2004 HSC Notes from the Marking Centre Mathematics

Published by Board of Studies NSW
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Sydney 2001
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ISBN 1 7414 7221 0

2005084

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2004 HSC NOTES FROM THE MARKING CENTRE MATHEMATICS

Introduction

This document has been produced for the teachers and candidates of the Stage 6 course, Mathematics. It is based on comments provided by markers on each of the questions from the Mathematics paper. The comments outline common sources of error and contain advice on examination technique and how best to present answers for certain types of questions.

It is essential for this document to be read in conjunction with the relevant syllabus, the 2004 Higher School Certificate Examination, the marking guidelines and other support documents that have been developed by the Board of Studies to assist in the teaching and learning of the Mathematics course.

As a general comment candidates need to read the questions carefully and set out their working clearly. It is unwise to do working on the question paper, and if a question part is worth more than 1 mark the examiners expect more than just a bald answer. Any rough working should be included in the answer booklet for the question to which it applies.

Question 1

Overall this question was well done. The vast majority of candidates obviously felt comfortable working with the basic principles involved in this question.

(a) This part was not handled well by many candidates who had trouble with either the exponent or the rounding. Common errors included 3.40×10^6, 3 400 000. In some cases negative indices were used. In a number of cases this was the only mark not awarded for this question.

(b) This part was generally well done by the majority of candidates. An error that occurred on a number of occasions was the confusion between differentiation and integration.

(c) The best responses on this part correctly formed the lowest common denominator (LCD) in the early stages of working, correctly simplifying and then solving. The majority of those who got this part incorrect fell into one or more of the following error categories:
- forgetting to multiply the right-hand side (RHS) by the LCD
- multiplying the RHS by 12 twice
- incorrect removal of brackets
  eg $4(x–5) – 3(x+1)$ becomes $4x–5–3x+1$ OR $4x–20–3x+3$.

(d) A significant number of candidates was not able to correctly expand $(3-\sqrt{2})^2$. A fair proportion of responses incorrectly interpreted the expansion as $(3-\sqrt{2})(3+\sqrt{2})$.

(e) Generally well done by candidates. A significant number of responses correctly identified the individual probabilities but failed to combine them. The most common errors involved the multiplication, rather than the addition, of probabilities and using the incorrect size of the sample space through basic addition of the number of jellybeans.

(f) Those candidates who interpreted the question as $-5 \le x+1 \le 5$ appeared to have the most success. Graphical solutions also tended to be correct in the majority of cases. Those who split the question into two equations often had problems with the negative case, eg $-x-1 \le 5$ became $-x \le 6$ and then $x \ge 6$.

A substantial number of responses retained the absolute value symbols as brackets within the solution.

Question 2

(a) (i) This part was well done. The most common errors were due to an incorrect formula, to not substituting correctly into the formula, or to not handling $2-(-1)$ correctly.

(ii) This part was well done. The most common errors were due to using the incorrect formula $m = \dfrac{x_2 - x_1}{y_2 - y_1}$ or to not handling $2-(-1)$ correctly.

(iii) The most common approach to solving this part was to first find $\angle ABO$ from the gradient of the line AB as found in part (ii) and then to calculate $90° - \angle ABO$ to find the required angle. The most common errors included not knowing which angle $\tan^{-1}(-1)$ gave or stopping after finding $\angle ABO$.

(iv) This part was well done. The most common error was to start with the required equation, rearrange it into the form $y = mx + b$ and then claim that this showed that this was the equation of the line AB since $m = -1$.

(v) The most common error in this part was to not shade in the entire half-plane. Shading which stopped at the line l or at the x-axis was common. In many cases it was difficult to tell whether the shading was meant to cover the entire half-plane.

(vi) This part was well done. The most common error was to try to determine the equation of l using the point-gradient formula for a line and using a gradient of either 0 or 1 for l. In a number of cases candidates found the coordinates of the point B as $y = 0$ and $x = 2$ but then did not make it clear that the equation of l was $x = 2$.

(vii) Most candidates attempted this question using coordinate geometry rather than plane geometry. The common errors that occurred in using the coordinate geometry approach included:

- finding that the gradient of AC was 1 but not knowing what to do from there
- drawing AC perpendicular to l instead of AB
- having the line pass through B or the midpoint of A and B instead of A.

(b) This part was very well done. The most common error was to not state the reasons associated with each step in the calculation.

(c) This part was not done well. The most common errors included:

- not knowing the correct form of the discriminant

- writing $-k^2 - 16$ instead of $(-k)^2 - 16$
- not being able to solve a quadratic inequality.

Candidates who used a diagrammatic approach to solve the quadratic inequality were generally successful.

Question 3

This question assessed competency in differentiating and integrating a variety of functions as well as applying trigonometry. The overall standard of responses was quite high.

(a) (i) Most candidates recognised the need to use the product rule, although some then had difficulty either differentiating $\log_e x$ or simplifying $\frac{x^2}{x}$. The other common incorrect answer was $2x \times \frac{1}{x}$.

(ii) The majority of candidates recognised that the chain rule was involved to obtain the $5(1+\sin x)^4$ part of the answer. Many candidates, however, did not differentiate sin x or incorrectly used – cos x. A limited number dropped the x out of the function/s and were penalised.

(b) (i) A significant number of candidates found the incorrect primitive, which suggests they did not use the standard integrals. In evaluating the answer, a common error was $e^6 - e^3 = e^3$.

(ii) A significant proportion of candidates struggled with this part or simply did not attempt it at all. Most other candidates realised the primitive involved $A\log(x^2 - 3)$, with a variety of expressions for A being presented. Some poor responses involved $\tan^{-1}$ or the use of the quotient rule on the integrand.

(c) (i) Not all candidates recognised that the cosine rule was required. Many struggled with using the sine rule, Pythagoras' theorem or assuming or constructing right triangles. The most common errors amongst those who used the cosine rule included poor calculation skills (eg 1044 – 720cos 20° = 324cos 20°) and the use of 30° as the angle rather than 20°. Other errors included not taking the square root, while constant rounding to integer values only compounded errors leading into part (ii).

(ii) Many candidates carried errors from (i) into this part and were able to calculate a value for $\angle RQP$ and the bearing as $270° + \angle RQP$. Most candidates used the sine rule, although there was still a significant proportion wanting to do everything by Pythagoras' theorem. Very few candidates gave their answer as a compass bearing ($N78°W$).

Question 4

(a) Most candidates performed well on this question with the usual response being an implementation of the formulae $l = r\theta$ and $A = \frac{1}{2}r^2\theta$. Some candidates correctly argued in terms of an appropriate ratio of the area of the complete circle. The need to choose appropriately between degree and radian measure was a source of some confusion. Some candidates did not give the area in exact form.

(b) (i) The majority of candidates were aware that stationary points are found by setting the first derivative to zero. However, many responses faltered on the technicalities of differentiating and solving the resulting quadratic equation. The second derivative test was the most popular testing instrument. However, arguments based on the first derivative and the function values themselves were also common. Many candidates had poor or incomplete presentation of standard tests. Some incorrectly classed a non-horizontal point of inflexion as a stationary point.

(ii) Many candidates did not indicate the x-intercept on their sketch.

(iii) Many candidates who effectively implemented the second derivative test could not then deal with concavity as an abstract concept in part (iii).

(c) Common errors included the claims that $(2\sec x)^2 = 2\sec^2 x$, $\int \sec^2 x = \tan^2 x$, $\tan(\frac{\pi}{3}) = \tan(\sqrt{3})$ and the omission of π in the volume formula. Many candidates did not present an exact volume as a solution.

Question 5

(a) (i) Most candidates correctly used the formula $T_n = a + (n-1)d$ to obtain the answer. A substantial number of candidates resorted to listing lengths of lesson times to solve this problem. Common errors included calculating and finding T_{19} or T_{20} or even T_{25} and/or using an incorrect formula.

(ii) Candidates who listed in (i) were able to add the first 21 terms to obtain the correct answer. Most common errors included an incorrect formula or incorrect addition of time intervals.

(iii) Some candidates used trial and error with the sum formula or the quadratic equation to arrive at the correct answer. Most candidates did not convert 50 hours to 3000 minutes, and some made numerous unsuccessful attempts to solve the resulting quadratic equation. The quadratic equation $n^2 + 11n - 1200 = 0$ was incorrectly factorised to $(n-30)(n+40) = 0$, leading to the answer 30. Some candidates did not support an answer with working. Many candidates used the formula for the nth term.

(b) (i) Most candidates found the correct answer of $x = 4$. Some candidates did not attempt to evaluate $\cos 0$. Others took $\cos 0 = 0$, and so obtained the incorrect answer $x = 1$.

(ii) The graph was poorly drawn by a large number of candidates. Axes were poorly labelled and the curve drawn did not resemble a sine curve. Many curves looked parabolic or simply V-shaped. The poorer sketches in most responses led to incorrect answers in later parts of the question. Correct responses clearly showed the amplitude, correct boundaries, period and stationary points at $t = 0, \frac{\pi}{2}, \pi$.

(iii) This was well done by candidates who had the correct graph in (ii). They were able to refer to the graph for their answers. Many errors were made with attempts using calculus. Differentiation was either incorrect or candidates could not solve a simple trigonometric equation such as $\sin 2t = 0$. Some candidates did not read the question carefully and did not give the 'first' time after $t = 0$ <u>and</u> the corresponding x-value as required.

(iv) Many candidates did not attempt this part. Errors in the calculus method by some candidates led to an incorrect interpretation of $\dot{x} = 0$, instead of $\ddot{x} = 0$ for maximum velocity, resulting in the incorrect calculation for time. Incorrect solutions of trigonometric equations were common. Some candidates solved the trigonometric equation in degrees instead of radians, even after differentiating the expression.

Question 6

Nearly all candidates attempted this question and obtained some marks. Better responses contained solutions that included explanations and reasons, diagrams, tables etc. This often enabled candidates to arrive at the correct solution or to be awarded marks for the progress made.

(a) Responses that recognised the equation as a quadratic were generally successful, obtaining the solutions $e^x = –5, 2$ to the resulting equation. Many candidates did not show that the only solution for x was $x = \log_e 2$. They did not realise that $e^x = –5$ has no solution.

(b) In geometry questions it is strongly recommended that candidates copy the diagram and then use their diagram in attempting a solution. This often helps in attaining some marks for a partial attempt and also helps in arriving at a complete solution. Many errors were made in naming angles.

(i) The congruence proof required correct statements with reasons. The correct congruence test (eg SAS) for the working needed to be shown.

(ii) Very few candidates gained full marks in this part. Candidates who began by letting $\angle BAC = 2\theta$ and then marking the information on their diagram were generally successful in gaining some marks. Some responses did not show the two answers required. Many showed a weakness in working with ratios and surds.

(c) Candidates who used a table to list the outcomes were overwhelmingly more successful in answering this question.

(i) There was a significant number of incorrect answers with no working shown. However, some candidates did not draw their table until part (ii) and then ignored their incorrect answer. It is recommended that even if a problem seems easy, a systematic counting technique be shown.

(ii) Nearly all candidates who employed a systematic counting technique obtained the correct solution.

(iii) Many responses did not explain what was being attempted. This meant that part marks could not be awarded when the correct answer was not obtained. Many candidates missed that the complementary event to {< 45} was {equal to 45 or greater than 45}. Many candidates did not realise the number in the sample space was 36^2.

Question 7

Overall the question was well done, with many candidates scoring high marks.

(a) Common errors included summing four terms instead of three, using the formula for the sum of an arithmetic series, and failing to total the three terms.

(b) Most candidates were able to handle standard exponential growth quite well.

(i) Most candidates successfully differentiated. Some correctly solved the differential equation by integration, which was not required. Candidates should note the marks allocated to a part and use this as a guide to the expected length or depth of the answer.

(ii) Most candidates could give the correct value of A.

(iii) Common errors included using t =12 or 14 instead of 13, and incorrectly evaluating $\ln\frac{20}{17}$. Candidates should show their calculator display and round appropriately.

(iv) The calculations in this part were done well, but many candidates did not answer the question, which was 'Predict the year during which …'. All too often they left their answer as 45.43 years, or incorrectly rounded up to 46 and gave the answer as 2037.

(c) Candidates are reminded to read the question carefully and number the parts clearly. Many began to answer part (ii) first and often the parts merged together.

(i) The main error was a failure to recognise that the interest was compounding monthly. Many candidates launched into a series and did not clearly specify the value of the first \$80 after 25 years.

(ii) The question required the writing down and summation of a geometric series. Candidates who treated each \$80 as a separate investment and then added them were the most successful. Those who tried to calculate the value at the end of each month made errors with nests of brackets and often included an extra \$80. Other errors included the wrong number of terms and an incorrect formula for S_n. Some candidates thought that a time payment method was required.

Question 8

(a) (i) Most candidates correctly used the trigonometric identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

(ii) Many candidates were unable to simplify the trigonometric equation to obtain $8\sin^3\theta = 1$. Candidates often made basic algebraic errors, or did not give both solutions to the equation.

(b) (i) This part was generally well done.

(ii) Candidates were required to use calculus to obtain the gradient of the tangent. A significant number unnecessarily used the quotient rule to differentiate $y = \frac{x^2}{16}$.

(iii) This part was found to be challenging for many candidates. Many did not realise that C was on the directrix.

(iv) The most common approach to this part was to subtract the area between the parabola $y = \frac{x^2}{16}$ and the x-axis from the area under the line AB. Errors were frequently made integrating the functions and selecting the limits of integration.

(v) This part was found to be quite challenging for most candidates. The most successful approach was to use (iv) and subtract this area from the area of ΔABC. Those who attempted to find the area of ΔABC often made an error applying the perpendicular distance formula or the cosine rule. Those who divided the area into subparts often forgot to evaluate one of the components.

Question 9

There was a substantial number of non-attempts for this question and many candidates only answered one of the three parts.

(a) (i) Many candidates correctly identified $a = 1$ and $r = -\tan^2\theta$ but stopped at $\frac{1}{1+\tan^2\theta}$ or $\frac{1}{\sec^2\theta}$. A significant number of candidates did not substitute into the correct formula.

(ii) A large number of candidates had difficulty solving an inequality involving a trigonometric ratio. Some candidates correctly solved $|r| < 1$ in part (i) but did not recognise this as the answer to part (ii).

(b) (i) This part was the only part attempted by many candidates.

(ii) The attempts at this part were generally quite poor. Better responses clearly indicated the relationship between area and velocity. Many candidates gave explanations which were long and hard to read. Common errors included choosing $t = 5$ and $t = 2$. Some gave a range of values for t.

(c) (i) This part was generally well done. Most candidates successfully used the quotient rule to find the derivative of $\frac{\log_e x}{x}$ and were able to solve $f'(x) = 0$ to obtain $x = e$ or to substitute $x = e$ to show $f'(x) = 0$.

(ii) Responses using the gradients either side of $x = e$, as suggested in the question, were well done. Candidates who used the second derivative test were often unable to correctly complete this part.

(iii) This part proved to be very challenging for most candidates. Successful candidates were those who recognised the link with (ii) and saw that $\frac{\log_e e}{e} \geq \frac{\log_e x}{x}$.

Question 10

(a) (i) This part assessed skills in an algebraic application of Simpson's rule and hence the manipulation of algebraic fractions. A common error was $4(\frac{1}{2a}) = 2a$. Better responses avoided this mistake but occasionally stopped short of the correct answer.

(ii) This part required candidates to recognise that the exact area is calculated from an integral and this value is approximated by Simpson's rule. Weaker responses ignored the 'show that' instruction. Better responses included an integral with correct mathematical notation and used the algebraic limits of integration, leading to an answer involving logarithms.

(b) (i) This part involved another 'show that' instruction and required candidates to use the information given to establish a relationship between the areas of ΔABC and ΔAST. Weaker responses avoided the formula $A = \frac{1}{2}ab\sin C$. Better responses did include the correct area formula but did not always use the letters provided in the diagram.

(ii) The better responses first stated the cosine rule as it applied to ΔAST and then proceeded to demonstrate the required substitutions. This was attempted by many candidates. As with the previous part, many responses indicated a lack of ability to apply a formula correctly to a given triangle.

(iii) This part required candidates to apply the calculus to z^2 rather than to z because it was the former which needed to be minimised. Many responses indicated that an attempt was being made to minimise the variable z. Another common error was to treat $bc\cos A$ as another variable rather than as a constant term. Poor algebraic skills were evident in those responses in which $\frac{1}{4x^2} = (4x)^{-2}$, and also those involving substitution errors when testing the value $x = \sqrt{\frac{bc}{2}}$ in the first and/or second derivatives. Better responses demonstrated the correct use of the notation required to complete this question efficiently.

(iv) This part required algebraic manipulation. There was a tendency to skip vital lines of working as if the instruction 'show that' had not been there.

Mathematics

2004 HSC Examination Mapping Grid

Question	Marks	Content	Syllabus outcomes
1 (a)	2	1.1	P3
1 (b)	2	8.7, 8.8	P7
1 (c)	2	1.4	P3
1 (d)	2	1.1	P3
1 (e)	2	3.1, 3.2	H5
1 (f)	2	1.2	P3
2 (a) (i)	1	6.5	P4
2 (a) (ii)	1	6.2	P4
2 (a) (iii)	1	2.3, 5.1, 6.2	P4
2 (a) (iv)	1	6.2	P4
2 (a) (v)	1	6.4	P4
2 (a) (vi)	1	6.2	P4
2 (a) (vii)	2	6.8	H5
2 (b)	2	2.3	P2, P4
2 (c)	2	9.2	P4
3 (a) (i)	2	12.5	H5, P8
3 (a) (ii)	2	13.5	H5, P8
3 (b) (i)	2	12.5	H5, P8
3 (b) (ii)	2	12.5	H5, P8
3 (c) (i)	2	5.5	P4, H1
3 (c) (ii)	2	5.4, 5.5	P4, H1
4 (a)	2	13.1	H5
4 (b) (i)	3	10.2	H5, H6
4 (b) (ii)	2	10.5	H5, H6
4 (b) (iii)	2	10.4	H5, H6
4 (c)	3	11.4, 13.6	H5, H8
5 (a) (i)	1	7.5	H1, H4, H5
5 (a) (ii)	2	7.5	H1, H4, H5
5 (a) (iii)	2	7.5, 9.1	H1, H4, H5
5 (b) (i)	1	14.3, 13.2	H5
5 (b) (ii)	2	13.2	H5
5 (b) (iii)	2	14.3	H1, H5, H7
5 (b) (iv)	2	14.3	H1, H5, H7
6 (a)	2	9.4, 12.1, 12.2	P4, H3

Question	Marks	Content	Syllabus outcomes
6 (b) (i)	2	2.3	P2, P4
6 (b) (ii)	3	2.5	H2, H5
6 (c) (i)	2	3.3, 3.1	H1, H5
6 (c) (ii)	1	3.3	H1, H5
6 (c) (iii)	2	3.3	H1, H5
7 (a)	1	7	H5
7 (b) (i)	1	14.2	H1, H3, H5
7 (b) (ii)	1	14.2	H1, H3, H5
7 (b) (iii)	2	14.2	H1, H3, H5
7 (b) (iv)	2	14.2	H1, H2, H3, H4, H5, H9
7 (c) (i)	2	7.5	H1, H5
7 (c) (ii)	3	7.5	H1, H2, H4, H5
8 (a) (i)	1	5.1	P3
8 (a) (ii)	2	5.2, 13.1	H5
8 (b) (i)	1	9.5	P5
8 (b) (ii)	2	10.7	H6, P6
8 (b) (iii)	1	6.3	P4
8 (b) (iv)	2	11.4	H8
8 (b) (v)	3	11.4, 6.8	H2, H5, H8
9 (a) (i)	2	7.3, 13.1	H2, H5
9 (a) (ii)	2	7.3, 13.1	H5
9 (b) (i)	1	14.3	H7, H9
9 (b) (ii)	2	14.3	H2, H7, H9
9 (c) (i)	2	10.2, 12.5	H6
9 (c) (ii)	1	10.2, 10.4	H2, H6
9 (c) (iii)	2	12.2, 10.6	H2, H3, H9
10 (a) (i)	2	11.3	H1, H5
10 (a) (ii)	1	11.3, 12.4	H1, H8
10 (b) (i)	1	5.5	P4
10 (b) (ii)	2	5.5	P4
10 (b) (iii)	4	10.6	H1, H6
10 (b) (iv)	2	10.6	H1, H2, H9

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2004 HSC Mathematics
Marking Guidelines

Question 1 (a)

Outcomes assessed: P3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Correct exponent or indication of understanding of rounding to 2 significant figures	1

Question 1 (b)

Outcomes assessed: P7

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Correct derivative of one term	1

Question 1 (c)

Outcomes assessed: P3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Multiplies to remove the denominators, or equivalent progress	1

Question 1 (d)

Outcomes assessed: P3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to expand $\left(3-\sqrt{2}\right)^2$ and replaces $\left(\sqrt{2}\right)^2$ by 2, or equivalent progress	1

Question 1 (e)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• An answer which shows evidence of an understanding of the sample space or the events which are favourable	1

Question 1 (f)

Outcomes assessed: P3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• An answer which displays some understanding of the meaning of absolute value in this context	1

Question 2 (a) (i)

Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 2 (a) (ii)

Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 2 (a) (iii)

Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (a) (iv)

Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 2 (a) (v)

Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Shades correct side of the line AB	1

Question 2 (a) (vi)

Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (a) (vii)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Answer shows an understanding of the geometry (eg 45° right triangle) OR attempts to find the equation of AC and solve simultaneously with ℓ OR equivalent progress	1

Question 2 (b)

Outcomes assessed: P2, P4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Correct answer without reasons or some progress with reasons	1

Question 2 (c)

Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Computes the discriminant OR completes the square OR equivalent merit	1

Question 3 (a) (i)

Outcomes assessed: H5, P8

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Shows understanding of product rule or knowledge that $\frac{d}{dx}\log_e x = \frac{1}{x}$	1

Question 3 (a) (ii)

Outcomes assessed: H5, P8

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Shows some understanding of the chain rule or knowledge that $\frac{d}{dx}\sin x = \cos x$	1

Question 3 (b) (i)

Outcomes assessed: H5, P8

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Correct primitive or equivalent merit	1

Question 3 (b) (ii)

Outcomes assessed: H5, P8

MARKING GUIDELINES

Criteria	Marks
• Correct primitive	2
• An answer of the form $a \log_e \left(x^2 - 3\right)$	1

Question 3 (c) (i)

Outcomes assessed: P4, H1

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Calculates $\angle RPQ$ and attempts to apply cosine rule	1

Question 3 (c) (ii)

Outcomes assessed: P4, H1

MARKING GUIDELINES

Criteria	Marks
• Correct solution from the answer given in (c) (i). The bearing must be given	2
• Applies sine or cosine rule appropriately OR computes the bearing from an incorrect $\angle RQP$	1

Question 4 (a)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Calculates $\angle AOB$ or demonstrates ability to calculate sector area based on an incorrect $\angle AOB$	1

Question 4 (b) (i)

Outcomes assessed: H5, H6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Finds both stationary points or correctly applies an appropriate test for the nature to a point at which the student believes $f'(x)=0$	2
• Shows understanding that stationary points occur when $f'(x)=0$ or equivalent merit	1

Question 4 (b) (ii)

Outcomes assessed: H5, H6

MARKING GUIDELINES

Criteria	Marks
• Sketch consistent with (i) showing both points on the axes	2
• Sketch consistent with (i) or with both points on the axes shown	1

Question 4 (b) (iii)

Outcomes assessed: H5, H6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Demonstrates understanding that concavity is determined by the sign of $f''(x)$, or demonstrates understanding of the meaning of concave up by giving an answer consistent with the sketch	1

Question 4 (c)

Outcomes assessed: H5, H8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Makes substantial progress towards evaluating the volume	2
• Shows knowledge of the formula for the volume	1

Question 5 (a) (i)

Outcomes assessed: H1, H4, H5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 5 (a) (ii)

Outcomes assessed: H1, H4, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution (not necessarily expressed in hours)	2
• Attempts to use one of the two formulae for the sum of an arithmetic series	1

Question 5 (a) (iii)

Outcomes assessed: H1, H4, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Reduces the problem to a quadratic equation/inequation, or equivalent progress	1

Question 5 (b) (i)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 5 (b) (ii)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Sketch with correct shape, period and range	2
• Cosine shape with correct period or amplitude	1

Question 5 (b) (iii)

Outcomes assessed: H1, H5, H7

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Correct time or position associated with first stationary point after $t = 0$	1

Question 5 (b) (iv)

Outcomes assessed: H1, H5, H7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Recognises that max speed occurs at a midpoint of the oscillation or calculates the velocity at an appropriate time	1

Question 6 (a)

Outcomes assessed: P4, H3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Recognises that this is a quadratic in e^x or equivalent progress	1

Question 6 (b) (i)

Outcomes assessed: P2, P4

MARKING GUIDELINES

Criteria	Marks
• Correct proof	2
• Applies the appropriate congruence test but without appropriate justification or equivalent merit	1

Question 6 (b) (ii)

Outcomes assessed: H2, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Finds $\angle YCM$ or equivalent merit	2
• Recognises the significance of $\angle BAY = \angle YAM = \angle MCY$	1

Question 6 (c) (i)

Outcomes assessed: H1, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Makes some progress towards computing the probability of rolling at least one zero	1

Question 6 (c) (ii)

Outcomes assessed: H1, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (c) (iii)

Outcomes assessed: H1, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Recognition that this is the complement of scores 45 and 50 or equivalent progress	1

Question 7 (a)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 7 (b) (i)

Outcomes assessed: H1, H3, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 7 (b) (ii)

Outcomes assessed: H1, H3, H5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 7 (b) (iii)

Outcomes assessed: H1, H3, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Obtains $20\,000\,000 = 17\,000\,000e^{13k}$ or equivalent progress	1

Question 7 (b) (iv)

Outcomes assessed: H1, H2, H3, H4, H5, H9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Deduces that $kt = \log_e\left(\frac{30}{17}\right)$ or equivalent progress	1

Question 7 (c) (i)

Outcomes assessed: H1, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to apply the compound interest formula appropriately	1

Question 7 (c) (ii)

Outcomes assessed: H1, H2, H4, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Writes down a substantially correct geometric series and makes an attempt to evaluate its sum	2
• Writes down a substantially correct geometric series or equivalent merit	1

Question 8 (a) (i)

Outcomes assessed: P3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 8 (a) (ii)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Deduces that $\sin^3\theta = \frac{1}{8}$ or equivalent	1

Question 8 (b) (i)

Outcomes assessed: P5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 8 (b) (ii)

Outcomes assessed: H6, P6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Finds the gradient of the parabola at A or the equation of a line through A	1

Question 8 (b) (iii)

Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 8 (b) (iv)

Outcomes assessed: H8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Expresses the area in terms of integral(s) or equivalent merit	1

Question 8 (b) (v)

Outcomes assessed: H2, H5, H8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Makes significant progress towards the solution of the problem	2
• Observes connection with (iv) and area of ΔABC or equivalent	1

Question 9 (a) (i)

Outcomes assessed: H2, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Applies the formula for the limiting sum of a geometric series or equivalent merit	1

Question 9 (a) (ii)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Recognition that the common ratio must have absolute value < 1	1

Question 9 (b) (i)

Outcomes assessed: H7, H9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 9 (b) (ii)

Outcomes assessed: H2, H7, H9

MARKING GUIDELINES

Criteria	Marks
• Correct answer with correct reason	2
• States that particle comes to rest when $t = 4$	1

Question 9 (c) (i)

Outcomes assessed: H6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Computes $f'(x)$ correctly or equivalent progress	1

Question 9 (c) (ii)

Outcomes assessed: H2, H6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 9 (c) (iii)

Outcomes assessed: H2, H3, H9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Observes that $\frac{\log_e x}{x} \le \frac{1}{e}$ for all $x > 0$	1

Question 10 (a) (i)

Outcomes assessed: H1, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Solution demonstrates an understanding of how to apply Simpson's rule	1

Question 10 (a) (ii)

Outcomes assessed: H1, H8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 10 (b) (i)

Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 10 (b) (ii)

Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Applies cosine rule to the ΔAST or equivalent	1

Question 10 (b) (iii)

Outcomes assessed: H1, H6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	4
• Establishes that there is a stationary point at $x = \sqrt{\frac{bc}{2}}$ and attempts to determine its nature	3
• Establishes that there is a stationary point at $x = \sqrt{\frac{bc}{2}}$	2
• Computes $\frac{d}{dx}\left(z^2\right)$ or equivalent	1

Question 10 (b) (iv)

Outcomes assessed: H1, H2, H9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Deduces that $z^2 = bc(1 - \cos A)$ or equivalent	1