2001 HSC Notes from the Examination Centre

Mathematics, Mathematics Extension 1 and Mathematics Extension 2

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2001 HSC NOTES FROM THE EXAMINATION CENTRE MATHEMATICS, MATHEMATICS EXTENSION 1 AND MATHEMATICS EXTENSION 2

Introduction

These notes from the examination centre are distilled from comments provided by markers on each of the questions from the Mathematics paper, together with comments on most questions from the two extension papers. The comments outline common sources of error and contain much good advice on examination preparation technique. Candidates and teachers should also be aware that past examination reports for 2 unit, 3 unit and 4 unit Mathematics are still relevant to the calculus-based Mathematics courses in the new HSC, and are an excellent source of advice on examination technique and presentation.

Mathematics

Question 1

This question was generally well done, with few zeros awarded and many candidates achieving full marks. However, in every question part there was evidence of candidate failure to read the question carefully and answer the actual question that was asked.

(a) There was confusion among many candidates between 3 significant figures and 3 decimal places. Calculation errors were common, especially not taking $\sqrt{87}$ in the denominator.

(b) Candidates had the most success if they solved $-2 < x + 3 < 2$. Various approaches to graphing were seen, many of which did not clearly indicate the required solution set. To achieve full marks, candidates had to deal with inequalities and not just critical points.

(c) This part was well done with the anticipated errors using the formula or incorrect factors $(x + 4)(x - 2)$ being commonly seen.

(d) The most common errors were either using $3 + \frac{1}{x} = 3x + 1$ or differentiating. Many did not recognise the answer $\ln x$ from the standard integrals.

(e) Poor algebra skills were evident from many candidates, such as dropping off the denominator, an inability to factorise $(x^2 - 4)$ and incorrect cancelling of supposed factors.

(f) Many candidates scored full marks. The most common error was finding 90% of \$979. Interestingly, a few candidates used the GST method of finding $\frac{10}{11}$ of \$979.

Question 2

Presumably set with the intention of providing straightforward marks for the majority of candidates, this question exposed major weaknesses in the abilities of many candidates.

(a) This was generally well done with most candidates scoring full marks. All but a handful were able to differentiate correctly to obtain $2x + 3$ but significant numbers went on to either equate this to zero, deduce that the gradient must be 2, or evaluate $2(1) + 3$ as 6.

(b) (i) The majority of candidates successfully found the gradient and correctly substituted into the point-gradient form. A small minority used substitution to verify that the coordinates of A and B satisfied the given equation.

(ii) This posed few problems. A surprisingly large number of candidates had scored full marks (or nearly so) at this point and then either failed to continue or did so without gaining further credit.

(iii) Many candidates did not know the perpendicular distance formula and, of those who quoted it correctly, a significant number made errors in substituting and/or simplifying or failed to correctly remove the absolute value signs. Common alternative approaches were to assume that the required distance was the distance from the origin to either A, the y intercept of AB, or the midpoint of AB. A small minority, with varying degrees of success, tried to find the equation of the perpendicular to AB from O, the coordinates of (P) the foot of this perpendicular, and hence the length of OP.

(iv) It was disturbing to see how many candidates did not know the formula for the area of a parallelogram. Some attempted the potentially correct method of regarding $ABCO$ as a trapezium, others condemned themselves by treating the figure as either a triangle or a rhombus. Of those who used a 'base times height' approach, a large number used a height that was not the perpendicular one – including many who had correctly answered part (iii)! Other approaches showed varying degrees of knowledge, ingenuity and desperation.

Of those who correctly saw that they needed only to multiply the results of parts (ii) and (iii), a surprisingly large number were unable to successfully simplify $2\sqrt{10}\times\frac{13}{\sqrt{10}}$.

(v) Relatively few candidates realised that the answer could be obtained by dividing the area by the length BC (or equivalently AO). Most proceeded to find the equation of BC, usually not by the most efficient manner, and then (again) employing the perpendicular distance formula – a repetition which should have signalled to more alert candidates that a better alternative existed.

All in all, the handling of the question showed that many candidates lacked a clear understanding of some basic techniques in coordinate geometry.

Question 3

This question consisted of four parts taken mainly from the calculus (logarithm and exponential functions) and trigonometry (cosine rule) areas of the syllabus. On the whole the question was reasonably well attempted with the majority of candidates gaining two-thirds or more of the marks allocated for the question.

(a) This proved difficult for candidates with many responses having no reference to the logarithm function in their answer. Incorrect simplifications of the correct answer $(\ln 5 - \ln 4)$ to $\ln 1$ and $\frac{\ln 5}{\ln 4}$ were common.

(b) This was an easy two marks for most candidates and proved to be the best answered of the four parts of the question. In fact, for some candidates, these were the only two marks they scored in the question. Some candidates using specific brands of calculators were not aware that they had to place the fractional exponent $\frac{2}{3}$ in brackets to obtain a correct approximation for $70^{\frac{2}{3}}$.

(c) (i) Of the two sub-parts in (c), this was by far the worst answered. Many attempted to apply the product rule stating that the derivative of ln was $\frac{1}{x}$. Others rewrote the expression as $\ln x^2 - \ln 9$ before taking a derivative.

(ii) This part was very well attempted. Candidates on the whole demonstrated their familiarity with the quotient rule and its application to the given function. However attempts to simplify the immediate answer obtained were poor with many instances of incorrect factorisation and 'cancelling' of terms in the numerator and denominator. The most common error was to write the denominator $(e^x)^2$ as e^{x^2}.

(d) The most common incorrect approach was to proceed as if the side length required was opposite the given angle of 60º, starting with x^2 as the subject of their expressed cosine rule and writing $x^2 = 7^2 + 13^2 - 2 \times 7 \times 13 \times \cos 60^\circ$. Having derived the correct quadratic equation, many candidates stopped there, thinking they had fulfilled all the requirements of the question. Others went straight from their opening statement of the cosine rule to the (given) quadratic equation, thus failing to satisfy the 'show' requirement of the question. It was good to see the number of candidates who attempted the latter part of the question (solving the quadratic equation) despite not knowing how to do the earlier part. The significant number of candidates who realised the need to discard their negative solution, because it was not relevant to the conditions of the question, was also pleasing.

Question 4

Many candidates lost no more than one or two marks in this question. But candidates at too many centres scored no marks in part (a), most not even attempting an answer. Candidates need to remember that Preliminary work is examinable. Similarly, candidates at too many centres scored negligible marks in part (c), demonstrating few skills in sketching sine curves and straight lines, and even less skills in using the table of standard integrals to integrate $3\sin 2x$ and $\frac{x}{4}$. Almost all candidates attempted part (b), some managing to fill up to two pages when 7 or 8 lines would have been sufficient. Many who scored full marks in part (b)(i) made no attempt at part (b)(ii); others scored full marks in the better done part (b)(ii) after losing one, or both, marks in part (b)(i).

(a) Generally well done by those candidates who used properties of the discriminant, the main source of error when solving for k being the minus sign in $4 - 12k < 0$. A significant portion of this candidature first solved $\Delta = 0$; over half then failed in their determination of values of k giving $\Delta < 0$.
Only a few of the candidates who attempted a calculus-based solution, or one involving the use of $x = -\frac{b}{2a}$, to find the turning point of a parabola, gained the first mark.

(b) A clear, sufficiently large diagram should always be drawn. As this question is about angles, these should be marked on the diagram. The question should be solved on the diagram. This then provides the basis for the logical sequence of steps that forms the written proof. All statements should have a reason, concise but understandable, attached in brackets.

(i) Many candidates who probably knew how to do this part lost marks by not providing sufficient information. The successive statements $\angle ACB = 180 - 2x^\circ$ (isos. Δ). $\therefore\ \angle ABC = 180 - 2x^\circ$ were not rewarded by markers, whereas $\angle ACB = 180 - 2x^\circ$ (angle sum isos. Δ CLM, base angles = x°) $\therefore \angle ABC = 180 - 2x^\circ$ (data) received full marks.

(ii) Candidates with poor algebra skills often did not simplify brackets or arrived at negative angles when simplifying, for example, $180 - (180 - 2x^\circ) = -2x^\circ$. They should be advised to simplify brackets at the earliest opportunity. Although the majority of these candidates indicated correct steps leading to the required answer, many 'fudged' to obtain it. They did not obtain full marks.

(c) (i) The best sketches of $y = 3\sin 2x$ were achieved by those candidates who used a template, and then fitted the appropriate scales. In freehand sketches care should be taken with the placement of maximum and minimum points, the symmetry and smoothness of the curve; $y = 3\sin 2x$ is not 'saw-tooth'. The main error was in the period; many candidates found the correct period but did not transfer it to their graph. A significant proportion of the candidature used 'software' coordinate axes, with the y–axis scale drawn on the far left–hand side of the diagram. Many of these candidates failed to gain the first mark in (ii) when they used the wrong origin for $y = \frac{x}{4}$.

(ii) Many candidates who ruled axes then sketched $y = \frac{x}{4}$ freehand, often with little regard to scale or the fact that the line passes through the origin. Incorrect shading resulted in the loss of the second mark, all too often being a region between the line and the x-axis, or the whole region bounded by $y = 3\sin 2x$ and $y = \frac{x}{4}$ including $x <$ 0. Many candidates made their line pass through the sine curve at $x = \frac{\pi}{4}$.

(iii) Candidates should show substitution into the primitive before proceeding with the evaluation. Even amongst those candidates who achieved a correct primitive $-\frac{3}{2}\cos 2x - \frac{1}{4}\cdot\frac{x^2}{2}$, many were unable to simplify fractions, $\frac{1}{4}\cdot\frac{x^2}{2}$ becoming $\frac{x^2}{2}$, $2x^2$ or $\frac{x^2}{6}$ and $\frac{1}{8}(\frac{\pi}{4})^2$ becoming $\frac{\pi}{4}$. Many replaced $\frac{\pi}{4}$ with 45° when evaluating $\frac{x^2}{8}$. Further errors that prevented the awarding of the second mark were failure to write down the exact value of $\cos\frac{\pi}{2}$, or evaluating $\cos(2 \times 0)$ as $2\cos 0$ and $\cos(2(\frac{\pi}{4}))$ as $(\cos\frac{\pi}{4})^2$.

A few candidates did evaluate the second part of the integral using its interpretation as the area of a right-angled triangle.

Question 5

This question was not answered well by the majority of candidates. The mean for this question was around 5 with very few gaining more than 8 marks. Improvement in showing working, knowing formulae and how to use them, and understanding how to calculate range and domain, are recommended.

(a) Candidates are advised to begin their answers to these types of questions by setting up the expression under the square root as greater than or equal to zero. They should then find the solution to this inequality, clearly identifying the resulting region as the domain. Many candidates incorrectly thought the 2 outside the square root sign caused the domain to change. Few realised there was both an upper and lower limit to the range. The technique, used by a number of candidates, of letting $x = 0$ and $y = 0$ rarely proved successful in identifying the correct regions for domain and range. A number of candidates showed the need for more practice in using inequalities.

(b) (i) Candidates who were able to identify the correct log law to use generally obtained full marks. Many candidates became sidetracked by using the change of base formula or the definition, neither of which helped with the solution.

(ii) Less than one percent were able to obtain a mark in this section. Candidates failed to attempt to use the result of part (i) to assist with their solution. Successful candidates generally listed the log and the number of digits for 2^{10}, 2^{20}, 2^{30}, and then identified the pattern.

(c) Successful candidates were able to identify the correct formula, change 30º to radians and then round their answers correctly to the nearest millimetre. There were significant errors in all of these areas. Many substituted 30º instead of $\frac{\pi}{6}$, left their answer as $\frac{48}{\pi}$ or incorrectly rounded 15.278 cm to the nearest mm. Candidates who converted to millimetres before substitution made fewer errors in rounding. Candidates need to check they have answered the question.

(d) (i) Candidates who found the area of each strip using the trapezoidal rule and then added the three areas had a much higher success rate than those who used the formula across the whole region. Candidates using the combined formula made many mistakes including miscalculating the strip width despite it being given ($\frac{b-a}{n}$ is not understood by many), not being able to read the first and last function values correctly, or confusing the formula with Simpson’s rule.

(ii) Being able to see the link to part (i) enabled candidates to achieve full marks in this section. Many did not see the link and errors included trying to integrate y^2 and then multiplying by π, or giving $0.5 \times 60 \times 60 = 1800$ as the answer. The concept of ‘volume of water’ was not well understood.

Question 6

This question was well done by the majority of candidates. More than 50% of candidates scored 8 or above. The modal score was 9, but many candidates scored 11 out of 12.

(a) This was very well done by most candidates. Those who lost marks used an incorrect formula. Most recognised that the series was arithmetic.

(b) This part was well done by candidates who knew their 'log laws'. Failure to manipulate the equations meant that this was a place where candidates lost 2 marks. It was obvious that many answers were lucky guesses. ('It must have something to do with logs, ln1.23 = 0.207') It is recommended that candidates show more than 2 decimal places in their answers. It was also obvious that some used 'trial and error' as their preferred method.

(c) (i) This part was also very well done. Most common errors arose from incorrect factorisation and solution of $y' = 0$. The correct substitution of $x = \frac{1}{3}$ to find y was pleasing, but too many candidates made errors when substituting $x = -1$ to find the ordinate of A.

(ii) This required that candidates solve an inequality. Too many merely substituted $x = \frac{1}{3}$ and $x = -1$ into y''. This was awarded no marks. Some found the point of inflexion but failed to test before and after to determine which side was concave up. Others gave the answer $x \geq -\frac{1}{3}$ to gain only one mark. Even though the graph was provided, the question required that candidates 'give reasons'. Failure to do so caused a loss of a mark. Very few candidates (2%) scored full marks.

(iii) This part was poorly done, as many incorrectly attempted to use the discriminant of a quadratic to find real solutions, while others attempted a solution involving α, β and γ. A clearer understanding of graphical properties is needed.

Question 7

Overall, the question was well done with many candidates gaining near full marks. Most candidates were able to answer one or two parts to gain some marks. Many candidates were hindered by poor algebra skills. Candidates were often unable to express their reasons in clear, unambiguous mathematical prose. The concept of a 'limiting velocity' was understood by very few candidates.

(a) Candidates often did not gain full marks because they did not make y^2 the subject of the equation correctly or through algebraic or arithmetic errors in calculating the intercepts. Others interpreted the equation as representing a circle and found a 'radius' that was used as one of their limits of integration.

(b) Candidates who produced tree diagrams were much more successful in both parts. The most common mistake in (b)(i) was to assume that Onslo connected on his first attempt as well. Others had the correct probabilities for each attempt, but added them instead of multiplying. Candidates should note that it is important that they record their unsimplified answer in questions such as this, as an answer to (b)(ii) such as 0.016 without any supporting working does not necessarily convince the examiner that the candidate has correctly computed the probability.

(c) Most candidates were able to gain the mark in (c)(i), although many found that $x = -1$ and then stated that the displacement was 1. Only a few were able to show the alternate form for the displacement in (c)(ii). Most merely showed that if it happened that they were equal, then it would follow that 1 = 1.

The majority of the candidature understood that 'at rest' meant that the velocity was 0. In (c)(iv), answers such as 'the velocity approaches 0 as *t* increases' were accepted.

Question 8

(a) Generally, candidates knew how to do this and a very high percentage scored 4 or 5 marks. The most common errors were failure to recognise that the value of N_0 was 18 and poor logarithmic or exponential calculations. While most candidates used their calculated value(s) through their work, a significant number inappropriately used rounded-off figures leading to a broad range of answers. Rounding-off is to be discouraged until the final stage of any calculation.

(b) (i) Almost all candidates scored this mark.

(ii) A significant number of candidates failed to understand the method required for non-replacement. This type of problem does not lend itself to a full tree diagram (far too many branches) and most who attempted one had failed to identify the pattern of choices.

(c) More than 70% of all candidates either failed to attempt this part or failed to score any marks.

(i) Most attempts showed clear confusion about the concept of maximum/minimum in relation to what is an unfamiliar problem. Many misinterpreted the maximum/minimum asked for as relating to the width of the flask rather than to the rate at which the depth was changing.

(ii) Many of the attempts were poorly planned and could have been improved by doing a rough sketch first. The flask could not hold a depth beyond 10 cm ('full') so the graph should not rise beyond the point (5, 10) and candidates should have recognised the domain $0 \leq t \leq 5$ and range $0 \leq y \leq 10$. It is important to know what an inflexion looks like (change of concavity).

Overall, it was pleasing to see only a few feather graphs, cross-outs or multiple arcs – a vast improvement. Attention must still be drawn to the need for clearly and carefully labelled (straight line) axes and the need for the graph to begin and end at particular points if the domain is limited.

Question 9

(a) Many candidates failed to recognise the link between the three sub-parts of part (a). A clearer understanding of terms such as *hence* and *deduce* needs to be encouraged. While there were a significant number of concise, well-argued responses, too many candidates provided far in excess of what was required. Candidates need to develop an improved understanding of the definition of similar triangles and to subsequently restrict the content of their responses. A common tendency was for candidates to assume what they were trying to prove.

(b) Candidates encountered great difficulty in their attempt to find the primitive of an exponential function. Of those who did find the correct expression, many failed to also consider the constant of integration. There is also an apparent inability of many to deal with negative indices, hence sub-part (ii) caused a real problem for candidates. Factorisation of the quadratic in e^t was problematic for a large proportion of the candidature. As was the case in part (a), candidates failed to establish a link between the sub-parts.

Question 10

(a) This part was an investment question, with interest compounded annually and equal amounts deducted annually to fund a prize. There were three sub-parts. Sub-part (i) confused many candidates and yet this was typically the only mark awarded in the entire question. This part asked for the balance at the beginning of the second year rather than at the end of the first year. Many candidates offered the balance at the end of both years but did not indicate which answer was actually appropriate. In sub-part (ii) candidates had to show that $B_n = 1200 - 200(1.06)^n$. It was pleasing that many candidates attempted to establish a pattern involving a geometric series. Unfortunately $(1 + 1.06 + ... + 1.06^n) = (\frac{1.06^n - 1}{0.06})$ was often seen, or a lack of terms in an otherwise correct series such as $(1 + .. + 1.06^{n-1}) = (\frac{1.06^n - 1}{0.06})$ or $(1 + 1.06 + 1.06^2 + ...) = (\frac{1.06^n - 1}{0.06})$. Candidates who quoted formulae were generally unsuccessful. Although sub-part (iii) was worth 3 marks, some candidates offered little to support their answer. The question asked how many years the investment would last, given that the amount deducted increased after 10 years. Poor use of notation caused problems for some candidates because the amount deducted changed after the tenth year. Isolating 1.06^m from $841.83(1.06)^m = \frac{90(1.06^m - 1)}{1.06 - 1}$ or from $841.83(1.06)^m - \frac{90(1.06^m - 1)}{1.06 - 1} \geq 0$ often resulted in algebraic errors. Candidates need to be reminded to avoid truncation errors during lengthy calculations. Candidates who did reach the stage of taking logarithms to find *m* were generally successful.

(b) (i) This required candidates to write two expressions for the time taken in terms of θ. Some candidates struggled with the trigonometry involved while others seemed unaware that $time = \frac{distance}{speed}$.

(ii) This part required candidates to work with the time difference. It followed logically from part (i). Unfortunately it was a waste of valuable time for most candidates. Some candidates used calculus in an effort to maximise or to minimise one of the expressions with no reward. Others equated the expressions for time and then attempted to solve a resulting trigonometric equation, again with no reward. Very few realised that the answer required the difference of the expressions to be maximised.

Mathematics Extension 1

Question 1

Overall, the question was fairly well done.

(a) Most candidates obtained an integral involving inverse sine. Nevertheless, despite the use of the table of standard integrals provided, there were mistakes including $\sin^{-1}\left(\frac{x}{2}\right)$ or $\frac{1}{4}\sin^{-1}\left(\frac{x}{4}\right)$. Giving the answer as 30 or 30º instead of $\frac{\pi}{6}$ was not uncommon.

(b) While many candidates showed knowledge of the product rule, errors in differentiating $\sin^2 x$ were common. There were many attempts, both correct and incorrect, to write $\sin^2 x$ as $\frac{1}{2}(1-\cos 2x)$. This proved more difficult to differentiate and mistakes involving negative signs or dividing by two instead of multiplying were prevalent. Some even managed to integrate one or both of the factors.

(c) The easiest method was simply to add up four numbers. Those candidates who resorted to formulae sometimes applied them incorrectly, for example by thinking that there were seven terms. Quite a few candidates tried to use combinations – as though this were a question about the binomial theorem.

(d) The simplest method was to use the formula, although there were a small number of successful responses that used gradients or similar triangles. It was quite common to substitute numbers into wrong places or not to use a negative sign for one of the numbers in the ratio.

(e) This was generally well done, although some candidates carried out the division rather than use the remainder theorem. Arithmetic and transcription errors were not uncommon. Predictably there were instances where $P(3)$ was found rather than $P(-3)$.

(f) Many candidates were able to use the substitution and change the limits. Of these, quite a few had difficulty in integrating $(u-1)\sqrt{u}$. Often the factors were integrated separately. Sloppy setting out, including omission of the brackets, was not uncommon.
There was some confusion about the change of limits – some didn't change them at all. Others changed their variable back to x but then evaluated using the new limits. A substantial number of candidates worried about the negative sign in the answer and inserted absolute value signs afterwards.

Question 3

(a) Most candidates understood Newton's method. However, answers were marred by a few common errors. A large number of candidates used degrees instead of radians, reaching an answer of –7.27. Disturbingly, very few candidates recognised the unreasonable nature of this result. Among those who did, only a small percentage managed to correct their error. Calculator work was often poor and, when intermediate working was not shown, it was difficult to award part marks.

(b) Answers to this part were poorly set out with inadequate justification and use of appropriate mathematical terms. Candidates need to distinguish memory aids (such as 'windsurfer theorem') and correct mathematical usage. The most common error in sub-part (ii) was a failure to link $\angle AOB$ and $\angle TAB$. A candidate whose complete answer was 'Alternate segment theorem' may have understood this link, but this was not sufficiently evident to gain the one mark. In sub-part (iii), many candidates attempted unsuccessfully to prove that triangles *AOP* and *AOB* were congruent, the common error being to assume without adequate proof that $\angle POA = 2\theta$. About 10% of candidates stated their final conclusion as $PA = PB$.

(c) Sub-part (i) was generally well done. Sub-part (ii) was poorly done, with only about one-third of candidates managing to gain full marks. Common errors included omission of the solution $\sin\theta = 0$. Around 25% of the total candidature reached the equation $\sin^2\theta = \frac{1}{4}$ but failed to conclude that $\sin\theta = \pm\frac{1}{2}$. Two-thirds of these reached the conclusion $\sin\theta = \frac{1}{2}$, while the remainder had $\sin\theta = \pm\frac{1}{\sqrt{2}}$. Candidates who found $\sin\theta = \pm\frac{1}{2}$ and/or $\sin\theta = 0$, were often not able to find corresponding values of θ in the range $0 \le \theta \le 2\pi$.

Question 4

This question was generally not handled well. Many of the candidates either chose an incorrect approach and/or an inappropriate approach to different parts of the question, leading them to difficulties in reaching the final solution.

(a) Many candidates did not exclude $x = 2$ from their solution. A variety of methods were attempted, with the most success gained from multiplying by the square of the denominator. Many candidates using the two-case method did so successfully. Testing regions, using the critical point method, was done poorly by many candidates.

(b) This part was poorly handled by most candidates. The common approach was to ignore the given equations for x and y and to derive the equations of motion, again ignoring the given initial conditions. There was a great deal of confusion between V as given in the question and V as the angle of projection from the origin. Candidates often equated $\frac{y}{x}$ to $\tan 135^\circ$ (or $\tan 45^\circ$). Those who did equate $\frac{\dot{y}}{\dot{x}}$ to the tangent of an angle frequently fudged the sign, as they could not interpret 45° to the vertical correctly.

(c) Many candidates approached this question by integrating the given differential equation rather than using $x = a\cos(nt + \alpha)$. The complex integration and algebra led to few correct solutions, although some candidates did manage to gain 4 marks. In many of the approaches taken, the candidates did not check the initial conditions for x and $\dot{x}$. Failure to do this resulted in the loss of one of the available marks.

Question 5

Overall, the response to this question was disappointing. Very few candidates scored 12, with potentially good candidates scoring about 7.

(a) (i) Most candidates were able to get at least 1 mark on this question. Candidates knew to substitute $x = 0$ but could not evaluate $\cos^{-1} 0$. The most common wrong response was: y-intercept = 2.

(ii) Candidates knew that to find the inverse function they should interchange the x and y, but their algebra skills let them down. On the whole, they were generally able to score at least 1 mark by either finding the correct inverse function or giving the correct domain, or the domain of their incorrect inverse function.

(iii) Most candidates knew that the area was $\int_0^3 \cos^{-1} \frac{x}{3}\, dx$ but could not integrate this. They did not link the inverse in (ii) with the required area. Most Extension 2 candidates who tried integrating by parts were unsuccessful.

(b) Most candidates could expand $(q + p)^n$ but to earn more than 1 mark they had to do much more than this. Some candidates seemed confused by the q being first. Many simply stated the last terms and this only scored 1 mark. A disappointing number of candidates wrote that the last term is 0 when n is even.

(c) (i) Candidates generally recognised this as binomial probability and were able to give the correct response. Candidates need to be clear in their writing, as it was very difficult to distinguish between n and r in many cases.

(ii) This was very badly done with many candidates not attempting it. Too many candidates could either not see the link with (b) or, if they did, they could not clearly show this. Some tried to manipulate the answer to part (i) to get the given expression.

Question 7

Given that this was the last question of a solid paper, most candidates had the inclination to 'have a go' at this challenging question, with far fewer non-attempts than might be expected.

(a) (i) Most candidates found the first 2 marks the easiest to gain, but too few immediately recognised that $\frac{dv}{dt} = \frac{d}{dx}\left(\frac{1}{2} v^2\right)$. Careless errors often resulted in the incorrect evaluation of the constant, if indeed the constant was noted.

(ii) This proved to be the hardest 2 marks to earn in this question. Most candidates failed to check the initial conditions and used $v = x - 1$ instead of $v = 1 - x$. The difficulty they then had with ln (–1) was dealt with in a variety of ways, invariably excluding them from gaining any marks.

(b) Generally candidates were not troubled with the 3-dimensional nature of this question, but inadequate knowledge of trigonometry and poor algebra skills let them down in many cases.

(i) Candidates who understood to use the cosine rule often had trouble using $h \cot\alpha$ correctly and/or noting that $\cos \frac{\pi}{3} = \frac{1}{2}$.

(ii) Candidates are advised to carefully state the cosine rule and the length of the sides for the triangle they are using. Many who initially used $\cos \theta$ as the subject of the formula had trouble manipulating the algebraic terms correctly. Those candidates who had used their initial statements for AP^2, rather than attempting to simplify first, more often gained the evaluation mark. Full marks were only allocated for correct working – if candidates realise they have made a mistake in one line of their working, all other lines need be corrected!

(iii) Many candidates mixed up θ and $\cos \theta$, degrees and radians, ratios and angles. Very few appreciated that their conclusion of 'maximum turning point' referred to $\cos \theta$ and that this implied a 'minimum turning point' for θ. Many conclusions contradicted the values found for θ at $\alpha = 0$, $\tan^{-1}2$ and $\frac{\pi}{2}$. Quite a few candidates recognised that the auxiliary angle method was an expedient way of approaching the question. However, in most cases, they made errors in the execution.

Mathematics Extension 2

Question 1

(a) This part was well done. Errors occurred when $\tan^3 x$ was broken into $\tan x\,(\sec^2 x - 1)$. Perhaps the most common error was letting $u = \sec^2 x$ leading to $du = \tan x\, dx$.

(b) Completing the square caused little problem. It was pleasing to see that the vast majority of candidates successfully used the table of standard integrals without any written direction. Some were successful with a substitution such as $x = 2 + \sqrt{3}\sec\theta$.

(c) Marks were lost through incorrect identification of u and v and also by confusing differentiation and integration processes. Candidates seemed to have a better chance of forming the correct integral expression if $u, u', v,$ and v' were stated separately at the beginning of their answer.

(d) This part relied mainly on the three basic processes required in integration involving substitution. Integrating the substitution expression was well done but incorporating this into the integral was where most errors in this question occurred. Making dx the subject usually led to success. Changing the limits and the body of the integral were quite well done. However recognising a common factor in expressions such as $\frac{(u^2+2)2u}{u}$ or successfully collecting terms such as $\frac{2\sqrt{2}}{3} + 2\sqrt{2}$ eluded many candidates.

(e) (i) Having been given the structure that the partial fractions would take, candidates found little difficulty in giving correct values for a and b. However, there were quite a few transcription errors.

(ii) Only a few candidates found difficulty in separating the first fraction into a 'ln' and $\tan^{-1}x$ result.

Question 2

All but two or three candidates attempted this question on complex numbers. Although many attempts earned at least 10 of the 15 marks available, the last part proved to be a stumbling block for many.

(a) This required candidates to calculate the product of two complex numbers as well as the reciprocal of one of them. In order to gain each of the two marks, a candidate had to arrive at the correct expression. To avoid being penalised for the frequent basic arithmetical errors, often committed as a result of nervousness in the early questions, candidates should check their answers once the initial nervousness has worn off.

(b) (i) This sub-part was successfully completed by most candidates in expressing $1+i\sqrt{3}$ as $2(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})$. A few failed to find the correct modulus, and a larger number determined the argument to be $\frac{\pi}{6}$.

(ii) Although the two marks were gained by over half the candidates, common errors included leaving the answer in mod-arg form as $1024(\cos\frac{10\pi}{3}+i\sin\frac{10\pi}{3})$, calculating 2^{10} as 20, and evaluating $\cos\frac{10\pi}{3}$ as $\frac{1}{2}$.

(c) Many candidates failed to secure the 3 marks because they drew a circle with either an incorrect centre or an incorrect radius, or rays with starting point at the centre of the circle. A few failed to indicate the region that represented the intersection. Although no marks were deducted for poorly drawn diagrams, some candidates whose circles were so badly drawn that it was impossible to determine the centre were not awarded the mark for the circle. The use of a template or a pair of compasses (or even a coin) for drawing a circle is to be encouraged.

(d) In solving the equation $z^4 = -1$, many candidates correctly wrote $cis \pm \frac{\pi}{4}, cis \pm \frac{3\pi}{4}$, but many confused $-cis\frac{\pi}{4}$ with $cis(-\frac{\pi}{4})$. Most realised that the four solutions were equally spaced around a circle (rewarded with a mark) but typically selected either 1 or *i* as the first solution.

(e) (i) The word 'explain' was taken to mean that a wordy (and often unconvincing) statement was necessary. Many dubious attempts were made to explain the presence of the negative sign, with vague mentions of different directions, different quadrants (although all points could be assumed to be in the first quadrant without loss of generality), and squaring techniques which produced negative signs where convenient. To gain the two marks, candidates were required only to note that the vector *BA* (or *AB*, since direction was ignored in this part) represented $z_1 - z_2$, and that the vector *BA* was equivalent to *iBC*. Many candidates realised that there was a rotation of $\frac{\pi}{2}$ involved, but those who wrote that $i(z_1 - z_2) = z_3 - z_2$ failed to gain a mark by squaring both sides since they had not demonstrated an understanding of the sense of the rotation.

(ii) The mark here proved to be the most elusive. The simple solution of $z_1 + z_3 - z_2$ was found by relatively few. Since this question was worth only 1 mark, the answer was required to be perfectly correct. Those who did not consider the direction of each vector often had the signs wrong and did not gain the mark. The most frequent error was to simply add $z_1 - z_2$ and $z_3 - z_2$. Five alternative solutions which did or did not include all of z_1, z_2 and z_3, such as $z_2 + i(z_3 - z_1)$ or $z_1 + i(z_2 - z_1)$, were also awarded the mark.

Question 4

(a) (i) This part was well done by the great majority of candidates. Most candidates could correctly identify the x values of the turning points and correctly classify them. A small percentage mixed up maximum and minimum turning points.

(ii) Most gained 1 or 2 marks for showing the point of inflexion and the smooth turning points. Relatively few gained a mark for correctly drawing the cusp at $x = 2$. Also few were able to sketch the function correctly as $x \to \infty$.

(b) (i) Most gained 1 mark for using a formula for the area between concentric circles. The second mark was gained for substituting $y^2 = R^2 - b^2$ (or similar) into a concentric circles area formula. The majority was able to do this.

(ii) Relatively few candidates gained 3 marks for this question, with the majority gaining 1 or 2 marks. Common errors included $\int db$, $\int_0^{2b}$, $\int dy$, $\pi \int_0^b$ and variations. Few were able to eliminate R and h since they could not make the connection that $R^2 - h^2 = b^2$.

(c) The differentiation of $y = \tan^{-1}\dfrac{x}{x+1} + \tan^{-1}\dfrac{1}{2x+1}$ was often poorly done with a large number of candidates not using the chain rule or quotient rule properly. Of those who differentiated properly, most could not draw the conclusion that y is a constant.

Question 5

The question was well done, with very few non-attempts or zero marks. The working was generally well set out and clear.

(a) Sub-parts (i) and (ii) were well done with most candidates gaining full marks. Sub-part (iii) caused a few problems. Many candidates tried to show that OP was perpendicular to QR using circle geometry. Others tried to find the point of intersection between OP and QR. This was often successful, but took time.

(b) (i) There was confusion between $F = ma$ and the F in the question. Successful candidates made the distinction clear by saying $\sum F = ma = F - kv^2$ or equivalent, or by using a diagram of forces.

(ii) This was again well done by those candidates who remembered that $\dfrac{dv}{dt} = v\dfrac{dv}{dx}$.

Candidates need to take more care with the constants of integration or specify the limits more clearly.

(c) This question was well done. Of those who were incorrect, many put addition signs between the terms. Some candidates used signs that were difficult to interpret.

Question 6

This question was very accessible in both parts and was generally well done, with few non-attempts and a number of candidates scoring full marks.

(a) (i) This was generally well done.

(ii) Those who got part (i) correct were generally able to continue, either by eliminating N implicitly using $\cos^2\theta + \sin^2\theta = 1$ or by explicitly making N the subject and substituting. A number of candidates who could not do part (i) resolved the forces along the road and were able to derive the required equation from there.

(iii) Many of those who were unable to successfully do parts (i) and (ii) nevertheless scored marks here by either using the given formula in part (ii) or quoting (from memory) tan $\alpha = \frac{v^2}{mg}$. The most common error was not converting the velocity to m/s.

(b) (i) This was generally well done. A reasonable number of candidates misunderstood the 'alternate segment theorem' and tried to apply it to $\angle TAF$ and $\angle TAE$. Many did not copy the diagram into their booklet, despite the instruction. This usually made it difficult to follow their answer. A reasonable number of candidates seemed to think that they had to prove all three angles equal in order to show similarity and wasted time doing this. A mark was deducted for not clearly stating the geometric results used. A worrying number of candidates gave incoherent and poorly expressed reasoning, such as 'tangent theory' or 'angle in arc' instead of 'alternate segment theorem'.

(ii) This was generally well done, even by those who could not completely do part (i).

(iii) This was not quite so well done. Many candidates tried to prove the triangles were similar using angle arguments, which did not work. A fair number seemed to believe that ratios of sides were sufficient, with no mention of the included angle.

(iv) To get this last mark reasons had to be given for all steps. Many candidates chased angles across the diagram and correctly proved the result, but a large number did not give reasons for the key steps and so did not get the mark. Candidates often did not draw the diagram in their booklets and then tacitly assumed certain angles (presumably marked on the diagram on their examination paper) were equal without stating what they were.

Question 8

Overall, this question proved far too daunting for the vast majority of candidates. Marks above 10 were extremely rare.

(a) Most candidates made some attempt at part (a). The first inequality in each sub-part should have provided most candidates with an opportunity to score 2 marks, but a surprising number were unable to do so. Candidates struggled with the second inequality in each sub-part, although neither is particularly difficult. Candidates would be well advised that an attempt to prove an inequality by starting with the statement that is to be proved is unlikely to be successful. It was very clear to the examiners that an ability to sustain a logical argument is not at all common.

(b) A reasonable number of candidates attempted sub-parts (i) and (ii). In sub-part (i) only a few candidates used the inequality $e < 3$ as a first step to show that the integral is less than $\int_0^1 3x^a\,dx$. This approach provided the easiest explanation. A number of candidates attempted a graphical argument, which was inappropriate. Sub-part (ii) was the most accessible part. A reasonable number of candidates were able to gain 1 mark for setting up the induction. Candidates should be warned against simply stating that the proposition holds for a base case without showing any working. There were very few attempts at parts (iii) and (iv), and virtually none that were totally successful.

Mathematics Extension 1

2001 HSC Examination Mapping Grid

Question	Marks	Content	Syllabus outcomes
1(a)	2	Integration	HE6
1(b)	2	The tangent to a curve and the derivative of a function; Trigonometric functions	P7, PE5
1(c)	1	Series and applications	H9
1(d)	2	Linear functions and lines	H5
1(e)	2	Polynomials	H2, PE3
1(f)	3	Integration	HE6
2(a)	2	Geometrical applications of differentiation	H5, H9
2(b)(i)	1	Integration, exponential functions	H8
2(b)(ii)	3	Integration, trigonometry functions	HE6
2(c)(i)	1	Permutations, combinations and further probability	PE3
2(c)(ii)	2	Permutations, combinations and further probability	PE3
2(d)	3	Binomial theorem	HE3
3(a)	3	Polynomials; Differentiation of Trigonometric functions	H5, HE4
3(b)(i)	1	Plane Geometry	PE3
3(b)(ii)	1	Plane Geometry	PE3
3(b)(iii)	2	Plane Geometry	PE3
3(c)(i)	2	Trigonometric ratios	H5, PE2
3(c)(ii)	3	Trigonometric ratios	H5
4(a)	3	Basic arithmetic and algebra	PE3
4(b)	4	Applications of calculus to the physical world	PE4, HE3
4(c)	5	Applications of calculus to the physical world.	HE3
5(a)(i)	1	Inverse functions and inverse trigonometric functions	P5, HE4
5(a)(ii)	2	Inverse functions and inverse trigonometric functions	P5, HE4
5(a)(iii)	2	Inverse functions and inverse trigonometric functions	H8
5(b)	3	Binomial theorem	PE3
5(c)(i)	2	Permutations, combinations and further probability	HE3
5(c)(ii)	2	Permutations, combinations and further probability	PE2, HE3
6(a)	3	Series and applications	HE2
6(b)(i)	2	The quadratic polynomial and the parabola	H5, PE3, PE4
6(b)(ii)	1	The quadratic polynomial and the parabola	H5, PE3
6(b)(iii)	4	The quadratic polynomial and the parabola	H5, PE3
6(b)(iv)	2	The quadratic polynomial and the parabola	H5, PE3
7(a)(i)	2	Applications of calculus to the physical world	HE3, HE5
7(a)(ii)	2	Applications of calculus to the physical world	H5, HE5
7(b)(i)	1	Trigonometric ratios	H5, PE2
7(b)(ii)	3	Trigonometric ratios	H5, PE2
7(b)(iii)	4	Trigonometric ratios; Geometrical applications of differentiation; Trigonometric functions	H5, H6, PE2, PE6

BOARD OF STUDIES
NEW SOUTH WALES

2001 HSC Mathematics Extension 1
Marking Guidelines

Question 1 (a) (2 marks)

Outcomes assessed: HE6

MARKING GUIDELINES

Criteria	Marks
• Gives the correct answer $\left(\frac{\pi}{6}\right)$, showing relevant working	2
• Gives the correct standard integral OR • Correctly evaluates an incorrect non-trivial integral	1

Question 1 (b) (2 marks)

Outcomes assessed: P7, PE5

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer $\left(\sin^2 x + 2x \sin x \cos x\right)$ or equivalent	2
• Uses the product rule OR • Correctly differentiates $\sin^2 x$	1

Question 1 (c) (1 mark)

Outcomes assessed: H9

MARKING GUIDELINES

Criteria	Marks
• Obtains numerical expression 11 + 13 + 15 +17 or equivalent	1

Question 1 (d) (2 marks)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Gives the coordinates of P as $(-5,9)$, showing relevant working	2
• Gives $x = -5$ OR • Gives $y = 9$ OR • Gives a correctly plotted diagram with an attempt to use similarity	1

Question 1 (e) (2 marks)

Outcomes assessed: H2, PE3

MARKING GUIDELINES

Criteria	Marks
• Evaluates $P(-3)$ and deduces that $P(-3) = 0$ implies $x + 3$ is a factor OR • Divides, obtaining the correct quotient and zero remainder	2
• Evaluates $P(-3)$ OR • Draws appropriate conclusion from incorrect working	1

Question 1 (f) (3 marks)

Outcomes assessed: HE6

MARKING GUIDELINES

Criteria	Marks
• Correctly evaluates the expression showing relevant working	3
• Shows $du = dx$ and $x = u - 1$ AND • Correctly changes the limits of integration or equivalent OR • Correctly evaluates apart from a single minor algebraic error	2
• Shows $du = dx$ and $x = u - 1$ OR • Correctly changes the limits of integration	1

Question 2 (a) (2 marks)

Outcomes assessed: H5, H9

MARKING GUIDELINES

Criteria	Marks
• Shows that $f'(a) = \lim\limits_{h \to 0} 6a + 3h + 1$	2
• Correctly substitutes $(a + h)$ for x in the expression for $f'(a)$	1

Question 2 (b) (i) (1 mark)

Outcomes assessed: H8

MARKING GUIDELINES

Criteria	Marks
• Gives $\log\left(1 + e^x\right)$ (ignore constant c)	1

Question 2 (b) (ii) (3 marks)

Outcomes assessed: HE6

MARKING GUIDELINES

Criteria	Marks
• Gives answer $\left(\frac{\pi}{2}\right)$, showing relevant working	3
• Correctly evaluates the integral, apart from one minor error	2
• Uses $\cos 6x = 2\cos^2 3x - 1$ OR • Correctly integrates a non-trivial expression used to replace $\cos^2 3x$	1

Question 2 (c) (i) (1 mark)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Gives answer $\left(\frac{9!}{2!}\right)$ or equivalent	1

Question 2 (c) (ii) (2 marks)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Gives answer $\left(\frac{4! \times 5!}{2!}\right)$ or equivalent	2
• Gives 4! OR 5! in numerator of answer	1

Question 2 (d) (3 marks)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Gives answer $\left({}^{9}C_{6}\right)$ or equivalent, showing relevant working	3
• Equates the exponent of x (in the general term) to 0 OR • Correctly carries out the computation apart from one minor error	2
• Gives an expression for the general term	1

Question 3 (a) (3 marks)

Outcomes assessed: H5, HE4

MARKING GUIDELINES

Criteria	Marks
• Correctly uses Newton's method to give answer (1.26)	3
• Gives correct derivative expression AND correctly evaluates $f(1.2)$ and $f'(1.2)$ OR • Correctly evaluates $f(1.2)$ and their $f'(1.2)$ AND correctly uses Newton's method	2
• Gives correct derivative expression OR • Correctly evaluates $f(1.2)$ and their $f'(1.2)$	1

Question 3 (b) (i) (1 mark)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Gives $\angle AOB = 2\theta$ plus reason	1

Question 3 (b) (ii) (1 mark)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Gives an explanation showing $\angle TAB = \angle AOB$	1

Question 3 (b) (iii) (2 marks)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Deduces that $\angle PBA = \theta$ and hence $PA = PB$	2
• Deduces that $\angle PBA = \theta$	1

Question 3 (c) (i) (2 marks)

Outcomes assessed: H5, PE2

MARKING GUIDELINES

Criteria	Marks
• Uses correct double angle formulae for both $\cos 2\theta$ and $\sin 2\theta$ AND • Replaces $\cos^2\theta$ with $1 - \sin^2\theta$	2
• Uses correct double angle formulae for both $\cos 2\theta$ and $\sin 2\theta$ OR • Replaces $\cos^2\theta$ with $1 - \sin^2\theta$	1

Question 3 (c) (ii) (3 marks)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correctly solves the equation, showing relevant working	3
• Shows $\sin\theta = 0 \Rightarrow \theta = 0,\ \pi,\ 2\pi$ OR • Shows $\sin\theta = \pm\frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	2
• Shows $4\sin^3\theta - \sin\theta = 0$	1

Question 4 (a) (3 marks)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Gives answer $-1 \le x < 2$ with relevant working	3
• Sketches an appropriate graph OR • Gives an expansion based on $3x(x-2) \le (x-2)^2$ OR • Equivalent	2
• Shows that $x \ne 2$	1

Question 4 (b) (4 marks)

Outcomes assessed: PE4, HE3

MARKING GUIDELINES

Criteria	Marks
• Gives answer $V = 200$ with relevant working	4
• Shows $\frac{dy}{dx} = -\frac{10x}{V^2}$ or $\frac{dy}{dx} = -\frac{10t}{V}$ AND • Equates $\frac{dy}{dx}$ to $\tan 135° = -1$, when $x = 4000$ OR • Solves for V with 1 error followed through	3
• Shows $\frac{dy}{dx} = -\frac{10x}{V^2}$ or $\frac{dy}{dx} = -\frac{10t}{V}$ OR • Gives incorrect expression for $\frac{dy}{dx}$ AND equates to $\tan 135° = -1$, when $x = 4000$	2
• Shows $y = \frac{-5x^2}{V^2}$ OR • Shows $\frac{dy}{dt} = -10t$ and $\frac{dx}{dt} = V$	1

Question 4 (c) (5 marks)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• 1 mark for showing $n = 2$ • 1 mark for using $t = 0,\ x = 3$ • 1 mark for differentiating to obtain $\dot{x}$ and using $t = 0,\ \dot{x} = -6\sqrt{3}$ to obtain correct second equation • 1 mark for evaluation of A • 1 mark for evaluation of α	5

Question 5 (a) (i) (1 mark)

Outcomes assessed: P5, HE4

MARKING GUIDELINES

Criteria	Marks
• Gives answer (π)	1

Question 5 (a) (ii) (2 marks)

Outcomes assessed: P5, HE4

MARKING GUIDELINES

Criteria	Marks
• Determines inverse function $y = 3\cos\frac{x}{2}$ and gives correct domain $(0 \le x \le 2\pi)$	2
• Determines inverse function $y = 3\cos\frac{x}{2}$ OR • Gives correct domain for a non-trivial inverse function OR • Gives domain consistent with part (i) via symmetry OR • Gives correct domain $(0 \le x \le 2\pi)$	1

Question 5 (a) (iii) (2 marks)

Outcomes assessed: H8

MARKING GUIDELINES

Criteria	Marks
• Correctly integrates and substitutes to give Area $= 6\left(\sin\frac{\pi}{2} - 0\right)$ (or equivalent from answer to 5(a)(ii))	2
• Gives Area $= \int_0^{\pi} 3\cos\frac{x}{2}\,dx$ or equivalent	1

Question 5 (b) (3 marks)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Gives the correct last terms, $2p^n$ and $2\binom{n}{1}qp^{n-1}$, as a result of correct working	3
• Gives either of the correct last terms based on $(q+p)^n$ and $(q-p)^n$	2
• Gives a correct expansion for either $(q+p)^n$ or $(q-p)^n$ OR • Gives both correct last terms without showing any working	1

Question 5 (c) (i) (2 marks)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Shows that the probability is $\binom{n}{r}\left(\frac{1}{6}\right)^r\left(\frac{5}{6}\right)^{n-r}$	2
• Identifies $\left(\frac{5}{6}+\frac{1}{6}\right)^n$ as the binomial probability function	1

Question 5 (c) (ii) (2 marks)

Outcomes assessed: PE2, HE3

MARKING GUIDELINES

Criteria	Marks
• Correctly applies the identity in (b) with $q=\frac{5}{6}$ and $p=\frac{1}{6}$	2
• Deduces a correct binomial expansion for the probability OR • Includes $q=\frac{5}{6}$ and $p=\frac{1}{6}$	1

Question 6 (a) (3 marks)

Outcomes assessed: HE2

MARKING GUIDELINES

Criteria	Marks
• Shows that 9 is a factor of the expansion for $S(k+1) - S(k)$, has appropriate induction hypothesis, and demonstrates that the result is true for $n = 1$	3
• Provides a correct statement of assumption, correct substitution of $k+1$ for k, and demonstrates that the result is true for $n = 1$	2
• Demonstrates that the result is true for a value of n OR • Provides statement of induction hypothesis and attempts to substitute into next statement	1

Question 6 (b) (i) (2 marks)

Outcomes assessed: H5, PE3, PE4

MARKING GUIDELINES

Criteria	Marks
• Shows that $\dfrac{dy}{dx} = t$ and that the equation of the normal is $y - at^2 = \dfrac{-1}{t}(x - 2at)$ or equivalent	2
• Shows that $\dfrac{dy}{dx} = t$ and that the normal's gradient is $\dfrac{-1}{t}$	1

Question 6 (b) (ii) (1 mark)

Outcomes assessed: H5, PE3

MARKING GUIDELINES

Criteria	Marks
• Identifies the coordinates of Q as $\left(\dfrac{-2a}{t}, \dfrac{a}{t^2}\right)$	1

Question 6 (b) (iii) (4 marks)

Outcomes assessed: H5, PE3

MARKING GUIDELINES

Criteria	Marks
• Shows that the required coordinates are true by solving the equations for the normals simultaneously (OR by another method)	4
• Shows that one of the required coordinates is true	3
• Writes a correct equation for the normal at Q AND • Attempts to solve simultaneously with the normal at P or equivalent	2
• Writes a correct equation for the normal at Q OR • Attempts to solve an incorrect equation for Q simultaneously with the normal at P or equivalent	1

Question 6 (b) (iv) (2 marks)

Outcomes assessed: H5, PE3

MARKING GUIDELINES

Criteria	Marks
• Gives the locus as $x^2 = a(y - 3a)$ or equivalent	2
• Squares $\frac{x}{a}$ or equivalent	1

Question 7 (a) (i) (2 marks)

Outcomes assessed: HE3, HE5

MARKING GUIDELINES

Criteria	Marks
• Correctly integrates $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ AND • Calculates the value $\left(\frac{1}{2}\right)$ of the constant	2
• Replaces $\frac{dv}{dt}$ *with* $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$	1

Question 7 (a) (ii) (2 marks)

Outcomes assessed: H5, HE5

MARKING GUIDELINES

Criteria	Marks
• Shows that $x = 1 - e^{-t}$	2
• Deduces that $v = 1 - x$	1

Question 7 (b) (i) (1 mark)

Outcomes assessed: H5, PE2

MARKING GUIDELINES

Criteria	Marks
• Uses $OA = h$ and $OP = h \cot \alpha$ in cosine rule	2

Question 7 (b) (ii) (3 marks)

Outcomes assessed: H5, PE2

MARKING GUIDELINES

Criteria	Marks
• Correctly uses ΔAPT, $PT = \dfrac{h}{\sin\alpha}$, $AT = \sqrt{2}h$ and cosine rule, and equates expression for AP^2 to give a correct expression for $\cos\theta$	3
• Correctly uses ΔAPT, $PT = \dfrac{h}{\sin\alpha}$, $AT = \sqrt{2}h$ and cosine rule, and equates expression for AP^2	2
• Uses ΔAPT and cosine rule	1

Question 7 (b) (iii) (4 marks)

Outcomes assessed: H5, H6, PE2, PE6

MARKING GUIDELINES

Criteria	Marks
• 1 mark for first derivative • 1 mark for $\alpha = 1.107$ radians or $63.4°$ and identifies this as a minimum with valid reason • 1 mark for $\alpha \to 0, \theta \to 69°$ $\alpha \to \frac{\pi}{2}, \theta \to 45°$ • 1 mark for a graph with axes θ and α and curve with a minimum consistent with (63°, 38°)	4