BOARD OF STUDIES
NEW SOUTH WALES

# 1997 HSC

# EXAMINATION REPORT

## Mathematics

Including:
- Marking criteria
- Sample responses
- Examiners' comments

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1997 HIGHER SCHOOL CERTIFICATE EXAMINATION

# MATHEMATICS

## ENHANCED EXAMINATION REPORT

## Contents

## General Comments

Each of the Mathematics examinations in 1997 provided good discrimination amongst the candidates, with a desirable spread of raw marks across almost the full range of available marks. Each paper contained questions that provided sufficient challenge for the ablest students. The average student in each candidature found that they were able to earn approximately half of the raw marks on their more difficult paper, in keeping with the standard of difficulty of the Mathematics examinations over many years.

The multiple-choice questions in Mathematics in Practice and Mathematics in Society were marked by machine, with correct responses earning 1 mark while incorrect responses, multiple responses and non-attempts scored 0 marks.

The free response questions in each paper were marked out of 12, with the exception of the 4 Unit (Additional) paper, where the questions were each marked out of 15. Only whole number marks are awarded, and, in all but the Mathematics in Practice and Mathematics in Society papers, the distribution of these marks between the major sections of each question was indicated on the paper as a guide to candidates. The marking process allocates each single mark to an ingredient which is required for the correct answer to the question asked, and candidates are awarded those marks if that ingredient can be identified in their answer. These marks, once earned, are not negated by subsequent errors. Thus, the marking process is one of awarding marks for knowledge displayed by the candidates, and is not a process of deducting marks for mistakes.

For the three related courses, a total score is computed by adding the raw marks for each question. In Mathematics in Society, the raw marks for the option questions are first scaled so that their distribution reflects the distribution of the candidates attempting that option on the compulsory sections of the paper, so as to avoid distortions which might arise if the option questions were not of equal difficulty. A total score is then computed. In Mathematics in Practice a total score is computed by applying a weighting of 1.2 to the total for the multiple choice questions and adding the marks from the free response section. This preserves the relative weightings of the two sections of the Mathematics in Practice paper. The Board scores for each candidate are then computed from these total scores in accordance with the published procedures.

The rest of this report consists of detailed comments on each of the free response questions for each paper in the 1997 examination. These comments are intended to provide insight into the way in which each question was marked, the difficulties encountered by students and common deficiencies in the preparation of candidates for the examination, and are written with a view to assisting teachers and candidates alike in preparing for future Higher School Certificate examinations.

Candidates should also be encouraged to bear in mind the following facts, which are essentially repeated from earlier examination reports.

1. At all times, answers should indicate in some way to examiners how they were derived. Hence, for example, candidates should not give single word or figure responses. If correct, such answers might receive full marks, but usually only in cases where the examiners are convinced that the correct answer is unlikely to be obtained from incorrect working. This is especially the case in the higher levels for questions which the paper indicates are worth several marks.

   Even in instances where the bare 'correct' answer would receive full marks, candidates should be aware that a bare incorrect answer will almost certainly receive no part marks, since the examiner will have no idea how the answer was arrived at. If the working is shown, trivial transcription errors and computational errors that can be clearly identified are often ignored in the marking process, and students who show their working may even be awarded full marks despite minor slips.

2. Graphs and diagrams should be clearly marked and reasonably executed to assist both the candidate and the examiner to know what they are doing. They should also be relatively large, for the same reason. When the paper instructs the candidate to copy the diagram into their writing booklet, the reproduction should be at least as large as the original. Candidates will not receive marks for such copying since it usually involves either tracing or a simple reproduction of the printed diagram.

   However, the instruction to copy the diagram is invariably an indication that the candidates will have to insert additional information, or make reference to the diagram in the course of completing the answer to the question. Those who neglect to copy the diagram as instructed place themselves at a severe disadvantage, as examiners (and especially candidates!) are very unlikely to be able to follow arguments involving non-existent diagrams.

3. Candidates should use pages in the booklet in which they are answering the given question for any rough work. Many candidates find the unruled left-hand pages useful for this purpose. All such work is read, and it is often the rough work of candidates that provides the evidence of the ingredients for the solution, which results in the candidate being awarded part marks.

   It must be emphasised that examiners do read everything written by every candidate, whether it is written on the backs of pages or booklets, whether it is crossed out, or even if it carries the explicit request 'Please do not mark this.' This is always to the candidate's advantage, as marks are never subtracted for errors, but are gained for appropriate work if it deserves it, no matter where it is displayed, unless, of course, the candidate's clear answer explicitly contradicts correct crossed out or rough work.

   For these reasons, candidates should also be discouraged from using 'white out' to obliterate their unwanted work, and advised to cross out in a way that leaves the underlying work legible.

Under no circumstances should candidates set aside a separate booklet for rough work for all questions. This practice makes it virtually impossible for examiners to locate the work on a specific question and relate it to the working shown in the writing booklet for that question. Candidates may needlessly miss out on marks that would otherwise have been awarded.

4. Candidates in the higher levels should be reminded not to forget the existence and usefulness of the tables of standard integrals printed on the back page of the examination booklet.

5. Candidates should write answers to different questions in different booklets. On the other hand, if they forget to do this under the stress of the examination, they should ensure that their working is clearly labelled with the question number as well as the part number so that it can be readily identified. A note on the cover of the writing booklets affected, such as 'also contains part of Q. 6' and 'part of Q. 6 is in Q. 5 booklet' will assist examiners in assembling the student's complete answer to the question. Candidates should be assured that all of their work will be read, and that they will not be penalised in any way for such slips.

6. Candidates should be discouraged from writing their answers in several columns on the page, as this can make it difficult for the examiner to check that everything has been read, and that all the appropriate marks have been awarded. Candidates who cram their work out of concern for the environment can be assured that all the paper in examination booklets will be recycled when the booklets are no longer required.

# Mathematics in Practice

## Question 31

A common occurrence was that monetary answers were inconsistently rounded; some to the nearest 5 cents (up or down), some to the nearest cent, some just had the tail lopped.

When a multi-stepped calculation was made, some students rounded at numerous points along the way, compounding their errors.

(a) (2 marks)
This question required the calculation of a sale price of an item after a 12% and then a 10% discount. Quite a high percentage of students were able to do this correctly.

(b) (i) (1 mark)
Students were asked to find the total cost of a small range of items. While this was generally well done, many went astray by misreading the question. For example, students answered '2 × \$2.90' in response to being asked the cost of '10 two-litre bottles of orange juice at \$2.90 per bottle'.

(ii) (1 mark)
Here students had to share the cost between 9 people. Most were able to either do this correctly or carry out the appropriate calculation on an incorrect total.

(c) (i) (1 mark)
A missing figure had to be provided for a table. The value was calculated by subtracting the sum of a collection of figures from a closing balance. Many students were unable to do this, while many others did so, but with calculation errors.

(ii) (1 mark)
Students were asked to indicate the due date for a credit card payment. The date was provided in the statement. Most answered correctly.

(iii) (2 marks)
In this part, very few scored both marks. It consisted of an interest calculation for 3 days. Most students made one or more errors.

(iv) (1 mark)
Candidates were asked to explain why a credit card transaction would have been rejected when an item costing \$1500 was to be purchased using a card with only \$1107 of available credit. Most students

earned the mark. However, a significant number did not indicate an understanding that the money was being spent on credit or that there was a credit limit.

(d) (3 marks)
These three parts worth 1 mark each involved the calculation of various costs and savings on family health insurance. Each calculation was quite involved, and frequent errors occurred because of misinterpretation of the question. Candidates need to read questions carefully.

**Question 32**

(a) Given a map, students had to give a direction and interpret scale with accurate measurement.

(i) (1 mark)
Well done by a majority of students who could recognise the direction 'south west'.

(ii) (1 mark)
Poorly done. Many students failed to round off to the nearest 100 m. There was also an obvious lack of understanding of scale.

(b) Students were required to interpret a timetable.

(i) (1 mark)
Well done. A common error was ignoring the pm.

(ii) (1 mark)
Well done. Common errors were 9.55 pm (misreading the table) and ignoring the pm.

(iii) (1 mark)
Not well done. The majority of students misread the table and wrote 3.35 am as their answer. A common error was ignoring the am.

(c) Students had to read a table correctly, and by calculation, compare accommodation costs.

(i) (2 marks)
This question was done reasonably well. One mark was awarded to students who gave the following answers:

$$\$247 + \$110 + \$56 = \$413$$
$$\$247 + \$110 + (\$56 \times 2) = \$469$$

Students who added 247 + 110 and neglected to consider the $56 per night additional rate were awarded 0 marks.

(ii) (2 marks)
Quite well done.

1 mark was awarded to students who calculated that \$387 was the cost of 'Suite Deluxe' and went no further to complete the question. A common error was to include combinations of \$56 with \$387. If this result was correctly subtracted from their answer in (i), they were awarded 1 mark.

(d) Students had to interpret a map representing an 8 day bus tour. They also had to convert Australian dollars into Irish pounds, given the conversion rate.

(i) (1 mark)
Very well done.

(ii) (1 mark)
Very well done.

(iii) (1 mark)
Well done. Common errors were $799 \div 0.488$ and neglecting to round to the nearest pound.

## Question 33

(a) This question involved interpreting a table, use of percentages and basic numerical calculations.

(i) (1 mark)
Reasonably well done. To gain the mark, students had to read the correct option and find

$$500 + \frac{1.5}{100} \times 180\,000 = \$3200\,.$$

Common reasons for not gaining this mark were the use of the wrong option, or an inability to find 1.5%. For example, students computed $1.5 \times 18\,000$ or $0.15 \times 180\,000$.

(ii) (2 marks)
Most students answered this well. If they read the correct section of the table, the common error then was to find 1% of \$180 000 instead of \$80 000. This was awarded 1 mark.

1 mark was also awarded if students found \$3 800, but did not complete the question by finding the 'extra' that was paid in fees.

(b) Given a floor plan of a flat, students had to find carpet laying costs and solve area questions.

(i) (1 mark)
Very poorly done. Frequent incorrect answers to this part included $\$125 \times 5 = \$625$ and $5 \times 6 \times \$125 = \$3750$.

(ii) (1 mark)
Reasonably well done. Common errors were 42 cm$^2$, 420 000 m$^2$, $7^2 \times 6^2$ and $7 + 6$.

(iii) (1 mark)
Poorly done when students gave their answer to (ii) in cm$^2$. They have a poor understanding of conversion of units.

Students who gave an answer to (ii) in m$^2$ usually went on to successfully answer this question.

(iv) (2 marks)
Reasonably well done.

1 mark was awarded if students made correct measurements but miscounted the number of windows.

A common incorrect answer was \$792, obtained by adding $132+165+220+275 = 792$.

(c) This question involved conversion of units, gaining correct information from a table and using it in numerical expressions.

(i) (1 mark)
This was not well done, mainly as a result of incorrectly getting information from the diagram provided. Students confused the bearers, joists and posts to come up with incorrect answers, or they misread the question and gave an answer of 19.4 m, which was the total amount of timber required.

(ii) (1 mark)
Students needed to convert their part (i) answer to metres to correctly answer this question. This is a skill the majority of candidates found very difficult, hence the question was poorly done.

The mark was also awarded if students rounded 10.4 m up to 11 m and found $11 \times \$8.50 = \$93.50$.

(iii) (2 marks)
Poorly done. The information given for the joists in the diagram below the pergola was frequently misused. For example:

$$5 \times 1800 \div 9000$$
$$5 \times 1800 \div 90$$
$$5 \times 1800 \div 45$$

1 mark was awarded if candidates correctly calculated that the joists cost \$54 and then neglected to add part (ii).

1 mark was also awarded for an otherwise correct attempt with one error, such as using 4 joists.

**Question 34**

(a) (2 marks)
This question required students to complete a pattern inside a rectangle. To achieve the accuracy needed to gain 2 marks, students needed to use a ruler. However, the majority of candidates attempted to complete the pattern freehand. If they did so correctly, they received 1 mark.

Students who showed evidence of congruent rectangles, or equally spaced parallel lines in both directions, also received 1 mark.

(b) (1 mark)
Quite well done. Students had to recognise a regular geometrical shape in a tiling pattern. Most students did recognise the hexagon although spelling was poor. (This was not penalised.)

(c) This question was poorly done. A large number of candidates did not attempt it.

(i) (2 marks)
The majority of students did not understand this question and had no idea where to begin. There were four calculations involved in finding the diameter of a bottle required to fit exactly into a partitioned cardboard box. The fact that both mm and cm were used in the question, and that units needed to be converted, caused endless problems.

Very few candidates answered this question correctly.

The correct answer of 16 cm was obtained from the numerical expression $(66 - 2) \div 4 = 16$.

1 mark was given if students did not convert the mm to cm and found $(66 - 20) \div 4 = 11.5$ cm

So few students made a correct attempt at the question that 1 mark was even awarded for candidates who found $66 \div 4 = 16.5$ cm.

(ii) (1 mark)
Students had to find the surface area of the 'outside' of a cardboard lid for a box. It was very poorly done. Many of those who did recognise the concept of surface area incorrectly found the surface area of all 6 faces. The concept of 'outside' was not understood.

(iii) (2 marks)
Many students did not realise this question related to part (ii). It was poorly done. Their answer to 1. was incorrectly found by measuring to get 2.5 mm. Part 2. was also attempted poorly. Even those students who correctly answered 5 cm to 1. then found $72 \times 72$ instead of $77 \times 77$ for the area.

(d) (2 marks)
Poorly attempted. Most students had no concept of adding or subtracting areas. Confusion arose when students used 20 cm for 0.2 m. The majority of candidates ignored the fact that there was a double addition of 0.04 $m^2$ and had an answer of 0.68 $m^2$ which was awarded 1 mark.

(e) (2 marks)
Students had to enlarge a design which involved the use of their pair of compasses. The question was poorly done.

The 'expected' confusion between diameter and radius caused a prevalent loss of 1 mark. A large number of candidates did not use compasses and a freehand sketch was awarded no marks. The accuracy of construction was poor and the method of marking equal radii around the circumference was not known to the majority of candidates.

**Question 35**

(a) (1 mark)
This question required the reading of a temperature from a scale. It was reasonably well answered, but a substantial percentage of students were unable to accurately complete the task.

(b) (i) (1 mark)
The task involved calculating the probability of a certain number winning in the spinning of a wheel. The fact that zero was one of the numbers caught out many students. However, a real problem was caused by students expressing their answers as ratios. For some students 1 : 19 meant '1 to 19', while others intended '1 in 19'. This became evident when examining subsequent answers in many cases. Answers were accepted in various forms, but ratios should either be avoided or qualified.

(ii) (1 mark)
When asked to calculate a probability where a range of favourable events could occur, many students were unable to make the calculation.

(iii) (1 mark)
In this question, students were asked to calculate the expected number of times a certain outcome might occur in 100 trials. This proved to be very difficult, with many students expressing their answer as a fraction or percentage.

(c) (1 mark)
Students were required to calculate the correct percentage discount relating to a sale, and thereby explain the fault with the advertising. This was successfully completed by most students, although a substantial number erred by not carefully reading the question.

(d) (i) (2 marks)
This question consisted of asking the students to complete a graph by adding in two further columns using data supplied in a table. A notable difficulty arose with students drawing up diagrams without the use of equipment. Inaccuracy caused the loss of marks in most cases.

(ii) (2 marks)
Having been asked to describe trends shown in the graph, students were awarded a mark for each meaningful trend supplied (up to 2). Most students scored either 1 or 2 marks. Some lost marks for making assumptions from the data. The range in the level of eloquence was extensive.

(e) (i) (1 mark)
From a diagram, students were to calculate the fraction of the area of a rectangle taken up by a smaller rectangle. Some students used estimation instead of measuring lengths. Some provided an unqualified ratio (eg 1 : 8, when the correct answer was 1/8).

(ii) (2 marks)
Candidates were asked compute the cost of an advertisement. In order to do this, the area of the advertisement had to be found, and this step required the use of a scale. This step was rarely correct. However, students commonly obtained one mark for multiplying the result of an area calculation by the cost per square centimetre.

# Mathematics in Society

## Question 21

(a) (1 mark)
This question was related to harder concepts in number. Many students answered the question well. Common errors included $7.5 \times 1.2 = 90$, $75 - 75 \times 0.2 = 60$, $75 \times 120 = 9000$ and giving a bald answer of 63 without showing working. These errors all scored zero marks.

(b) This question was related to harder concepts in measurement.

(i) (1 mark)
This question was reasonably well answered with errors restricted mainly to incorrect substitutions. It was common to see $V = \frac{4}{3}\pi r^2$, indicating a lack of familiarity with the formula. Another common error that scored zero was to substitute the diameter rather than the radius.

(ii) (2 marks)
Approximately 50% of students were able to answer this question correctly. Many students tried to use their answer from part (i) in the calculation or added 25 000 to their previous answer. There were many mistakes made in rearranging the equation formed after substituting the volume. A common error in this regard was to take the cube root first and then divide by $\frac{4}{3}\pi$. The division by $\frac{4}{3}\pi$ was not well done. Many variations on $\pi$ were used by students. Given that $\pi$ can be obtained from the calculator, students should use the value it supplies.

(iii) (1 mark)
Generally well done. A common error involved division by 6, presumably because a cube has 6 faces.

(c) This question related to probability.

(i) (1 mark)
This was generally well done, although common errors involved the use of $\frac{2}{3}$ or $\frac{3}{2}$. Students who used of the latter showed that they did not know that $P \leq 1$. Other students ignored the bias and used $\frac{1}{2}$.

(ii) (2 marks)
Many students were unable to see the connection between parts (i) and (ii), and reverted to the concept of a fair coin. Other students ignored the words 'at least'. Students who used a tree diagram were generally able to score at least one mark. Very few students used $1 - (3/5)^2$ to find the answer.

(d) This question was related to probability.

(i) (1 mark)
Many students counted the number of times the event could occur rather than giving the probability. This was not enough to score the mark. However, this question was answered very well by the majority of students.

(ii) 1. (1 mark)
Generally a well-answered question with 11 being the most common incorrect answer.

2. (2 marks)
This question was poorly done with the majority of students receiving zero marks. Most failed to consider $P(2\&5)$ as well as $P(5\&2)$, and $P(4\&3)$ as well as $P(3\&4)$. Students who did not consider order, but who showed working were able to score 1 mark. Generally, students who did not show working were disadvantaged when the answer given was incorrect. This caused significant differences for some students. Students should always be encouraged to show working at all times.

**Question 22**

This question consisted of three parts taken from the areas of harder work in numeration, measurement and algebra (graph reading and interpretation) and from extensions in trigonometry (sine and cosine rules).

(a) (i) (1 mark)
Candidates were asked to locate the position of a town given a distance and bearing from a fixed point. Many students were able to locate the 'approximate' position of Lati, but a significant number of students (15–20%) did not understand bearings.

Common errors were measuring the bearings from $S$, $E$ and $W$; measuring the bearings in an anticlockwise direction; not labelling or incorrectly labelling diagrams and measuring the bearings from Jadphur or from the middle of the line joining Kimbala and Jadphur.

(ii) (2 marks)
This question required students to find a distance using the cosine rule. Many students answered this question well, with about 70% scoring at least 1 mark.

Most students were able to evaluate an expression from the substitution they had made. Common errors involved order of operations and failing to take the square root. A large number of students had difficulty in substituting the right numbers and common errors came from using 37.5 instead of 75 or from using 125°, 45°, 135°, 60° or 120° instead of 55°.

(b) (i) (1 mark)
This question involved obtaining information from a graph and approximately 60–70% of the students were able to give the correct response of 4h 36min (or 4 : 36 or 4.36).

Incorrect responses included 4h 3min, 4h 6min, 4h 45min, etc, which shows that students have difficulty converting 'decimal time' to hours and minutes.

(ii) (1 mark)
The majority of students (99% or more) were unable to do this question, with many of the candidates not understanding the meaning of rate and not being able to relate it to the slope of a tangent. Common errors were the responses of 440 or $\frac{440}{3}$. A large number of students also substituted $t = 3$ into the formula.

(c) Students often abandoned this part of the question when they found they could not do a section.

Some students could not store all the information that was required when answering a new part. In many cases they would calculate results they had obtained in a previous part. Students who drew the whole diagram or the relevant triangles performed better.

(i) (1 mark)
This part of the question was well answered by most students. However, a significant number of students thought there was more in the question and became lost in complicated trigonometric calculations. Students who obtained answers $\leq$ 200 metres failed to see the significance of their result.

(ii) (2 marks)
This question required students to show $TX = 49$ metres. Many students used the 49 metres to show that $TX = 49$ metres and were not awarded the mark. The second part of the question required students to find the height of the Seng Office Block. Most did this successfully.

(iii) (1 mark)
The majority of students who used a diagram in their explanation answered the question correctly and were awarded a mark. However, explanations were generally hard to comprehend. Some students even used the cosine rule to show $\angle TSV = 153°$. The term 'alternate' was used by a number of students, but some used the term 'corresponding' instead.

A number of students used the angle sum of the quadrilateral $TUVS$ to show the required result.

(iv) (2 marks)
Carelessness in using the appropriate lengths and angles for substitution in the sine rule cost many students one or more marks in this part. Students who substituted into the given form of the sine rule failed to take the reciprocal of both sides. They then had difficulty in calculating the answer.

(v) (1 mark)
This part of the question was not well answered, with less than 50% of the candidature obtaining the mark. Many students failed to realise that the answer to part (iv) could be used to obtain the correct result. Many students did not understand the meaning of the term 'angle of depression' and just repeated their part (iv) result or gave the answer $63°$. Answers greater than $90°$ were also common. A number of students used the tangent ratio to find the angle of depression.

**Question 23**

(a) (2 marks)
This equation caused great difficulties for the vast majority of students. The inability to cope with the pronumerals in the denominator was the main area of concern. Even more simple algebraic manipulations were often done poorly, eg $4x = 7$ becoming $x = \frac{4}{7}$.

(b) (1 mark)
Although often answered correctly, many responses simply spoke about consistent scores rather than how the fact that Andrew's scores were more consistent could be deduced from the statistics.

(c) (i) (1 mark)
There was a clear distinction between those who had learned the meaning of 'almost certainly' and those who had not. Arithmetic mistakes were also cause for concern for a significant number of students.

(ii) (1 mark)
Most students realised there was a connection between the previous response and this one, and gained the mark, albeit only as a result of being a correct deduction from the previous answer. Many responses displayed an inability to present the required answer or explanation in clear mathematical terms.

This comment is equally applicable to the four parts of this question that required a written response.

(d) (i) (1 mark)
Reasons for the different sizes were generally well understood but often not explained concisely.

(ii) (1 mark)
Most students realised that there was deception involved, but found it hard to put into words in many cases. The use of words such as 'much more', 'a significant increase', 'better by far', and even 'heaps more popular' was required.

(e) (i) (1 mark)
Most students answered this correctly.

(ii) 1. (2 marks)
Many bald correct and incorrect answers given. For a 2 mark question this is a risky approach as the possibility of gaining part marks is highly unlikely. There was evidence of competency in the use of tree diagrams in many cases, but this was frequently undermined by misunderstanding about 'replacement' and multiplication rather than addition for successive events.

2. (2 marks)
The comments for 1. above apply equally here. The marking scheme took account of the two possible ways of interpreting this question, either using the original set of counters or those in the event space for 1.

**Question 24**

This question contained three parts taken from five areas of the syllabus, namely the use of properties of the ellipse, Kepler's laws, units used in astronomy, distance, velocity and time and the distance of planets from Earth. The marks obtained were evenly distributed, with as many students scoring zero as twelve. On the whole, the question was reasonably well answered. The main weaknesses were failing to read the question carefully and thus not using information given; poor calculator work; poor working out and frequent transcription errors.

(a) (i) (1 mark)
The majority of students were awarded the mark. A few gave $b/a$ as a negative number, thus creating problems in working through the next two parts. Some used the diagram measurement, even though the diagram was labelled 'not to scale'.

(ii) (1 mark)
Again, this was well done. Some students created unnecessary work

by using $e^2 = 1 - \frac{b^2}{a^2}$. Others generated more work by changing $DC$ from A.U. to km.

(iii) (1 mark)
Generally well done with the exception of a few who truncated the decimals before arriving at the answer.

(iv) (1 mark)
Many students stopped after working out $CS$ instead of proceeding on to calculate $AS$. Once again, some students created more working by expressing $AS$ in km.

(v) (1 mark)
Many students obtained the correct answer. Incorrect answers were due to incorrect substitution for $R$.

(b) (2 marks)
A significant number of students scored two marks. Some failed to halve the time taken, thus losing a mark. A few students also had problems with scientific notation. The conversion from km to m was sometimes confused.

(c) (i) (1 mark)
This part was well done. Many believed light years were expressed in units of time or in units of speed. There were those who managed to remember that a light year is $9.46 \times 10^{12}$ but expressed it as speed.

(ii) (2 marks)
This part of the question was poorly done. Those who recognised 1 light year $= 9.46 \times 10^{12}$ km found it a very easy question. Unfortunately, many were unable to convert seconds to years. Most scored zero for this part.

(iii) (2 marks)
This part of the question was answered reasonably well. Many students were able to write the time in hours but failed to convert it to years. A number interchanged $8.19 \times 10^{14}$ with $8.19 \times 10^{4}$ and $5.6 \times 10^{4}$ with $5.6 \times 10^{14}$.

**Question 25**

This question contained four parts taken from three areas of the syllabus, namely the language of chance, counting techniques and mathematical expectation.

(a) (i) (1 mark)
This was well answered. There were two legitimate interpretations of the question, depending on whether one assumed the mathematics books were identical or different. This created two different answers, 5! or $5!/4! = 5$, which were both accepted.

(ii) (1 mark)
This was also well answered. Once again, there were two legitimate interpretations yielding either $2 \times 4!$ or 2.

(b) (i) (1 mark)
This part of the question was poorly done with many students scoring zero. Most students wrote down 2/12 while a small percentage treated the problem as ordered arrangements. Only very few students were able to write 1/66 as the answer.

(ii) (2 marks)
Very few students understood the word 'on'. They treated it as 'odds against' rather than 'odds on', thus giving the answer as \$480. Only better candidates answered \$160.

(c) (i) (1 mark)
This was the best answered part of the question.

(ii) (1 mark)
This was the worst answered part of the question. The most common answer was 4/6. Students who understood the question still had trouble with the concept of 'adjoining rooms'.

(d) Overall, a handful of students scored well in this part. Most students had little idea of the concept of expectation.

(i) (1 mark)
The common answer was 3/30. Of those who wrote down $1/10 \times 1/10 \times 1/10$, very few simplified it to 1/1000. Their poor concept of operations on fractions was reflected in such answers as 3/1000.

(ii) (1 mark)
The popular answer was 3 ways. Some wrote 9 ways. Those who got 27, the correct answer, usually scored well in the other parts of (d).

(iii) (1 mark)
Once again, this was poorly done. Very few saw the connection between (ii) and (iii), thus providing all sorts of answers.

(iv) (2 marks)
Marking this part was made very difficult by the numbers that were used. It had two subparts, and the answer to one of the subparts was also the number of that subpart. This made it very difficult to work out if the candidate was writing the question number or answering the question. Most students scored zero. Those students who were correct usually scored more than 9 for the complete question.

## Question 26

The question was not difficult but still achieved a spread of marks from 0 to 12. Most students were able to gain marks throughout the parts by substitution of values into formulae. The fact that the sine rule and Simpson's rule have had a great deal of emphasis in teaching was demonstrated in the overall quality of the answers. Time, small circles, and bearings are still the weaker parts of this option and this was very apparent, considering that the questions asked in part (d) were very straightforward.

(a) (i) (1 mark)
Generally well done, though some students found problems in using the scale correctly. Common errors were measurements of 6 m and 7 m or 600 m and 700 m.

(ii) (1 mark)
For some the lack of a protractor was a problem. A common error was 120°.

(iii) (1 mark)
Well done, as students were able to substitute values and do the calculations. There may have been some confusion in the question as to whether the area to be calculated was on the scale drawing or the actual field. A possible wording of this question to avoid this confusion could have been 'using the answers in parts (i) and (ii), calculate the area in square metres'.

(b) (2 marks)
The Simpson's rule question was done quite well, with most students able to substitute some values and obtain answers. Common errors involved $h = 400$, 600, 800 or 470; misunderstanding the meaning of the symbols for the left, middle and right lengths, leading to incorrect substitutions; lack of understanding of the order in which the operations should be done on the calculator to get answers for each area; applying the rule to incorrect sections (eg using 410, 300 and 380) and using only one application involving 500, 300 and 470.

Transcription errors were a major concern for the markers in this question as the correct areas were 162 666.666 66 and 152 666.666 6. These were then entered as either 16 266.666 or 15 266.666, leading to an incorrect total.

(c) (i) (1 mark)
Getting the size of the angle was a significant problem showing a lack of understanding of bearings. A common error was 18° although other angle sizes were also obtained, eg 52°, 162°, 342°, 38°, 142°, 322°, 128° and 110°.

(ii) (2 marks)
The sine rule was done very well by most students as they substituted and manipulated correctly, despite any error they may have made in (i).

(d) (i) (1 mark)
This question caused difficulties for students in converting the 7.6 hours to hours and minutes initially and then in deciding whether to add or subtract these from midnight. The question was, in fact, easier than in previous years because midnight was used as the original time. Their difficulties highlight students' confusion in this topic.

(ii) (1 mark)
Students had difficulty with this question as they did not know the formula for the radius of a small circle and it was not given in the question. Another common error was to use the arc length of a small circle with 6400 as the radius.

(iii) (2 marks)
A significant number of students had difficulty with this question. Common errors included using some number other than 5800 or their answer in (ii) for the radius of the small circle and using incorrect angles for the fraction of the circumference (eg $152°$, $25°$, $114°$ etc). Others used 6400 as the radius of the small circle, made use of the great circle or did calculations based on $1° = 60$ nautical miles. It was also noted that the formula for the circumference was not known well enough, with parts of it often being omitted.

**Question 27**

This question was well done by most students. Some problems in calculating percentages were evident, and part (d), involving the graph, was poorly attempted. Students who showed working were often able to be awarded marks despite making transcription errors.

(a) (2 marks)
A salary of \$38 000 was to be increased by 8% and a further 7%. This was extremely well done, with the majority of students gaining full marks. The most common error was to increase by 15%.

A significant number of students found the answer by first finding 8% and adding, then finding 7% and adding. This resulted in a greater frequency of transcription and algorithmic errors. Students who found 108%, then 107%, were more successful.

Some students were unable to find a percentage correctly, with their working showing multiplication by 0.8, 1.8 etc or division by 8. A large number of students failed to express the cents correctly, giving their answer to one decimal place only.

(b) (i) (1 mark)
The purchase price and weekly rental for a computer were given, with the number of weeks before the rent equalled the purchase price to be calculated. This was extremely well done.

(ii) (1 mark)
The purchase price of a second computer was given, with the weekly rental to be calculated so as to be in the same ratio as in part (i). This was also extremely well done. Nearly all students correctly interpreted this as requiring a division by the number of weeks calculated in the previous part. Students who set up a ratio were generally not as successful.

(c) (i) (1 mark)
The interest only on a loan of \$100 000 at 9.5% pa fixed for 3 years was to be paid. Students were to calculate the total interest for the 3 years. This was very well done, with the majority of students answering correctly. A common incorrect response used compound interest, while other students subtracted the interest from the amount each year for 3 years.

(ii) (2 marks)
The amount saved by dropping the interest rate to 7.8% for the third year in exchange for paying a charge of \$500 was to be calculated. This was generally well done. Finding the total charge for 3 years was as common an approach as simply finding the charge for the third year. Errors occurred in accounting for the charge of \$500. Some of these were addition to, rather than subtraction from, the difference between the two interest charges, or addition of the \$500 to the principal before performing the percentage calculation.

A large number of transcription errors occurred. For instance, students wrote 7.5% instead of 7.8%. Where working was shown, students were not penalised.

(d) A taxation table was given.

(i) (1 mark)
The tax payable on \$60 000 was to be calculated. This was well done by the majority of students.

(ii) (1 mark)
A graph of taxable income against tax payable was to be completed. This was very poorly done, with few students responding correctly. Points corresponding to changes in tax brackets were not plotted, with

$38 000 generally being omitted. Many joined the graph to the top right hand corner of the graph paper rather than to the tax payable on $60 000 calculated in (i). Many students calculated points at intervals of $5000 or $10 000. A large number of students merely extended the line drawn on the graph paper.

The scale on the axes were not interpreted correctly. The most common error was to use $200 or $2000 as the value for each unit on the horizontal axis. Points were not plotted accurately and many students did not join the points with a ruler, resulting in the students not receiving the mark for this part.

Many students drew the graph in pen. This made it difficult to determine the final answer when students corrected their first attempt.

(iii) (1 mark)
The taxable income corresponding to $11 400 tax payable was to be read from the graph. This was not well done. Errors in reading the scale on the axes were common. Some students calculated the income from the table rather than reading from the graph. Other students read the tax payable on an income of $11 400 from the graph.

(iv) (1 mark)
The percentage of taxable income paid as tax was to be calculated. This was generally well done, although students often recorded their calculation incorrectly. For instance, students inverted the fraction, but still found the correct answer. There were also frequent errors in rounding off.

(v) (1 mark)
The tax bracket applying to taxpayers who pay 20% of their total income in tax was to be found. This was fairly well done, although not many calculations were shown. Many selected the 20c tax bracket, ignoring the tax free threshold.

**Question 28**

This question required candidates to read and interpret a house plan, to apply their knowledge of three-dimensional geometry and to use trigonometry to solve problems. The question was generally well done, but many candidates lost marks simply because they did not show sufficient working to provide the examiner with sufficient evidence to award the mark.

(a) (i) (1 mark)
This part required candidates to measure part of the floor plan and to use the measurement to determine the scale that was used. Many candidates did it well. However, some candidates stated that there

was no scale as they could not find it written on the examination pages and many gave the scale in two ways that contradicted each other.

(ii) (1 mark)
This part required candidates to determine the actual width of the house. Most candidates did this well. However, a few had little understanding of the need to apply their scale or of the units required.

(iii) (2 marks)
This part required candidates to calculate the total area of the house. Many candidates did this part well, but almost a third gave no indication of the calculations and measurements they used to produce their answer. These students lost marks as a result.

(iv) (2 mark)
This part required candidates to determine the cost of carpeting Bedroom 1. Only about 50% of the students could do the calculations correctly. Many made basic arithmetic errors and many were unable to correctly convert from one metric unit to another.

(v) (1 mark)
This part required candidates to explain a problem with the design of the linen cupboard. This was very well done, with most candidates explaining the problem sufficiently well to gain the mark.

(b) (i) (1 mark)
This part required candidates to name a particular roof shape. This was poorly done, with many candidates giving two different roof names as their answer.

(ii) (1 mark)
This part required candidates to calculate the height of the ridge line above the ceiling height. It was well done. However, many candidates used the wrong trigonometric ratio or found the wrong side of the triangle.

(iii) (2 marks)
This part required candidates to calculate a length which represented the width of the roof. It was poorly done. Many were unable to start the question or correctly apply Pythagoras' theorem. However, some candidates showed a very good understanding of the calculations required while using a more complex trigonometrical method.

# Mathematics 2/3 Unit (Common)

## Question 1

There were seven parts in this question, dealing with arithmetic (calculator skills and surds), algebra (simplification of an algebraic expression), converting from degrees to radians, calculation of arc length, use of standard integrals and probability.

Most candidates handled the question well — the majority scoring over 8. It was disconcerting to note that a significant (although small) number of candidates scored fewer than 4 marks.

(a) (2 marks)
This part required candidates to calculate the value of a fraction using order of operations, then convert their answer to a decimal correct to three significant figures. Most candidates displayed correct calculator usage in obtaining 0.04545... However, too many students made errors in the order of operations, resulting in their obtaining $\frac{1}{36}$, $\frac{1}{105}$ or $15\frac{3}{7}$ as the answer to $\frac{1}{7+5\times3}$ instead of $\frac{1}{22}$.

A large number of candidates obtained 0.045 as the answer correct to 3 significant figures — many students confused 3 significant figures with 3 decimal places. Candidates who used scientific notation, ie $4.55 \times 10^{-2}$, did not have this problem.

Candidates who scored 11 out of 12 on the question usually lost their one mark in this part, giving their answer to 3 decimal places rather than 3 significant figures.

(b) (2 marks)
This part asked candidates to remove parentheses and collect like terms. While this part was generally well done, a surprising number of mechanical errors occurred. These included failure to handle the double negative or an inability to calculate $2-5$ or $-3x+4x$ correctly. A number of candidates simply ignored the minus sign and proceeded to multiply either $(2-3x)(5-4x)$ or $(2-3x)(-5+4x)$. A smaller number of students attempted to form and then solve an equation, even after obtaining the correct answer.

(c) (1 mark)
This part required the candidates to convert degrees to the exact equivalent in radians. The majority of candidates answered this question correctly, but a significant number then proceeded to provide a decimal approximation. The most common incorrect response was to simply write 2.356... radians — a large number of students had no concept of the meaning of 'exact'.

(d) (1 mark)
This part required the candidates to calculate an arc length. Most were able to quote the correct formula and apply it correctly. Unfortunately, a number of students then changed $\frac{5\pi}{6}$ to $150°$ and then proceeded to multiply (using $l = r\theta$) to obtain 450 cm as the answer, showing a real lack of understanding of the concept of arc length. A number of students correctly used the formula $\frac{\theta}{360} \times 2\pi r$ to obtain the answer.

(e) (1 mark)
This part required the candidates to use the standard integrals supplied on page 12. Most candidates answered this question correctly. A small percentage of candidates left one '$a$' unsubstituted. There were attempts to find the answer using integration by substitution and integration by parts — some students had obviously never used the table of standard integrals, or had no idea where to find them. A large number of the candidates, approximately 50%, failed to add the constant of integration. On this occasion, no marks were lost through its omission.

(f) (2 marks)
Candidates were asked to find the probability that a number selected from the numbers 1 to 45 was even. This question was answered correctly by over 90% of candidates. A common error was to correctly determine that there were 22 even numbers, then interpret this to mean that $P(\text{even}) = \frac{1}{22}$.

Other mistakes occurred because of some candidates' inability to determine the correct number of even numbers between 1 and 45, with students' claims ranging from 1 to 46. A common response was to simply state that the probability was $\frac{1}{2}$. Presumably, such students do not actually believe that there are $22\frac{1}{2}$ even numbers between 1 and 45.

(g) (3 marks)
This part required candidates to rationalise the denominator and to express their answer in the form $a + b\sqrt{5}$. This part was well done by the majority. The most common errors were in carrying out the product of the conjugate surds in the denominator (eg obtaining $3 - 5$, $9 - 25$, $9 + 5$ or $6 - 5$) and not correctly carrying out the final step in the simplification of the surdic fraction — for instance, claiming that $\frac{24+8\sqrt{5}}{4} = 6 + 8\sqrt{5}$. Those students who had been taught to leave the numerator in factored form as $\frac{8(3+\sqrt{5})}{4}$ did best at this stage.

A small number of students did not attempt to rationalise the denominator or chose to multiply by a variation of $\frac{3\pm\sqrt{5}}{3\pm\sqrt{5}}$ that failed to simplify easily.

## Question 2

The four parts of this question respectively involved finding the coordinates of significant points on the graph of a cubic function, establishing a complete primitive, writing a primitive involving the logarithmic function, and determining the coordinates of the vertex and focus of a parabola from its equation.

Many candidates obtained full marks, many lost one or two marks, a large number missed out only on the part (d) marks, and most scored at least one mark. Indeed, it appeared that the work examined in (d) either had not been treated in some centres, or had not been extended beyond the standard parabola $x^2 = 4Ay$.

(a) Many of the candidates who obtained full marks on this part wasted valuable time proving $M$ and $N$ were as stated. A significant proportion of the other candidates worked through a well-rehearsed procedure that first involved setting $y' = 0$, and had little understanding of how to relate any answers obtained to the questions asked. Many of these candidates arrived at $L(4, 0)$, or $L(2, 0)$ and $M(2, 16)$, or $M(2, 12)$, and for (iii) either left it blank or repeated their (ii) working. Some candidates plotted points and answered all subparts of this question via such calculations.

(i) (2 marks)
This subpart required candidates to find the coordinates of the point where the graph of a cubic function crossed the horizontal axis. Only a small number of candidates failed to gain the mark allocated to setting $y = 0$. A significant percentage of the candidates who arrived at the resultant equation $6x^2 - x^3 = 0$ could neither factorise the cubic nor solve the equation to obtain $x = 6$ (and $x = 0$), and hence $L(6, 0)$. Some, though, obtained the answers via the formulae for the roots of a quadratic equation. A similar percentage gave $(0, 6)$ or $L = 6$ as their answer. Students who merely wrote these bare answers were awarded only one mark.

(ii) (2 marks)
Those candidates who correctly differentiated their $y$, and equated $y'$ to zero, obtained the first mark. As in (i), several of these candidates could not solve $12x - 3x^2 = 0$, to obtain $x = 4$ and hence $M(4, 32)$ to be awarded the second mark.

Many candidates confused the requirement for finding where the function has a stationary point ($y' = 0$) with that for a possible point of inflection ($y'' = 0$); others thought all that was required for a (maximum) turning point was $y'' < 0$ (or $y' < 0$), leading to an inequality that miraculously produced one single value for $x$. Candidates who plotted points had no way of justifying $(4, 32)$ as the maximum turning point.

(iii) (2 marks)
Those candidates who correctly differentiated their $y'$, and equated $y''$ to zero, were awarded the first mark. Most candidates who arrived at $12 - 6x = 0$ obtained $x = 2$, but a small percentage then substituted into their expression for $y'$ (rather than $y$) and were not awarded the second mark.

The property of a cubic function that both coordinates of its point of inflection lie midway between the corresponding coordinates of its turning points was used by a small percentage of the candidature to obtain $N(2, 16)$, often without stating a reason. Frequently, these candidates obtained the abscissa of $N$ from an incorrect one for $M$, found the ordinate either similarly, or from $y$, but then failed to check that it satisfied both constraints. However, these answers were still awarded 2 marks. Those candidates who repeated their $y' = 0$ approach in this part received no marks.

(b) (2 marks)
This question paraphrases Question 5(a) of the 1996 paper. Those who marked both these questions are pleased to report a significant improvement in responses this year, though far too many candidates still could not correctly use both parts of the given ordered pair $(1, 4)$ to find the constant of integration in $f(x) = x^2 + 7x + c$. Common errors were to find a function that passed through $(1, 0)$ or $(4, 1)$ or to set $c = \int_1^4 2x + 7\,dx$. Similarly, far too many candidates still thought the question required them to find the line through $(1, 4)$ with gradient $2 \times 1 + 7$, or even $2x + 7$. These answers attracted no marks.

(c) (1 mark)
The only answer awarded the mark was $\ln(x^2 + 1)(+c)$, with log and $\log_e$ accepted as synonyms for ln, and $|x^2 + 1|$ a most welcome replacement for $x^2 + 1$. The lack of parentheses was not penalised, but those very few candidates who wrote $\ln(x^2+1+c)$ were not awarded the mark. Candidates from centres using the 'teaching' technique $\frac{2x\ln(x^2+1)}{2x}$ did not receive the mark unless they proceeded to $\ln(x^2 + 1)$.

The majority of candidates who recognised that $\int \frac{2x}{x^2+1}\,dx$ is of the form $\int \frac{f'(x)}{f(x)}\,dx$ proceeded to the correct answer. Far too many candidates separately integrated the numerator and denominator, or 'managed' to first simplify the rational function; some worked the inverse tangent function into their answer, and a few integrated $\ln(x^2+1)$ to $\ln(\frac{x^3}{3}+x)$. Of more concern was the large number of candidates who equate primitive with derivative and treated the question as the derivative of a quotient.

(d) This was the least well-done part. Although many candidates wrote down bare, or almost bare, correct answers to (i) and (ii) from the given equation, most had to resort to either comparing the given equation explicitly with $(x - h)^2 = 4A(y - k)$, using calculus to find the turning point, or working from their formula for the axis of symmetry of the parabola. The first method met with the greatest success, the last with the least. Many candidates drew helpful sketches. Others, sketching through axial intercepts, were led astray when they solved $x^2 = -4$ to obtain $x = \pm 2$. Too many candidates failed to label either the focus or vertex. Such sketches did not aid the awarding of marks. All too often, candidates with most acceptable sketches interchanged the ordinate and abscissa values of their labelled points. In one version, the vertex was plotted at $(1, 0)$, leading to a focus $(1, 1)$.

Some candidates provided answers which indicated they had little understanding of any of the properties of any parabola. Many did not attempt (ii).

(i) (1 mark)
Only a minority of the 'formula-comparison' group of candidates failed to give $(0, 1)$ as the vertex, the main error being the reading of $k = -1$. A large proportion of the 'calculus' group either could not proceed beyond finding $y'$, or showed poor algebraic skills in making $y$ the subject of the formula and so were unable to arrive at $(0, 1)$ from their flawed equation. The 'axis of symmetry' group often took $x^2 - 4y + 4$ as the quadratic in $x$, arriving at $(2, 2)$ or $(-2, 2)$ for the vertex. Other members of this group, with $\frac{1}{4}x^2 + 1$ as the quadratic, failed to correctly identify the coefficients $a$, $b$ and $c$.

Too many candidates thought that the directrix, or even the axis of symmetry, was the vertex despite the clear request in the question to 'find the coordinates of . . . '.

(ii) (2 marks)
The two steps required in finding the coordinates of the focus of the parabola were recognising that the focal length was 1, and adding this amount to the appropriate coordinate of the vertex to arrive at $(0, 2)$. The main error in the 'formula-comparison' group was the identification of the focal length. All too often these candidates wrote $4Ay = 4(y - 1)$, arriving at $A = 1 - \frac{1}{y}$, and so $A = 0$ (substituting $y = 1$) or $A = 1$ (perhaps from substituting $y = 0$). Somewhat less than 50% of the 'calculus' group who were successful in (i) had any idea of how to proceed from vertex to focus. Many members of the 'axis of symmetry' group identified the coefficient of their $x^2$ with the focal length.

A significant percentage of the candidature thought that $(0, A)$ was the focus of the parabola. Many of these candidates could not identify $A$, though others, giving $(0, 1)$ as their focus, appeared unconcerned that it coincided with their vertex. Others confused the focal length with the focus, or referred to the focus as a line ($x = \ldots$, or $y = \ldots$).

**Question 3**

This question consisted of two parts each worth 6 marks taken from two separate areas of the syllabus. Part (a) required students to perform three differentiations where the product rule, the quotient rule and the chain rule could be applied. It involved a knowledge of differentiation of trigonometric and logarithmic functions. Part (b) was a coordinate geometry question related to the concepts of midpoint, gradient, perpendicular lines and simultaneous equations.

The question was, on the whole, extremely well done, with the majority of candidates scoring more than half marks and less than 1% of candidates scoring zero marks. The stronger candidates found the question quite easy and were able to gain full marks or near full marks. In general, student responses were well set out with the necessary rules being quoted at the beginning of each answer and working being sequentially shown in each step of the response.

(a) (i) (2 marks)
The majority of candidates recognised the need to use the chain rule (composite function rule) and applied this rule correctly. Those who had difficulty commonly gave incorrect responses of $3(x^2+5)^2 \cdot 2$ or $3(x^2+5) \cdot 2x$. One mark was awarded for incorrect answers of the form $3P(x) * 2x$, where $*$ was any one of the 4 arithmetic operations.

Few students chose to expand $(x^2+5)^3$ first. Those who did, more often than not, made algebraic errors in their expansion. The marking scheme awarded students who used this method one mark for a correct expansion and one mark for the correct derivative of their expansion.

(ii) (2 marks)
This was the least well answered of these three subparts. Most students recognised the need for the quotient rule, but many got signs confused in the numerator of this rule. The other common error encountered was for students to write $\frac{d}{dx}(\cos x) = \sin x$. Incorrect answers were awarded one mark provided that they were in one of the forms

$$\frac{\pm x\sin x \pm \cos x}{x^2} \qquad \text{or} \qquad \frac{-x\sin x - \cos x}{f(x)}$$

with $f(x) \neq 1$. Perhaps the worst feature was students' attempts to simplify their answers after correct substitution in the quotient rule. The expression $x\sin x$ often became $\sin x^2$ while $\frac{-x\sin x - \cos x}{x^2}$ was reduced to $\frac{-\sin x - \cos x}{x}$ and even $\frac{-x\sin - \cos}{x}$ after cancelling.

Some students rewrote the question in the form $\cos x \cdot x^{-1}$ and used the product rule, often with success. Incorrect responses in the form $Ax^{-1} + B\cos x$ received one mark.

(iii) (2 marks)
Most candidates realised the product rule was required here, and were able to correctly state the rule and substitute into it. The most common incorrect response was to ignore the need to use the product rule and simply 'differentiate term by term' to obtain

$$\frac{d}{dx}x^2\ln x = 2x \cdot \frac{1}{x} = 2\,.$$

Once again, simplification errors were frequent with $(\log x) \cdot 2x$ often becoming $\log 2x^2$. The marking scheme awarded incorrect answers in the form $A \ln x + Bx^2$ one mark.

(b) (i) (1 mark)
This part was answered correctly by almost all the candidates. The only incorrect responses came from the use of incorrect formulae such as

$$\text{midpoint} = (\frac{x_2 - x_1}{2}, \frac{y_2 - y_1}{2})$$

or

$$\text{midpoint} = (\frac{y_1 + y_2}{2}, \frac{x_1 + x_2}{2})\,.$$

The mark was awarded for a correct substitution into the correct formula and subsequent numerical errors were ignored.

(ii) (1 mark)
The gradient was also found correctly by most students. The marking team paid particular attention to whether the answer, which was 1, was obtained correctly from substitution into the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

or incorrectly from the formula

$$m = \frac{x_2 - x_1}{y_2 - y_1}\,.$$

However, students who merely stated the correct answer were, on this occasion, awarded the mark.

(iii) (2 marks)
This part proved more difficult for candidates than the preceding two parts. Having found the gradient of $AB$ in part (ii), many students then went on to find the equation of $AB$, write it in the form $y = mx + b$ and conclude from this that the gradient was 1. Some students wrote down $m_2 = -1$, perhaps not really appreciating that the condition for lines to be perpendicular is $m_1 m_2 = -1$.

Many students failed to use the midpoint coordinates, not realising the implication of the word 'bisector'. Thus, a common response was to find that the equation of $AB$ was $y = x + 1$ and then state that the equation of the perpendicular bisector must be $y = -x + 1$.

Students who did not gain full marks could be awarded 1 mark for obtaining a gradient which was $-1$ or the negative reciprocal of their answer in part (ii). If they did not receive this mark, they could still receive one mark for correctly substituting their midpoint in part (i) into a correct form for the equation of a straight line.

A very small number of students chose a locus approach in this part by letting $P(x, y)$ be a point satisfying the condition $PA = PB$. Attempts using this method were often marred by algebraic errors in the substitution and simplification.

(iv) (2 marks)
This part was the least well answered of all parts of the question. The most noteworthy comment is that many students failed to recognise the connection between parts (iii) and (iv). Rather, they used the locus approach here ($PA = PB$ etc) to find the equation of the line that they had already found in part (iii). Having obtained their answer in this way, most realised the need to solve simultaneously, which was done, on the whole, with considerable success.

An alternative approach was to choose points by trial and error on $y = 2x - 9$ and test using the distance formula to see if the point was equidistant from $A$ and $B$. Quite a few thought that the formula for the perpendicular distance of a point from a line had to be used somewhere in the question, while the answer $(4\frac{1}{2}, -9)$ obtained from the intercepts on the axes was not uncommon. Some students tried a graphical approach, but few succeeded.

In general, the marking scheme awarded students one mark for recognition of the need to solve $y = 2x - 9$ and their answer to part (iii) simultaneously and a second mark for correctly finding either the $x$ or $y$ coordinate of $P$ from their two equations.

**Question 4**

The question contained three parts. Part (a) involved using Simpson's rule to evaluate a definite integral with the function values supplied in table form. Part (b) (i) and (ii) required the graphs of a quadratic and an absolute value function, followed by (iii) finding the points of intersection of the graphs and (iv) solving an inequality. Parts (iii) and (iv) could be done either graphically or algebraically. The final part (c) required the evaluation of the area under two curves where the diagram and point of intersection of the curves were given. The question rewarded students with a good basic understanding of the required concepts. Unfortunately, a significant number of students were only able to show limited knowledge and consequently gained few marks.

(a) (3 marks)
About two-thirds of the students gained 3 marks. Students who used a table with given weights were usually successful. The following mistakes were most frequent.

- Not calculating the 'width' of their application correctly (eg $\frac{b-a}{5}$ or $\frac{1}{3}$).
- Using a mixture of $x$ and $y$ values or only their $x$ values.
- Treating the 5 function values together and reversing the pattern (ie (1 2 4 2 1) instead of (1 4 2 4 1)).
- Only using 3 function values or trying to 'create' 9 function values.
- Quoting a correct formula but being unable to apply it correctly. These students were often confused by 'odd and even values' and formula subscripts.
- Using the trapezoidal rule.

(b) (i) (2 marks)
Most students were able to gain at least one mark for a concave up parabola. Many students used a template successfully while others supplied an accurate graph by plotting points. Most supplied the $y$-intercept but many ignored the request to label both $x$ and $y$ intercepts. The markers were pleased to note that the typical size of the students' graphs is increasing, with many providing accurate half-page graphs. Unfortunately, there were still some centres where the majority of the graphs were much smaller and poorly drawn.

(ii) (1 mark)
Well done, with the common mistakes being the graphs of $y = \pm x$, $x = |y|$, or $y = x$ for $x \geq 0$. Students who used a ruler to draw the correct graph always gained the mark, while freehand attempts were often incorrect as the graph took on the features of $y = x^2$ near the origin or for $x > 3$ or $x < -3$.

(iii) (2 marks)
Students who had drawn accurate graphs in parts (i) and (ii) were often able to write down correct values directly (the better candidates checking their solutions by substitution). Alternatively, the better students who used an algebraic approach were able to reject the values $x = \pm 2$ by observing their graphs. Unfortunately, many students only considered $|x|$ to equal $x$ and were happy to provide solutions of $x = 3$, $x = -2$ even though $x = -2$ contradicted their graph.

(iv) (1 mark)
This part was not well done, with students being successful by reading straight from their graphs or considering $x^2 - 6 \leq \pm x$ and testing various values for $x$.

Common mistakes were to shade the region on their graph enclosed by the functions or to not clearly indicate their final solution.

Very few students used the simple definition of absolute value, namely $|x| = x$ for $x \geq 0$ and $|x| = -x$ for $x < 0$. Many students attempted (iii) and (iv) using only algebraic techniques and did not see the connection or attempt to use their graphs from (i) and (ii).

(c) (3 marks)
About one-third of the students gained full marks. There were some very poor attempts. Although the diagram was provided, many students were unable to interpret it successfully, believing that they needed to evaluate $\int f(x) - g(x)\,dx$ for a particular domain. Many students did not make use of the table of standard integrals. Some students were unable to find exact values on substituting into their trigonometric functions, believing the values were in degrees rather than radians. Many students arrived at zero or a negative area and tried to remedy this by taking the absolute value. Better students, however, realised the contradiction and were able to recheck their solution successfully. Setting out and correct use of notation were often poor and it was sometimes difficult for examiners to know which parts of the student's solution were involved in obtaining the final answer. Some students spread their work out too much with their solution written over several pages. The following mistakes were common.

- Area $= \int_0^{\pi/2} \cos x \pm \sin 2x\,dx$ or $\int_0^{\pi/6} \cos x - \sin 2x\,dx + \int_{\pi/6}^{\pi/2} \sin 2x - \cos x\,dx$.
- $\int \sin 2x\,dx = -2\cos 2x,\ 2\cos 2x$, or $\frac{1}{2}\cos 2x$.
- $\int \cos x\,dx = -\sin x$ (quite often after successfully integrating $\sin 2x$).
- Incorrect substitutions into their definite integrals (eg $\cos 0 = 0$), ignoring $\frac{1}{2}$ etc in part of their evaluation, lack of care with negatives involved in subtractions.
- $\sin\frac{\pi}{2} = 0.0274\ldots$ etc.

## Question 5

This question contained four parts, which required students to be able to evaluate an integral, prove two triangles were congruent, demonstrate an understanding of first and second derivatives and to recognise and evaluate a limiting sum.

(a) (2 marks)
The majority of students were able to find the primitive of $e^{-x}$. However, many students were unable to handle the substitution of the limits and the consequent evaluation. Some students left their answer as $1 - e^{-\ln 7}$, failing to see that $e^{-\ln 7}$ is $\frac{1}{e^{\ln 7}} = \frac{1}{7}$.

A common error was to assume that $e^0 = 0$.

(b) Students should redraw the diagram in their answer booklet when attempting a question such as this.

(i) (2 marks)
This part of the question was generally well done. Students who followed a SAS proof tended to obtain full marks. Those who attempted a SSS or RHS proof often failed to give sufficient information to support their case and gain full marks. Some students failed to quote the test they were using.

Many students made careless transcription errors in naming angles, eg $\angle BPQ$ was often written as $\angle PPQ$. They must check their work carefully for this type of error.

(ii) (2 marks)
Students needed to see the connection with part (i) and be able to deduce a pair of corresponding equal angles in the congruent triangles.

This question was often left unattempted. Some students demonstrated correct logic but failed to gain full marks because they did not give any reasons for their statements.

(iii) (2 marks)
Many students misunderstood this question, and did not see the connection with calculus. The question called for an increasing concave down curve. Many students mistakenly drew two curves: one for price and one for inflation.

(c) (i) (1 mark)
Most students scored the mark in this question. Some students were confused by the 'after the third bounce'. Careful reading of the question was necessary to see which term was required. Candidates should also check that they have copied the numbers in the question carefully to their writing booklet. A common error was to write 1.125 as 1.25.

(ii) (1 mark)
This question was generally well answered. However, a few students who were using correct geometric formulae in parts (i) and (iii) wrote A.P.

(iii) (2 marks)
Few students gained full marks for this part of the question. Many students had difficulty associating 'coming to rest' with a limiting sum. Those who realised they had to calculate a limiting sum often then didn't take into account the 'up and down'. Students often quoted the formula incorrectly or even if using $S_\infty = \frac{a}{1-r}$ interpreted it to mean the number of bounces rather than the distance travelled. Many students wasted time by doing unnecessary exhaustive calculations.

## Question 6

This question consisted of two parts. The first part dealt with a piece of wire bent to form two sides of a right-angled triangle, and the second explored a logo made of squares and shaded in a repeated pattern. Despite the difficulty of the question, the quality of responses was good, with the vast majority attempting a substantial amount of the question. This question rewarded perseverance, as both parts (a) and (b) had difficult sections that students needed to work beyond or simply use in later parts to gain relatively easy marks. Once again, there were multiple 'pathways' to the correct answers. Students are well advised to consider these alternatives before launching into what may well be 'the hardest pathway'.

(a) (i) (1 mark)
This part required the students to find an expression for the hypotenuse in terms of the length of the wire (5 metres) and the other side, labelled $x$. It was well done by most students. Some felt obliged, since it was a right-angled triangle, to use Pythagoras' theorem and complicated the question and confused themselves. Others felt that since there was an $x$ in the diagram, they should solve for $x$ or find an expression for $x$.

(ii) (2 marks)
Whilst many students clearly set out their working and went from 1/2 base times height to $\frac{1}{2} \cdot x\sqrt{25 - 10x}$, there were some students who failed to fully show where $\sqrt{25 - 10x}$ came from, or 'fudged' their own expression to fit the given area formula. Full justification was expected here with the answer provided. Those who clearly started with a correct Pythagorean expression did well.

(iii) (3 marks)
Many students found this part particularly difficult, with only about 1 in 7 candidates scoring full marks. Most candidates recognised that differentiating was appropriate. Some students did not use the product rule, but simply differentiated each term separately. For those students who did attempt to differentiate using the product rule, attention to detail was critical. Minus signs, halves and $x$'s appeared in the wrong place or were forgotten altogether. Having obtained the correct expression for $\frac{dA}{dx}$, many students found solving $\frac{dA}{dx} = 0$ very difficult because of the amount of algebraic simplification with fractions and radicals involved. Some students, dismayed by their expression for $\frac{dA}{dx}$, went no further. Others made multiple attempts, filling several pages.

Unfortunately, too many students launched into their calculation of the second derivative immediately after calculating $\frac{dA}{dx}$, and usually found this even more difficult than the first derivative. More thought by candidates as to the best way to test for the nature of the stationary point would have helped.

Despite earlier mistakes, this was a question that rewarded perseverance. Students were given credit for solving their (incorrect) $\frac{dA}{dx} = 0$ correctly, provided it was of equal difficulty to the correct expression. They also received marks for substituting their (often incorrect) solution to $\frac{dA}{dx} = 0$ into $\frac{1}{2} \cdot x\sqrt{25 - 10x}$. However, it was disappointing to see that a substantial number of students failed to recognise that negative answers for $x$ or values of $x > 2.5$ were physically impossible.

(b) This part of the question required students to consider a logo that had stripes whose areas formed an arithmetic progression.

(i) (1 mark)
Students found this part difficult and the concept of the $n^{\text{th}}$ stripe too abstract. Students could approach the problem in a number of ways. The most common approach was to consider the difference in area of the $n^{\text{th}}$ and $n + 1^{\text{th}}$ square. In addition to being conceptually difficult, many students stumbled on the algebraic expansions. An alternative, though uncommon, geometrical approach was to find the area of two rectangles or trapezia. This method had the advantage of being algebraically simpler, though the onus was on the student to 'show' where $8n + 20$ came from.

It is possible some students were discouraged from doing parts (ii) and (iii) because they couldn't understand the concepts in part (i).

(ii) (2 marks)
Most students scored at least part marks here, usually for the area of the first stripe. Far fewer students managed to find the area of the last stripe.

(iii) (3 marks)
Many students recognised that they could substitute their answers from part (ii) straight into $\frac{n}{2}(a + l)$. However, many used $n = 19$ or 20 and so lost part marks.

Students who manually calculated all 10 shaded stripes were also successful.

## Question 7

This question touched on several topics in the syllabus. It began with a proof based on trigonometric identities. Then followed a calculus-based question using a trigonometric function which related to the original identity. The last part of the question was an application of calculus in the physical world. The candidates needed a knowledge of exponentials and logarithms as well as calculus. Nearly every candidate attempted this question.

Most candidates presented their solutions clearly and logically. Very few used liquid paper, so only a small number of candidates lost marks through failing to re-insert values they were changing. The question was generally well answered, with many students gaining either almost full, or full marks.

(a) (2 marks)
Many candidates showed that, although they were familiar with trigonometric relationships, they did not grasp the concept of proof. Students did not score the second mark if they failed to show the necessary link in the proof. Some candidates failed to acknowledge the angles and stated that $\sec = 1/\cos$ and $\tan = \sin/\cos$.

(b) (i) (2 marks)
Most candidates knew the required volume formula but were unable to link it with the given form using part (a).

(ii) (3 marks)
Nearly every student knew that $\int \sec^2 2x\,dx$ was $\frac{1}{2}\tan 2x$. Not all the students integrated the constant and some changed the question from $\pi \int \sec^2 2x - 1\,dx$ to $\pi \int \sec^2(2x-1)\,dx$. Few gained the final mark for finding the exact volume of the solid. A common error was to take out 1/2 as a common factor from a correct expression without adjusting the second term to $\frac{\pi}{3}$.

(c) (i) (1 mark)
The first part was not answered well. One method that inevitably failed was to begin with $\frac{dv}{dt} = -kv$ and to use integration. Few candidates succeeded using this approach.

(ii) (1 mark)
Many students recognised that $C = 100$ without showing any working and then correctly incorporated the value into part (iii) to find $k$.

(iii) (2 marks)
In this section many candidates revealed their lack of knowledge of the relationship between exponentials and logarithms.

(iv) (1 mark)
If a candidate incorrectly gained or lost a negative sign when substituting into $v = 100e^{-kt}$, then the answer was greater than the initial velocity. It was good to see that many students, who had a value for $v$ greater than 85, stated that their answer was incorrect and also identified the reason for this.

## Question 8

There were six parts in this question, dealing with geometry (exterior angle property, angles in an isosceles triangle and angle sums in a triangle) and calculus (derivatives of trigonometric functions, interpretation of motion, curve sketching). Algebraic skills were very poor, especially with regard to expanding expressions involving grouping symbols and even writing expressions that should have had grouping symbols. The average raw mark scored on this question was 4.3 (out of a possible 12) and many candidates (more than one in seven) were unable to score even one mark for the question, including many complete non-attempts.

(a) It should be noted that virtually no candidates copied the diagram into their answer booklet as part of their answer to this part of the question. This step should be automatic in any geometry problem.

(i) (1 mark)
Generally well done. A large number of candidates were clearly unfamiliar with the term 'exterior angle' while others did not know the difference between 'exterior' and 'interior.' Many candidates didn't realise that a simple reason was all that the question required. There were many long proofs (full page and more) involving working that answered (ii) and (iii) on the way to an answer to this part. On the positive side, there were some refreshingly innovative answers that involved drawing a line through $A$ parallel to $BC$ or through $D$ parallel to $AB$.

(ii) (1 mark)
Generally well done. Most errors involved incorrect removal of grouping symbols or failure to put grouping symbols in when subtracting an expression.

(iii) (2 marks)
Only the better candidates understood how to get $\angle EDC$ and, generally, they went on to do well in the whole question. Poor algebraic skills were the main problem here. Typical was trying to simplify

$$\alpha + \beta - \frac{2\alpha + \beta}{2}$$

and getting $\alpha + \beta - \alpha + \frac{\beta}{2}$ or $2\alpha + 2\beta - 2\alpha \pm \beta$. There were also many attempts to turn an expression for an earlier angle into an equation from which they obtained $\alpha$ (eg $\alpha = 180° - \beta$). These students then substituted this back to get $\angle EDC$ in terms of $\beta$.

(b) There were many graphical solutions to the trigonometric equation $\cos t = -1$ (eg $v = \cos t$ reading where $v = -1$ or reading where $v = 0$). It was common to see variables substituted or interchanged (eg $v = 1 + \cos x$ or $v = 1 + \cos \theta$). While it was disappointing that so many candidates reversed the signs of the derivatives of $\sin t$ and $\cos t$ in both parts (i) and (ii), it was encouraging to see that very few candidates integrated to find what should have been derivatives. The significance of the interval $0 < t < 3\pi$ was missed by most candidates in all 3 parts of the question.

(i) (2 marks)
Poorly done. Many who attempted this part were unaware that a particle at rest has zero velocity and that they should have used a derivative to find when this occurs. Of those who used the derivative, many decided that their 'graph' from (iii) should determine their answer to this part and promptly overruled their (correct) $t = \pi$, $(3\pi)$ and replaced it with $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$. It was common to see either the incorrect derivative $\dot{x} = \cos t$ (omitting the $+1$) or to see the correct derivative followed by an incorrect equation ($\dot{x} = 1 + \cos t$ then $\cos t = 1$).

(ii) (2 marks)
Equally poorly done, though more candidates used derivatives here (but not in (i)). There were many trivial solutions ($t = 0$) without any attempt to find others in the domain. There were many cases where the candidate found both derivatives here, having recognised that 'acceleration' required a second derivative without recognising that 'stationary' implied that the first derivative is zero.

(iii) (4 marks)
In general, the graph of the function $t + \sin t$ was completely unfamiliar to the vast majority of 2 Unit candidates. Also, many failed to make any connection between this and parts (i) and (ii). The concept of curvature and its relation to points of inflection or the sign of the first derivative was obviously not understood by most candidates. While there were very few completely correct solutions, the better candidates presented very good sketches with evidence of a clear understanding of the information from (i) and (ii) and of the requirements of the question. Overall, many candidates plotted points for this part of the question and those who obtained decimal values for the $x$-values finished with good sketches.

A sketch like this requires labelling in words (horizontal inflection, stationary point, inflection) to clearly indicate the nature of the point and also requires that these types of points are correctly drawn. This was often not the case. Many otherwise good sketches had no labelling or had correct labelling but no attempt to make the curve horizontal at $(\pi, \pi)$. The most common incorrect sketches involved a simple sine curve or a sine curve that was translated up (or down) by 1 unit or '$t$' units and this may or may not have been consistent with (i) and (ii).

Finally, the limited domain should have led candidates to work out the coordinates of the two endpoints (at $t = 0$ and $t = 3\pi$) as one of their first steps, but far too often there was no attempt to do this at all.

## Question 9

This question contained two sections: part (a) dealt with probability and part (b) with an application of calculus. Nearly all students attempted this question. There were very few non-attempts. Students found it easy to get some marks whilst the majority of better students were unable to gain full marks. Good attempts in (a) were sometimes followed by poor attempts in (b) and vice versa.

(a) This probability question centred on two draws without replacement from a bag containing 4 balls: 2 red, 1 black and 1 white.

(i) (1 mark)
The students were asked to draw a tree diagram. Generally, this was done well. However, about 20% did not get the mark, either because they didn't know how to draw a tree diagram or, if they could, for allowing the possibility of $WW$ or $BB$. The latter indicated that they did not realise there was no replacement.

(ii) (1 mark)
The students were asked for the probability of $RR$. This was generally well done. Most mistakes occurred when students assumed that all outcomes from their probability tree were equally likely. Therefore, common answers were 1/7 and 1/9 instead of 1/6. Trees showing 12 outcomes seemed to handle this part better. Interestingly, many students who lost the mark in (i) got the correct answer here.

(iii) (1 mark)
The students were asked for the probability of at least one red. Generally, this was not done as well as (ii). Again, most problems faced by students resulted from their tree and from assuming that all outcomes in their tree were equally likely.

(iv) (2 marks)
Students found this very hard. The fact that the sample space had changed to 5 (as only 5 of the original outcomes contained a red) eluded nearly all students. Very few showed any working. Again, students whose tree contained 12 outcomes did better. A very small number of candidates displayed a knowledge of conditional probability.

(b) This question tested students' knowledge in applying calculus to a situation where a pen is moving back and forth in a straight line.

(i) (2 marks)
The students had to find the displacement function $x(t)$ from a given velocity function $v(t)$. This was done well by the majority of students. The usual mistakes of integration and substitution for the constant appeared. Many differentiated. Some floundered here because they did not understand the question, but found the correct expression for $x(t)$ later.

(ii) (1 mark)
The students had to recognise that the point where $t = 1$ on the velocity curve corresponded to a local maximum on the displacement curve. Most students attempting this answered that $x = 6$ when $t = 1$, which missed the point entirely. Better students stated that it was a stationary point, horizontal point of inflection or turning point.

(iii) (4 marks)
Students had to find the distance travelled in the first two seconds and hence the amount of ink used. Most students evaluated their displacement expression at $t = 0$. Others evaluated at $t = 0$ and $t = 2$. Better students used $t = 0$, $t = 1$ and $t = 2$ but then had difficulty determining the total distance. Students who could integrate using $t = 0$, 1, 2 did well, as did students who could draw either a linear representation of $x = f(t)$ or part of the graph of $x = f(t)$.

## Question 10

This question contained two sections. Part (a) was a short but unusual inequality question, worth 2 marks. Part (b), worth 10 marks, required students, through guided questions, to prove the relationship for $\tan 2\beta$ in terms of $\tan\beta$. The large majority of students attempted this question, with most of these getting some marks for part (a) and part (b) (i).

Overall, time was obviously a factor here in that a large number of students did not go on from (b) (i), but many students also had little idea what to do with so many variables. It was pleasing, however, to see so few total non-attempts. Students must have received the message that they can get some marks quite easily even in Question 10. It was also pleasing to see the work of students who were able to answer the question perfectly, as well as that of those who provided the more unusual methods of solution.

(a) (2 marks)
This question asked students to graph the solution of $4x \le 15 \le -9x$ on a number line.

While this was a relatively simple looking inequality, it was obvious that students had rarely seen similar questions, and had great difficulty making the decision about the resulting solution. Very few students gained the full

2 marks here. Although many students were able to split the inequality into two parts and gain the correct solution algebraically, they lacked the confidence to graph the solution, $x \le 3\frac{3}{4}$ and $x \le -1\frac{2}{3}$ correctly. Most decided that their solution must have been wrong and instead graphed $-1\frac{2}{3} \le x \le 3\frac{3}{4}$.

Many other students did not break up the original inequality into two parts, and used reciprocals to try to obtain their solution, usually without consideration of the sign of $x$. The negative sign also confused many students, who then failed to reverse the direction of the inequality when dividing by $-9$. A number of students found extremities and tested cases, but were usually unsuccessful, since they either missed testing some regions or did not believe their results.

(b) This question led students through the use of familiar geometrical relationships to the development and solution of a quadratic equation in terms of $x$ and $m$ (the gradient). From this they were led to deduce the $\tan 2\beta$ relationship. It used 'hence' in two parts, and required students to find the coordinates of $x$ by solving the given equation, thus guiding the students down a single path to the discovery of the relationship.

(i) (2 marks)
This part asked the students to explain why a triangle within a circle, with one vertex at the centre of the circle, is isosceles, and then to deduce that the outside angle $\alpha$ is equal to twice the interior angle $\beta$. One mark was awarded for each of these results. Most students gained marks in this question, although some students obviously missed the second part. Some 3 Unit students quoted the 'angle at the centre' result, ignoring the 'hence' in the question.

Students' explanations about the isosceles triangle and reasons for $\alpha$ equal to twice $\beta$ ranged from very concise to wordy, circular arguments, or explanations that lacked clarity in the naming of sides and angles.

Many students (probably close to half) used the angle sum of a triangle and supplementary angle argument, rather than the exterior angle theorem. Most were successful using this method.

(ii) (1 mark)
This part asked students to find the equation of the line joining $P$ to $Q$, given that $\tan\beta = m$.

Most students who continued beyond part (i) used the point-gradient form of the straight line, $y - y_1 = m(x - x_1)$. Of those who gained the equation $y = m(x+1)$, many went on to rewrite this immediately as $y = mx + 1$. A small number of students used the coordinates $(\cos\alpha, \sin\alpha)$ for the coordinates of $P$, while a small number used $(x_1, y_1)$. It was clear that many students had difficulty leaving $m$ in the equation and gave $m$ a value, the most common of which was 1, 'since $\beta = 45°$'.

(iii) (2 marks)
This part required students to show that the $x$ coordinates of $P$ and $Q$ were solutions of the given equation.

For many students who had the correct equation of $PQ$ in part (ii), this presented little difficulty, since they were easily able to solve $y = mx + m$ simultaneously with the equation of the circle. However, less than half of the students who attempted part (b) went on from parts (i) or (ii). Many students were able to substitute the $x$ coordinate of $Q$ to show that it satisfied the equation. A number of these had difficulty with negative signs and ended up with results such as $2m^2 = 0$.

There was a tendency for some students to commence with the result and make all sorts of substitutions, spending a great deal of time getting nowhere with a lot of algebra. A little time spent thinking about the question may have saved them the trouble. It was clear that many students did not realise the significance of what they were doing, and that they were not referring back to the diagram to justify their method of solution.

(iv) (3 marks)
Students who applied the quadratic formula here generally made a reasonable attempt at simplifying the algebra and getting a result for the $x$ coordinates of $Q$ and $P$. A small number used the sum or product of the roots. However, most did not recognise that one of the roots was $-1$. Most students who got this far (less than 20%) were able to interpret their answers and then substitute the $x$ value of $P$ into their $PQ$ equation to find the $y$ coordinate.

Some students, with varying degrees of success, attempted to factorise the quadratic expression to $(x+1)((m^2+1)x + m^2 - 1)$, while others tried to solve for $m$. Solving for $m$ gave $m^2 = \frac{1-x^2}{(1+x)^2}$, which could be simplified to $m^2 = \frac{1-x}{1+x}$, since $x = -1$ gives the $x$ value of $Q$. This could then be substituted back to find the $x$ coordinate of $P$ correctly, although few students were ultimately successful using this method.

(v) (2 marks)
The few students who had the correct answer to (iv) often realised that they needed to use the gradient of $OP$ for $\tan\alpha$. However, this was only a very small proportion of the candidature. Some tried to 'fudge' the values of the $x$ and $y$ coordinates of $P$ as $(1 - m^2, 2m)$. Many students (obviously 3 Unit students) saw the result as a trigonometric identity and quoted the result for $\tan(\beta+\beta)$. No marks were gained for this method, since the question asked students to 'hence deduce' the result from the previous working.

## Mathematics 3 Unit (Additional) and 3/4 Unit (Common)

### Question 1

This question consisted of five unrelated parts. It was generally well done although there were some common errors.

(a) (2 marks)
The students were required to differentiate the product of an exponential and a trigonometric function. The marking scale awarded one mark for each correct term. The question was well done, with most students scoring 2 marks. Only a few failed to recognise the need to use the product rule.

(b) (2 marks)
Surprisingly, this part was poorly done considering that it involved the use of a straightforward formula from the 2 Unit course, namely the formula for the perpendicular distance from a point to a line. Many students could not remember it correctly, failing, for example, to include the squares or the square root sign. Those who did have the correct formula often substituted numbers into the wrong places. This included putting the coordinates of the point into the denominator. The fact that the equation of the line was not in general form also caused problems with some using the gradient, the $y$-intercept and zero as their coefficients. The standard of arithmetic was less than impressive with working such as $\sqrt{3^2 + -1^2} = \sqrt{8}$ not uncommon. Students who failed to take the absolute value for their distance were not penalised.

(c) (1 mark)
This was a straightforward use of one of the laws of logarithms. Students were awarded the mark for subtracting the two decimal values. Many students scored the mark although dividing the logarithms was not uncommon.

(d) (3 marks)
The students were required to evaluate a definite integral in which the primitive involved the inverse tangent function. The response from the students was disappointing, with many failing to recognise this as one of the standard integrals on the back of the paper. In any case this is an important part of the 3 Unit course. Attempts that involved the log function were not uncommon and those that did use $\tan^{-1}(\frac{x}{2})$ often omitted the coefficient $\frac{1}{2}$. For full marks the student needed to evaluate $\tan^{-1} 1$ correctly in radians, preferably in terms of $\pi$ (although the decimal was accepted).

(e) (4 marks)
This required the students to evaluate a definite integral using a given substitution, although good students were able to successfully achieve this simply by algebraic rearrangement. Essentially the four marks were

awarded for differentiating in order to relate $dx$ and $du$, correctly forming the integrand, finding the primitive and evaluating with correct limits. Whilst most students showed familiarity with the substitution process and were able to score the first mark, algebraic errors thereafter were common. Many candidates correctly obtained

$$\int \frac{\frac{u-1}{2}}{u} \cdot \frac{du}{2}$$

but were then unable to simplify. Atrocities such as

$$\int \frac{u-1}{u}\, du = \frac{\frac{1}{2}u^2 - u}{\frac{1}{2}u^2}$$

were quite frequent, as were attempts to integrate mixtures of the variables $x$ and $u$. To obtain the final mark, students were able to evaluate using the limits 1 and 3 for functions of $u$, or using 0 and 1 by returning to the original variable $x$.

## Question 2

On the whole this question was quite well done. In general, students are not astute in discerning the amount of work required for 1 mark. While a number produced some very good work, they went on for pages, in some cases for 1 mark only.

(a) (i) (1 mark)
Even though they were instructed to copy the diagram, many students did not do so.

The students did not understand the implication of 'state'. Some tried to prove that $\angle AOB = 52°$. While on the whole most students earned this mark, many need more practice in stating geometrical reasons.

(ii) (2 marks)
Most students scored full marks for this part. Those who only scored 1 mark usually did not provide justification for their answer: namely that $\angle OAD = \angle BCD$, alternate angles.

(b) Students, generally, did a lot of work for this question and in some cases got nowhere. Some treated the question purely as a simple harmonic motion question and drew no parallels from their knowledge of trig functions. Very few students simply stated answers.

(i) (1 mark)
Most students were able to gain this mark. Those who gave the answer in degrees scored 0.

(ii) (1 mark)
Some students actually used calculus to try to find the maximum acceleration. This was too laborious and in most cases unsuccessful. The most successful method was to realise that the maximum of this trig function is 12. There seemed to be little understanding of acceleration as a vector, and many students were too ready to accept $-12$ as the maximum.

(iii) (2 marks)
This question might have been fairer if it had been $x = 3\sin(2t-5)$ as students tried to find the value of $t$ when $x = 2$ and then substitute it into $v$. This method lent itself to many incidental errors (arithmetic, sign of the derivative, etc). They also had difficulties when they encountered a negative time, which made this method much harder than using $v^2 = n^2(x^2 - a^2)$. This latter method was by far the easiest and most successful. While some students tried to derive this formula, they were only expected to quote it. Even though some knew the formula, they used incorrect values for $n$, $x$ and $a$.

Some students substituted $t = 2$ (instead of $x = 2$) into the velocity equation. This earned 0 marks.

Rounding off during the question led to quite imprecise solutions, which were exacerbated by the use of degrees rather than radians.

(c) Students, on the whole, managed this question quite well but it was disappointing that many missed the connection between the parts.

(i) (2 marks)
The most popular method was substituting the roots in for $x$ and then solving the resulting equations simultaneously. This, however, was not the most successful method as it led to arithmetic errors. The easiest and most successful method was realising that $x(x-3)(x+3)$ must be the polynomial and then equating the coefficients. Using sum and product of roots, etc, was usually successful except when students said that $0 \times 3 \times -3 = 9$. This was quite a common error.

(ii) (1 mark)
Even though students were instructed not to use calculus, some still did and wasted a lot of time. Some ignored the given information and used their wrong values of $b$, $c$, $d$ from (i) to sketch a totally irrelevant graph. This was given 0 marks. As the polynomial was cubic, some students drew a point of inflexion at the origin, as well as the other two roots, not realising that this would no longer be a polynomial of

degree 3. Students were not penalised if they only drew the graph in the domain $-3 \le x \le 3$. The students scored full marks if they indicated the roots as given in the question and had the right shape.

(iii) (2 marks)
Many students do not appreciate the significance of the word 'hence'. If they had, the easiest method of solution was multiplying both sides by $x^2$. Those students who used this method were generally successful. If they had only drawn the graph between 3 and $-3$, they tended to have lost part of the solution.

The method of looking at both cases was generally not very well done. Testing regions was usually very successful and quite easy.

If students drew the wrong graph, they could still earn full marks if their solution was correct from their graph and if it consisted of two regions.

**Question 3**

This question produced a complete range of marks, from zero to 12, with an average of just over 7. The most common scores were 7, 8 and 9.

Parts (a) and (b) requested three different treatments of $2\sin\theta = \theta$ with (b) (ii) being the only part requiring any 3 Unit knowledge.

Part (c) was a question on probability, which could be answered with either 2 or 3 Unit techniques.

(a) (i) (2 marks)
Candidates were asked to sketch the graphs of $y = 2\sin\theta$ and $y = \theta$ for $-\pi \le \theta \le \pi$ on the same set of axes.

Students were awarded 1 mark for drawing $y = 2\sin\theta$ with amplitude of 2, period of $2\pi$ shown, and extending at least from $-\pi$ to $\pi$.

The other mark was for the sketch of $y = \theta$ which needed to be a straight line, with positive slope, passing through $(0, 0)$ and extending at least past the two outer intersection points with $y = 2\sin\theta$.

This was generally an easy two marks for students. However, very few really gave a clear indication of a scale on the $y$-axis to indicate that a point such as $(\pi, \pi)$ lies on $y = \theta$. Because it was a 'sketch' students were not penalised in this instance.

A common error was to think $y = \theta$ was either a horizontal line (eg $y = 0$ or $y = 1$) or even a family of horizontal lines.

Too many students were not sure which function had the larger gradient at $(0, 0)$, which led to varieties of points of intersection.

(ii) (1 mark)
Students were asked to determine the number of solutions of $2\sin\theta = \theta$ lying in the range $-\pi \leq \theta \leq \pi$.

The mark was awarded for the correct answer of 3 or even three values written down as attempts at a solution. The mark was also awarded if the answer was consistent with the number of points of intersection shown on the graphs in (a).

This was generally well done. However, far too many students spent valuable time trying to find the actual solutions (which were not required by the question) and students often stated that there were two solutions, omitting the more obvious solution of $\theta = 0$.

The question was often not attempted, perhaps being overlooked by students after drawing their graphs, or not understanding that their points of intersection provided the solutions.

(b) The question supplied a diagram in which the point $P$ lies on the circumference of a semicircle of radius $r$ and diameter $AB$. The point $C$ lies on $AB$ and $PC$ is perpendicular to $AB$. The arc $AP$ subtends an angle $\theta$ at the centre $O$, and the length of the arc $AP$ is twice the length of $PC$.

(i) (1 mark)
Students were asked to show that $2\sin\theta = \theta$. This could be done by observing that

1. $AP = r\theta$
2. In $\triangle POC$, $\sin(\pi - \theta) = r\sin\theta$, and so $PC = r\sin\theta$.
3. Since $AP = 2PC$, $r\theta = 2r\sin\theta$.

Hence $\theta = 2\sin\theta$.

One mark was awarded if all 3 statements (1, 2 and 3), or equivalent, were presented or if 1. and 2. were correctly linked (eg. $r\theta = 2r\sin\theta$, or equivalent). However, if incorrect information was used, then no mark was awarded. Common errors were letting the radius be 1 unit or assuming $\angle POC = \theta$.

Students often presented a page or more for an attempt at this proof, in the hope they would score a mark somehow. Because only 1 mark was allocated the proof needed to be accurate, and brevity often led to success. (Use of Pythagoras' rule often led to lengthy attempts.)

The fact that $\sin\theta = \frac{PC}{r}$ was freely quoted, and was to be accepted unless students explicitly wrote $\angle POC = \theta$.

Many students used the expansion of $\sin(x - y)$ to evaluate $\sin(\pi - \theta)$.

This question was often omitted and was more commonly given 0 than 1.

(ii) (3 marks)
Starting with $\theta = 1.8$ as an approximation for the solution to the equation $2 \sin \theta = \theta$ between $\pi/2$ and $\pi$, candidates were asked to use one application of Newton's method to give a better approximation.

One mark was awarded for each of three separate steps:

1. finding the correct function and its derivative;
2. use of correct, or stated, values of $f(\theta)$ and $f'(\theta)$ into a correct Newton's formula;
3. correct use of the calculator, in 'radian' mode, throughout all three steps and gaining an answer between $\pi/2$ and $\pi$.

The most common errors were to use 'degrees' mode on the calculator, or substitute values such as $\pi/2$ and $3\pi/4$ instead of 1.8.

As students were told that this provided a 'better approximation', they did not need to use up time showing this to be true.

The formula for Newton's approximation was often misquoted as

$$\theta_2 = \theta_1 - \frac{f'(\theta_1)}{f(\theta_1)},$$

probably from confusion with integrals leading to logarithmic functions.

Students should be taught to state their formula and values clearly, before substitution, so that they may still earn the mark for correctly processing their information.

This was a difficult question for students, with the mode being 1, when attempted, because they failed to see that Newton's formula was finding the zeros of a function, and so the step: $2 \sin \theta - \theta = 0$ was often not appreciated. As a result, the most common solution was effectively to apply one application of Newton's formula to approximate the solution of $2 \sin \theta = 0$, starting with 1.8. This yielded an approximation of $6.086\ldots$, which was awarded one mark.

More application of Newton's formula to non-polynomial functions and equations requiring manipulation to equal zero is obviously desirable in future teaching and study.

(c) Students were asked about the game of Sic Bo in which three regular, six-sided dice are thrown once. This is a real game, played in casinos.

(i) (1 mark)
They were asked what is the probability that, in a single game, all three dice show 2. Almost all students obtained the correct answer of 1/216.

(ii) (1 mark)
Next, they were asked what is the probability that exactly two of the dice show 2? The answer is $3\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)$, but students often multiplied by 6 rather than 3.

(iii) (1 mark)
Students were asked what is the probability that exactly two of the dice show the same number? This was poorly done, even though the mark here was awarded for an answer that was 6 times the answer to part (ii).

(iv) (2 marks)
Finally, students were asked to determine whether the claim that you expect to see three different numbers on the dice in at least half of the games is correct and to justify their answer. Here, one mark could be obtained for using a correct method to calculate the probability that the three numbers were different from the answers given in previous parts, or for giving a consistent conclusion for whether the claim was correct based on whether the probability had been calculated to be more or less than $\frac{1}{2}$.

Many students appeared to be somewhat confused by the familiar probability words 'expect' and 'at least' appearing in the same sentence. The argument needed to be based on probability in order to be successful. Most students did not understand how to justify their argument, even if the correct probability of $\frac{5}{9}$ for the numbers being different had been calculated.

Some students answered that the claim was incorrect because the probability of the three numbers being different was more than $\frac{1}{2}$, whereas 'at least half' would require $p = \frac{1}{2}$.

Long essays wasted time and usually gained no marks.

**Question 4**

This question related to trigonometric functions and identities. Part (a) used geometry and the sine rule to establish that $\sin 2\alpha = 2\sin\alpha\cos\alpha$ and used this to find the limiting sum of a G.P. Part (b) involved finding the value of a definite integral using a trig substitution, while part (c) required the computation of a rate of change related to a right-angled triangle.

There were many students who scored 12, but many also scored zero despite attempting several parts of the question.

(a) (i) (1 mark)
Most students correctly quoted the ratios of the relevant sides, and some used the complementary angle results. It is disturbing that some 3 Unit students cannot correctly quote a basic ratio.

(ii) (2 marks)
Most quoted the relevant sine rule for the given triangle correctly, but too many used $\triangle ADB$ or $\triangle BCD$, or used area formulae. Students who correctly used area formulae to develop the required result scored 1 out of 2.

Too many students still do not successfully show the result. A bald $\frac{AC}{BC} = 2\sin\alpha$ was not considered adequate.

(iii) (2 marks)
A very high proportion of candidates scored 2 marks. Too many of them wasted time by developing their answer for $S_\infty$ from $S_n$ and taking quite some time to do so (occasionally using some very woolly logic!).

Students should be advised that the 3 Unit paper is difficult to finish in the given time, and only necessary working should be shown. Perhaps more notice should be taken of the part marks for each question.

(b) (3 marks)
This was not very well done. Many scored zero by ignoring the suggested substitution and treating $1 - x^2$ as a linear function, hence obtaining

$$\int (1-x^2)^{\frac{1}{2}}\,dx = \frac{(1-x^2)^{\frac{3}{2}}}{3/2}\,.$$

It was common for those who did the first part of the substitution correctly to treat $\cos^2\theta$ in the same way, leading to $\int \cos^2\theta\,d\theta = \frac{\cos^3\theta}{3}$.

As expected, there were many misquotes for the appropriate double angle formula, and incorrect integration of $\cos 2\theta$.

Students should be encouraged to show all steps involved. Too many candidates attempted to quote $\int \cos^2 t\,dt = \frac{t}{2} + \frac{1}{4}\sin 2t$, and if this result is misquoted they cannot be credited with the steps in between.

Those who attempted to use a relatively formal approach and to write $\int f(t)\frac{dt}{dx}\,dx = \int f(t)\,dt$ very frequently got confused and ended up integrating $\int 1\,dx$.

Another common error was to say $t = \sin(\frac{1}{2})$ when $x = \frac{1}{2}$.

Relatively few candidates attempted to obtain the answer using areas. Those who did were usually successful (and with much less working).

(c) (i) (2 marks)
Apparently because students were asked to show that $\frac{dx}{d\theta} = -\frac{0.9}{\sin^2\theta}$ they assumed that they had to start with the ratio $\sin\theta$. Many incorrect ratios followed. There were also many very long answers which eventually correctly got to $x = \frac{0.9}{\tan\theta}$. The subsequent differentiation was reasonably well done by those who were correct to that stage.

The $x(t)$ and $\theta(t)$ confused some students, who tried to work with $x$ as a function of time, such as $x = 240t + c$. Occasionally these students were successful.

(ii) (2 marks)
Many did not make any connection with time in this part, and even though the correct answer was given, those correctly using the chain rule often did not realise that a simple change of units was required. A remarkable diversity of calculations managed to (supposedly) equal $\frac{1}{27}$!

The most common error amongst relatively good attempts in this part was to fail to invert $\frac{dx}{d\theta}$ to obtain $\frac{d\theta}{dx}$.

## Question 5

In general, this question highlighted the need for students to be reminded that in a 'show' question, there needs to be as much working as possible shown in order to convince the marker that the student knows the work (ie they have not just copied the result from the question paper).

Candidates should also realise that the result is there to guide them. If they do not get the given result, students should go back and look for an error, or even for a different approach to the question. If they cannot show the required result, they may still assume it to be true for subsequent questions.

Students should also be advised that, when checking for possible errors, they should look for simple arithmetic and algebraic errors. When such an error is found, students should be encouraged to rewrite the relevant parts rather than write over the original work.

(a) (i) (2 marks)
A significant number of students found $\frac{dv}{dx}$ for the acceleration. Others who wrote $a = \frac{d(\frac{1}{2}v^2)}{dx}$ had difficulty finding $\frac{1}{2}v^2$ correctly, commonly writing $\frac{1}{2}v^2 = xe^{-x^4}$, $\frac{1}{2}v^2 = x\frac{e^{-2x^2}}{2}$ or $\frac{1}{2}v^2 = 4xe^{-2x^2}$.

(ii) (2 marks)
A number of students mentioned 'the centre of motion', implying some sort of simple harmonic motion. Some tried to solve $e^{-2x^2} = 0$. Others could not successfully solve the simple quadratic equation $1-4x^2 = 0$, for example, obtaining $x^2 = \frac{1}{4}$, but then concluding that $x = \frac{1}{\sqrt{2}}$. Substitution of the value of $x$ was also a problem for some, with a common result being $v = \sqrt{2 \cdot \frac{1}{2} e^{-\frac{1}{2}^2}} = e^{\frac{1}{4}}$.

(iii) (1 mark)
A significant number of students mentioned the 'area under the curve'. Many, despite having the result in front of them, wrote the integral without the limits and/or the $dx$.

(iv) (2 marks)
A surprisingly large number of students did not write any formula at all. Many wrote very informal ones, for example, $\frac{h}{2}(\text{first} + \text{last} + 2 \times \text{middle})$. The value of $h$ was often incorrectly evaluated, and a large number found $f(\frac{1}{2})$ for the middle term. Of particular interest was the number of students who, having written an informal formula such as the one above, and having correctly evaluated $f(1)$, $f(1.5)$, $f(2)$ (in that order), then substituted for $f(1) + f(1.5) + 2f(2)$.

(b) (i) (2 marks)
Students in general did not appear to be competent in working with factorials. Even though $\binom{n}{r}$ was given, the required terms were not correctly expressed in factorial notation. Many did not show that they know of the relationship between $r!$ and $(r+1)!$, or that $(n-r)!$ is larger than $(n-r-1)!$

(ii) (3 marks)
The biggest problem for students seemed to be the number of different pronumerals used. Many students seemed to think that the pronumerals were interchangeable, and many assumed the statement to be true for $j = k$ as their induction hypothesis. Some, in verifying the statement true for $n = 3$, worked with fractions, writing $\binom{3-1}{2}$ as $\left(\frac{3-1}{2}\right)$. Some kept with the notion of fractions in the next step, while others changed to the correct notation. Many did not appear to understand the sigma notation (or even that there was a sequence involved), writing 'assume true for $n = k$, ie $\binom{k-1}{2} = \binom{k}{3} \ldots$' Some, however, then went on to write $S_{k+1} = S_k + T_{k+1}$ and substituted correctly for $S_k$ and $T_{k+1}$.

A significant number did not connect the two parts of the question.

## Question 6

The examiners had expected a much better response to this question at both ends of the spectrum. They expected more professional responses at the top end, and a little more easy mark-gathering at the other.

(a) (i) (1 mark)
This question, which asked for the domain of the inverse function to the function $f(x) = \sec x$ defined on $0 \leq x \leq \pi/2$, was not at all well done. Those students who realised that the domain of $f^{-1}(x)$ corresponded to the range of $f(x)$ tried substituting $x = \frac{\pi}{2}$ into $\sec x$ and wrote statements such as:

$$1 \leq x \leq 0$$
$$1 \leq x < \sec\frac{\pi}{2},$$

while others ignored the given domain of $f(x)$, and so gave $x \leq -1$ or $x \geq 1$. The simple correct diagram showing the relationship between $y = f(x)$ and $y = f^{-1}(x)$ was rare ($< 1\%$), although incorrect diagrams abounded.

(ii) (1 mark)
By way of contrast there were many good solutions to this part, which asked students to show that $f^{-1}(x) = \cos^{-1}\frac{1}{x}$. A typical correct response would argue

$$x = \sec y$$
$$\text{ie} \qquad \cos y = \frac{1}{x}.$$
$$\text{Therefore} \qquad y = \cos^{-1}\frac{1}{x}.$$

There was also, however, much confusion between $(\ )^{-1}$ meaning reciprocal and $(\ )^{-1}$ meaning inverse function. This led to incorrect responses such as

$$\frac{1}{\cos x} = \cos^{-1} x$$
$$\text{ie} \quad \sec^{-1} x = \frac{1}{\cos^{-1} x} = \cos^{-1}\frac{1}{x}.$$

(iii) (2 marks)
The computation of $\frac{d}{dx}f^{-1}(x)$ was well done by a minority, though there were many incorrect computations of $\frac{d}{du}\cos^{-1} u$, and far too many who did not complete (or recognise) the chain rule.

(b) There was a surprisingly large number of students who did not attempt (b), which was worth close to 10% of the marks on the entire paper. This was upsetting in light of the fact that many of these had shown quite sophisticated mathematical skills in (a).

(i) (3 marks)
Most candidates chose the following method:

$$\begin{aligned} B_1 &= AR - M \\ B_2 &= B_1R - M = AR^2 - M(R+1) \\ &\vdots \\ B_n &= AR^n - M(R^{n-1} + \cdots + R + 1) \\ B_n &= AR^n - M\frac{R^n - 1}{R - 1} \end{aligned}$$

The sequencing of the solution was frequently confused, and the question could have made it clear that the first repayment would occur at the end of the first month. However, many centres had clearly been taught this procedure as a matter of drill, although far more had not. Many sign mistakes were made, leaving students with sums that were not geometric. Usually, this did not deter students. They simply claimed that their sequence was geometric and that it led to the expression given in the question. The other main offence was that half the students who started correctly $B_1 = AR - M$, and showed correct sequence to $B_3$, still wrote $B_n = AR^n - M(R^n + R^{n-1} + \cdots + R + 1)$. This error did not prevent students from (incorrectly) claiming that this was equal to the given answer.

Verbal explanations were primitive, and often ended up giving entirely the opposite impression to that intended. Often students' explanations showed that they did not understand what the terms in the expression represented, or that they did not know how they were derived from the given information.

(ii) (1 marks)
This part required students to show that the monthly repayment should be \$622.75. The substitution was generally well done, although the number of candidates who had incorrect values for $R$ or $n$ and still gave the given answer left it as a matter of conjecture as to how many actually did use their calculators.

(iii) (4 marks)
Students were asked to find the number of repayments remaining after a lump sum payment had been made with the twelfth repayment. At this stage many students did prove to be deficient in the operation of their calculators by getting $B_{12}$ wrong from a correct expression.

Very few used the equivalent of

$$0 = (B_{12} - 5000)R^n - M\frac{R^n - 1}{R - 1}$$

to give $n = 37$.

More wrote $(B_{12} - 5000) = AR^n - M\frac{R^n-1}{R-1}$, which gives $n = 23$. The correct answer is $60 - 23$, but few realised the reason for that.

Making $R^n$ the subject of the equation was too tough for most, but those who persevered knew to take logarithms to find $n$, albeit that many were trying to take logs of a negative number!

A small number of candidates used trial and error and occasionally this was set out in a methodical fashion.

## Question 7

(a) The projectile motion question was attempted by most candidates but it was clear that many ran out of time.

(i) (4 marks)
Most students attempted this part, integrating correctly to obtain $\dot{x}$, $x$, $\dot{y}$ and $y$. A large number of candidates introduced an angle $\theta$ (or $\alpha$) (generally undefined), needlessly complicating their answer. A few candidates tried to quote formulae from physics rather than using calculus. These candidates rarely scored any marks for this part.

(ii) (2 marks)
This part was done very well. It was pleasing to see a large number of candidates leaving their answer as $2\sqrt{6}$ seconds.

(iii) (3 marks)
Students had a lot of difficulty dealing with units in this part, that is, converting km/h into m/s. Many paid no attention to units or converted only one of 3.6 km/h and 216 km/h into metres per second. In general the candidates did not handle the drift very well. Those who considered the velocity of the plane relative to the sailor were more likely to complete this part correctly.

(b) (i) (2 marks)
Very few candidates made a serious attempt at this part. Of those who tried to use

$$2^{n-1} = \binom{n-1}{0} + \binom{n-1}{1} + \cdots + \binom{n-1}{n-1}$$

most did not notice that $\binom{n-1}{0}$ was not included in the expression in the question. Even many of those who did correctly adjust for the missing final term did not notice that the first term in the sum of binomial coefficients was also missing.

(ii) (1 mark)
The mark for this part was given if the candidate answered the question correctly given their response to (b) (i). A few candidates successfully answered this part without attempting (b) (i) by using their calculators to evaluate the left-hand side of the expression in the question for various values of $n$.

# Mathematics 4 Unit (Additional)

## Question 1

This question was well answered by a large number of candidates, with many scoring at least 12 of the 15 marks available. A handful of non-attempts appeared among the few scripts that failed to gain any marks at all. At the other end of the scale, many candidates scored the maximum number of marks, showing that they had been well drilled in the methods of integration.

This question gave those candidates who had practised methods of integration a chance to boost their confidence in tackling the rest of the paper, while also providing a challenge by expecting them to identify and use the method most suitable for an efficient solution of the problem.

(a) (2 marks)
Many candidates employed the method of substitution of $u^2$ for $x + 4$ to evaluate $\int_0^5 \frac{2}{\sqrt{x+4}}\,dx$. Those who integrated directly by writing $\frac{1}{\sqrt{x+4}}$ as $(x+4)^{-\frac{1}{2}}$ were equally successful. Through nerves or carelessness, a surprising number of candidates added the values of the primitive function, $f(5)$ and $f(0)$, instead of subtracting them.

(b) (3 marks)
This question, which required candidates to evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin\theta}{\cos^4\theta}\,d\theta$, resulted in a large number of errors. Typical responses included the substitution of $u$ for $\cos\theta$, but the failure to recall that the derivative of $\cos\theta$ is $-\sin\theta$ resulted in the answer being of the wrong sign. Other common errors included arriving at integrals such as $\cos^{-5}\theta$, $\cos^5\theta$, or $\cos^3\theta$. The value of $\cos\frac{\pi}{4}$ as $\frac{1}{\sqrt{2}}$ caused a number of problems, as candidates frequently forgot to use the reciprocal in their calculations. A number of candidates also interpreted the index of $-\frac{1}{3}$ as the cube root. A few candidates attempted to use the $t$ substitutions, but most of them had to resort to 'fudging' to obtain a manageable integrand.

(c) (2 marks)
This question asked candidates to find $\int \frac{1}{x^2+2x+3}\,dx$.

Many candidates attempted to factorise and then completed the square to obtain the correct integral involving inverse tan. Some were unsure how to treat the '2' and either ignored it totally or omitted to take its square root. Thus common answers included $\tan^{-1}(x+1)$ and $\tan^{-1}\frac{x+1}{2}$. Those who attempted to factorise the denominator over the complex field soon gave up.

(d) (4 marks)
Most candidates recognised $\int \frac{4t-6}{(t+1)(2t^2+3)}\,dt$ as an integral involving partial fractions and proceeded accordingly, though many did fail to evaluate $A$, $B$ and $C$ correctly. One pleasing aspect of the responses was the ability of candidates to 'tidy up' the logs to arrive at a simplified answer of

$$\ln \frac{2t^2+3}{(t+1)^2} + C\,.$$

(e) (4 marks)
Again many candidates recognised $\int_0^{\frac{\pi}{3}} x \sec^2 x\,dx$ as an integral which could be solved by the method of integration by parts. Many, however, had difficulty in setting up the required line of $x \tan x - \int \tan x\,dx$ and, of those who did, many could not integrate $\tan x$. The final step of inserting the limits and evaluating was only successfully completed by those candidates who knew their exact values, as the final answer was required to be free of any trigonometric expressions.

**Question 2**

This was a relatively straightforward question on the topic of complex numbers, which was well attempted by the great majority of the candidates. Many gained full marks. However, it must be noted that candidates lost marks for careless numerical errors and (at times) poor algebraic manipulation. It appeared that many candidates may not have read parts of the question carefully.

(a) This part related to expressing a complex number in the 'mod-arg' form and applying de Moivre's theorem.

(i) (2 marks)
One of the marks was awarded for finding the modulus of $\sqrt{3}-i$, and the other for finding the correct argument for $\sqrt{3}-i$.

Many were accustomed to using the notation $\operatorname{cis}\theta$ in place of $\cos\theta + i\sin\theta$, which was perfectly acceptable.

There were frequent numerical errors such as $\tan^{-1}(-\frac{1}{\sqrt{3}}) = \pi - \frac{\pi}{6}$ and $\sqrt{(\sqrt{3})^2+(-1)^2} = \sqrt{3-1} = \sqrt{2}$. Such mistakes often occurred in spite of the fact that they were inconsistent with the student's correct sketch.

(ii) (2 marks)
Here, one mark was awarded for obtaining 64, while the second mark was essentially for deducing that the answer was a negative real number.

Almost everyone used de Moivre's theorem, with only a very few using the binomial theorem or binomial products.

It was noticeable that quite a few left their answer to $(\sqrt{3} - i)^6$ as $64 \operatorname{cis} \pi$, which scored the first but not the second mark.

(b) This part was related to solving a cubic equation with complex coefficients and complex roots.

(i) (1 mark)
Almost all students were able to find that the answer was $8i$. Many showed their arithmetic calculations clearly. Only a few used mod-arg form and de Moivre's theorem.

(ii) (3 marks)
The three marks were allocated for having a suitable method to solve $z^3 = 8i$, for indicating there were 3 roots to this equation and for obtaining the correct roots.

A variety of methods were used to solve the equation. Some students used the mod-arg approach, writing $(r \operatorname{cis} \theta_k)^3 = 8i$, while others plotted $-2i$ and rotated by $\pm\frac{2\pi}{3}$. Some used a mixture of both these approaches. Still others factored $z^3 - 8i$, or resorted to the algebraic expansion and solution of $(x + iy)^3 = 8i$.

Common mistakes were to simply state that $z = -2i$ by using (b) (i). Students using the mod-arg approach often mistakenly applied the method to find the square root, and obtained $r = 2\sqrt{2}$ and $\theta = \frac{\pi}{2} + 2k\pi$.

Those attempting the factorisation method sometimes had problems handling $z^3 - 8i$ as $z^3 - (-2i)^3$ correctly. There were many variations on $(z + 2i)(z^2 - 2iz - 4)$, and subsequent errors in handling the quadratic, such as stating $(-2i)^2 = 4$ in the calculation of the discriminant.

(c) (3 marks)
This part involved shading a region on the Argand plane from 2 modular inequalities. One mark was awarded for representing $|z - 3 + i| \leq 5$ as the interior and boundary of a circle centred at $(3, -1)$ and radius 5. Another mark was awarded for representing $|z + 1| \leq |z - 1|$ as the half plane region $x \leq 0$. The final mark was for finding the 'correct' shaded minor segment region based on the candidates' representation of the solution of two individual inequalities.

It was good to see candidates use a pair of compasses to assist in sketching the circle.

Common errors included having a circle with the wrong centre, or radius $r = \sqrt{5}$. The analysis of $|z + 1| \leq |z - 1|$ was not handled well. A number tried to use vectors and geometry. Even those who could arrive at $x = 0$ algebraically, often shaded as if it was the $\frac{3}{4}$ plane $y \leq 0$ or $x \leq 0$. Others used the idea of perpendicular bisector, but missed the $\leq$ and so arrived at $x = 0$ only. A common incorrect response involved shading the part of the vertical strip with real part between 1 and $-1$, which lies inside the circle centred at $3 - i$ with radius 5.

(d) This involved an application of complex numbers to a question on number theory and Pythagorean triads.

(i) (2 marks)
There was a very good response to this part. Students were awarded one mark each for obtaining $wz = \frac{-33+56i}{65}$ and $w\bar{z} = \frac{63-16i}{65}$.

Carelessness in the arithmetic, such as $3 \times 5 = 8$ or $4i \times 5 = 9i$, cost students marks, as did mistakes in handling the fractions. Many candidates expended considerable time by converting $w = \frac{3+4i}{5}$ to $\operatorname{cis}\alpha$ where $\tan\alpha = \frac{4}{3}$, and $z = \frac{5+12i}{13}$ to $\operatorname{cis}\beta$ where $\tan\beta = \frac{12}{5}$. Some of these were successful with the manipulation for $wz$ and $w\bar{z}$ via $\operatorname{cis}(\alpha + \beta)$, but it is hardly an efficient approach.

A few calculated $wz$ correctly, but then incorrectly stated that $w\bar{z}$ was the conjugate. These students obtained one mark but created difficulties for themselves with the remaining part of the question.

It should also be noted that many candidates needlessly wasted time verifying the information that was given in the question, namely that $|z| = |w| = 1$ before they even started (d) (i).

(ii) (2 marks)
This was a different style of complex number question, and many who scored 13 lost their 2 marks here. It is not clear that the question correctly differentiated between the standard of the candidates, as only a few worked out how to arrive at the correct pairs from the data. Many just saw their answers to (d) (i), used their calculator and stated the answers. Some of the more able candidates may have been in the group who thought it must be more difficult than it was and simply left it.

One mark was awarded for each correct pair from $(a, b) = (16, 63)$ $(25, 60)$ $(33, 56)$ and $(39, 52)$.

## Question 3

(a) (4 marks)
This part was reasonably well done if the student chose the appropriate method. Students using the shell method usually got the wrong sign for the function. (It was the lower part of the circle $x^2 + (y - 2)^2 = 9$.) Many

students showed their lack of understanding of basic algebraic properties. For instance, such working as

$$\begin{aligned} x^2 &= \sqrt{9-(y-2)^2} \\ &= 3-(y-2) \\ &= 5-y \end{aligned}$$

was very common.

(b) (i) (2 marks)
This part was not well done. It was difficult to follow the logic of the solution from the evidence provided by what the students had written. The best method of solution involved completing the square by writing $f'(x) = 15(x^2-1)^2+1$. The next most common (correct) method was employed by candidates who drew a graph of $f'(x)$ and showed that the minimum points were at $(1,1)$ and $(-1,1)$. A common incorrect answer showed that the discriminant of $f'(x)$ was not real, so $f'(x) \geq 0$ and so $f'(x) \geq 1$.

(ii) (2 marks)
This was reasonably well done. A common error was to get one region, $x > 1$. Many students drew a sketch of $f''(x)$ and these students generally answered the question correctly.

(iii) (3 marks)
The graphs were marked very strictly, and the answers were generally disappointing. Many students did not identify the points of inflection. A large number put the points $(1,9)$ and $(-1,-9)$ on the graph but did not label them as inflection points or show any change in concavity at those points. This resulted in marks not being awarded.

Most students did not use parts (i) and (ii) to draw their graph for (iii). This was probably an advantage – many had errors in (ii) but somehow got the correct concavity in (iii).

Having said that, many other students had excellent graphs, well labelled and showing inflection points and concavity clearly.

(c) (i) (1 mark)
Very well done.

(ii) (1 mark)
Also very well done. Two answers, $\frac{3}{5} \times \frac{3}{5}$ and $\frac{3}{5}$ were accepted as this part of the question could be read in two ways. Students who answered $\frac{3}{5}$ here needed to calculate the probability of the event corresponding to the other interpretation of the question to successfully answer the remaining parts.

(iii) & (iv) (3 marks)
These two parts were marked together. They were not done well. Students used inappropriate combinatorial methods. Marks were awarded for recognition of a series solution.

**Question 4**

Overall this question was well handled by students, with between 5% and 10% of the candidates scoring full marks. Few candidates scored low marks on this question.

(a) This part tested knowledge of the hyperbola.

(i) (2 marks)
Students were required to find the foci of the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{7} = 1.$$

Whilst most students could do this correctly, a significant number did not know the correct formula:

$$b^2 = a^2(e^2 - 1),$$

others used wrong values for $a$ and $b$, eg $a = 9$, and some weaker students made mistakes in solving $7 = 9(e^2 - 1)$.

(ii) (1 mark)
Students were asked to find the equations of the directrices of the hyperbola. Those who had incorrectly computed $a$ or $e$ in the earlier part were awarded the mark here if their answer was consistent with their incorrect values.

(iii) (1 mark)
This part asked students to show that the point where the tangent at $P(x_0, y_0)$ on the hyperbola crosses the $x$ axis is $T(\frac{9}{x_0}, 0)$. Almost all could do this but some did not realise you merely had to substitute $y = 0$.

(iv) (3 marks)
This part required students to show that $\frac{PS}{PS'} = \frac{TS}{TS'}$, where $S$ and $S'$ are the focii, and suggested using the focus-directrix definition. Those who used this approach were usually successful in proving the result. However, a large number of students did not know the focus-directrix definition and those who used other methods usually got bogged down in the algebra.

(b) (i) (1 mark)
Students were required to express $\cot 2A$ in terms of $\tan A$. Most were able to do so.

(ii) (2 marks)
This part was well done by most students. It required students to show that $\tan A$ and $-\cot A$ are solutions to the equation

$$x^2 + 2x \cot 2A - 1 = 0\,.$$

The most successful method was to substitute $\tan A$ and $-\cot A$. Other methods applied by students included factorisation, expansion, sums and products of roots and the application of the quadratic formula. Students who used the quadratic formula were generally less successful than those who used other methods.

(iii) (3 marks)
Students were required to find the exact value of $\tan \frac{\pi}{8}$, which is $\sqrt{2} - 1$. Most of those who used the sum of roots were successful, while those who used the quadratic formula again got caught up in the arithmetic.

(iv) (2 marks)
This part asked candidates to deduce the value of $\tan \frac{\pi}{16} - \cot \frac{\pi}{16}$, which is $-2 - 2\sqrt{2}$. Those who had answered the previous part correctly usually had little difficulty.

## Question 5

Many answers to this question were disappointing. There were quite a few very easy marks, which were often thrown away through laziness and thoughtlessness. Quite a few students scored marks in the range 10–14, but very few students were awarded full marks.

Many candidates appeared to have had the ability to have done better. Students would profit from reading the question carefully and thinking before rushing in.

(a) This part involved sketching $y = -x^2$, $y = e^{-x^2}$, and then calculating the volume of revolution obtained by rotating the region between the curve $y = e^{-x^2}$, the $x$ axis, the $y$ axis, and the line $x = 2$, around the $y$ axis. Use of the method of cylindrical shells was required.

(i) (1 mark)
It was disappointing that many students drew very poor sketches. To earn their mark for sketching the parabola, students had to draw something approaching the right shape, and indicate the region $-2 \leq x \leq 2$. Many lost marks for failing to indicate the scale,

or the endpoints, for drawing 'parabolas' with vertical tangents or with 'points' at the vertex, or for placing the vertex away from the origin. The best sketches were done by drawing the parabola carefully (sometimes with a template) and then adding the axes to pass through the vertex, using a ruler. Many of the students who scored 14 out of 15 lost their mark for a poor parabola.

(ii) (1 mark)
To earn their mark for sketching $y = e^{-x^2}$, students had to draw an even function with a maximum at $(0, 1)$, increasing to the left of the $y$ axis, and decreasing to the right. As this function is differentiable, it must be flat at the maximum—some students even drew cusps, which is surprising. Although use of calculus was not required, some students found the inflection points. This led to nicer graphs, but no extra marks, and may be considered to be a waste of time.

(iii) (3 marks)
Those students who knew the method of cylindrical shells found the volume of revolution fairly straightforward. The first mark was awarded for writing the integral expression

$$\int_0^2 2\pi x e^{-x^2}\, dx,$$

or a similar expression (eg with the wrong limits, or with 2 or the $\pi$ missing), the second mark for making an appropriate substitution (eg $u = \pm x^2$, or $u = e^{-x^2}$), and the third mark for the correct numerical answer $\pi(1 - e^{-4})$ (or $3.08\ldots$). Common errors included writing the wrong limits ($-2$ to 2, $e^{-4}$ to 1, 0 to 1), or making simple arithmetic mistakes, which lost one mark, taking $x - 2$ as the axis of rotation (in which case one mark was still possible for integrating $\int xe^{-x^2}$ by substitution), and trying to integrate by parts, which often led to a lot of time being wasted. Students who did not use the method of cylindrical shells could not get the mark for applying this method, nor for integrating $\int xe^{-x^2} dx$. The very few who did obtain the correct answer by another method were awarded full marks.

(b) (3 marks)
This part required the students to resolve forces and find an expression for $F/N$ in terms of $\theta$, where $F$ is the frictional force and $N$ is the normal reaction force on a particle at rest on a plane inclined at an angle $\theta$. To get full marks, students had to resolve in two different directions, indicating these clearly (either in words such as 'horizontally' and 'vertically', or 'perpendicular to' and 'parallel to the plane', or by means of a legible and complete diagram). They then had to derive the formula $F/N = \tan\theta$ for the third mark.

Many students were completely lost with this part and filled the page (or pages) with wrongly remembered formulae. Students should understand how to resolve forces—it's just simple trigonometry—and then this would not happen. Further, they should read the question: many students included $m\omega^2 r$ or $\frac{mv^2}{r}$ terms, appropriate to a moving particle. Unless they subsequently set $v = 0$ or $\omega = 0$, they could not express $F/N$ in terms of $\theta$. Moreover, many students who included this centripetal term gave it the wrong sign, considering it as an extra force acting rather than as a resultant force leading to acceleration.

(c) This part dealt with a biquadratic polynomial $P(z) = z^4 + bz^2 + d$.

(i) (1 mark)
Students were first required to show $P'(z)$ is odd. About 10% of students failed to calculate $P'(z)$ correctly, and others got horribly confused about the number of minus signs required, but most found this an easy mark.

(ii) (2 marks)
Next, students were required to show that, if $\alpha$ is a double root of $P(z)$, then $-\alpha$ is too. The most common, and perhaps best, approach to this was to observe first that $P(\alpha) = 0$ and $P'(\alpha) = 0$ (since $\alpha$ is a double root), then, using the facts that $P(z)$ is even and $P'(z)$ is odd, deduce that $P(-\alpha) = 0$ and $P'(-\alpha) = 0$, and finally conclude that $-\alpha$ is a double root of $P$. When this method was employed, the two marks were awarded for showing that $P(-\alpha) = 0$ and for showing that $P'(-\alpha) = 0$. Many students proved only one of these, and many were not awarded the mark for one or the other part because they did not express their argument clearly. For example: 'Let $-\alpha$ be a double root. Then $P'(-\alpha) = 0$. But $P(-\alpha) = 0$. Therefore $-\alpha$ is a double root.' This 'solution' was awarded 0. While it contains the two crucial statements $P(-\alpha) = 0$ and $P'(-\alpha) = 0$, no valid reasons are given. Other methods tried, usually unsuccessfully, involved relating the roots of the equation $P(z) = 0$ to the coefficients.

(iii) (2 marks)
this part required candidates to show that $d = b^2/4$. At least twenty different methods were used. The most common, and most likely to be awarded the two marks available, involved using the sum of products of the roots in pairs to show that $b = -2\alpha^2$ and using the product of all the four roots to show that $d = \alpha^4$. The desired result follows by eliminating $\alpha$. With this (and other) methods of solution, the first mark was given for exhibiting a viable method, and the second for justifying the key step, eg writing 'the sum of the products of the roots in pairs is equal to the coefficient of $z^2$ divided by the coefficient of $z^4$', or an abbreviated version of this.

(iv) (1 mark)
The two final subparts of this equation involved deducing information about the coefficient $b$ from information about the roots. In this case,

students were asked to find $b$ given that $i\sqrt{3}$ is a root. The students who showed in (iii) that $b = -2\alpha^2$ should have had little difficulty in showing that $b = 6$.

(v) (1 mark)
Given that the roots are real, students were asked to find what range of values for $b$ is possible. Again, the students who showed in (iii) that $b = -2\alpha^2$ should have had little difficulty in showing that $b \leq 0$ in the second. (Actually, $b < 0$ is the correct response, as $d = b^2/4$ and $d \neq 0$ in the data, but $b \leq 0$ was also awarded the mark.) However, many of the students who scored 14 out of 15 lost their mark on the last part – the words 'real roots' set off an automatic response '$\Delta \geq 0$, whence $b^2 - 4d \geq 0$, so $b^2 \geq 4d$.' In fact, $b^2 = 4d$ from (iii), so this was automatically satisfied. The point was that the solutions to $z^4 + bz^2 + d = 0$ are the square roots of the solutions to the quadratic $u^2 + bu + d = 0$, ie the square roots of $\dfrac{-b \pm \sqrt{b^2 - 4d}}{2}$. As $b^2 - 4d = 0$, the solutions are the square roots of $-b/2$.

## Question 6

(a) This involved summing a geometric progression, proving an inequality using the sum, integrating the inequality and deducing an approximation for $\pi/4$. Each step was reasonably simple. The marks for part (a) were, however, quite low. Almost half the candidates did not attempt parts (ii), (iii) and (iv).

(i) (1 mark)
This was most simply answered by stating that the series is a GP with common ratio $-x^2$, and using the formula for the sum of a GP to obtain the result. Alternatively, one could show that $(1 + x^2)(1 - x^2 + x^4 \cdots + x^{4n}) = 1 + x^{4n+2}$. Most of those who attempted this part gained the mark. There were a surprising number of non-attempts, some moderately successful attempts at long division, and a few attempts at proof by induction.

(ii) (2 marks)
Using part (ii), this amounted to showing that

$$\frac{1}{1+x^2} \leq \frac{1+x^{4n+2}}{1+x^2} \leq \frac{1}{1+x^2} + x^{4n+2}.$$

Of course, this is glaringly obvious. However, the question asked candidates to show that it is true, and so marks were not awarded unless adequate reasons were stated. For 2 marks, reasons had to include the statement that $4n + 2$ is even, so that $x^{4n+2} \geq 0$ for all

$x$, and either $1 + x^2 \geq 1$, or $1 + x^2 \geq 0$ (in the case where the right-hand side was re-written as $\frac{1+x^{4n+2}+x^{4n+4}}{1+x^2}$). Only a handful of students scored 2 marks. Unsuccessful attempts included illogical arguments, wrong reasons (for example, $x^{4n+2} \geq 0$ because $x$ is positive, or because $n$ is positive), or no reasons at all. It needs to be emphasised to students that no matter how obvious the result, their job is to convince the examiner that they understand why it is true.

(iii) (3 marks)
This involved recognising that the result could be obtained by integrating the inequality in (ii) between 0 and $y$, and using the fact that $0 \leq y \leq 1$ to obtain the right-hand inequality. Most candidates who attempted this part took indefinite integrals of the functions in (ii), and then switched variables from $x$ to $y$ (sometimes adding constants of integration in various places). Since this does not really make any sense, such responses were not given full marks. Students also had problems with the term $\frac{1}{4n+3}$ on the right-hand side. Many tried to avoid the problem by claiming that the integral of the right-hand side in part (ii) was equal to $\tan^{-1} x + \frac{1}{4n+3}$.

(iv) (1 mark)
The result is obtained by substituting $y = 1$ and $n = 250$, and knowing $\tan^{-1} 1 = \pi/4$. Students who made it this far through part (a) were generally a little more successful here than on parts (ii) and (iii). Most of the unsuccessful attempts were due to an inability to find an appropriate value for $n$. Some students substituted $y = 0$ on the left-hand side, and $y = 1$ elsewhere (perhaps because they did not believe that it was legitimate to subtract $\pi/4$ from all three expressions).

(b) This was a standard resisted motion question, involving finding the equation of motion of a ball thrown vertically upwards, finding the maximum height reached and the time taken to reach that height, and the velocity of the ball when it returned to the origin. Most candidates attempted this part quite successfully.

(i) (2 marks)
Students were given $\ddot{y} = v\frac{dv}{dy} = -\frac{v^2}{10} - 10$, and asked to derive the equation $v^2 = 164e^{-y/5} - 100$. The initial condition $v = 8$ when $y = 0$ was given in the preamble. Most students were able to obtain $\frac{dy}{dv} = -\frac{10v}{v^2+100}$ (or something equivalent), integrate correctly, and use the initial condition to find their constant of integration. Re-arranging $y = -5\ln(v^2 + 100) + 5\ln 164$ into the required form proved difficult for quite a few students. The significant number of students who wrote (correctly) $\frac{dy}{dv} = \frac{10v}{-v^2-100}$, followed by $y = -5\ln(-v^2 - 100) + C$, omitting the absolute value signs, obviously got into difficulty.

(ii) (1 mark)
This was successfully answered by almost all those who attempted it. It simply involved recognising that the maximum height is reached when $v = 0$, and then solving the equation $164e^{-y/5} = 100$. Some candidates failed to realise that $\ln \frac{100}{164}$ is a negative number, and so thought that their answer of $-5 \ln \frac{100}{164}$ was negative. This caused some of them to write things like $y = \left|-5 \ln \frac{100}{164}\right| = 5 \ln \frac{100}{164}$.

(iii) (2 marks)
This part was very similar to (b) (i). It involved solving the differential equation

$$\frac{dt}{dv} = -\frac{10}{v^2 + 100},$$

using the initial condition as in (i), and finding $t$ when $v = 0$. The integration involves a simple standard integral, and this part of the question was well done by most candidates. An alarmingly large number of students, however, wrote the answer which is $\tan^{-1}(0.8)$ as 38.66.

(iv) (3 marks)
This was not as well done as the other parts of (b). Basically, this part is a repeat of part (i), using the equation $\ddot{y} = 10 - \frac{v^2}{10}$, with downwards as the positive $y$-direction (or $\ddot{y} = -10 + \frac{v^2}{10}$ with upwards as the positive $y$-direction). In the first case, the initial condition is $y = 0$ when $v = 0$, and the value of $v$ when $y = 5 \ln \frac{164}{100}$ is to be found. (With upwards as the positive $y$-direction, $y = 5 \ln \frac{164}{100}$ when $v = 0$, and $v$ is to be found when $y = 0$.) An extremely common error was to take down as the positive direction, with $y = 5 \ln \frac{164}{100}$ when $v = 0$, and to find $v$ when $y = 0$. (This leads to $v^2 = -64$.) Other errors included simply writing down the number 8 as an answer, or using the equation given for $v^2$ in part (i), and substituting either $y = 0$ or $y = 5 \ln \frac{164}{100}$. Students who started with $v\frac{dv}{dy} = -10 + \frac{v^2}{10}$ invariably integrated to obtain $y = 5 \ln(v^2 - 100) + C$, again with absolute value signs missing (and, of course, $v^2 < 100$). Either there are a lot of students who don't know that $\int \frac{dx}{x} = \ln |x|$, or else they think the absolute value signs are unimportant.

## Question 7

In general this question appeared quite difficult for many students. Perhaps being confronted with three diagrams in the one question caused anxiety to those who were not very proficient in geometry. However, many other syllabus topics were interwoven in the three main sections to this question, and students who persevered found it challenging and rewarding.

There were some relatively easy marks to be found. However, a sound knowledge of content and a proficient level of mathematical ability seemed necessary to obtain high marks.

(a) (2 marks)
To show that $OU \times OT = 1$ implies some form of ratio manipulation and students who started the question using similar triangles had a chance of success. Choosing the relevant triangles caused problems. Many combinations were tried. This could have been avoided by using the triangles involving $OU$ and $OT$.

Using the sine or cosine ratio was another method, surprisingly not commonly used. The use of Pythagoras' theorem was long and involved and for many led nowhere, using up valuable time. A few candidates showed $SORT$ was a cyclic quadrilateral and used $OU \times UT = SU \times UR$ to gain the result. Whichever method was chosen, poor letter formation often caused confusion between $O$ and $U$. Students who took the time to copy the diagram usually achieved the correct result more economically.

(b) The above comment also applies to this section. A large diagram with all the relevant information avoided circle and point confusion.

(i) (1 mark)
The main objective here was to state that $\angle QPR = 90°$ as $PR$ is a tangent to the inner circle. Many then went on to laboriously prove $\angle OUP = 90°$, a fact stated in the question.

Another less common method to show $OU \parallel PQ$ involved the use of similar triangles where $UR$ was extended to the $x$ axis.

(ii) (2 marks)
Showing that $RS$ has the equation $x\cos\theta + y\sin\theta = r(1+\cos\theta)$ was one of the best answered sections of the question. Realising that the gradient of $RS$ was $-\frac{1}{\tan\theta}$ avoided implicit differentiation of the inner circle, or the chance of algebraic error in handling

$$\frac{-(r + r\cos\theta - r)}{r\sin\theta - 0}.$$

(iii) (1 mark)
Using the perpendicular distance formula with the point $(90, 0)$ and the abovementioned line gave the result simply and quickly. This was rarely seen. A more common method was to solve $OT$ ($y = x\tan\theta$) with $RS$ ($x\cos\theta + y\sin\theta = r(1+\cos\theta)$) and use the distance formula with points $O$ and $U$, inviting much scope for error, and again using valuable time.

Ratios of corresponding sides of formed similar triangles were attempted with no success. One extremely efficient way was demonstrated only a few times. It involved dropping a perpendicular from

$Q$ to meet $OU$ at $B$, giving $OB = r\cos\theta$ as $\angle UOQ = \theta$ from part (i), then realising that $OU = OB + BU$ ($BU$ being $r$).

(iv) (3 marks)
Only the more able students completed this locus question. Most stopped at $OT \times r(1+\cos\theta) = 1$. Those who proceeded with $OT = \sqrt{x^2+y^2}$ found difficulty in eliminating $\theta$.

Another method was to drop a perpendicular from $U$ to the $x$ axis and find expressions for the $x$ and $y$ coordinates of $T$ in terms of $\theta$, knowing that $OT = \frac{1}{r(1+\cos\theta)}$ from part (a). Although this required one tricky factorisation step, it avoided dealing with the square root. Of the few that attempted this section, this latter method was the more common.

(c) (i) (1 mark)
Substituting $y = \frac{x^2}{4a}$ in $x^2 + y^2 + 2gx + 2fy + c = 0$ was successfully done by those who attempted this section, although quite a few found a quartic equation in $y$.

(ii) (3 marks)
Students had to realise that a double root existed at $J$, that the sum of the roots one at a time equalled zero (from the quartic equation) and then deduce that $M$ and $J$ were equidistant from $N$ ($x$ coordinates). Expressing the last condition proved difficult even though the result appeared obvious from the preceding work.

If the $x$ coordinate of $J$ is $\alpha$ and the $x$ coordinate of $M$ is $-\alpha$ (since $-\alpha = \frac{\gamma+\beta}{2}$) then since the $x$ coordinate of $N$ is $O$, $JN = NM$. A common false premise was that $N$ was the centre of the circle.

(iii) (2 marks)
Students who realised that the perpendicular heights of $\triangle JNK$ and $\triangle NMK$ were the same and also that $\triangle JKM$ and $\triangle JML$ had equal perpendicular heights and chose the correct bases had little difficulty in deriving the fractional explanation required. If the wrong bases were chosen, then constructions involving congruent triangles followed.

A method involving use of the formula Area $= \frac{1}{2}ab\sin C$, was efficient though rarely done correctly. Another method, which was not seen, is to construct parallels and find equal heights by the ratio-intercept theorem.

## Question 8

(a) Throughout this part, the claim for a similar proof was often made, usually wrongly, and is obviously not a safe procedure for most candidates. For example, one can accept that $CXAE$ is similarly cyclic, but after marking many scripts, it was clear that the claim that $\angle CXE$ and $\angle AXE$ are similarly $\frac{\pi}{3}$ is dubious in the absence of the cyclicity of $CXAE$. It is certainly invalid to claim that the quadrilateral $AFBX$ is similarly cyclic, because that fails to realise that $X$ was defined as the join of $AD$ and $BE$. Finally, whereas one may reasonably claim that similarly $\triangle CFA \equiv \triangle EBA$, the bare claim that because $BE = AD$, then similarly $BE = CF$, is dubious in the absence of any statement of the preceding congruence (and nearly half of those who did state a justifying congruence gave a wrong congruence).

A minority of candidates understood most of the issues well and were able to explain their logic in the clear and neatly written prose conventionally used in geometry. However, most seemed to have little real understanding of collinearity, concurrency or concyclicity, and most of those who ventured beyond part (ii) could not explain their ideas clearly. With theorems and their converses constantly alternating, the continual confusion in written solutions between premise and conclusion was often costly, and one must question the standard procedure of many candidates in first writing down the result to be proven with no indication that it is their aim rather than a fact to be used. Many of the stated reasons were off the point, ambiguous, unreadable, or plain wrong. With so many intervals for angles to stand on, the frequent absence of any named intervals in the reasons was often dangerous. Chasing angles is also a dangerous occupation, because large numbers of wrong vertices were written in places where it was impossible to establish that the candidate had made a forgivable transcription error. Finally, handwriting was routinely a serious problem, with illegible words, illegible vertices in named angles, ambiguous overwriting, and many signs whose meaning remained unclear. Such things are of course accentuated in the last question, but this was obviously not the principal reason for the many problems mentioned here.

(i) (1 mark)
Most who attempted to do so were able to establish the equality of the overlapping angles.

(ii) (1 mark)
Again, the congruence was shown by most who attempted this part, although quite a few omitted mentioning the congruence test, and a significant minority had no real understanding of congruence or confused it with similarity.

(iii) (2 marks)
By contrast, most candidates found this rather difficult. Many seemed only to know the opposite angles test for a cyclic quadrilateral, which is difficult to apply here, and far too many assumed the quadrilateral cyclic in order to prove it so.

(iv) (2 marks)
Again, this proved rather difficult. Many who established correctly that $\angle BXC = \frac{2\pi}{3}$ were then unable to move on to find the individual angles $BXD$ and $CXD$, and there was often confusion about whether the sizes of all four angles had been securely established.

(v) (2 marks)
This result was correctly proven only by a handful of students. Most who attempted it could write down a reasonable condition for collinearity, but then inadvertently assumed collinearity, for example by claiming that $\angle BXF$ was an external angle of the cyclic quadrilateral $BDCX$, or more blatantly, by claiming that $\angle BXF$ was vertically opposite to $\angle CXE$.

(vi) (2 marks)
This straightforward part was usually forgotten. If this was a result of the difficulty of part (v), it demonstrates poor examination technique. Many who attempted it claimed a false pair of congruent triangles for the second equality.

(b) (i) (1 mark)
The examiners wanted a clear statement that $\omega$ was an anticlockwise rotation of $\frac{\pi}{3}$, and that such a rotation was needed because the vectors $t = \overrightarrow{OT}$ and $u = \overrightarrow{OU}$ are of equal length and inclined at $\frac{\pi}{3}$. The explanations ranged from three or four words to page-long verbosity, and although most who attempted it scored the mark, few seemed competent in handling the prose required.

(ii) (1 mark)
There were very many possible forms of the answer to this part, but it was a concern that a substantial number who understood that the rotation involved was now clockwise gave the answer $r = -\omega s$.

(iii) (3 marks)
Most got into a huge mess trying to prove that $|t - \overline{\omega} s| = |\omega t - s|$, a situation easily avoided by rewriting the result of part (ii) as $s = \omega r$. The large variety of routine errors included claiming that $RT = |r+t|$, claiming that $|t - r| = |t| - |r|$, claiming that because $|t| = |r|$ and $|u| = |s|$ then $|t-r| = |u-s|$, claiming that $|\omega t-s| = |t-s|$, mistaking the length $RT$ for the product $rt$, and claiming that $|\omega t - s|^2$ was equal to $(\frac{1}{2}t - s)^2 + (\frac{1}{2}t\sqrt{3}\,)^2$. A few did resolve correctly into real and imaginary parts before using Pythagoras to find distances, but only a handful were then able to push the resulting difficult calculation through. Many solved the problem by using either congruence or the cosine rule, or by realising the connection with part (a). Such solutions could gain at most 2 of the 3 marks because the question required the use of complex numbers.