2015 Notes from the Marking Centre – Mathematics

Introduction

This document has been produced for the teachers and candidates of the Stage 6 Mathematics course. It contains comments on candidate responses to the 2015 Higher School Certificate examination, highlighting their strengths in particular parts of the examination and indicating where candidates need to improve.

This document should be read along with:

- the *Mathematics Stage 6 Syllabus*
- the 2015 Higher School Certificate Mathematics examination
- the marking guidelines
- *Advice for HSC students about examinations*
- other support documents developed by the Board of Studies, Teaching and Educational Standards NSW to assist in the teaching and learning of Mathematics in Stage 6.

Question 11

(a) This part was generally done well.

A common problem was:

- changing the given expression into an equation and solving to find a value for x.

(b) Most candidates correctly factorised the quadratic expression.

Common problems were:

- changing the question to a quadratic equation and solving it
- only recognising the common factor of 3
- dividing the expression by 3.

(c) This part was done well and correctly set out by most candidates.

Common problems were:

- multiplying only the denominator by the conjugate
- multiplying the numerator and denominator by $2 + \sqrt{7}$ or by $2\sqrt{7}$ or by $\sqrt{7}$
- incorrectly expanding the binomial product.

(d) This part was done well. In better responses, candidates showed how they calculated the value of r by using $\frac{T_2}{T_1}$ and then substituted into the formula for the limiting sum of a geometric series.

Common problems were:

- using the absolute value of r
- using an incorrect formula.

(e) This part was challenging.

Common problems were:

- the incorrect use or omission of brackets
- incorrectly differentiating $e^x + x$.

(f) This part was generally done well. In better responses, candidates often listed u, u', v and v' before they substituted into the product rule.

Common problems were:

- stating the product rule as $u'v - v'u$ or $v'u - u'v$
- incorrectly differentiating u and/or v.

(g) This part was attempted well. Evaluating $sin2x$ when $x = \frac{\pi}{4}$ was difficult for some candidates. They often calculated $2sinx$ or only $sinx$.

Common problems were:

- using an incorrect primitive function
- substituting limits incorrectly
- not evaluating and leaving the solution in terms of $\sin\frac{\pi}{2}$
- evaluating in degrees rather than radians.

(h) The majority of candidates recognised that the primitive was a log function.

Common problems were:

- incorrectly setting up $\int \frac{f'(x)}{f(x)} dx$, for example, using $2\int \frac{2x}{x^2-3} dx$ or $2\int \frac{x}{x^2-3} dx$
- incorrectly using or omitting brackets, for example $\frac{1}{2}(\ln x^2 - 3) + c$ or $\frac{1}{2}\ln x^2 - 3 + c$.

Question 12

(a) This part was generally answered well by most candidates.

Common problems were:

- giving the answer in degrees rather than radians
- only finding the acute angle
- finding angles from all four quadrants.

(b)(i) Most candidates used the fact that the diagonals are perpendicular and then applied the point-gradient formula to arrive at the required equation.

Common problems were:

- substituting the coordinates of the point $A(7,11)$ into the given equation for the line l_1
- finding an incorrect gradient.

(b)(ii) The majority of candidates recognised the need to use simultaneous equations for this part. Some candidates who tried using the fact that D was the midpoint of AC, made little progress.

Common problems were:

- not using the correct equation for l_1 provided in part (i)
- making arithmetical and algebraic errors when solving equations simultaneously.

(c) The majority of candidates used the quotient rule to successfully find the derivative. Those candidates who used the product rule often made careless algebraic errors.

Common problems were:

- using an incorrect formula, for example $\frac{uv' \pm vu'}{v^2}$
- not including brackets in the answer, for example $\frac{2x(x-1)-x^2+3}{(x-1)^2}$
- making algebraic errors when expanding and/or simplifying
- using incorrect derivatives for u and v.

(d) This part was challenging. Most candidates who attempted this part realised that they needed to use the discriminant.

Common problems were:

- not using the correct expression for the discriminant
- not recognising that a quadratic equation has real roots when $\Delta \geq 0$
- making careless algebraic errors when solving an inequality.

(e)(i) The majority of candidates were able to correctly find the derivative and use the point-gradient form to find the equation of the tangent. Some candidates complicated this part by unsuccessfully attempting to use the quotient rule to find the derivative.

Common problems were:

- finding an incorrect derivative of $y = \frac{x^2}{2}$
- not substituting the x coordinate of P into the derivative to find the gradient.

(e)(ii) The majority of candidates scored full marks for this part.

Common problems were:

- incorrectly labelling the directrix as $x = -\frac{1}{2}$ or $d = -\frac{1}{2}$
- incorrectly stating that the point $Q(0, -\frac{1}{2})$ was the equation of the directrix.

(e)(iii) Most candidates realised that they needed to solve simultaneously the equations found in (e)(i) and (e)(ii). Those candidates who were not successful in finding the correct equation of the tangent in (e)(i) generally struggled to complete this part. Some candidates did not make the link between their coordinates for Q and the fact that Q lies on the *y*-axis.

Common problems were:

- finding the y-intercept of the tangent without linking it to the directrix
- showing that $Q(0, -\frac{1}{2})$ lies on the tangent instead of the y-axis.

(e)(iv) Most candidates were able to use their diagram to establish that $PS = QS = 1$.

Common problems were:

- finding the lengths of PQ and PS and stating that they were equal.
- making arithmetic errors when using the distance formula.

Question 13

(a)(i) This part was generally done well by most candidates.

Common problems were:

- using an incorrect formula for the cosine rule
- incorrectly substituting into the correct formula
- attempting to find cos A using right triangle trigonometry.

(a)(ii) In better responses, candidates used the results from (a)(i), formed a right-angled triangle and used Pythagoras's Theorem to obtain the third side, allowing them to find the exact value of $\sin A$.

Common problems were:

- using an incorrect formula for area
- correctly finding the exact value of sin A as $\frac{\sqrt{15}}{8}$ and then using this value as angle A in the area of a triangle formula
- finding the value of angle A and sin A using the calculator and giving an approximation for the area of the triangle
- not being able to find the exact value of sin A
- interpreting an exact value to mean 'round off to the nearest whole number'.

(b)(i) This part was challenging. Candidates who sketched $y = \sqrt{9 - x^2}$ were generally more successful in finding the correct domain and range.

Common problems were:

- incorrectly stating their solution as $x \geq -3, x \leq 3$ instead of $-3 \leq x \leq 3$
- not recognising the function as a semicircle and using the same values for the domain and range
- having the inequality signs reversed or using $<$ instead of $\leq$
- only stating the domain
- giving the range as $y \geq 0$ since a square root is always positive.

(b)(ii) This part was challenging. Many candidates correctly graphed the semicircle but were less successful adding the graph of $y = x$ and correctly shading the required region. Candidates are reminded to draw neat, clear diagrams, use a ruler to draw lines and show a scale on each axis. The size of each diagram should be at least one-third of a page.

Common problems were:

- not drawing the vertical boundaries of $|x| = 3$ for the semicircular region
- not shading any region at all or not continuing the region below the x-axis between the lines $x = -3$ and $y = x$
- sketching the semicircle only and not the line $y = x$
- drawing an incorrect semicircle, for example $y = -\sqrt{9 - x^2}$
- graphing the solution as an inequality on a number line.

(c)(i) Most candidates answered this part very well, setting out their work in clear, logical steps. Responses in which the second derivative test was used to determine concavity were generally more successful than those in which a table and the first derivative test were used.

Common problems were:

- inability to correctly factorise the quadratic derivative
- finding an incorrect y value
- attempting to determine the nature of the stationary points using a table but not indicating whether they were using y, y' or y'' and often not showing the resulting value
- using the second derivative test to determine the nature but incorrectly identifying the condition for a maximum or minimum turning point.

(c)(ii) Most candidates successfully solved $y'' = 0$ to find the point of inflection and then used the second derivative to show a change in concavity.

(c)(iii) In the majority of responses, candidates used their previous results to sketch the cubic curve. Candidates are again reminded to draw large diagrams of at least one-third of a page and clearly label the required features.

Common problems were:

- not finding the y-intercept
- not labelling the stationary points, point of inflexion and y-intercept as required
- not graphing their correct points with relative position
- graphing the point of inflexion as a horizontal point of inflexion.

Question 14

(a)(i) This part was generally done well, with most candidates correctly integrating and making the appropriate substitutions to arrive at the required equation for displacement.

Common problems were:

- not showing the required substitutions to find the constants of integration
- integrating with respect to x instead of t

- very poor use of notation, especially $\int$ and dx
- differentiating the given equation to show that acceleration is $\ddot{x} = -10$.

(a)(ii) The vast majority of candidates found the correct time but did not realise that they needed to find the distance that the chair had fallen and so merely found its height at that time.

Common problems were:
- not subtracting the height of the chair from the original height of 110 m to find the distance fallen
- using a velocity of $37\ \text{ms}^{-1}$ when the brakes were applied which resulted in a negative value for time.

(b)(i) In the better responses, candidates correctly stated $P(Saturday\ dry) = P(WD) + P(DD)$ and then wrote the correct combination of fractions and operations using the probability tree. Candidates are advised to always use probability notation so that it is very clear how they obtained each fraction in their working.

Common problems were:
- writing any combination of fractions from the tree diagram that gave an answer of $\frac{2}{3}$
- not showing full setting out
- not using addition or multiplication of fractions appropriately.

(b)(ii) Many candidates first wrote the branches needed to have both Saturday and Sunday wet, namely $P(both\ Saturday\ \&\ Sunday\ wet) = P(WWW) + P(DWW)$, before attempting to write the associated numerical expression.

Common problems were:
- numerical errors in simple calculations involving fractions
- only calculating one half of the required probability
- ignoring Friday's probability in calculations
- not completing and not explaining working.

(b)(iii) In better responses, candidates realised that part (b)(ii) directly related to part (b)(iii) and required the use of the complement. Candidates are advised to show working for all questions including writing the complement statement explicitly. Candidates who opted for the alternate method of using six tree branches to find the probability of at least one of Saturday or Sunday dry generally made numerical errors or omitted some of the required branches.

Common problems were:
- writing an answer without any working
- not linking (b)(iii) with (b)(ii) and using the longer approach.

(c)(i) This part was generally done well, with many candidates clearly writing the expression for A_1 and then showing the working and steps needed to generate the expression for A_2. Candidates are reminded that all 'show' questions require working with logical steps which lead to the required answer and writing the last step before the given answer is crucial.

Common problems were:
- finding an expression for A_1 and then going straight to the given expression for A_2 without showing how A_2 is actually obtained
- incorrect use or omission of brackets when writing the expressions for A_1 and A_2
- omitting zeros with 1.006 changing to 1.06 at some stage in the solution
- attempting to work backwards from the given expression for A_2.

(c)(ii) In the better responses, candidates showed the expression for A_3 and then generalised the pattern to show $A_n = 100\,000(1.006)^n - M[1 + (1.006) + (1.006)^2 + \cdots + (1.006)^{n-1}]$. Using the first three terms of their series, they deduced that it was geometric. Correct substitution into the GP sum formula and simplification, led to the given result.

Common problems were:

- using a rote-learned formula instead of deriving A_n
- only showing two terms in the series
- incorrect use of brackets
- writing the last term of the series to the power n instead of $n - 1$
- not showing the substitution of values into the GP sum formula and merely stating the given answer.

(c)(iii) This part was done very well, with most candidates substituting $M = 780$ and $n = 120$ into the result provided in (c)(ii) to arrive at the answer \$68 499.46 which could then be shown to be equal to \$68 500 correct to the nearest \$100.

Common problems were:

- making the substitution but not writing the calculator display in their working
- performing their calculation in sections, rounding off each result and combining them to obtain an incorrect answer.

(c)(iv) Candidates who removed the denominator of 0.006 by dividing 780 by 0.006 were usually the most successful at achieving a correct solution. Many candidates did not link the equation given in (c)(ii) to part (c)(iv) and so spent considerable time and effort re-establishing a pattern and a formula to use.

Common problems were:

- using incorrect values for A_n, P and M
- being unable to perform the algebraic steps necessary to isolate $(1.006)^n$ on one side of the equation
- inability to use logs to solve an exponential equation
- using a trial and error method to solve the exponential equation but not showing any or sufficient evidence of the values of n tested and the associated answers.

Question 15

(a)(i) The most successful approach was to start with $C = Ae^{-0 \cdot 14t}$, correctly differentiate and substitute to show that the expression for C was a solution to $\frac{dC}{dt} = -0 \cdot 14C$. Candidates who used the more complex process of integration starting with $\frac{dt}{dC}$, were generally less successful as they often did not deal correctly with the constants or logarithmic/exponential rearrangements.

(a)(ii) This part was answered extremely well.

Common problems were:

- substituting incorrectly
- assuming e^0 was e^1.

(a)(iii) Most of the responses were correct, with $A = 130$ and $t = 7$ being substituted into the formula given in (a) (i).

(a)(iv) This part was challenging, especially in using logarithms to solve an exponential equation.

Common problems were:

- using half of the answer from (a)(iii)
- using 70 or 75 as half of 130
- making algebraic errors when solving the equation
- using logarithms in base 10 instead of e in the calculation.

(b)(i) This similarity proof was found to be quite challenging. Most candidates were able to identify $\angle ACB = \angle DCF$ and provide a correct reason. Showing $\angle BAC = \angle ADE = \angle CDF$ proved to be difficult.

Common problems were:

- writing incorrect reasons; for example, stating that angle C was a common angle or stating that a pair of angles were alternate when they were vertically opposite
- labelling angles incorrectly
- using an incorrect test for similarity
- poor setting out with little or no reasoning
- using congruency tests to prove similarity.

(b)(ii) Most candidates recognised the need to use the similar triangle result from (b)(i) to identify the pair of corresponding equal angles.

Common problems were:

- using incorrect reasoning or no reasoning
- assuming all angles are equal in similar triangles.

(b)(iii) This part was found to be quite challenging. A popular method was to prove $AB = 2FD$ and then use the result of (b)(ii) to find $EF = EB$. Other successful approaches included constructions and trigonometry.

Common problems were:

- using incorrect reasoning or no reasoning
- using incorrect proportion statements.

(c)(i) In many responses, candidates started by attempting to integrate $\frac{dV}{dt}$. For many, this resulted in them using V rather than $\frac{dV}{dt}$ to find when the water started to decrease. The better responses included a sketch of the function $\frac{dV}{dt}$ and this was used to determine the solution.

Common problems were:

- incorrectly solving $\sin(0 \cdot 5t) < 0$; for example finding $t < 0,\ t < 2\pi$
- misreading the solution when using the graphical approach; for example finding $t = \pi$
- using the solution for (c) (ii)
- writing the answer as $t = 360$ hours
- integrating $80 \sin(0 \cdot 5t)$.

(c)(ii) In many responses, candidates obtained the correct primitive function and used $V = 1200$ when $t = 0$ to find the correct expression for $V(t)$.

Common problems were:

- differentiating instead of integrating
- not using a constant of integration
- using the calculator in degree mode instead of radian mode
- not realising $\cos 0 = 1$ and hence not correctly evaluating the constant of integration
- poor setting out, misuse of brackets.

(c)(iii) Many candidates found it difficult to link (c)(i) and(c)(ii) and completed the same working twice. In the better responses, candidates stated and used the fact that $-1 \leq \cos t \leq 1$. Only a small percentage of candidates used a graph to identify the maximum volume.

Common problems were:

- substituting/calculating in degrees when t was given in radians
- unnecessarily calculating the second derivative.

Question 16

(a)(i) This part was done well by the majority of candidates. A small percentage of students factorised incorrectly.

(a)(ii) This part was done well by the majority of candidates.

Common problems were:

- making an incorrect substitution for $y = 10$
- stating the x and y co-ordinates in the wrong order.

(a)(iii) This part was done well by the majority of candidates.

Common problems were:

- differentiating instead of integrating
- substituting the limits incorrectly
- including a constant of integration.

(a)(iv) A variety of methods was used in this part. In the better responses, candidates recognised the symmetry of the parabola and used the answer from (a)(iii).

Common problems were:

- incorrectly calculating the area between the line $y = 2x - 4$ and the curve
- finding the incorrect equation of the line
- evaluating a single integral involving a line and a parabola
- incorrectly using a complex approach involving rectangle, trapezium and integrals.

(b) This part was found to be challenging.

Common problems were:

- finding the volume rotated about the x-axis
- omitting π
- not correctly expressing x as the subject of the equation.
- not correctly squaring an expression involving an exponential
- not correctly integrating the expression $e^{\frac{y}{4}} + 2e^{\frac{y}{8}} + 1$
- incorrect use of limits , for example, 0 to 2
- not showing the substitution of limits and making a calculator error
- not correctly evaluating after substituting 0 into the exponential expression.

(c)(i) This part was found to be challenging. In the better responses, candidates used relationships of similar triangles.

Common problems were:

- working backwards from the equation given in the question to arrive at an expression for y and then using it to 'show' the given expression
- not recognising the correct matching sides of similar triangles
- subtracting the volume of the cylinder from the volume of the cone
- using Pythagoras's theorem
- assuming that $R = 2x$ and/or $H = 2y$.

(c)(ii) This part was found to be challenging. Candidates are reminded that expanding the expression, where possible, before differentiating, is often easier than using the product rule.

Common problems were:

- using the product rule incorrectly to differentiate
- incorrectly solving $\frac{dV}{dx} = 0$
- omitting a test to establish a maximum
- not comparing the volumes of the cylinder and the cone.