2015 Notes from the Marking Centre – Mathematics Extension 1

Introduction

This document has been produced for the teachers and candidates of the Stage 6 Mathematics Extension 1 course. It contains comments on candidate responses to the 2015 Higher School Certificate examination, highlighting their strengths in particular parts of the examination and indicating where candidates need to improve.

This document should be read along with:

- the *Mathematics Extension 1 Stage 6 Syllabus*
- the 2015 Higher School Certificate Mathematics Extension 1 examination
- the marking guidelines
- *Advice for HSC students about examinations*
- other support documents developed by the Board of Studies, Teaching and Educational Standards NSW to assist in the teaching and learning of Mathematics in Stage 6.

Question 11

(a)
The question was generally very well done by a large number of candidates.

In the better responses, candidates approached the question by first stating a correct relationship between $\sin^2 x$ and $\cos 2x$ before then giving the correct primitive.

Common problems were:

- attempting to quote the primitive directly, but making an error
- incorrectly making $\sin^2 x$ the subject of their correct identity
- giving the incorrect primitive for $\cos 2x$

(b)
In the better responses, candidates approached the question by stating a correct formula before showing the correct substitution of the gradients.
Common problems were:

- using incorrect variations of the formula, particularly
$$\tan\theta = \left|\frac{m_1 + m_2}{1 - m_1 m_2}\right|$$
- making arithmetic errors inside the absolute value signs
- giving geometric responses without including a diagram to assist with the reasoning.

(c)
In the better responses, candidates approached the question by identifying the two critical values $x = -3$ and $x = 1$ before giving the correct algebraic solution: $-3 < x \le 1$

There was a wide range of solution types. A significant feature of the responses was not so much the method used to arrive at the two critical values, but in analysing the correct section of the number line.

Common problems were:

- including $x = -3$ in the solution, ie incorrectly writing $-3 \le x \le 1$
- factorising the quadratic expression $x^2 + 2x - 3$ incorrectly
- using incorrect reasoning from a diagram.

(d)
The question was attempted well by the majority of candidates.

In the better responses, candidates used appropriate formulae to directly obtain the values of A and α.

Common problems were:

- not using the information, $0 \le \alpha \le \frac{\pi}{2}$, which was given in the question. In such cases, responses typically stated that $\tan\alpha = \frac{-12}{5}$
- incorrect reasoning after expanding $A\cos(x+\alpha)$
- giving α in degrees, even though the question asked for α in radians.

(e)
In the better responses, candidates correctly substituted to obtain the correct definite integral in terms of the variable.
Common problems were:

- incorrectly manipulating the substitution in making x the subject, often writing $u = 2x - 1 \rightarrow x = \frac{u-1}{2}$
- not separating the integrand correctly so as to find the correct primitive
- incorrectly simplifying the definite integral, for example

$$\int_1^3 \left(\frac{\frac{u+1}{2}}{u^2} \right) \frac{du}{2} = \int_1^3 \frac{u+1}{u^2}\, du.$$

(f)(i)
The majority of students answered this part well.
There were no common problems with this part, although a few candidates found that $k \neq 6$.

(ii)
In the better responses, candidates factorised $P(x) = x^3 - 6x^2 + 5x + 12$ by using their result from part (i) and found the remaining quadratic factor $x^2 - 3x - 4$. Hence, factorising this quadratic, the correct zeroes of -1, 3 and 4 were obtained.
In many responses, the fact that the zeroes would be factors of the constant term 12 was used. Fortunately the zeroes were integers and the question was answered correctly.

Common problems were:

- incorrectly factorising $x^2 - 3x - 4$
- testing all of the factors of 12, including $x = 3$, and not realising the significance of the information provided in part (i)
- not showing any working for solutions and/or guessing values.

Question 12

(a)(i)

A large number of candidates found $\angle ACB = 60°$ using the fact that $\angle ACD$ was the angle in a semicircle. Almost all candidates gave reasons even though it was not required in the question.

(ii)

Many candidates used the appropriate theorem (ie the angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment) in one step to correctly find $\angle ADX = 30°$.

A small number of candidates did not find the correct value of $\angle ADX$ after numerous steps and quoted many circle geometry theorems, when reasoning was not required.

(iii)
A large number of candidates found $\angle CAB = 70°$, correctly quoting or stating some circle geometry theorems.
A small number of candidates executed an efficient solution in two steps, stating the appropriate theorems.

Common problems were:

- finding almost every angle in the pursuit of $\angle CAB$ and quoting irrelevant theorems
- writing every theorem in the hope that one may be correct, rather than composing a deductive argument.

(b)(i)

This part highlighted the need to understand the difference between $A \Rightarrow B$ and $B \Rightarrow A$. Candidates were given that PQ was a focal chord and asked to show that $pq = -1$. Candidates who substituted $(0, a)$ into the equation of the chord were able to derive the result.

A common problem was:

- substituting $pq = -1$ or calculating gradients with spurious justifications, suggesting that what was required was not understood.

(ii)

Candidates who recognised that $2ap = 8a \Rightarrow p = 4$ and then used $pq = -1$ to derive $q = -\frac{1}{4}$ quickly arrived at the correct coordinates $\left(\frac{-a}{2}, \frac{a}{16}\right)$.

(c)(i)
Many candidates were able to show that $OA = h\cot 15°$.

A small number of candidates did not provide a solution to this part, some demonstrating a lack of understanding of the notation by writing expressions such as $15° = \cot\left(\frac{OA}{h}\right)$.

(ii)

This part required the use of Pythagoras's theorem with the substitution of some trigonometric ratios, and candidates with good algebra skills executed a correct solution in a few lines.

Common problems were:

- choosing the incorrect triangle; substituting incorrectly into Pythagoras's theorem; misreading which angle was the right angle (it was marked on the diagram); and both poor calculator skills and poor algebraic manipulation.
- using radian mode although the angles were given in degrees.

It was possible to find the correct answer without the direct use of Pythagoras's theorem via utilising trigonometric ratios with all three triangles. Only a small number of candidates were successful using this approach.

(d)(i)

A large number of candidates quoted the cosine rule and substituted correctly to arrive at the required result.

A common problem was:

- not quoting the formula and/or not showing the substitutions

(ii)
Many candidates were able to execute an accurate and efficient solution by recognising that $r = \frac{200}{\theta}$, then substituting this into the equation from part (i) to eliminate r.

Common problems were:

- making algebraic slips when trying to simplify in the intermediate steps
- not relating to the information given in the question or not knowing the necessary formula $l = r\theta$

(iii)
Most candidates quoted the correct formula for Newton's method and correctly substituted their values.

A common problem was:

- not correctly differentiating $\cos\theta$

Question 13

- (a)(i) Most candidates stated the values of $x = 3$ and $x = 7$.

A common problem was:

- giving the answer as $x = 0, x = 5,\ x = \sqrt{3}$ or $x = \sqrt{7}$.

(ii)
Many candidates gave the correct value.

Common problems were:

- giving an answer of $v = 11$
- not recognising that speed is positive (ie giving answers of plus or minus a value).

(iii)
The most common method was to use simultaneous equations to first find *c*, then use this to find *a* and *n*. Some candidates used the data from the given parabola to manipulate the equation into the form given in the question. They were then able to read the required values easily from the equation.

Common problems were:

- not recognising the significance of the variables in relation to SHM
- trying to use data from the parabola in standard trig formula in SHM
- poor calculation and algebraic skills when solving simultaneous equations.

(b)(i)
Many candidates recognised and used terms of a binomial expansion, in particular the general term to help solve this question.

Common problems were:

- calculating the incorrect term, such as the second term
- using $k = 3$ instead of $k = 2$ in their expressions
- not recognising that only the coefficient was required.

(ii)
Some candidates who could not complete (i) had some success in this part.

(c)
Most candidates worked through the steps of an induction proof and were able to complete the proof.

Common problems were:

- not clearly setting out the proof, particularly in the inductive step
- not recognising what was the correct sum to prove
- not finding the correct denominator, and a poor understanding of factorials
- crossing out and making multiple attempts at the $n = k + 1$ stage.

(d)(i)
Candidates are encouraged to take care with writing plus and minus signs.

Common problems were:

- ignoring the question and trying to solve the problem without differentiation
- calculating the derivative incorrectly
- not understanding that a constant function has a gradient of 0.

(ii)

Candidates were generally more successful when they took the simplest approach of substituting into $f(x)$ an appropriate value (in the correct domain).

Common problems were:

- not reading the question (the word 'hence') to deduce the solution from part (i) not showing any proof
- ignoring the requirement to deduce the result by substituting into the required equation and showing the result was true.

Question 14

(a)(i)
Finding the time when the particle hits the ground and then substituting this time into the horizontal displacement proved to be the most successful way of approaching this problem.

Some candidates use the symmetry of the problem, and found the time of the greatest height and then doubled this time to find the flight time.

Some candidates chose a longer solution, creating a Cartesian equation by eliminating the parameter '*t*' from the displacement equations.

(ii)
In contrast to part (i), part (ii) proved to be challenging for many candidates.

Candidates who recognised that $\tan\alpha = \frac{\dot{y}}{\dot{x}}$, were generally successful.

Common problems were:

- using the displacement equations instead of the velocity equations to find the required angle
- confusing the fixed angle of projection, with the variable angle the particle makes with the horizontal
- using the exact values for $30°$ instead of the given $\frac{\pi}{3}$.

(iii)
Generally candidates who were successful in part (ii) were successful in using it to justify their answer to part (iii).

The other successful approach was to compare the given time with the time at the greatest height, and then correctly concluding that it was after this time, so the particle must be going down.

A common problem was:

- using incorrect logic such as 'as the angle is less than the projection angle, the particle must be travelling downwards' or 'as acceleration due to gravity was negative, the particle must be travelling downwards'.

(b)(i)
Those candidates who used $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ or $v\frac{dv}{dx}$ usually proceeded to write v^2 as a perfect square in terms of *x*.

Common problems were:

- not recognising that acceleration was given as a function of displacement (x) and not as a function of time (t), and simply differentiating with respect to x

- not using the initial conditions to justify why $\dot{x} = 1 - x$ (and not $\dot{x} = x - 1$), just assuming it to be so.

Responses using definite integrals were generally more successful than those using indefinite integrals.

(ii)
Common problems were:
- simply integrating the velocity equation to get a displacement equation, seemingly unconcerned with a function involving a mixture of x and t as the independent variables
- not understanding what ‘x as a function of t’ means, and leaving answers, with t being the subject of the formula
- not finding the constant of integration when using indefinite integrals.

(iii)
Those candidates who successfully completed part (b)(ii), found a limiting position by realising that $\lim_{t \to \infty} e^{-t} = 0$.
Another successful method was to solve $\dot{x} = 0$, which is valid for a continually increasing function.

Many candidates who left their answer in terms of *x* in part (b)(ii) commenced this part by successfully making *x* the subject.

Common problems were:
- simply giving the range of the displacement function
- not realising a limiting position must be a finite number.

(c)(i)
In general, the meaning of the word ‘explain’ was not understood and quite often responses did not have enough detail to demonstrate appropriate understanding of the problem.

Common problems were:
- While it was understood that that player *A* had to win 5 games out of 7, candidates could not explain the need for $\binom{6}{4}$ and not $\binom{7}{5}$
- incorrectly stating that A had to win the first game.

(ii)
Many candidates successfully applied the pattern from (c)(i) to this part to get the correct expression. Some candidates who did not provide a correct explanation in part (c)(i) were able to provide the correct expression in part (ii), demonstrating a weakness in being able to articulate mathematical logic.

From the phrase ‘at most’ many candidates incorrectly concluded that the answer must be 1 – [part (c)(i) answer] or some other similar expression.

(iii)
Most candidates struggled with this part, with only a small number of responses being awarded full marks.

Generally it was the candidates who were successful in part (c)(ii) who could generalise their answer and go on to prove the required result.

Many incorrect responses involved attempts to manipulate the Binomial theorem to force it into the form of the given expression. The requirement ‘by considering the probability’ seemed to have been ignored.