2004 HSC Notes from the Marking Centre Mathematics Extension 2

Published by Board of Studies NSW
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ISBN 1 7414 7235 0

2005105

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2004 HSC NOTES FROM THE MARKING CENTRE MATHEMATICS EXTENSION 2

Introduction

This document has been produced for the teachers and candidates of the Stage 6 course, Mathematics Extension 2. It is based on comments provided by markers on each of the questions from the Mathematics Extension 2 paper. The comments outline common sources of error and contain advice on examination technique and how best to present answers for certain types of questions.

It is essential for this document to be read in conjunction with the relevant syllabus, the 2004 Higher School Certificate Examination, the marking guidelines and other support documents that have been developed by the Board of Studies to assist in the teaching and learning of the Mathematics Extension 2 course.

Many parts in the Extension 2 paper require candidates to prove, show or deduce a result. Candidates are reminded of the need to give clear, concise reasons in their answers to convince the examiners in such questions.

Question 1

(a) This part was well done. Nearly all candidates knew how to integrate by parts. However, some candidates had difficulty as a result of not choosing $u = x$ and $dv = e^{3x}dx$, while other candidates chose $dv = e^{3x}dx$ but differentiated instead of integrating.

(b) This was generally well done. Most candidates (successfully) attempted this by substituting $u = \cos x$. Other successful techniques involved using:

- the reverse of the function of a function rule for integrating $\sin x(\cos x)^{-3}$ or $\tan x \sec^2$ x
- the substitutions $u = \sin x$ or $u = \cos^2 x$ or $u = \cos^3 x$ or $u = \tan x$ or $u = \sec x$ or u = $\sec^2 x$.

Common errors included leaving out the minus sign when differentiating $\cos x$ or errors in determining limits and/or in substituting the limits.

(c) Many candidates had difficulty with the negative coefficient of x^2 when trying to complete the square. Common incorrect responses included $1-(x-2)^2$ and $(x-2)^2-9$. Most candidates were able to correctly integrate upon completing the square.

(d) (i) This part was generally well done. Most successful responses either equated coefficients or substituted and $x = -1$ after having determined the identity $x^2 - 7x + 4 \equiv a(x-1)^2 + b(x+1)(x-1) - (x+1)$. A number of candidates made minor errors in equating coefficients or in substituting into the identity.

(ii) Most candidates were able to correctly find $\int\left(\frac{a}{x+1}+\frac{b}{x-1}-\frac{1}{(x-1)^2}\right)dx$ using their values of *a* and *b*. Most incorrect responses occurred when candidates were unable to integrate $(x-1)^{-2}$.

(e) Most candidates were able to correctly substitute $x = 2\sin\theta$ to find the new expression and limits. Common errors included:

- incorrect conversion of limits
- not squaring the constant when substituting
- not changing dx when substituting
- incorrectly integrating $2(1-\cos 2\theta)$
- incorrect substitution into the primitive.

Question 2

(a) (i) Generally well done.

(ii) The result was achieved by taking the conjugate at the start or the end of the question.

(b) (i) After realising the denominator, the real and imaginary parts needed to be grouped. Some errors occurred through incorrect bracketing or by the omission of the denominator.

(ii) This was answered very well by almost all candidates.

(iii) Division of moduli and subtraction of arguments was the most common and efficient method used. Realising the denominator introduced some errors and may have taken additional valuable time.

(iv) This part required the equating of the imaginary parts from (i) and (iii). This proved difficult for those who were unable to obtain an argument of $\frac{\pi}{12}$ in (iii). Some ignored 'hence' and expanded $\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$ to obtain the correct answer. A few incorrectly thought $\sin\frac{\pi}{12} = \sin\frac{\pi}{3}-\sin\frac{\pi}{4}$.

(c) While some took the centre of the circle at (0, – *i*), the region given by $|z-i| \leq 1$ was very well understood. The other region, $|z+\bar{z}| \leq 1$ proved difficult for many, even when their working reached the correct $|x| \leq \frac{1}{2}$.

(d) (i) Candidates preferred one of two methods: (1) establishing two equilateral triangles by noting that the vector sum forms a parallelogram and hence a rhombus, or (2) using 'the angle at the centre is twice the angle at the circumference standing on the same arc' theorem, and the opposite angles of a parallelogram (or rhombus) are equal. Most errors occurred in (2), with confusion between the obtuse $\angle BOA$ and the reflex $\angle BOA$; or when $OACB$ was incorrectly thought to be a cyclic quadrilateral.

(ii) The most common method was to write $w = z\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$, cube both sides using de Moivre's theorem and note that cis $2\pi = 1$. An alternate method used arguments, stating arg (w) = arg $(z) + \frac{2\pi}{3}$, multiplying by 3 and using 3 arg (w) = arg (w^3) to obtain the result. Most assumed $|z^3| = |w^3|$, obvious from the figure.

(iii) Most arrived at the result by recognising that $z^2 + zw + w^2$ was a factor of $z^3 - w^3$, or that $z^2 + w^2 + zw = (z + w)^2 - zw$. Those who substituted $w = z$ cis $\frac{2\pi}{3}$ into $z^2 + zw + w^2$ were rarely successful.

Question 3

In sketching graphs, candidates need to clearly make the distinction between preliminary or working graphs and the final answer. Marks may be awarded for progress towards the correct answer. It is important to clearly mark all axes, intercepts, asymptotes, or any other significant feature of a graph.

(a) This part was well done by most candidates, who generally showed a sound understanding of asymptotes. However, a significant number of candidates failed to find the horizontal asymptote, assuming that as $x \to \pm\infty$, $y \to 0$. Candidates had the option of using calculus but it was not essential as it yielded little extra information about the graph. Some candidates were unaware of the fact that the function was even or that it passed through the origin.

(b) This question was generally well done by most candidates. Some candidates were under the impression that a graph could not pass through its own asymptote. Some responses showed confusion between stationary points and cusps.

(c) A significant number of candidates did not appear to be sufficiently familiar with the process of implicit differentiation. The term involving $\frac{d}{dx}(xy)$ caused the most trouble, with many candidates not realising that the product rule was required. Poor arithmetic skills were often displayed in evaluating the derivative, especially where $\pm$ signs were used in the algebraic substitution.

A significant number of candidates did not complete the question, finding only the gradient of the tangent and not its equation.

(d) (i) Candidates needed to take extra care that they did indeed 'show' that the area of the triangle was $\sqrt{3}h^8$. In some cases too many steps of explanation were omitted. It was often beneficial to present a labelled diagram of the triangle.

(ii) Finding the volume by integration was generally well done. Some candidates were under the impression that the answer to such questions must contain π.

Question 4

(a) (i) Most candidates completed this part successfully.

(ii) This also was very well done. There were a few methods used to evaluate $\alpha^2+\beta^2+\gamma^2$, for example:

- $\Sigma\alpha^2 = \left(\Sigma\alpha\right)^2 - 2\times\Sigma\alpha\beta$.
- form a new equation, $9x^3+17x^2-593x-2601=0$, whose roots are α^2, β^2 and γ^2.

(iii) This was reasonably well done with many arguing their case well. Some found it particularly challenging and did not attempt this part, while others needed clear justification in their 'hence' approach. The 'otherwise' approach was generally via writing $P(x)=0$ as $(x+3)(3x^2-2x+17)=0$ and analysing the result. This method was generally successful.

(b) The main concern with this question was that candidates' 'reasons' in (i), (ii) and (iv) were not appropriate or clearly stated and often were only implied.

(i) This was quite well attempted. Copying the diagram was of great assistance. Candidates used a variety of approaches:

- similar triangles and/or angle sum of triangles (180°)
- quadrilateral angle sum (360°) and using the straight angle at H
- cyclic quadrilaterals and their properties.

(ii) Better responses used (i) and the angles on the minor arc AB to show the result via isosceles triangles or similar triangles.

(iii) Candidates provided proofs for this part when they were only required to state the result. Candidates need to read the question carefully, check the wording and note the number of marks allocated.

(iv) Many candidates did not attempt this part. Typical responses proved that $\angle BAC = \frac{1}{2}\angle MAL$ and then did not satisfactorily prove the relationship with arcs, or did not prove angle relationships but satisfactorily proved relationships with arcs.

(c) The candidates overall found part (c) very challenging.

(i) Typical responses substituted the focus into the equation to achieve the required result. Many thought they could substitute $x=\frac{a}{e}$ into the equation of PQ to show the result. Some wrote sentences to explain the result while many others gave complicated algebraic expressions.

(ii) This was not well done. Better responses found the coordinates of P, found SP and then used $b^2=a^2(1-e^2)$ to obtain the ratio.

(iii) This part was challenging for almost all candidates, and many did not attempt it. Many of those who answered (ii) failed to see the relationship to this part. Many assumed *PQ* was perpendicular to the *x*-axis without proof.

(iv) Many candidates did not attempt this part. Some were able to answer this part without (ii). Some had correct expressions that could have led to the result but confused their relationships between *a, b* and *e*.

Question 5

(a) (i) Generally well done. A few candidates ignored the solution $x = 0$.

(ii) Generally well done.

(iii) Candidates generally made a reasonable attempt at this question. Very few candidates tried to rotate about the wrong line. A number of candidates successfully completed this part even though they were unsuccessful in parts (i) and (ii).

(b) (i) This part was not well done and there were many who did not attempt it. Many candidates gave the answer as a sum of binomial coefficients rather than $2^n - 2$, but they often included one or both of the terms $\binom{n}{1}$ and $\binom{n}{n}$.

(ii) This part was poorly done. Only a small number of candidates saw the connection with (i). Most who attempted this part tried to look at cases, and quite a number did obtain the correct answer via counting arguments.

(c) (i) Generally well done with many candidates drawing diagrams to assist in their explanation.

(ii) Most candidates established that $\cos\theta = \dfrac{g}{r\omega^2}\sin\theta$ but many did not know how to handle the $\sin\theta$ term. Very few dealt with the $\theta = 0$ case.

(iii) Generally well done but many attempts were not set out clearly. There were many candidates claiming (incorrectly) that $\cos\theta > 1$ in order to obtain the required inequality. Many missed the significance of the strict inequality caused by $\theta \neq 0$.

Question 6

The parts to this question provided a guided path to obtain successive results in relatively few steps. However, many candidates chose other routes which often involved longer, circuitous and sometimes repetitive and/or tortuous algebra. A significant number of candidates did not attempt (b).

(a) (i) The majority of candidates handled this part well with the most common errors being not choosing a suitable substitution, mismatching values with the differential, losing a minus sign or claiming that $\tan^{-1}(-1) = \dfrac{3\pi}{4}$.

(ii) Most candidates successfully completed the stipulated substitution, most using a step-by-step process. Only a minority saw that the result could be expressed in terms of both the original expression and that in the previous part. Common errors included the omission of grouping symbols, limits and/or differentials.

(b) (i) Most candidates used the fact that terminal velocity is achieved when the acceleration is zero, to quickly obtain the required result. However, quite a lot of candidates, ignoring the fact that this question was only worth one mark, adopted the standard approach of drawing a diagram, stating a sign convention, obtaining an equation of motion and deriving everything from first principles.

(ii) This part produced the poorest responses with very few candidates correctly applying the product rule to expand the left hand side, and a significant number asserting that $\frac{d}{dt}(ve^{kt}) = kve^{kt}$. Alternative approaches included obtaining an expression for ve^{kt} and directly differentiating, which is a time-consuming process.

(iii) Many candidates completed this part, but instead of directly integrating the result of the previous part, most reverted to the equation of motion and integrated (with varying degrees of success) with respect to *t*. Having thus deduced that $e^{kt} = \frac{g - kA}{g - kv}$, they then went on to make v the subject of the formula. The timing of the substitution of $\frac{g}{B}$ for k (and vice versa) often made manipulations unnecessarily cumbersome.

(iv) The majority of candidates noted the clear connection with the previous part. Errors that did arise usually stemmed from poor integration of the exponential or omission of the constant of integration.

(v) Expanding rather than collecting the exponential terms was the more common technique, but this sometimes led to errors in sign.

(vi) The better responses showed that for $T = \ln\frac{2AB}{2AB - gh} > 0$ required $\frac{2AB}{2AB - gh} > 1$. However, instead of going on to state that for this to happen the denominator must be positive, and hence obtaining the required inequality, most assumed this to be true and multiplied through by the denominator to find that $gh > 0$. By contrast a lot of other candidates started with the assertion that $2AB - gh > 0$ and went on to give the correct inequality.

Question 7

Most candidates made a reasonable attempt at this question.

(a) (i) This was generally well done.

(ii) Most candidates knew what was required to establish the inequality by induction. However many had difficulty expanding the expression

$$(a_1 + \ldots + a_k + a_{k+1})\left(\frac{1}{a_1} + \ldots + \frac{1}{a_k} + \frac{1}{a_{k+1}}\right).$$

After checking an initial case and showing that the result for $n = k + 1$ follows from the assumed result for $n = k$, it is sufficient to conclude the proof with something like 'So by mathematical induction, the statement is true for every *n*'. A lengthy paragraph justifying induction as a method is not necessary.

(iii) Most who attempted this part tried to simplify the expression $\operatorname{cosec}^2\theta + \sec^2\theta + \cot^2\theta$, rather than note the word 'hence' and therefore apply (ii).

(b) (i) The most common error was to forget the '*i*' when solving the quadratic equation $z^2 - 2z\cos\alpha + 1 = 0$.

(ii) Generally well done.

(iii) When solving $\cos\alpha + \cos 2\alpha + \cos 3\alpha = 0$, many candidates found $\cos\alpha = -\frac{1}{2}$, but then only found one solution to this equation. It is sometimes helpful to consider a point on the unit circle with coordinates $(\cos\alpha, \sin\alpha)$ to find all solutions in cases like this.

Question 8

Many candidates did not attempt this question.

(a) (i) Generally well done.

(ii) Weaker responses included attempts to integrate $\frac{1}{x}$ from zero, or claims that $OPQ = QPP'Q' + \frac{1}{2} - \frac{1}{2}$ without any indication of what areas the $\frac{1}{2}$ values represented.

(iii) Many candidates were unable to find a suitable pair of similar triangles. Those who used the triangles ROR' and MOM' (where M' is on the *x*-axis) were generally successful. Equally successful were those who found the equation of OM and used the fact that R lies on OM to give the result. Poor responses often included the claim that the triangles ROR', POP' and QOQ' are similar.

(iv) Most responses to this part included a correct evaluation of at least one of the definite integrals required. Some of the weaker responses included the claim that the result followed from the fact that $Q'QRR' + R'RPP' = Q'QPP'$.

(v) Very few candidates were successful in this part.

(b) (i) The most successful responses were those that began by looking at I_{n+2}, or $I_n + I_{n+2}$. Many candidates who began with I_n were also successful, although not all candidates who took this approach realised that their conclusion was equivalent to that required.

(ii) Poor responses were those that included an incorrect expression for J_{n-1}, and/or those that did not correctly apply the result from (i).

(iii) Only a small number of candidates were successful in this part. Very few were able to sum both sides of the identity in (ii).

(iv) This was the only part of (b) attempted by many candidates. Many candidates began with I_n in terms of *u,* and used the substitution to obtain I_n in terms of *x*.

(v) Weak responses had insufficient or incorrect reasons.

Mathematics Extension 2

2004 HSC Examination Mapping Grid

Question	Marks	Content	Syllabus outcomes
1 (a)	2	4	E8
1 (b)	3	4	E8
1 (c)	2	4	E8
1 (d) (i)	2	4, 7.6	E8
1 (d) (ii)	2	4	E8
1 (e)	4	4	HE6, E8
2 (a) (i)	1	2.1	E3
2 (a) (ii)	1	2.1	E3
2 (b) (i)	1	2.1	E3
2 (b) (ii)	2	2.2	E3
2 (b) (iii)	1	2.2	E3
2 (b) (iv)	1	2.2	E3
2 (c)	3	2.5	E3
2 (d) (i)	2	2.3	E2, E3
2 (d) (ii)	2	2.4	E3
2 (d) (iii)	1	7.4	E4
3 (a)	3	1.8	E6
3 (b) (i)	2	1.3	E6
3 (b) (ii)	2	1.6	E6
3 (b) (iii)	2	1.5, 1.7	E6
3 (c)	3	1.8	E6
3 (d) (i)	1	5.1	E7
3 (d) (ii)	2	5.1	E7
4 (a) (i)	1	7.5	E4
4 (a) (ii)	2	7.5	E4
4 (a) (iii)	1	7.4	E4
4 (b) (i)	2	8.1	PE3, E2
4 (b) (ii)	1	8.1	PE3, E2
4 (b) (iii)	1	8.1	PE3, E2
4 (b) (iv)	2	8.1	PE3, E2
4 (c) (i)	1	3.1	E3
4 (c) (ii)	2	3.1	E3
4 (c) (iii)	1	3.1	E3
4 (c) (iv)	1	3.1	E3

Question	Marks	Content	Syllabus outcomes
5 (a) (i)	1	1.4 (Mathematics)	P3
5 (a) (ii)	1	4.4 (Mathematics)	P4
5 (a) (iii)	4	5	E7
5 (b) (i)	1	8	PE3, E2
5 (b) (ii)	2	8	PE3, E2
5 (c) (i)	2	6.3	E2, E5
5 (c) (ii)	3	6.3	E5
5 (c) (iii)	1	6.3	E5
6 (a) (i)	2	4	E8
6 (a) (ii)	3	4	HE6, E8
6 (b) (i)	1	6.2	E5
6 (b) (ii)	2	6.2	E5
6 (b) (iii)	2	6.2	E5
6 (b) (iv)	2	6.2	E5
6 (b) (v)	2	6.2	E5
6 (b) (vi)	1	6.2	E5
7 (a) (i)	2	8.3	E2, E9
7 (a) (ii)	4	8.2, 8.3	HE2, E2, E9
7 (a) (iii)	1	5.2 (Mathematics), 8	P4, H9, E2, E9
7 (b) (i)	3	2.4	E3
7 (b) (ii)	2	2.4, 8	E3, E4
7 (b) (iii)	3	8	E4
8 (a) (i)	1	3.3	P4, E4
8 (a) (ii)	1	2.3, 8	P4, E2, E9
8 (a) (iii)	2	2.3, 8	P4, E2, E9
8 (a) (iv)	2	3.3, 8, 4	H8, E2, E9
8 (a) (v)	1	3.3, 8	E2, E9
8 (b) (i)	2	4	E8
8 (b) (ii)	1	4	E8
8 (b) (iii)	2	4	E2, E8
8 (b) (iv)	1	4	E8
8 (b) (v)	2	4	E2, E8

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2004 HSC Mathematics Extension 2
Marking Guidelines

Question 1 (a)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct primitive	2
• Solution demonstrates some understanding of the method of integration by parts	1

Question 1 (b)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Obtains correct primitive (with consistent limits of integration if substitution is made) or equivalent merit	2
• Attempts to make an appropriate substitution or equivalent progress	1

Question 1 (c)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct primitive	2
• Completes the square or equivalent merit	1

Question 1 (d) (i)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Uses an appropriate technique to attempt to evaluate a and b	1

Question 1 (d) (ii)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct primitive (consistent with a and b claimed in (i))	2
• Applies the partial fraction decomposition in (i) and correctly evaluates at least one of the 3 integrals	1

Question 1 (e)

Outcomes assessed: HE6, E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	4
• Makes the given substitution correctly (including limits) and attempts to apply double angle formula or equivalent progress	3
• Makes the given substitution correctly (including limits) or equivalent progress	2
• Makes the given substitution and obtains correct integrand or equivalent merit	1

Question 2 (a) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (a) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (b) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (b) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Correct modulus OR argument OR equivalent merit	1

Question 2 (b) (iii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (b) (iv)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct deduction from (i) and (iii)	1

Question 2 (c)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct region	3
• Graph indicates region corresponding to $\lvert \operatorname{Re}(z) \rvert \le \frac{1}{2}$ or equivalent merit	2
• Graph shows circle centre $\pm\, i$ of radius 1 or equivalent merit	1

Question 2 (d) (i)

Outcomes assessed: E2, E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Makes some progress (e.g. attempts to use some facts which follow from *OACB* a parallelogram)	1

Question 2 (d) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Solution displays knowledge of the geometrical interpretation of multiplication	1

Question 2 (d) (iii)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 3 (a)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct even shape passing through the origin with all 3 asymptotes in correct positions	3
• Has most of the features of a correct graph	2
• Shows some understanding of the asymptotic behaviour or equivalent merit	1

Question 3 (b) (i)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct sketch	2
• Sketch has correct shape, but asymptotes and intercepts are not indicated or are incorrect OR equivalent merit	1

Question 3 (b) (ii)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct sketch	2
• Sketch has correct shape, but asymptotes and intercepts are not indicated or are incorrect OR equivalent merit	1

Question 3 (b) (iii)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct sketch	2
• Correct shape, but asymptotes and intercepts are not indicated or are incorrect OR equivalent merit	1

Question 3 (c)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Finds the slope of the tangent at (2, –1) or equivalent progress	2
• Attempts to use implicit differentiation or expresses x as a function of y and attempts to find $\frac{dx}{dy}$ or equivalent progress	1

Question 3 (d) (i)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 3 (d) (ii)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Writes down an integral which corresponds to this volume or equivalent merit	1

Question 4 (a) (i)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 4 (a) (ii)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to rewrite in terms of $(\alpha+\beta+\gamma)$ and $(\alpha\beta+\alpha\gamma+\beta\gamma)$ OR equivalent merit	1

Question 4 (a) (iii)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct answer with appropriate justification	1

Question 4 (b) (i)

Outcomes assessed: PE3, E2

MARKING GUIDELINES

Criteria	Marks
• Correct proof	2
• Recognises that *DHEC* is a cyclic quadrilateral, or provides proof with insufficient justification	1

Question 4 (b) (ii)

Outcomes assessed: PE3, E2

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 4 (b) (iii)

Outcomes assessed: PE3, E2

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 4 (b) (iv)

Outcomes assessed: PE3, E2

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Shows $\angle BAC = \frac{1}{2}\angle MAL$ or equivalent progress	1

Question 4 (c) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 4 (c) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Recognises that PQ is vertical or equivalent progress	1

Question 4 (c) (iii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 4 (c) (iv)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 5 (a) (i)

Outcomes assessed: P3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 5 (a) (ii)

Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Correct region sketched	1

Question 5 (a) (iii)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	4
• Shows that $V = 2\pi\int_0^{2a}(4a^2x - x^3)dx$ or equivalent	3
• Writes down an integral for the volume using the correct height and radius	2
• Finds the height or radius of the shell	1

Question 5 (b) (i)

Outcomes assessed: PE3, E2

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 5 (b) (ii)

Outcomes assessed: PE3, E2

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to consider the cases with 0, 1 or 2 rooms empty OR equivalent merit	1

Question 5 (c) (i)

Outcomes assessed: E2, E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to resolve forces in the horizontal plane and the vertical direction OR equivalent merit	1

Question 5 (c) (ii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Establishes that $\cos\theta = \dfrac{g}{R\omega^2}$ provides a solution	2
• Uses $r = R\sin\theta$ OR shows $\theta = 0$ is a solution OR equivalent merit	1

Question 5 (c) (iii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (a) (i)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to substitute $u = \cos x$ or equivalent merit	1

Question 6 (a) (ii)

Outcomes assessed: HE6, E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Rewrites the integral, after substitution, in terms of $\sin u$ and $\cos u$	2
• Makes the given substitution correctly	1

Question 6 (b) (i)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (b) (ii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Applies product rule to LHS or equivalent progress	1

Question 6 (b) (iii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Shows $ve^{kt} = \frac{g}{k}e^{kt}(+C)$ or equivalent	1

Question 6 (b) (iv)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Correct primitive or equivalent progress	1

Question 6 (b) (v)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Uses the fact that the two expressions have the same value at T	1

Question 6 (b) (vi)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 7 (a) (i)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to expand an appropriate square OR • Shows $x+\frac{1}{x}$ has a stationary point at $x = 1$ or equivalent progress	1

Question 7 (a) (ii)

Outcomes assessed: HE2, E2, E9

MARKING GUIDELINES

Criteria	Marks
• Establishes the initial case and proves the inductive step	4
• Proves the inductive step or gives proof with minor algebraic errors	3
• Attempts to relate the case when $n = k + 1$ to the case when $n = k$	2
• Verifies the case $n = 1$ or equivalent merit	1

Question 7 (a) (iii)

Outcomes assessed: P4, H9, E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 7 (b) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Applies de Moivre's theorem correctly to a solution of $z+\frac{1}{z}=2\cos\alpha$ or equivalent merit	2
• Rewrites $z+\frac{1}{z}=2\cos\alpha$ as a quadratic	1

Question 7 (b) (ii)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Correct proof	2
• Substitutes $z+\frac{1}{z}$ for w or equivalent merit	1

Question 7 (b) (iii)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Finds all 6 solutions	3
• Applies the results in (ii) and makes progress towards solving the cubic	2
• Applies the result in (i) or equivalent merit	1

Question 8 (a) (i)

Outcomes assessed: P4, E4

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 8 (a) (ii)

Outcomes assessed: P4, E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 8 (a) (iii)

Outcomes assessed: P4, E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Applies similar triangles or equivalent merit	1

Question 8 (a) (iv)

Outcomes assessed: H8, E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Finds area of one piece or equivalent merit	1

Question 8 (a) (v)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 8 (b) (i)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Replaces $1+\tan^2 x$ by $\sec^2 x$	1

Question 8 (b) (ii)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 8 (b) (iii)

Outcomes assessed: E2, E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Evaluates J_0 or recognises telescoping sum or equivalent merit	1

Question 8 (b) (iv)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 8 (b) (v)

Outcomes assessed: E2, E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Applies $0 \le \dfrac{u^n}{1+u^2} \le u^n$ or equivalent merit	1