

2014 Notes from the Marking Centre – Mathematics

Introduction

This document has been produced for the teachers and candidates of the Stage 6 Mathematics course. It contains comments on candidate responses to particular parts of the 2014 Higher School Certificate examination, highlighting candidates' strengths and indicating where they need to improve.

This document should be read along with:

- the [Mathematics Stage 6 Syllabus](#)
- the [2014 Higher School Certificate Mathematics examination](#)
- the [marking guidelines](#)
- [Advice for students attempting HSC Mathematics examinations](#)
- [Advice for HSC students about examinations](#)
- other support documents developed by the Board of Studies, Teaching and Educational Standards NSW to assist in the teaching and learning of Mathematics in Stage 6.

Question 11

(a) This part was done well by most candidates.

Common problems were:

- multiplying only either the numerator or the denominator by the conjugate
- multiplying the numerator and denominator by $\frac{\sqrt{5}-2}{\sqrt{5}-2}$ or $\frac{\sqrt{5}}{\sqrt{5}}$
- incorrectly expanding $(\sqrt{5}-2)(\sqrt{5}+2)$ to give either 3 or 9
- not using conjugates at all
- expressing the answer as a decimal, indicating the direct use of a calculator.

(b) A variety of methods to factorise the given expression was used successfully.

Common problems were:

- solving the equation $(3x-2)(x+1)=0$ after factorising correctly
- giving an answer in the form $(x-\frac{2}{3})(x+1)$
- using the quadratic formula to solve $3x^2+x-2=0$, leading to $(x-\frac{2}{3})(x+1)=0$
- mixing signs and terms resulting in answers such as $(3x-1)(x+2)$ or $-(3x+2)(x-1)$.

(c) This part was generally done well with most candidates correctly using the quotient rule. The use of the product rule was less successful as often errors were made in manipulating indices.

Common problems were:

- using an incorrect formula for example $\frac{uv' \pm vu'}{v^2}$.
- using incorrect derivatives of u and v .

(d) Quite a few candidates found this part difficult.

Some of those expressed $\frac{1}{(x+3)^2}$ as $(x+3)^{-2}$ then integrated incorrectly.

Common problems were:

- dividing by a positive power rather than a negative
- differentiating instead of integrating
- obtaining a logarithmic function for the primitive.

(e) Integration involving trigonometric functions proved to be difficult for some candidates. In better responses, candidates used the table of standard integrals to obtain a primitive of the form $a \cos \frac{x}{2}$ and then substituted the limits correctly.

Common problems were:

- getting $\cos \pi$ instead of $\cos \frac{\pi}{4}$ when substituting $\frac{\pi}{2}$ into $\cos \frac{x}{2}$
- incorrectly calculating $\cos \frac{\pi}{4}$ and/or $\cos 0$
- using the calculator in degree mode instead in radian mode.

(f) Candidates who found the primitive function $f(x) = 2x^2 - 5x + C$ were generally able to complete this part correctly.

Common problems were:

- omitting the constant of integration
- incorrectly substituting to find a value for C , for example using $x = 2$ and $f(x) = 0$
- finding the equation of the tangent at $x = 2$.

(g) This part was generally done well. Most candidates used the formula $l = r\theta$ to find the length of the arc to be $\frac{8\pi}{7}$ and added twice the length of the radius. Attempts were also made by some candidates to find the arc length using a fraction of the circumference.

Common problems were:

- interpreting $\frac{8\pi}{7}$ to be the perimeter of the sector
- expressing the answer as a decimal instead of in exact form
- finding a perimeter using an incorrect arc length
- using trigonometry to find the arc length.

Question 12

(a) The majority of candidates scored full marks for this part.

Common problems were:

- using incorrect formulae for S_n and T_n
- confusing S_n , T_n and d in their substitution
- only finding the value of n
- making calculator entry errors.

(b)(i) This part was answered quite well with many candidates scoring full marks. Most candidates chose to use either the point-gradient formula or the two-point formula.

Common problems were:

- using incorrect formulae for the gradient and/or point-gradient form of a line
- miscalculating the gradient by using incorrect coordinates
- neglecting to show working and not writing the equation in general form as required.

(b) (ii) A high percentage of candidates correctly used the ‘perpendicular distance formula’.

Common problems were:

- substituting incorrect values into the perpendicular distance formula
- leaving out the square root sign in the denominator and/or the absolute value signs in the formula.

(b)(iii) Most candidates used their answer to (b)(ii) in their solution. The area of the triangle was then easily found after finding the length of AC .

Common problems were:

- not using the result from (b)(ii) as the perpendicular height of the triangle
- using $A = \frac{1}{2} ab \sin C$ with angle $\angle ABC = 90^\circ$
- making arithmetic errors in the distance formula calculation
- using an incorrect area formula.

(c) (i) Most candidates answered this part correctly and showed that they had a good understanding of how a tree diagram should be drawn.

Common problems were:

- not recognising that this was a two stage experiment
- not writing the probabilities on tree branches
- using replacement
- drawing tree diagrams that were too small or untidy with writing that was difficult to decipher.

(c) (ii) This part was done well and most candidates were able to successfully use their diagram from (c)(i) to find the correct probability.

Common problems were:

- adding and multiplying the probabilities incorrectly
- finding only $P(RG)$ or $P(GR)$

(d)(i) Most candidates equated the two functions $f(x)$ and $g(x)$ and set up a quadratic equation leading to the correct answer $x = 3$. The use of the quadratic formula was usually successful here.

Common problems were:

- incorrectly factorising the quadratic equation
- dividing by x and eliminating the solution $x = 0$.

(d)(ii) Most candidates found a primitive function and used $x = 0$ and their x -value from (d)(i) as limits for their definite integral.

Common problems were:

- finding an incorrect primitive function
- incorrectly simplifying $f(x) - g(x)$ before integrating
- using incorrect limits or making calculation errors
- differentiating instead of integrating.

Question 13

(a)(i) Most candidates completed this part, correctly recognising that $2\cos 2x$ is the derivative of $\sin 2x$.

Common problems were:

- incorrectly adjusting the constant multiplier using $\frac{1}{2}$ or -2
- misinterpreting the question and differentiating using the product rule $uv' + vu'$ with $u = 3$ and $v = \sin 2x$
- attempting to integrate instead of differentiating
- making errors when transcribing from line to line, for example $\sin 2x$ changing to $\sin x$.

(a)(ii) In successful responses, candidates recognised that they were working with a log function, and showed the required adjustment to the numerator, linking their response from (i).

Common problems were:

- not recognising that the integral was in the form $\frac{f'(x)}{f(x)}$ and the primitive was a logarithmic function
- incorrect value of constant multiplier
- incorrect use of brackets or no brackets at all
- using the quotient rule and differentiating.

(b)(i) The most successful candidates were those who started with $O = Cg^{-m}$ and then correctly differentiated and substituted to show that the expression for M was a solution to $\frac{fO}{fv} = -nO$.

Candidates who attempted to use the more complex process of integration starting with $\frac{fv}{fO}$ were generally less successful, as they often did not deal correctly with the constants or the logarithmic/exponential rearrangements.

(b)(ii) This part was done very well with a majority of candidates earning full marks. Candidates are reminded to consider the logical correctness of their answer, as poor use of negative signs often led to a final solution larger than initial value of A .

Common problems were:

- misusing negative signs
- careless rounding of values that were needed in subsequent working
- incorrectly substituting for M and A , often in reverse order.

(c)(i) A significant number of candidates found this part quite challenging. Some made the question more difficult by first expressing displacement as a single fraction and then differentiating twice using the quotient rule. Poor algebraic skills and incorrect use of the quotient rule made this a time consuming and inefficient method to use.

Common problems were:

- mislabelling expressions for velocity and acceleration
- integrating to obtain $v = \ln(1+t)$, and then differentiating to return to the original expression with $a = \frac{1}{1+t}$
- stating the derivative of t to be 0 instead of 1
- not verifying that $\frac{d^2x}{dt^2} < 0$.

(c)(ii) A significant number of candidates also had difficulty with this part and there was a clear lack of familiarity with limits and the concept of infinity.

Common problems were:

- substituting $t = 0$ into an expression for velocity
- attempting to solve $v = 0$.

(d)(i) Better responses included a large neat diagram with all information given in the question clearly labelled.

Common problems were:

- incorrectly stating and using the cosine rule
- correctly substituting into a form of the cosine rule and then just stating the given answer
- using the given value of $AC=210$ and the sine rule to find $\angle CDE = 89^\circ$
- using $\angle CDE = 90^\circ$ due of a lack of understanding of bearings and poorly drawn diagrams.

(d)(ii) Candidates who drew a diagram had much greater success with this part. In better responses, candidates used the sine rule to find $\angle ACB$ using sides AC and AB and $\angle ABC$, then correctly calculated the size of the angle between north and AC at C . The correct bearing was then easily found by calculating $582^\circ - 5^\circ - 33^\circ$.

Common problems were:

- not finding the bearing
- not using the given value of AC from d(i)
- mislabelling angles and confusing the A , B and C from the question with the A , B and C in their sine and cosine rule formulae

- incorrectly rounding angles and not using brackets correctly when calculating the final bearing
- citing new angles and sides lengths drawn perhaps on the diagram in the question booklet, but not visible in the answer booklet.
- finding $\angle CED = 5:2$ and just subtracting from 582° without considering the bearing from north.

Question 14

(a) This part was generally well attempted with most candidates finding a stationary point. The first derivative test was a more popular test for the nature of the point. Candidates are encouraged to use calculators to check values when testing for positive, negative and zero gradients.

Common problems were:

- using the product rule on the function ex and treating e as a function, rather than a constant
- omitting the test to determine the nature
- only finding the x coordinate and not the y coordinate.

(b)(i) Most candidates stated the correct value.

A common problem was:

- using $\alpha + \beta = \frac{b}{a}$

(b)(ii) Most candidates realised the need to find a value for $\alpha\beta$ and attempted to factorise and combine (b)(i).

A common problem was:

- after correctly factorising, and finding correct values for $\alpha\beta$ and $\alpha + \beta$, many candidates made errors solving the resulting equation.

(c) Most candidates were able to demonstrate an understanding of the method for finding a volume of revolution.

Common problems were:

- writing an incorrect expression for y^2
- not correctly integrating the term with the fractional index
- rotating about the y -axis.

(d)(i) Most candidates gained full marks for this part.

(d)(ii) Most candidates recognised that a geometric series and a limiting sum were required.

Common problems were:

- incorrectly stating the limiting sum formula
- using an exponential growth formula

(e) Most candidates were able to correctly engage with at least one of the important features of the derivative graph, usually point C from the given function. In better responses, candidates copied the original function into their answer booklet and completed their derivative graph directly below, indicating important features.

Common problems were:

- poor graphing skills
- using incorrect x intercepts and nature of turning points.

Question 15

(a) Many candidates had difficulty with this part.

Common problems were:

- not recognising or incorrectly using the substitution $\sin^2 x + \cos^2 x = 1$
- dividing both sides of their equation by $\cos x$
- incorrectly factorising the trigonometric quadratic expression
- expressing answers in degrees rather than radians.

(b)(i) This part was generally done well, with the majority of candidates using ‘equiangular’ to prove similarity. A few candidates stated the intercept properties of transversals and parallel lines to show that the sides about the equal angle were in proportion.

Common problems were:

- not recognising the two pairs of corresponding angles, or indicating an equal pair of angles without appropriate reasoning
- using tests for congruence instead of similarity
- poor setting out with little or no reasoning, and omitting a concluding statement.

(b)(ii) This part was generally done well. Most candidates used the result in (b)(i) to explain part (ii). Some candidates wrote a full paragraph of justification when a brief statement was all that was required for 1 mark. Most candidates knew about similar triangles having corresponding sides in the same ratio and used relevant terminology.

A common problem was:

- stating pairs of equal sides and writing $DS = DR$, $DE = DF$, then $\frac{DR}{DF} = \frac{x}{x+y}$.

(b)(iii) Most candidates struggled with this part. Better responses were well set out with a series of logical steps. The candidates who achieved full marks for this part either performed algebraic manipulation involving $\frac{A_1}{A}$, or applied ‘ratios of areas is equal to the square of the ratios of side lengths’ in similar figures to achieve the result.

Common problems were:

- incorrectly using the formula $\text{Area} = \frac{1}{2}ab \sin C$, often with incorrect sides
- assuming the triangles were right angled
- assuming that $DR = DS = x$, $DE = DF = y$.

(b)(iv) In the better responses candidates appreciated the link between (iii) and (iv) presenting a succinct and accurate answer. Most candidates indicated that $\sqrt{\frac{A_2}{A}} = \frac{y}{x+y}$, using their expression from (b) (iii). However, a considerable number of candidates struggled to provide the working to deduce $\sqrt{A} = \sqrt{A_1} + \sqrt{A_2}$.

Common problems were:

- not realising how to use the result from (b) (iii), some candidates unnecessarily completed a further similar triangle proof to establish the result.
- after obtaining $\sqrt{\frac{A_1}{A}} = \frac{x}{x+y}$ and $\sqrt{\frac{A_2}{A}} = \frac{y}{x+y}$, some candidates assumed that $\sqrt{A_1} = x$, $\sqrt{A_2} = y$ and $\sqrt{A_1} + \sqrt{A_2} = x + y$.

(c)(i) This part was generally done very well. The majority of candidates correctly sketched an exponential graph and tangent.

Common problems were:

- not recognising that $y = mx$ passes through the origin
- omitting the sketch of the tangent .

(c)(ii) Candidates found this part quite challenging. The majority of candidates correctly equated $mx = e^{2x}$ and/or differentiated $y = e^{2x}$.

Common problems were:

- not interpreting the question correctly
- making algebraic errors when solving the equations
- leaving the answer in terms of m or a logarithmic expression.

(c)(iii) This part was generally done well with most candidates substituting their co-ordinates for P into an appropriate equation.

A common problem was:

- not having a point P to use from c(ii)

Question 16

(a) This part was generally attempted well with most candidates recognising the appropriate form of Simpson's Rule and using five function values. The most successful attempts started with candidates filling out a table of values and using correct weightings.

Common problems were:

- not correctly evaluating the sec function values, particularly $\sec\left(-\frac{\pi}{3}\right)$ and $\sec\left(-\frac{\pi}{6}\right)$
- mixing up the order of the weightings
- finding the value of 'h' in the table by identifying correct x values, but then attempting to use $h = \frac{b-a}{n}$ incorrectly to find a different value for 'h'
- using values for 'h' expressed in degrees.

(b)(i) Most candidates were able to explain at least one of the terms in the expression given for the bank balance at the end of the second month. The explanation was expressed mathematically with excellent use of brackets to show cause and effect or using words to explain.

Common problems were:

- re-writing or expanding the given answer
- explaining only one of the terms in the given expression.

(b)(ii) Most candidates attempted this question, realising that the response involved the summation of a geometric series.

Common problems were:

- not finding the common ratio from a correct series for A_{60}
- incorrectly splitting the series for A_{60} into two separate series, one involving $(1.01)^n$ and the other $(1.003)^n$
- incorrectly identifying the value of 'a' from their series
- omitting the factor of $(1.01)^n$ in their series

(c)(i) Most candidates were able to find an expression for the length of the frame and then show appropriate logical steps to obtain the required result.

Common problems were:

- ignoring the instruction 'show' and moving directly from a correct expression for the length of the frame to the given answer
- not being able to find the correct expression for the arc length part of the frame.

(c)(ii) This part was often omitted or poorly attempted by candidates

Common problems were:

- incorrectly squaring the $\frac{x}{2}$ value used for the radius of the semi-circle
- not understanding how to apply the 'units of light'
- algebraic errors when expanding $3 \left[5x - x^2 \left(1 + \frac{\pi}{4} \right) \right]$.

(c)(iii) Most candidates realised that this part could be attempted by using a standard calculus maxima process.

Common problems were:

- incorrectly expanding terms in $L = 15x - x^2 \left(3 + \frac{5\pi}{8} \right)$
- poor attempts at simplifying the expression for the value of x
- finding the maximum light that came through the window rather than the dimensions of the frame that allowed for maximum light to pass through
- omitting one of the required maxima tests or not evaluating both dimensions of the frame
- stating a value for y (often incorrect) without showing necessary working.