2002 HSC Notes from the Marking Centre Mathematics Extension 2

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Contents

2002 HSC NOTES FROM THE MARKING CENTRE MATHEMATICS EXTENSION 2

Introduction

This document has been produced for the teachers and candidates of the Stage 6 course, Mathematics Extension 2. It is based on comments provided by markers on each of the questions from the Mathematics Extension 2 paper. The comments outline common sources of error and contain advice on examination technique and how best to present answers for certain types of questions.

It is essential for this document to be read in conjunction with the relevant syllabus, the 2002 Higher School Certificate Examination, the Marking Guidelines and other support documents that have been developed by the Board of Studies to assist in the teaching and learning of the Mathematics Extension 2 course.

Question 1

Question 1 was generally completed well. Overall, the two main problems were remembering or developing the *t*-formulae and inaccurate algebra often linked to not showing all necessary working.

A number of common errors were identified for each part.

(a) Many candidates, including some who achieved high marks overall for the question made a nervous start by making elementary errors. Often sec x was not differentiated correctly, while some thought the integral of tan x was $\sec^2 x$. Candidates would often forget to substitute back to get an expression in terms of x.

(b) A common error here was to incorrectly complete the square. Often $(x+2)^2 - 3$ was given.

(c) Mistakes were often associated with no working shown for the partial fractions; possibly completed mentally, or perhaps using other methods appropriate for a constant numerator. Quite a few candidates incorrectly used partial fractions with $\dfrac{Ax}{x+3}$ as a term.

(d) A major source of error was to incorrectly integrate or differentiate $\sin x$ and $\cos x$ by dropping or including a negative. Some candidates left their final answers as a function in x rather than applying the limits. The constant 1 was often given instead of –1. Some candidates clearly did not understand the recursion idea.

(e) A significant number of candidates had not learnt the *t*-formulae or how to develop them from a right-angled triangle. This was the major problem in the whole question. Furthermore, many candidates had poor algebra skills following the *t*-substitutions. In a number of these cases, candidates did not provide enough working to correctly transform the algebraic fractions. Some candidates left their answers as an inverse tan, instead of an expression in terms of π.

Question 2

This question was assessing the outcomes associated with the operations involving complex numbers, with applications in polynomials and induction. Considering it examined the more straightforward aspects of these topics it was not particularly well done.

(a) Both parts involved basic arithmetic operations with complex numbers and were generally answered correctly.

(b) Most candidates recognised that the required region consisted of the overlap of a strip and a circle. Common errors included finding the incorrect centre of the circle or an incorrect strip. There were a few responses involving other shapes such as wedges indicating a possible confusion between moduli and arguments.

(c) Poorly answered. Only a minority of candidates recognised how this question could be done simply.

(i) Many candidates simply stated that complex roots occurred in conjugate pairs without any reference to the necessity for the coefficients to be real.

(ii) The simplest method involved multiplying out the two given linear factors from (i) and finding the third by inspection.

The multiplication of $[z-(2+i)][z-(2-i)]$ to z^2-4z+5 caused many problems. Another simple method involved using the product of the roots. Unfortunately it was not uncommon to see up to 2 or 3 pages of calculations trying to find the coefficients r and s, often for no reward. The time lost here could have had serious ramifications for the attempts at later questions.

(d) This induction was a simple variation on De Moivre's theorem. However, some candidates felt that it was acceptable to use this instead of actually using induction. While there was a fair amount of success in this question it is clear that a large number of candidates have merely memorised the process without necessarily having a full understanding of why it works. There were many cases of eloquent and well-memorised conclusions stating why the result had been shown when the supporting mathematics was meaningless. Under the marking scale the conclusion, whether consisting of one line or a half-page, was not allocated any marks. Some setting out was less than impressive – especially when there were manipulations on both the left hand and right hand sides at the same time.

(e) Again, candidates struggled in this part. This involved finding the conjugate and reciprocal of $1-z$ where $z = 2(\cos\theta + i\sin\theta)$. Despite the fact that part (i) could be used to find the answer to (ii), many candidates did not seem to realise the connection. Quite often they were correct in (i) but not (ii), or vice versa. Most errors arose from a misconception that $\overline{1-z} = 1+z$ even when z is complex.

In part (iii) there was confusion as to whether or not the imaginary part included *i,* even amongst some candidates who had found little difficulty in answering the previous parts. Clearly candidates could benefit from a greater understanding of the properties of conjugates, real and imaginary parts.

Question 3

Most candidates handled this question rather well.

(a) It was pleasing to see that almost all candidates drew large, clear graphs. Candidates could be encouraged to pay more attention to labelling turning points, asymptotes etc in questions like this.

(i) Most candidates gained two marks for this part.

(ii) Many candidates did not include the point (0,0) on their sketch and thereby did not gain two marks. Another common error was to draw only the graph of $y = +\sqrt{f(x)}$.

(iii) Candidates had more trouble with this graph, although many gained 2 marks. A common error was to draw the points (2,0) and (–2, 0) as smooth turning points, and/or the point (0,0) as a cusp. Many drew the graph of $y = f(|x|)$, or the correct graph for positive x only.

(iv) This part was well done with most candidates knowing that the function exists only for $f(x) > 0$.

(b) (i) Most candidates had no problem with this part.

(ii) Similarly, this part was straightforward for the majority of candidates.

(iii) This part was done rather poorly. Many candidates obtained $q = 2p$, but were unable to use this result successfully. A good number obtained the correct parametric form for T but did not recognise this as a hyperbola. Some good candidates successfully showed that T lies on a hyperbola but could not recall its eccentricity.

Question 4

Only about 1% of candidates received full marks for this question. The question proved to be difficult for the majority of candidates.

(a) Most candidates attempted this part of the question.

(i) This was well done. Most candidates clearly identified the required coordinate by stating that it must lie in the first quadrant from the graph.

(ii) The main problem in this part was with the rotation around $x = -1$. Many candidates ignored this and rotated around the y-axis. Many used the term $(x - 1)$ instead of $(x + 1)$. A few candidates successfully shifted the graphs across by 1 unit to the right. Those who broke the areas into up to four parts were rarely successful.

(iii) Generally well done. Those who were incorrect in part (ii) and simplified the integration only scored one mark.

(b) Many candidates did not attempt this part. Those that did achieved mixed results. There were excellent responses, presenting beautiful work and others using circular arguments.

(i) Well done. Candidates typically gave the reason for *DRAS* being a cyclic quadrilateral. They then identified the angles standing on the same arc.

(ii) This was poorly done. Many candidates assumed that *R, S*, and *T* were collinear in their arguments. Others stated that *DSTC* was a cyclic quadrilateral but did not give the reason with sufficient clarity, or did not give reasons at all.

(iii) Candidates who were not successful in part (ii) could gain full marks for part (iii) by using the information from (i) and (ii). Again some candidates assumed the conclusion in their arguments.

(c) Many candidates did not attempt this part of the question.

(i) This was well done by those who attempted it. Many candidates wrote $\frac{6}{9} = \frac{1}{3}$ (?).

Some candidates gave the number of possibilities but did not express their answer as a probability.

(ii) Very few candidates received marks for this part. There was some misinterpretation of the question. Candidates thought that descending was in numerical order and so got the answer of $\frac{7}{504}$. A few successful candidates made a complete list of the possibilities.

Question 5

This question was reasonably well done.

(a) This was generally well done. The majority of candidates received full marks for this part. However there was a surprising number of candidates who did not simplify their values for k (giving answers such as $k = \pm 18\sqrt{\frac{27}{12}}$ which simplifies to the correct answer of $k = \pm 27$).

(b) The two main methods used in this part were employed by about equal numbers of candidates. Those who used the transformation techniques usually had more success than those who used the symmetric functions of the roots. The transformation techniques also required much less work.

(i) The majority of candidates got full marks for this part. The best method was using the result that if the given equation was $P(x) = 0$ with roots α, β, γ then the required equation was $P(x+1) = 0$ which then simplified to $x^3 - 2x^2 - 7x + 1 = 0$.

(ii) The majority of candidates also got full marks for this part. The best method was using the result that the required equation was $P(\sqrt{x}) = 0$ which then simplified to $x^3 - 25x^2 + 50x - 25 = 0$.

(iii) This part caused more difficulty with the most common score being 1 mark. Those candidates who tried to find an expression for $\alpha^3 + \beta^3 + \gamma^3$ by expanding out $(\alpha + \beta + \gamma)^3$ went through a maze of algebra and could not get the result. The best method was to write the given equation as $x^3 = 5x^2 - 5$, so that $\alpha^3 + \beta^3 + \gamma^3 = 5(\alpha^2 + \beta^2 + \gamma^2) - 15$ and then use part (ii) to see that $\alpha^2 + \beta^2 + \gamma^2 = 25$ giving the result of 110.

(c) (i) The vast majority of candidates got full marks for this part.

(ii) This part was by far the worst part completed of the whole question, with the majority of candidates not scoring any marks. The proof here is short but subtle and most candidates did not explain clearly what was needed. Only about 10% of candidates managed to get full marks for this part.

(iii) This part had about an equal number of candidates get each of the four possible marks (0, 1, 2, 3). Many candidates spent a large amount of time going through several attempts with the algebra needed to simplify the expressions. However, many candidates showed strong algebraic skills in this part.

Question 6

This question was assessing the outcomes associated with circular motion and integration (reduction). Many candidates did not attempt part (a) but tried part (b).

(a) (i) Poorly done. Candidates often did not show the correct component and the required angle α. Diagrams and basic geometry were poor at times. Many candidates thought that the vertical component of *N* is the weight *mg* ie $N \sin\alpha = mg$

(ii) Surprisingly, a large number of candidates earned full marks, but gained 0 marks in part (i).

(iii) Several candidates lost marks through algebraic errors. Many did not read the question and did not express ω in terms of *l, g* and α.

(b) (i) There were five (correct) methods for finding $\int \tan x \, dx$:

(1) $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = -\ln(\cos x)$

(2) $\int \frac{\tan x \sec x}{\sec x} dx = \ln(\sec x)$

(3) Let $u = \tan x$, $\therefore \int \tan x \, dx = \int \frac{u \, du}{1+u^2} = \frac{1}{2}\ln(1+u^2) = \frac{1}{2}\ln(1+\tan^2 x)$

(4) Let $u = \sin x$, $\therefore \int \frac{\sin x\, dx}{\cos x} = \int \frac{u\, du}{1-u^2} = \frac{-1}{2}\ln\left(1-u^2\right) = \frac{-1}{2}\ln\left(1-\sin^2 x\right)$

(5) Let $t = \tan\frac{x}{2}$, $\therefore \int \tan x\, dx = \int \frac{2t}{1-t^2}\frac{dt}{1+t^2} = \int\left(\frac{t}{1-t^2} + \frac{t}{1+t^2}\right)dt = \frac{1}{2}\ln\frac{1+t^2}{1-t^2}$

Some candidates used their calculator to try and show $\left[\ln(\cos\theta)\right]_0^{\pi/4} = \frac{1}{2}\ln 2$.

Two common incorrect 'methods', attempting to get $\frac{1}{2}\ln 2$ were:

(1) $\int_0^{\pi/4} \tan x\, dx = \left[\ln(-\cos x)\right]_0^{\pi/4} = \ln\left(-\frac{1}{\sqrt{2}}\right) = -\ln\frac{1}{\sqrt{2}}$

(2) $\int_0^{\pi/4} \frac{\tan x(\sec x + \tan x)}{\sec x + \tan x}dx = \left[\ln(\sec x + \tan x)\right]_0^{\pi/4} = \ln\left[\left(\sqrt{2}+1\right) - (1+0)\right] = \ln\sqrt{2}$.

(ii) This was a standard question, so it was surprising that it was poorly done. Many of the candidates chose to prove the reduction formula using integration by parts and became very tangled. Quite a few candidates thought the question was meant to be done by induction. Unfortunately, no one was successful.

(iii) Many candidates did not attempt this part. Only a small number of candidates earned full marks. Some could give a proof for the inequality but could not give the explanation.

(iv) The most common problem was the careless manipulation of signs when evaluating. However, many gained the 2 marks, although they were unable to do parts (ii) and (iii). Many started with $I_7 < I_5 < I_3$ and were successful.

Question 7

Most candidates attempted part (a) but less than half attempted part (b).

(a) (i) Most recognised that the chain rule gave the best approach but many did not justify or comment on why $\frac{dV}{dy} = A$.

(ii) Almost all candidates obtained some marks for this part. Common mistakes were incorrect integrals, using the condition $y = y_0$ when $t = T$, and careless algebra.

(iii) Many candidates could not start this part as they could not reach the point of expressing the given information as $y = \frac{y_0}{2}$ when $t = 10$. Those who obtained the correct quadratic often did not bother to justify their choice of solution from the two possibilities.

(b) Candidates should be warned that in questions like this marks are generally not awarded without valid reasons. Use of the symbol ∵ should be discouraged as many see it as a substitute for giving a reason.

(i) Hardly any candidates obtained the two marks for this part. Almost all candidates who attempted this part just gave a calculation based on the difference between the arguments of the complex numbers. Most did not mention how these arguments related to the angles shown on the diagram.

(ii) Most candidates attempting this part obtained a mark for observing that as the indicated angles are equal and stand on the line segment P_0P_1 the four points form a cyclic quadrilateral. Sometimes this was all they did. Explaining why the angles are equal caused more problems. A common mistake was to assume that OP_2 and P_0P_1 are parallel without proof.

(iii) A large number of candidates effectively only said 'similarly' in answer to the first part. Many assumed that as $\angle P_1OP_2 = \angle P_0P_2P_1 = \angle P_1P_3P_2$ and they are standing on equal segments, we must have a cyclic quadrilateral, with no mention of the need to state that the given segments are already in a circle. Many did not even mention segments at all, just assuming that as the angles are equal the points are concyclic.

(iv) Many candidates just observed that as we had a pentagon then $\beta = \frac{2\pi}{5}$. Some assumed that the final figure was an octagon, as the diagram suggests. The most common mistake was to assume that as $z_0 + z_1 + z_2 + z_3 + z_4 = 0$ the complex numbers are the fifth roots of unity and hence $\beta = \frac{2\pi}{5}$. About half the candidates attempting this part noted why we have a regular pentagon or successfully calculated β.

Question 8

Only about half the candidature attempted this question.

(a) (i) Most of those who attempted Question 8 completed this part. In general they realised that the result followed from equating the imaginary parts of $\cos(2m+1)\theta + i\sin(2m+1)\theta$ and the binomial expansion $(\cos\theta + i\sin\theta)^{2m+1}$, but many were very careless in their attempts at showing this.

(ii) Very few candidates attempted this part. A small number were able to obtain $p(x)$ by dividing the equation in part (i) by $\sin^{2m+1}\theta$ and letting $x = \cot^2\theta$ but few realised that $\sin^{2m+1}\theta$ could not equal 0. Virtually no candidates explained why the values 1, 2, …, m for k gave m distinct roots.

(iii) Candidates who attempted this part usually gained 2 marks. Some attempted to prove the result by induction and others thought they were summing a geometric progression.

(iv) The small number who persevered to this part were quite often successful.

(b) (i) An inability to interpret the diagram may have caused the many poor or non-attempts. Candidates who were able to locate a line of length *KL* in triangle *ABE* were usually able to deduce $KL = a - x$ successfully. Those who ignored the advice to consider triangle *ABE* had great difficulties, as did those who used triangle *ACD* in order to find *LM*. A couple of candidates realised that the length plus the breadth of the rectangle is always $2a$.

(ii) Most candidates who obtained an expression for the area of the rectangle understood that the volume was obtained by integrating. Finding the limits for the integration eluded many.

Mathematics Extension 2

2002 HSC Examination Mapping Grid

Question	Marks	Content	Syllabus outcomes
1(a)	2	Integration	HE6
1(b)	2	Integration	E8
1(c)	3	Integration	E8
1(d)	4	Integration	E8
1(e)	4	Integration	HE6
2(a)(i)	1	Complex numbers	E3
2(a)(ii)	1	Complex numbers	E3
2(b)	3	Complex numbers	E3
2(c)(i)	1	Complex numbers; Polynomials	E3, E4, E9
2(c)(ii)	2	Complex numbers; Polynomials	E3, E4
2(d)	3	Complex numbers	HE2, E3
2(e)(i)	1	Complex numbers	E3
2(e)(ii)	2	Complex numbers	E2, E3
2(e)(iii)	1	Complex numbers	E3
3(a)(i)	2	Graphs	E6
3(a)(ii)	2	Graphs	E6
3(a)(iii)	2	Graphs	E6
3(a)(iv)	2	Graphs	E6
3(b)(i)	2	Conics	E3, E4
3(b)(ii)	2	Conics	E3, E4
3(b)(iii)	3	Conics	E3, E4
4(a)(i)	1	Basic arithmetic and algebra	P4
4(a)(ii)	3	Volumes	E7
4(a)(iii)	2	Integration	E8
4(b)(i)	2	Circle geometry	E2, E9
4(b)(ii)	2	Circle geometry	E2, E9
4(b)(iii)	2	Circle geometry	E2, E9
4(c)(i)	1	Probability	HE3
4(c)(ii)	2	Probability	HE3
5(a)	2	Polynomials	E4
5(b)(i)	2	Polynomials	E4
5(b)(ii)	2	Polynomials	E4
5(b)(iii)	2	Polynomials	E4
5(c)(i)	2	Conics	E3, E4

Question	Marks	Content	Syllabus outcomes
5(c)(ii)	2	Conics	E3, E4
5(c)(iii)	3	Conics	E3, E4
6(a)(i)	1	Mechanics	E5
6(a)(ii)	3	Mechanics	E5
6(a)(iii)	2	Mechanics	E5
6(b)(i)	1	Integration	E2, E8, E9
6(b)(ii)	3	Integration	E2, E8, E9
6(b)(iii)	3	Inequalities; Integration	E2, E8, E9
6(b)(iv)	2	Inequalities; Integration	E8, E9
7(a)(i)	1	Applications of calculus to the physical world	HE5
7(a)(ii)	4	Mechanics	E2, E5
7(a)(iii)	2	Mechanics	E2
7(b)(i)	2	Complex numbers	E3
7 (b)(ii)	2	Geometry; Complex numbers	E3
7(b)(iii)	2	Complex numbers	E2, E3, E9
7(b)(iv)	2	Complex numbers	E2, E3, E9
8(a)(i)	2	Complex numbers	E3, E9
8(a)(ii)	3	Complex numbers; Polynomials	E4, E9
8(a)(iii)	2	Polynomials	E2
8(a)(iv)	2	Inequalities	E2, E9
8(b)(i)	4	Geometry	E2, E7
8(b)(ii)	2	Volumes	E7

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2002 HSC Mathematics Extension 2
Marking Guidelines

Question 1 (a)

Outcomes assessed: HE6

MARKING GUIDELINES

Criteria	Marks
• Correctly evaluates integral, possibly omitting the constant of integration	2
• Obtains a relation equivalent to $du = \sec x \tan x dx$ OR • Obtains the expression $\int u^2 du$	1

Question 1 (b)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correctly evaluates integral by completing the square, possibly omitting the constant of integration	2
• Completes the square correctly $\left(\text{ie } x^2 + 2x + 2 = (x+1)^2 + 1\right)$	1

Question 1 (c)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correctly evaluates integral, possibly omitting the constant of integration • Obtains result $\frac{3}{4}\ln(x+3)+\frac{1}{4}\ln(x-1)(+c)$	3
• Obtains partial fraction expression $\frac{3}{4}\left(\frac{1}{x+3}\right)+\frac{1}{4}\left(\frac{1}{x-1}\right)$	2
• Gives partial fraction format $\frac{A}{x+3}+\frac{B}{x-1}$	1

Question 1 (d)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Obtains correct answer	4
• Correctly integrates by parts twice, using correct choice for u, dv but fails to obtain final result	3
• Correctly integrates by parts once, handling limits correctly	2
• Integrates by parts once. Gets correct result for integral except for a change of sign or failure to handle limits of integration	1

Question 1 (e)

Outcomes assessed: HE6

MARKING GUIDELINES

Criteria	Marks
• Obtains correct result	4
• Obtains correct primitive and correct limits of integration	3
• Obtains expression $\int_0^1 \frac{2dt}{t^3+3}$ or equivalent (in terms of t) or obtains correct primitive but with incorrect (or no) limits of integration	2
• Obtains expressions $d\theta=\frac{2dt}{1+t^2}$, $\cos\theta=\frac{1-t^2}{1+t^2}$	1

Question 2 (a) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	1

Question 2 (a) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	1

Question 2 (b)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Shades strip between 0 and 2 • Sketches circle centred at (1, –1), radius 2 units • Restricts strip to inside circle	3
• Two of above	2
• One of above	1

Question 2 (c) (i)

Outcomes assessed: E3, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Gives appropriate explanation	1

Question 2 (c) (ii)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	2
• Identifies either $(z+4)$ or (z^2-4z+5) OR • Finds third root	1

Question 2 (d)

Outcomes assessed: HE2, E3

MARKING GUIDELINES

Criteria	Marks
• Includes test for $n=1$ • Uses induction assumption • Applies correct trigonometric identity	3
• Two of above	2
• One of above	1

Question 2 (e) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	1

Question 2 (e) (ii)

Outcomes assessed: E2, E3

MARKING GUIDELINES

Criteria	Marks
• Multiplies by correct conjugate and simplifies correctly	2
• Multiplies by correct conjugate, but does not use $\cos^2\theta + \sin^2\theta = 1$ to simplify correctly OR • Multiplies by incorrect conjugate and uses $\sin^2\theta + \cos^2\theta = 1$ to simplify denominator	1

Question 2 (e) (iii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer consistent with working in (ii)	1

Question 3 (a) (i)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Draws reasonable graph (should show asymptotes and turning point)	2
• Makes no more than one significant error	1

Question 3 (a) (ii)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
Draws reasonable graph, showing the following features: – the isolated point at the origin – the two branches meeting at (2, 0)	2
• Misses isolated point at the origin, but otherwise correct OR • Omits one of the branches meeting at (2, 0), but otherwise correct (including isolated point)	1

Question 3 (a) (iii)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
Draws correct graph, showing: – symmetry about $x = 0$ – cusps – turning points	2
Draws correct graph for $x \geq 0$ OR • Incorrectly shows corners as smooth, but otherwise correct OR • Shows symmetry about $x = 0$	1

Question 3 (a) (iv)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Draws correct graph, showing graph cuts x–axis at $x = 3$ and asymptote	2
• Shows domain of function implicitly on sketch	1

Question 3 (b) (i)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Correctly obtains equation $y - \frac{c}{p} = \frac{-c^2}{c^2 p^2}(x - cp)$	2
• Obtains gradient $\frac{-c^2}{c^2 p^2}$ at P	1

Question 3 (b) (ii)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Correctly demonstrates T is given point	2
• Attempts to solve $x + p^2 y = 2cp$ $x + q^2 y = 2cq$	1

Question 3 (b) (iii)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Obtains $q = 2p$ • Correctly uses their expression for T to show the locus is a hyperbola • Correctly finds the eccentricity of their hyperbola	3
• Two of the above	2
• One of the above	1

Question 4 (a) (i)

Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Finds correct coordinate	1

Question 4 (a) (ii)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Obtains correct integrand with limits consistent with (i)	3
• Obtains correct integrand OR • Gives correct limits with simple mistake in integrand	2
• Clearly indicates understanding of method of cylindrical shells	1

Question 4 (a) (iii)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Obtains correct answer	2
• Correctly obtains indefinite integral, but fails to deal correctly with limits OR • Makes one error in otherwise correct working	1

Question 4 (b) (i)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Gives correct reasoning to show result	2
• Gives first significant step in correct argument eg showing *DSAR* in cyclic	1

Question 4 (b) (ii)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Writes correct argument showing $\angle DST = \pi - \angle DCT$	2
• Gives first significant step in correct argument eg showing *DSTC* is cyclic	1

Question 4 (b) (iii)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Provides any correct argument that R, S, T are collinear	2
• Gives first significant step in correct argument eg showing $\angle DCT + \angle DAB = \pi$	1

Question 4 (c) (i)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	1

Question 4 (c) (ii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	2
• Recognises that there are 6 ways of ordering 3 numbers	1

Question 5 (a)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Obtains correct answer	2
• Uses a correct method but obtains incorrect answer	1

Question 5 (b) (i)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Finds correct equation	2
• Uses a correct method but obtains incorrect answer	1

Question 5 (b) (ii)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Finds correct equation	2
• Uses a correct method but obtains incorrect answer	1

Question 5 (b) (iii)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Finds correct value	2
• Uses a correct method but obtains incorrect answer OR correctly evaluates one of $\alpha + \beta + \gamma$, $\alpha\beta + \alpha\gamma + \beta\gamma$	1

Question 5 (c) (i)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Gives correct gradient for tangent • Correctly substitutes coordinates of *P* into point-gradient form of equation of tangent (or equivalent)	2
• Obtains correct derivative	1

Question 5 (c) (ii)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Correctly derives equation	2
• Obtains conditions that *T* lies on tangents at *P* and *Q*	1

Question 5 (c) (iii)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Provides a correct argument to prove *TS* and *SR* are perpendicular	3
• Finds the coordinates of *R*, OR the gradient of *TS*, and indicates the product of the gradients is –1	2
• Gives coordinates of *S* OR equation of directrix OR $b^2 = a^2\left(1 - e^2\right)$	1

Question 6 (a) (i)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Provides correct demonstration	1

Question 6 (a) (ii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Attempts to resolve forces vertically and horizontally • Includes $mr\omega^2$ in equation for horizontal forces • Provides any correct expression for $T - N$	3
• Two of above	2
• One of above	1

Question 6 (a) (iii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Substitutes $N = 0$ into equations of (ii) • Obtains correct expression for ω or ω^2	2
• One of above	1

Question 6 (b) (i)

Outcomes assessed: E2, E8, E9

MARKING GUIDELINES

Criteria	Marks
• Provides correct demonstration	1

Question 6 (b) (ii)

Outcomes assessed: E2, E8, E9

MARKING GUIDELINES

Criteria	Marks
• Uses $\tan^2\theta + 1 = \sec^2\theta$ • Substitutes $u = \tan\theta$, $du = \sec^2\theta d\theta$ (or equivalent) • Obtains $\int_0^1 u^{n-2} du = \frac{1}{n-1}$	3
• Two of above	2
• One of above	1

Question 6 (b) (iii)

Outcomes assessed: E2, E8, E9

MARKING GUIDELINES

Criteria	Marks
• Provides correct explanation of $I_{n+1} < I_n$ • Correctly deduces $I_n < \frac{1}{2(n-1)}$ • Correctly deduces $I_n > \frac{1}{2(n+1)}$	3
• Two of above	2
• One of above	1

Question 6 (b) (iv)

Outcomes assessed: E8, E9

MARKING GUIDELINES

Criteria	Marks
• Correctly demonstrates result	2
• Obtains $I_5 = \frac{1}{2}\ln 2 - \frac{1}{4}$ OR • Obtains $\frac{1}{12} < I_5 < \frac{1}{8}$	1

Question 7 (a) (i)

Outcomes assessed: HE5

MARKING GUIDELINES

Criteria	Marks
• Uses $V = Ay$	1

Question 7 (a) (ii)

Outcomes assessed: E2, E5

MARKING GUIDELINES

Criteria	Marks
• Provides correct derivation	4
• Provides unfinished correct derivation or derivation with a single mistake	3
• Provides derivation of relationship between t and y showing constants	2
• Separates variables	1

Question 7 (a) (iii)

Outcomes assessed: E2

MARKING GUIDELINES

Criteria	Marks
• Obtains correct answer	2
• Substitutes either $\frac{y}{y_0} = \frac{1}{2}$ or $t = 10$ with incorrect answer	1

Question 7 (b) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Provides correct explanation	2
• Provides partial explanation that shows understanding of vector addition	1

Question 7 (b) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Provides correct argument	2
• Shows $\angle P_0OP_1 = \angle P_0P_2P_1$	1

Question 7 (b) (iii)

Outcomes assessed: E2, E3, E9

MARKING GUIDELINES

Criteria	Marks
• Provides correct argument	2
• Shows that $P_0P_1P_2P_3$ is cyclic	1

Question 7 (b) (iv)

Outcomes assessed: E2, E3, E9

MARKING GUIDELINES

Criteria	Marks
• Provides correct argument	2
• Shows that vertices O, P_0, P_1, P_2, P_3 form a regular pentagon	1

Question 8 (a) (i)

Outcomes assessed: E3, E9

MARKING GUIDELINES

Criteria	Marks
• Puts $n = 2m + 1$ into de Moivre's theorem and indicates the need to take the imaginary part • Gives binomial expansion of $(\cos\theta + i\sin\theta)^{2m+1}$	2
• One of above	1

Question 8 (a) (ii)

Outcomes assessed: E4, E9

MARKING GUIDELINES

Criteria	Marks
• Puts $x = \cot^2\theta$ to obtain $p(x)$ from (i) • Solves $\sin(2m+1)\theta = 0$, $\sin\theta \neq 0$ • Explains why $k = 1, 2, \ldots, m$ gives solution	3
• Two of above	2
• One of above	1

Question 8 (a) (iii)

Outcomes assessed: E2

MARKING GUIDELINES

Criteria	Marks
• Indicates LHS is sum of roots • Shows RHS is $\binom{2m+1}{3} \Big/ \binom{2m+1}{1}$	2
• One of above	1

Question 8 (a) (iv)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Applies $\cot\theta < \frac{1}{\theta}$ in (iii) • Rearranges $\frac{m(2m-1)}{3} < \frac{(2m+1)^2}{\pi^2}\left[\frac{1}{1}+\frac{1}{2^2}+\ldots+\frac{1}{m^2}\right]$	2
• One of above	1

Question 8 (b) (i)

Outcomes assessed: E2, E7

MARKING GUIDELINES

Criteria	Marks
• Provides diagram or equivalent relating KL to ΔABE • Provides reasons for $KL = a - x$ • Uses correct method to find LM • Obtains $LM = a + x$ AND area $= a^2 - x^2$	4
• Three of above	3
• Two of above	2
• One of above	1

Question 8 (b) (ii)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Obtains correct answer, given their answer to (i), provided the answer is positive	2
• Obtains correct expression for the volume as an integral	1