, having associated terminal velocity with $\ddot{x} = 0$.

(d) (ii) Most candidates started this part with the statement that $m\ddot{x} = mg - kv^2$ and then proceeded to show that $x_{max} = \frac{V_T^2}{2g}\left(1 - \frac{u^2}{V_T^2}\right)$, instead of using $m\ddot{x} = -mg - kv^2$ to obtain the appropriate result. In almost all situations, candidates successfully integrated and substituted $v = 0$ to find their expression for the maximum height.

(d) (iii) Most candidates who successfully completed (ii) were also able to show the required result for this part.

Common problems were:

- not applying the appropriate limits of integration
- using incorrect formulae
- making careless errors.

Question 16

(a) (i) In successful responses, candidates carefully considered the restriction on the domain, and in particular the value of P at $x = 0$. Some candidates showed that the minimum occurs at $x = 4$ without stating the actual minimum value of $P(x)$. Some candidates approached the problem using calculus, although factorising $P(x)$ led to an alternative solution.

(a) (ii) Responses which recognised the connection with part (i) were generally successful. Less successful responses merely showed that the inequality holds for some values of x, typically just $x = 4$, rather than for all $x \geq 0$.

(a) (iii) Substitution of x with $m + n$ in the inequation of part (ii) was a successful first step in the solution of this part.

Common problems were:

- expanding the left-hand side of the inequation
- assuming the result before proceeding.

(b) (i) This part was well done by candidates who made an attempt.

(b) (ii) This part was generally well done. Candidates used the focus–directrix definition of an ellipse. Others calculated SP directly from ΔQPS, using Pythagoras' theorem and $r^2 = a^2(1-e^2)\sin^2\theta$.

(b) (iii) This part was also generally well done, with candidates explicitly showing each necessary step in the calculation of the numerator $S'Q$ and denominator $S'P$.

(b) (iv) Better responses included clear algebraic manipulation of the expression for the forces acting on P in the vertical direction, leading to the desired result.

Common problems were:

- confusing θ and the angle β
- assuming $\angle S'PS$ to be a right angle.

(b) (v) Better responses included a correct calculation of $r = QP$, with care shown in algebraic manipulation leading to the final result.

(b) (vi) This part was very well done. Candidates considered the quotient of the expressions from parts (iv) and (v).