BOARD OF STUDIES
NEW SOUTH WALES

HIGHER SCHOOL CERTIFICATE EXAMINATION

2000
MATHEMATICS
2/3 UNIT (COMMON)

Time allowed—Three hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on page 16.
- Board-approved calculators may be used.
- Answer each question in a SEPARATE Writing Booklet.
- You may ask for extra Writing Booklets if you need them.

QUESTION 1 Use a SEPARATE Writing Booklet. **Marks**

(a) Find the value of $\log_e 8$ correct to two decimal places. **2**

(b) Solve $x + 7 \geq 3$ and graph the solution on the number line. **2**

(c) What is the exact value of $\cos \dfrac{\pi}{6}$? **1**

(d) A bag contains red marbles and blue marbles in the ratio 2 : 3. **1**

A marble is selected at random.

What is the probability that the marble is blue?

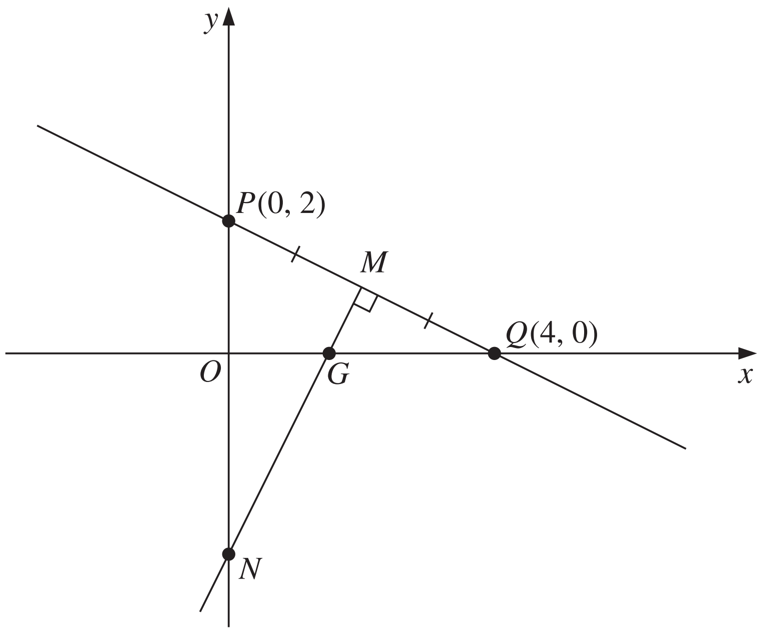
(e) Solve the pair of simultaneous equations: **2**

$$x - y = 2$$
$$3x + 2y = 1 \, .$$

(f) Solve $|x - 5| = 3$. **2**

(g) Sketch the line $y = 2x + 3$ in the Cartesian plane. **2**

QUESTION 2 Use a SEPARATE Writing Booklet. **Marks**



The diagram shows the points $P(0, 2)$ and $Q(4, 0)$. The point M is the midpoint of PQ. The line MN is perpendicular to PQ and meets the x axis at G and the y axis at N.

(a) Show that the gradient of PQ is $-\dfrac{1}{2}$. **1**

(b) Find the coordinates of M. **2**

(c) Find the equation of the line MN. **2**

(d) Show that N has coordinates $(0, -3)$. **1**

(e) Find the distance NQ. **1**

(f) Find the equation of the circle with centre N and radius NQ. **2**

(g) Hence show that the circle in part (f) passes through the point P. **1**

(h) The point R lies in the first quadrant, and $PNQR$ is a rhombus. Find the coordinates of R. **2**

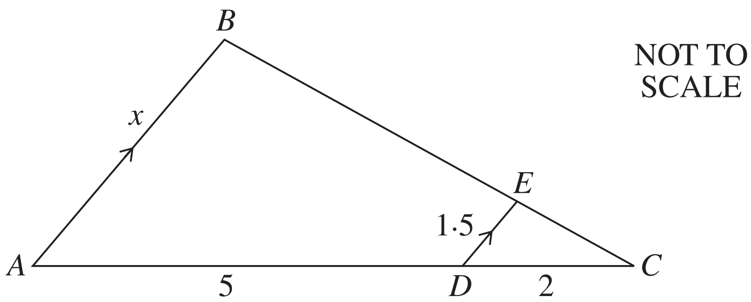
QUESTION 3 Use a SEPARATE Writing Booklet. **Marks**

(a) Differentiate the following: **4**

(i) $3xe^x$

(ii) $\sin(x^2 + 1)$

(b) **2**



In the diagram, AB is parallel to DE, AD is 5 cm, DC is 2 cm and DE is 1·5 cm.

Find the length of AB.

(c) Find: **3**

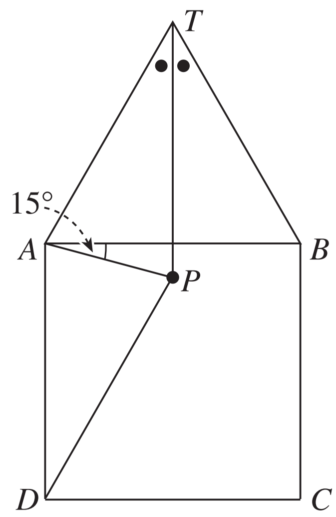
(i) $\int \sec^2 5x \, dx$

(ii) $\int_{-2}^{1} \frac{2}{x+3} \, dx$

(d) Find the equation of the tangent to the curve $y = 2\log_e x$ at (1, 0). **3**

QUESTION 4 Use a SEPARATE Writing Booklet. **Marks**

(a) **6**



In the diagram, $ABCD$ is a square and ABT is an equilateral triangle. The line TP bisects $\angle ATB$, and $\angle PAB = 15°$.

(i) Copy the diagram into your Writing Booklet and explain why $\angle PAT = 75°$.

(ii) Prove that $\Delta TAP \equiv \Delta DAP$.

(iii) Prove that triangle DAP is isosceles.

(b) In the construction of a 5 km expressway a truck delivers materials from a base. After depositing each load, the truck returns to the base to collect the next load. The first load is deposited 200 m from the base, the second 350 m from the base, the third 500 m from the base. Each subsequent load is deposited 150 m from the previous one. **6**

(i) How far is the fifteenth load deposited from the base?

(ii) How many loads are deposited along the total length of the 5 km expressway? (The last load is deposited at the end of the expressway.)

(iii) How many kilometres has the truck travelled in order to make all the deposits and then return to the base?

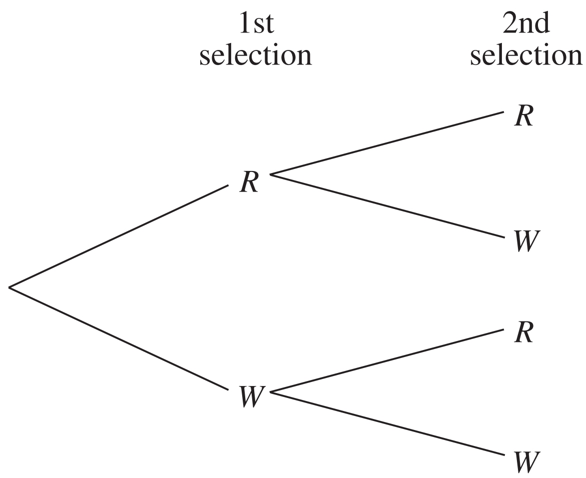
QUESTION 5 Use a SEPARATE Writing Booklet. **Marks**

(a) Solve $\tan x = 2$ for $0 < x < 2\pi$. **2**

Express your answer in radian measure correct to two decimal places.

(b) Four white (W) balls and two red (R) balls are placed in a bag. One ball is selected at random, removed and replaced by a ball of the other colour. The bag is then shaken and another ball is randomly selected. **5**

(i) Copy the tree diagram into your Writing Booklet. Complete the tree diagram, showing the probability on each branch.



(ii) Find the probability that both balls selected are white.

(iii) Find the probability that the second ball selected is white.

(c) The population of a certain insect is growing exponentially according to $N = 200e^{kt}$, where t is the time in weeks after the insects are first counted. **5**

At the end of three weeks the insect population has doubled.

(i) Calculate the value of the constant k.

(ii) How many insects will there be after 12 weeks?

(iii) At what rate is the population increasing after three weeks?

QUESTION 6 Use a SEPARATE Writing Booklet. **Marks**

(a) Sketch the curve $y = 1 - \sin 2x$ for $0 \le x \le \pi$. **3**

(b) The number N of students logged onto a website at any time over a five-hour period is approximated by the formula **9**

$$N = 175 + 18t^2 - t^4, \quad 0 \le t \le 5.$$

(i) What was the initial number of students logged onto the website?

(ii) How many students were logged onto the website at the end of the five hours?

(iii) What was the maximum number of students logged onto the website?

(iv) When were the students logging onto the website most rapidly?

(v) Sketch the curve $N = 175 + 18t^2 - t^4$ for $0 \le t \le 5$.

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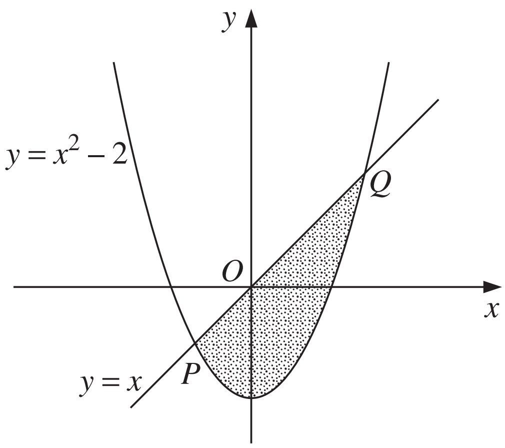
QUESTION 7 Use a SEPARATE Writing Booklet. **Marks**

(a) The area under the curve $y = \dfrac{1}{\sqrt{x}}$, for $1 \le x \le e^2$, is rotated about the x axis. **4**

Find the exact volume of the solid of revolution.

(b) Estimate $\displaystyle\int_0^1 \sin\left(1 + x^2\right) dx$ by using Simpson's rule with three function values. **3**

(c) The diagram shows the graphs of $y = x^2 - 2$ and $y = x$. **5**



(i) Find the x values of the points of intersection, P and Q.

(ii) Calculate the area of the shaded region.

QUESTION 8 Use a SEPARATE Writing Booklet. **Marks**

(a) A particle is moving in a straight line, starting from the origin. At time t seconds the particle has a displacement of x metres from the origin and a velocity v m s^{-1}. The displacement is given by $x = 2t - 3\log_e(t+1)$. **7**

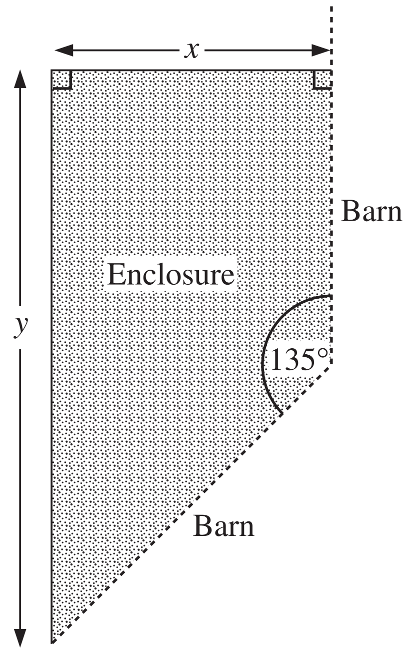
(i) Find an expression for v.

(ii) Find the initial velocity.

(iii) Find when the particle comes to rest.

(iv) Find the distance travelled by the particle in the first three seconds.

(b) An enclosure is to be built adjoining a barn, as in the diagram. The walls of the barn meet at 135°, and 117 metres of fencing is available for the enclosure, so that $x + y = 117$ where x and y are as shown in the diagram. **5**



(i) Show that the shaded area of the enclosure in square metres is given by

$$A = 117x - \frac{3}{2}x^2 .$$

(ii) Show that the largest area of the enclosure occurs when $y = 2x$.

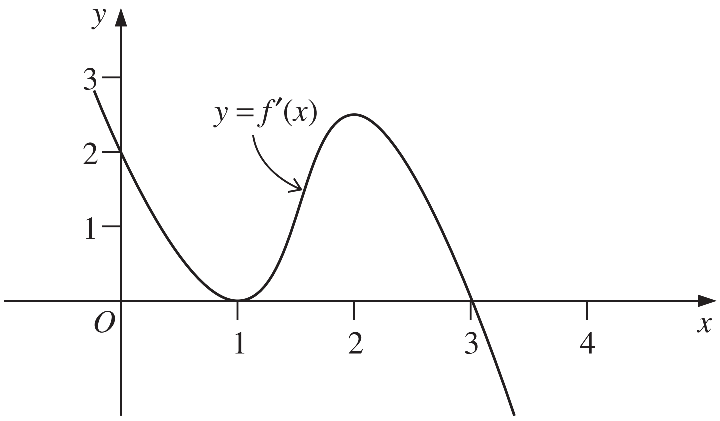
QUESTION 9 Use a SEPARATE Writing Booklet. **Marks**

(a) (i) Without using calculus, sketch $y = \log_e x$. **3**

(ii) On the same sketch, find, graphically, the number of solutions of the equation

$$\log_e x - x = -2.$$

(b) **2**

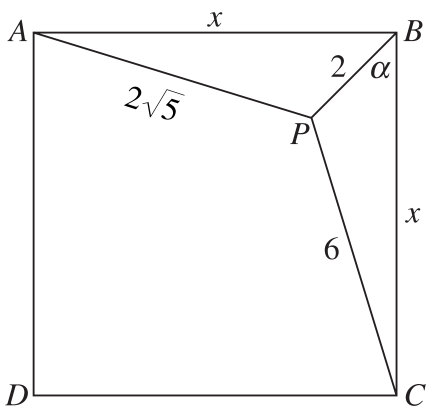


The above diagram shows a sketch of the gradient function of the curve $y = f(x)$.

In your Writing Booklet, draw a sketch of the function $y = f(x)$ given that $f(0) = 0$.

QUESTION 9 (Continued) **Marks**

(c) **7**



NOT TO SCALE

The diagram shows a square $ABCD$ of side x cm, with a point P within the square, such that $PC = 6$ cm, $PB = 2$ cm and $AP = 2\sqrt{5}$ cm.

Let $\angle PBC = \alpha$.

(i) Using the cosine rule in triangle PBC, show that $\cos\alpha = \dfrac{x^2 - 32}{4x}$.

(ii) By considering triangle PBA, show that $\sin\alpha = \dfrac{x^2 - 16}{4x}$.

(iii) Hence, or otherwise, show that the value of x is a solution of

$$x^4 - 56x^2 + 640 = 0.$$

(iv) Find x. Give reasons for your answer.

QUESTION 10 Use a SEPARATE Writing Booklet. **Marks**

(a) A store offers a loan of \$5000 on a computer for which it charges interest at the rate of 1% per month. As a special deal, the store does not charge interest for the first three months however, the first repayment is due at the end of the first month. **6**

A customer takes out the loan and agrees to repay the loan over three years by making 36 equal monthly repayments of $\$M$.

Let $\$A_n$ be the amount owing at the end of the nth repayment.

(i) Find an expression for A_3.

(ii) Show that $A_5 = (5000 - 3M)1{\cdot}01^2 - M(1 + 1{\cdot}01)$

(iii) Find an expression for A_{36}.

(iv) Find the value of M.

(b) The first snow of the season begins to fall during the night. The depth of the snow, h, increases at a constant rate through the night and the following day. At 6 am a snow plough begins to clear the road of snow. The speed, v km/h, of the snow plough is inversely proportional to the depth of snow. (This means $v = \dfrac{A}{h}$ where A is a constant.) **6**

Let x km be the distance the snow plough has cleared and let t be the time in hours from the beginning of the snowfall. Let $t = T$ correspond to 6 am.

(i) Explain carefully why, for $t \geq T$,

$$\frac{dx}{dt} = \frac{k}{t}, \text{ where } k \text{ is a constant.}$$

(ii) In the period from 6 am to 8 am the snow plough clears 1 km of road, but it takes a further 3·5 hours to clear the next kilometre.

At what time did it begin snowing?

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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