2003 HSC Notes from the Marking Centre
Mathematics Extension 1

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2003 HSC NOTES FROM THE MARKING CENTRE
MATHEMATICS EXTENSION 1

Introduction

This document has been produced for the teachers and candidates of the Stage 6 course, Mathematics Extension 1. It is based on comments provided by markers on each of the questions from the Mathematics Extension 1 paper. The comments outline common sources of error and contain advice on examination technique and how best to present answers for certain types of questions.

It is essential for this document to be read in conjunction with the relevant syllabus, the 2003 Higher School Certificate Examination, the Marking Guidelines and other support documents that have been developed by the Board of Studies to assist in the teaching and learning of the Mathematics Extension 1 course.

Question 1

This question tested knowledge and skills from different areas of the course and was answered well by the majority of candidates. Part (c) caused candidates the most trouble.

(a) Most candidates knew the formula for the division of an interval and applied it well. Common errors included: dividing the interval in the ratio 3:1; dividing the interval externally; poor arithmetic. It is important to show working since credit was given for the correct numerical expression even if there was a subsequent error in the arithmetic.

(b) Many candidates appeared to be oblivious of the unknown in the denominator and did not consider that the denominator may be negative. A sizable minority confused the inequality with $\left|\frac{3}{x-2}\right| \leq 1$. Another fairly common mistake was to square both sides of the inequality.
Several different legitimate methods were used to solve the inequality. Methods used:

- Multiplying both sides of the inequality by $(x–2)^2$. This was the most popular method. However, careless algebraic mistakes, as well as not being able to correctly solve a quadratic inequality, prevented many candidates from obtaining the correct solution.
- Critical points method. Solving $\frac{3}{x-2} = 1$ (noting $x \neq 2$), hence obtaining critical values 2 and 5 and testing on number line for the solution. This was generally the method that was most successfully implemented since the algebra was simple to handle.
- Considering the cases $x < 2$ and $x > 2$ and solving the two sets of inequalities. This method was poorly done. Candidates had difficulty drawing the correct conclusions from the resulting inequalities.
- Using the graphs of $y = \frac{3}{x-2}$ and $y = 1$. This was not a common approach. However, those candidates who chose this method were generally successful in finding the correct solution.

(c) Many candidates arrived at the correct answer of $\frac{3}{2}$ without reasonable justification. Many were uncomfortable dealing with $\lim_{x \to 0} \frac{x}{\sin x}$ rather than $\lim_{x \to 0} \frac{\sin x}{x}$.

(d) This was well done. However there were candidates who did not appear to know the meaning of 'Cartesian equation' and found the equation of the tangent to $x^2 = 4ay$ at the point $\left(\frac{t}{2},\ 3t^2\right)$.

(e) There were some candidates who clearly did not understand the process of integration by substitution and were unable to obtain an integral involving only the variable *u*.

Common mistakes were: incorrectly dealing with the 2 in $\frac{du}{dx} = 2x$; incorrectly integrating $\int \frac{du}{2u^3}$; not changing the limits of integration.

Question 2

This question produced a good standard of responses, however some candidates used very inefficient methods to obtain relatively few marks.

(a) There was some confusion between domain and range. A number of candidates confused inverse sine and cosine functions. The graphs were often in conflict with the candidate's stated range and domain.

(b) Generally well done.

(c) Generally well done. Many did not recognise this as a standard integral. Attempts using substitution methods frequently led to incorrect solutions.

(d) Candidates who used the expression for the general term in a binomial expansion were usually successful. Those who wrote out the complete expansion of $(2 + x^2)^5$ frequently made algebraic errors.

(e) (i) Candidates who started by stating that $R\cos(x+\alpha) = R\cos x\cos\alpha - R\sin x\sin\alpha$ and then equated coefficients were generally successful.

(ii) Many candidates failed to obtain full marks because of careless errors. Common errors included: failure to label intercepts and place scales on both axes; sketches with $f(0) \neq f(2\pi)$; the amplitude not indicated on the graph.

Candidates who used addition or subtraction of ordinates frequently became confused, resulting in some ordinates being added and others subtracted.

Question 3

Overall, this question was answered well by the majority of candidates.

(a) This part was correctly answered by most candidates.

(b) (i) Candidates were first required to show that the motion was in fact simple harmonic motion. It was clear from the responses that many candidates believed, or have been taught, that recognition of the general equation $x = a \sin (nt + \alpha)$ was sufficient proof. Those candidates who knew that they needed to show that acceleration was proportional to displacement usually managed to differentiate correctly to find expressions for velocity and acceleration. However, many were then unable to correctly express acceleration as a function of displacement.

(ii) This part was handled well by the majority of candidates.

(iii) A large proportion of candidates started their response by stating that maximum speed occurs when $v = 0$. Candidates who correctly stated that acceleration needed to be 0 often obtained a negative result and then either ignored the negative or took the absolute value. A number of candidates calculated the velocity at $t = \frac{\pi}{3}$ and then discarded this result because the result was negative, confusing the terms 'velocity' and 'speed'.

(c) Most candidates were able to correctly show by means of a list or a table that there are four outcomes that give a sum of 5 from a total of 36 possible outcomes when a pair of fair dice is tossed. Candidates need to be reminded that when asked to explain a particular result they need to actually explain in words or symbols what they are doing. It is not sufficient to restate the question.
In part (ii) the phrase 'at least two' was misunderstood by many candidates. Candidates who used the complement $1 - \left[P(0) + P(1)\right]$ were generally more successful in writing the correct numerical expression.

(d) This was found to be the most difficult part of this question, although quite a considerable number presented perfect answers. Many responses were consistently well written and demonstrated good understanding of the concept of mathematical induction and practice in using it.

Poorer responses often displayed confusion about what was being proved. 'Show true for $n = 1$' requires some sort of substitution to demonstrate the result. Many candidates confused the nth term with the sum to n terms and/or did not really understand that there was a sum involved at all.

Question 4

This question was quite well done by most students, but very few candidates were able to score full marks. Candidates need to be reminded to show all working, particularly substitution into formulae.

(a) Almost all candidates successfully gained this mark by giving the answer as ${}^{14}C_6$.

(b) Candidates were required to use Newton's method to find a better approximation to the root of the function $f(x) = \sin x - \frac{2x}{3}$. Most students knew the correct formula, differentiated correctly and showed their substitution into the formula. However, common errors included: working in degrees; not working to a sufficient level of accuracy; lack of facility on the calculator. Candidates should be advised not to round the results of caclulations until the final step.

(c) Candidates were required to use the relationship between the roots and coefficients of a polynomial to find a missing coefficient in a polynomial equation, given that two of the roots were reciprocals. Most candidates understood correctly what the roots being reciprocals meant, but some students used α and $(-\alpha)$ or more commonly α and $(-\frac{1}{\alpha})$. Many candidates correctly stated one of the relationships between the roots and their coefficients, written in terms of α and $\frac{1}{\alpha}$. Only a few candidates made an efficient approach to finding k by substituting the root into the equation. Most candidates unsuccessfully attempted to solve simultaneous equations. Candidates are advised to note the number of marks assigned to the question and consider what approach might be most efficient.

(d) In general this part was poorly done with non-attempts by some candidates on all or some of the parts. Lack of facility with geometric notation and terminology affected some candidates. Candidates are advised not to introduce their own private notation eg *d* presumably for diameter. Concise reasons are also required rather than cumbersome explanations, which are often inaccurate and ambiguous.

(i) Candidates were required to give a valid reason, such as 'angles in the same segment are equal'.

(ii) Many candidates could successfully answer only this part out of the four parts in (d). A valid reason, such as 'the opposite angles are supplementary in a cyclic quadrilateral', was required.

(iii) Candidates were required to show that two angles were equal by considering each of the cyclic quadrilaterals separately to show that angles in the same segment were equal. Many candidates presumed right-angled triangles such as ΔTCR.

(iv) Candidates were required to show that $AR \perp CB$. Although many candidates had incorrectly assumed this result to prove (iii) they were still able to convincingly gain marks for this part. The majority of candidates used similar triangles to show this result, either considering ΔATQ and ΔCTR or ΔARB and ΔBCQ. Some candidates showed that $ACRQ$ was a cyclic quadrilateral and thus were able to show that $\angle AQC = \angle ARC = 90°$.

Question 5

(a) This part was well done. Candidates demonstrated that they knew the identity and were able to integrate a trigonometric function. The major problems were: rote learning the integral and then not being able to apply the $3x$; relating $2\cos^2 x = 1 + \cos 2x$ to $\cos^2 3x$; losing the $6x$ after the integration step.

(b) It is quite clear that inverse functions are not well understood by many candidates. Candidates did not interrelate the parts of the question. Many had a correct graph or expression but an incorrect domain.

(i) Some candidates confused their responses and did not make it clear whether they were referring to $g(x)$ or $g^{-1}(x)$. Drawing a diagram to assist explanation definitely helped.

(ii) Sketches were generally not very good. Some candidates tried to find the equation first and then could not draw the corresponding graph. Many graphs that were close to a correct attempt had the following problems: the end point (1, 2) was mislabelled as (2, 1); $g^{-1}(x)$ was not drawn to continuously decrease, or to be concave up; where $g(x)$ and $g^{-1}(x)$ had been drawn on the same axes the endpoints of each were unclear.

(iii) There was confusion between domain and range. Most candidates were able to give the domain of their graph in (ii).

(iv) The vast majority of candidates knew to 'swap' the x and y. Many had no idea how to then make y the subject. Some who did change the subject to y had difficulty making the correct choice for $g^{-1}(x)$.

(c) (i) This part was generally well done. Candidates only needed to verify that $T = A + Be^{kt}$ satisfies the differential equation. Those who integrated were not always successful.

(ii) The neatest and most economical solution changed the time-frame to $t = 0$ at $T = 80$, arriving at the value of k most efficiently. Candidates who did this were then able to adapt the conditions to answer (iii) correctly. Using a definite integral was a rare but also neat approach. Many candidates thought the initial temperature was $100°$. Some defined 'after a further 2 minutes' as $t = 2$ even though they had started at $t = 6$.

(iii) Nearly all candidates recognised that 'initially' means $t = 0$.

Question 6

(a) This part was reasonably well done, although more care needs to be taken when evaluating the constants of integration.

(i) This part was well done by those who remembered that acceleration was equal to $\frac{d}{dx}\left(\frac{v^2}{2}\right)$.

(ii) The key to this part was to invert the answer to part (i) and integrate. Many were not able to use the table of standard integrals.

(b) (i) Many candidates struggled to recognise the right-angled triangle BAE and instead used the sine rule. As a result answers for α were sometimes left in the form $p = \sin(\frac{\pi}{2} - \alpha)/\sin(\alpha)$.

Most candidates did not realise that the triangles AGE and CFH were similar, resulting in a lot of unnecessary calculations.

(ii) Very few candidates realised that $\angle EAG = \frac{\pi}{4}$.

(iii) Many candidates were able to develop an expression for the area of the quadrilateral but were unable to obtain the required expression.

(iv) There were four things required in this part: the derivative of the area, the values of p which makes this zero, an explanation as to why one of these yields a maximum, and the consequent maximum value of the area.

Question 7

Careless calculation and/or notation errors meant that many candidates failed to maximise their marks in this question.

(a) Some candidates had difficulty with the 3-dimensional nature of this question. Most candidates who attempted the question were able to calculate the lengths of the sides of the base triangle, although many experienced difficulty calculating the angle between these sides. When attempted, the use of the cosine rule to calculate the required distance was generally well done. The bearing between the two points was rarely calculated correctly. Candidates are reminded of the necessity to justify explicitly whether an angle is acute or obtuse when using the sine rule. Use of the cosine rule to calculate an angle in the base triangle enabled some candidates to calculate the bearing correctly.

(b) (i) Generally well done.

(ii) This part was also generally well done, with most candidates correctly calculating the time when $y = 0$ (time of flight) and substituting this time into the equation for x (range).

(iii) Very few candidates indicated an understanding of the maximum range occurring when $\alpha = 45^{\circ}$ or the restriction imposed by the height of the ceiling. Candidates needed to explain this boundary between the two cases.

Mathematics Extension 1

2003 HSC Examination Mapping Grid

Question	Marks	Content	Syllabus outcomes
1 (a)	2	6.7 E	P4
1 (b)	3	1.4 E	P4, PE3
1 (c)	2	13.5 E	H5
1 (d)	2	9.6 E	PE3
1 (e)	3	11.5	HE6
2 (a)	2	15.2 E	HE4
2 (b)	2	15.5	P7, HE4
2 (c)	2	15.5	HE4
2 (d)	2	17.3 E	PE3, HE7
2 (e) (i)	2	5.9	H5
2 (e) (ii)	2	13.3	H5, PE6
3 (a)	2	18.1	PE3
3 (b) (i)	2	14.4	HE3
3 (b) (ii)	1	14.4	HE3
3 (b) (iii)	1	14.4	HE3
3 (c) (i)	1	3.2	H5
3 (c) (ii)	2	18.2	HE3
3 (d)	3	7.4	HE2
4 (a)	1	18.1	PE3
4 (b)	3	16.4	HE7
4 (c)	2	16.2	PE3
4 (d) (i)	1	2.10	PE2, PE3, PE6
4 (d) (ii)	1	2.10	PE2, PE3, PE6
4 (d) (iii)	2	2.10	PE2, PE3, PE6
4 (d) (iv)	2	2.10	PE2, PE3, PE6

Question	Marks	Content	Syllabus outcomes
5 (a)	2	13.6	HE6
5 (b) (i)	1	15.1	HE4
5 (b) (ii)	1	15.1	HE4
5 (b) (iii)	1	15.1	HE4
5 (b) (iv)	2	15.1	HE3, HE4
5 (c) (i)	1	14.2	H3, HE3
5 (c) (ii)	3	14.2	H3, HE3
5 (c) (iii)	1	14.2	HE3, HE7
6 (a) (i)	3	14.3	HE5
6 (a) (ii)	2	14.3	HE5
6 (b) (i)	2	5.2	H5, PE2
6 (b) (ii)	2	5.7	H5, PE2
6 (b) (iii)	1	5	H5, PE2
6 (b) (iv)	2	10.6	H5, PE2
7 (a)	4	5.6	H5
7 (b) (i)	2	14.3 E	HE3
7 (b) (ii)	2	14.3 E	HE3
7 (b) (iii)	4	14.3 E	HE3

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2003 HSC Mathematics Extension 1
Marking Guidelines

Question 1 (a)
Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Correct answer or equivalent expression	2
• One coordinate correct	1

Question 1 (b)
Outcomes assessed: P4, PE3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• A correct approach with a single error	2
• Identifies one of the end points $x = 2$ or $x = 5$ or notes $x \neq 2$	1

Question 1 (c)
Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Applies $\lim_{x \to 0} \frac{\sin x}{x} = 1$	1

Question 1 (d)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Reasonable attempt to eliminate *t* from the equation	1

Question 1 (e)

Outcomes assessed: HE6

MARKING GUIDELINES

Criteria	Marks
• Correct answer or equivalent numerical expression	3
• Correct apart from a single error such as not changing the limits, or incorrectly dealing with the 2 in $\frac{du}{dx} = 2x$	2
• Demonstrates some understanding of the process of integration by substitution (eg correct change of limits or use of $\frac{du}{dx} = 2x$)	1

Question 2 (a)

Outcomes assessed: HE4

MARKING GUIDELINES

Criteria	Marks
• Correct graph	2
• Any of: correct shape, correct domain, correct range	1

Question 2 (b)

Outcomes assessed: P7, HE4

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Clear attempt to apply product rule or evidence that $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$	1

Question 2 (c)

Outcomes assessed: HE4

MARKING GUIDELINES

Criteria	Marks
• $\frac{\pi}{4}$ or decimal approximation	2
• Correct evaluation after incorrectly obtaining $\sin^{-1}\frac{x}{2}$ or similar as primitive OR • An answer left in form $\sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1} 0$	1

Question 2 (d)

Outcomes assessed: PE3, HE7

MARKING GUIDELINES

Criteria	Marks
• ${}^5C_2\ 2^3$ or equivalent	2
• Includes ${}^5C_3$ or ${}^5C_2$ or equivalent	1

Question 2 (e) (i)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Obtains $R\cos\alpha = 1$ and $R\sin\alpha = 1$ or equivalent progress	1

Question 2 (e) (ii)

Outcomes assessed: PE6, H5

MARKING GUIDELINES

Criteria	Marks
• Correct graph	2
• Attempt to shift a cosine curve by $\frac{\pi}{4}$ or equivalent	1

Question 3 (a)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Numerical expression for correct answer	2
• Demonstrates an understanding that there would be 9! arrangements if letters were not repeated OR • Displays knowledge of how to deal with repeated letters	1

Question 3 (b) (i)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Reasonable attempt to find both derivatives or equivalent	1

Question 3 (b) (ii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 3 (b) (iii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 3 (c) (i)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• An answer which shows evidence that the student understands that the four outcomes (1, 4), (2, 3), (3, 2) and (4, 1) give a sum of 5 and that there are 36 possible outcomes	1

Question 3 (c) (ii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Correct solution — A numerical expression is sufficient (evaluation is not required)	2
• Identification of the complementary event or correct numerical probability for one of the events $n = 2, \ldots, n = 6$, $n = 7$ as an ingredient of their answer	1

Question 3 (d)

Outcomes assessed: HE2

MARKING GUIDELINES

Criteria	Marks
• A solution which verifies the statement when $n = 1$, and correctly deduces that if the statement holds for a positive integer n, it also holds for the next integer $n + 1$	3
• A proof which is correct apart from the verification of the initial case or a proof with verification of the initial case and which shows the evidence of the structure of an induction proof, and a reasonable attempt at the algebra in the induction step	2
• Verifies the initial case, or makes an appropriate beginning to the induction step	1

Question 4 (a)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Correct numerical coefficient	1

Question 4 (b)

Outcomes assessed: HE7

MARKING GUIDELINES

Criteria	Marks
• Obtains a correct numerical expression for the second approximation	3
• Correctly computes $f(1.5)$ and $f'(1.5)$ OR • Makes a single error in the process (eg works in degrees, or $x_2 = x_1 + \dfrac{f(x_1)}{f'(x_2)}$, or $f'(x) = -\cos x - \dfrac{2}{3}$)	2
• Computes $f(1.5)$ and $f'(1.5)$ using degrees rather than radians OR • Substitutes incorrect information into the correct formula for Newton's method OR • Correctly computes $f'(1.5)$	1

Question 4 (c)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Finds one of the roots or makes equivalent progress	1

Question 4 (d) (i)

Outcomes assessed: PE2, PE3, PE6

MARKING GUIDELINES

Criteria	Marks
• Valid reason	1

Question 4 (d) (ii)

Outcomes assessed: PE2, PE3, PE6

MARKING GUIDELINES

Criteria	Marks
• Valid reason	1

Question 4 (d) (iii)

Outcomes assessed: PE2, PE3, PE6

MARKING GUIDELINES

Criteria	Marks
• Correct proof with reasons	2
• Correct proof with some reasons omitted or progress equivalent to establishing –*TPQ* = –*TAQ*	1

Question 4 (d) (iv)

Outcomes assessed: PE2, PE3, PE6

MARKING GUIDELINES

Criteria	Marks
• Correct proof with reasons	2
• Shows that an appropriate angle is complementary to –*TAQ* or –*QCB* or equivalent progress	1

Question 5 (a)

Outcomes assessed: HE6

MARKING GUIDELINES

Criteria	Marks
• Correct answer (+ *C* may be omitted)	2
• Makes appropriate double angle formula substitution OR • Correctly integrates an incorrect expression derived from a reasonable attempt to use the double angle formula	1

Question 5 (b) (i)
Outcomes assessed: HE4

MARKING GUIDELINES

Criteria	Marks
• An answer which essentially asserts that $f(x)$ is not 1 – 1	1

Question 5 (b) (ii)
Outcomes assessed: HE4

MARKING GUIDELINES

Criteria	Marks
• Correct shape and position of graph	**1**

Question 5 (b) (iii)
Outcomes assessed: HE4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 5 (b) (iv)
Outcomes assessed: HE3, HE4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Obtains $x = y^2 - 4y + 5$	1

Question 5 (c) (i)
Outcomes assessed: H3, HE3

MARKING GUIDELINES

Criteria	Marks
• Shows that the expression satisfies the differential equation OR • Finds the general solution of the *DE*	1

Question 5 (c) (ii)

Outcomes assessed: H3, HE3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Obtains one value by legitimate means OR • Uses a correct method with a single mistake (eg $T = 50$ at $t = 2$ leading to $k = \dfrac{\log_e 2}{4}$ and $B = 15\sqrt{2}$)	2
• Substitutes both initial conditions into the equation OR • Uses one initial condition and the given value of k to find B	1

Question 5 (c) (iii)

Outcomes assessed: HE3, HE7

MARKING GUIDELINES

Criteria	Marks
• Correct solution consistent with B found in part (ii)	1

Question 6 (a) (i)

Outcomes assessed: HE5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Obtains $\dfrac{1}{2}v^2 = 2x^4 + 16x^2 + C$ or equivalent	2
• Rewrites as $\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = 8x\left(x^2 + 4\right)$ or equivalent	1

Question 6 (a) (ii)

Outcomes assessed: HE5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Obtains $t = \int \frac{dx}{2\left(x^2 + 4\right)}$ or equivalent	1

Question 6 (b) (i)

Outcomes assessed: H5, PE2

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Correct solution for either *a* or *b*	1

Question 6 (b) (ii)

Outcomes assessed: H5, PE2

MARKING GUIDELINES

Criteria	Marks
• Correct proof	2
• Observes that $\alpha + \beta = \frac{3\pi}{4}$ or attempts to use angle sum formula for tangent or cotangent or equivalent	1

Question 6 (b) (iii)

Outcomes assessed: H5, PE2

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 6 (b) (iv)

Outcomes assessed: H5, PE2

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Obtains $p = -1 \pm \sqrt{2}$ or equivalent progress	1

Question 7 (a)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution. If sine rule used in D*ABD*, justification for obtuse or acute angle is required. (NB –*DAB* is obtuse)	4
• Correct solution apart from one error, or calculates *AD* correctly	3
• Uses the information to correctly compute –*ABD* and at least one of *AB* or *BD*	2
• Uses the information to compute one piece of data about D*ABD*	1

Question 7 (b) (i)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Finds $\frac{dy}{dt}$ and attempts to solve $\frac{dy}{dt} = 0$ or equivalent	1

Question 7 (b) (ii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Deduces that $t = \frac{2v \sin\alpha}{g}$	1

Question 7 (b) (iii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	4
• Shows that $v^2 = 4g(H-S)$ is the boundary between the two cases, and – determines that $\cos\alpha = \dfrac{\sqrt{v^2 - 2g(H-S)}}{v}$ in the case $v^2 \geq 4g(H-S)$ or equivalent progress OR – deals with one case completely and determines the value of α for maximum range in the other case	3
• Shows range $= \dfrac{v^2}{g}$ if $v^2 \leq 4g(H-S)$ or shows $\cos\alpha = \dfrac{\sqrt{v^2 - 2g(H-S)}}{v}$ when skims ceiling or equivalent progress	2
• Shows that the range $= \dfrac{v^2}{g}$ if ball is thrown at 45° without hitting ceiling OR • Shows that the ball can be thrown at 45° just touching the ceiling if $v^2 = 4g(H-S)$ OR • Shows that it skims ceiling when $\alpha = \sin^{-1}\left(\dfrac{\sqrt{2g(H-S)}}{v}\right)$	1