2006 HSC Notes from the Marking Centre Mathematics Extension 2

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2006 HSC NOTES FROM THE MARKING CENTRE
MATHEMATICS EXTENSION 2

Introduction

This document has been produced for the teachers and candidates of the course Mathematics Extension 2. It is based on comments provided by markers on each of the questions from the Mathematics Extension 2 paper. The comments outline common sources of error and contain advice on examination technique and how best to present answers for certain types of questions.

It is essential for this document to be read in conjunction with the relevant syllabus, the 2006 Higher School Certificate examination, the marking guidelines and other support documents that have been developed by the Board of Studies to assist in the teaching and learning of the Mathematics Extension 2 course.

As a general comment candidates need to read the questions carefully and set out their working clearly. In answering parts of questions candidates should always state the relevant formulae and the information they use to substitute into the formulae. In general, candidates who do this make fewer mistakes and, when mistakes are made, marks are able to be awarded for the working shown. It is unwise to do working on the question paper, and if a question part is worth more than 1 mark the examiners expect more than just a bald answer. Any rough working should be included in the answer booklet for the question to which it applies.

Many parts in the Extension 2 paper require candidates to prove, show or deduce a result. Candidates are reminded of the need to give clear, concise reasons in their answers to convince the examiners in such questions.

Question 1

Candidates are reminded that in cases of change-of-variable or substitution in integrals such as parts (a) and (e), the du for the new variable u should not be omitted and that any limits of integration should become limits in the new variable u.

(a) The vast majority of candidates were able to make an appropriate substitution, which simplified the integral, and most of them then correctly evaluated the integral. Common errors were substitution errors for the dx term, loss of sign or missing a sign in the integration of $\int (9-u)^{-\frac{1}{2}}\,du$, failure to substitute correctly for x^2 in substitutions like $u = x^2$, or changing $(9-4x^2)^{-\frac{1}{2}}$ in the integral to $u^{\frac{1}{2}}$ for the substitution $u = 9-4x^2$. Some candidates chose the more difficult substitution $x = \frac{3}{2}\sin\theta$ and arrived at a correct answer in terms of θ; however, they usually did not express this in terms of x. Errors were more common with this substitution. Those responses where no explicit substitution was made fared worst. A small number of candidates attempted integration by parts, but were generally unsuccessful in using this method.

(b) This was the best done part, as almost all candidates could successfully complete the square and then use the standard integral sheet to find the correct primitive.

(c) (i) Many candidates could find the values of a, b, c efficiently in a few lines by finding the suitable polynomial equation for $16x - 43$, substituting $x = -2$ and 3 to find c and a, and then reading off the equation for the coefficient of x^2, namely $b + c = 0$, to find b. However, many other candidates took the more laborious approach of expanding the polynomial equation, finding all coefficient equations, or other equations from other substitutions for x such as $x = 1$, and then solving. This approach was open to many errors. There were also many errors in clearing denominators to reach a polynomial equation, with an extra factor of $x - 3$ in each term on the right-hand side the most common error, or the omission of a linear factor in one of the terms.

(ii) Most candidates had no problem integrating the three terms from part (i), with most errors occurring in the first term, typically a missed sign change after integration.

(d) Nearly all candidates recognized that integration by parts was required and many were successful in evaluating this definite integral. Common errors were an incorrect primitive for e^{-t} or loss of a sign, and evaluation errors for $\left[-e^{-t}\right]_0^2$. Those candidates who made the wrong choice and integrated the *t* factor first were rewarded for a correct integration by parts, but they could not complete the integration from there.

(e) Most candidates could quote the formulae $\sin\theta = \dfrac{2t}{1+t^2}$ and $\dfrac{d\theta}{dt} = \dfrac{2}{1+t^2}$ and then go on to show this definite integral has the value $\frac{1}{2}\log 3$. A common error was neglecting to change the limits of integration or to change them incorrectly.

Question 2

(a) The only notable feature about this part was that a number of candidates made basic errors, either in expanding the brackets or failing to deal with i^2.

(b) (i) Most candidates understood the requirement to put the expression into mod-arg form. Most common errors involved calculating the wrong angle.

(ii) De Moivre's theorem was usually correctly applied. Errors often occurred when candidates tried to transform the angle $-\dfrac{7\pi}{6}$ into a simpler equivalent angle.

(iii) Usually this part was done well from answers above, with occasional slips.

(c) This part posed more problems than the previous parts. The two solution strategies that worked most effectively were either to use De Moivre's theorem, and correctly match up the corresponding parts, or to use the unit circle and three equal angles generated from –1. The circle method was far less frequent than might be expected at this level. Candidates tended to go wrong when they tried to use/remember a more formula-driven approach for solving the equation. Some candidates used the factorised approach, but often incorrectly solved the quadratic, or failed to convert to mod-arg form.

(d) A significant proportion of candidates could not do this question at all. It was clear that they could not see the connection between the foci and the length $2a$. Many had no appreciation of the ellipse as a locus. Many candidates tried using $x + iy$ and finding the moduli squared. A few were successful in this approach, ending up with a correct factorised version of the ellipse. However, most were unsuccessful with this multi-step solution, more often than not making the elementary error of not squaring the whole of the left-hand side and finishing with a circle.

Some candidates did not get the last mark because they made a slip, which meant that the ellipse did not touch one or both axes. Even though some good candidates realised that tangents needed to be drawn to get the correct argument in these situations, they could not calculate these values.

Question 3

(a) (i) The quality of responses was generally good. Better responses included the horizontal left-hand asymptote at $y = 4$ and minimum turning points at $x = 0$ and 3 on the x-axis. Incorrect responses included graphs with sections drawn below the x-axis, some without the horizontal asymptote, or with an asymptote incorrectly positioned (typically at $y = 2$). Cusps were drawn by some candidates, mistakenly, at the two local minimum points. Candidates are reminded to clearly indicate essential features of a graph, including any x or y intercepts.

(ii) The two vertical asymptotes, $x = 0$ and $x = 3$, were included in most responses. The better responses included a left-hand horizontal asymptote of $y = \frac{1}{2}$ and a right-hand asymptote of $y = 0$. Errors included having only one horizontal asymptote for the entire graph, usually $y = 0$, but sometimes $y = \frac{1}{2}$. Some responses included an asymptote of $y = 2$ in the graph of the reciprocal function.

(iii) Better responses included a local maximum turning point at the origin as well as the realisation that $y = 2x$ is an asymptote of the graph as $x \to -\infty$.

(b) The quality of responses was generally good.

(i) Nearly all responses were successful in establishing the correct intercepts of $(\pm 12, 0)$.

(ii) Nearly all responses indicated the necessity of calculating the coordinates $(\pm ae, 0)$. The better responses calculated e correctly. Other responses made errors either in the calculation or the establishment of the formula $b^2 = a^2(e^2 - 1)$, or assigning 5 or 13 as the value of a.

(iii) The better responses included the respective equations as $x = \pm\frac{a}{e}$ and $y = \pm\frac{b}{a}x$. Other responses often included $y = \pm\frac{a}{e}$, or $\left(\pm\frac{a}{e}, 0\right)$ for the directrices and/or $y = \pm\frac{a}{b}x$ for the asymptotes. Some candidates gave answers for the equations of the directrices but not the asymptotes.

(c) (i) The better responses noted that the complex zeros of the real polynomial $P(x)$ occurred in conjugate pairs, these zeros being $a \pm ib$ and $a \pm 2ib$. (The candidates who failed to make this observation did not make any significant progress in this part of the question.) The values of a and b were then found using the relationships between the zeros and coefficients of $P(x)$. The value of b was found by first solving a quadratic equation in b^2, which proved problematic for some candidates.

(ii) The better responses expressed the real polynomial $P(x)$ as a product of four complex linear factors leading to two real quadratic factors. Other responses offered only the complex linear factors or included calculation errors. Some candidates experienced difficulty having not recognised the existence of complex conjugate pairs in part (i).

Question 4

The first three parts of this question were generally done very well.

(a) This part was answered well with most responses obtaining some marks, either by noting that $p(1) = 0$ or noting that $p(-1) = 4$. Most candidates also observed that $p'(1) = 0$ and were then able to attempt the solution of the resulting set of equations, mostly successfully. A small number of responses argued that as 1 was a repeated root and the coefficient of x^2 was zero, 1 was a double root and the other root was –2. Unfortunately, most of these responses immediately stated that $p(x) = (x-1)^2(x+2)$ rather than deducing that $p(x) = a(x-1)^2(x+2)$ and then using the fact that $p(-1) = 4$ to show that $a = 1$.

(b) A substantial minority of responses obtained full marks for this part with a few lines of working. A much larger group of responses treated this part as a solid of revolution question and struggled to gain any marks at all. A few placed the squares parallel to the y-axis rather than the x-axis. A small but significant proportion of responses that might otherwise have obtained full marks, included a π in the calculation to obtain an answer of 2π.

(c) Apart from a very few candidates who apparently could not find the equation of a straight line, the major problem was poorly-managed algebra.

(i) Most responses successfully calculated the gradient of PQ and then deduced the equation of the required line. Many candidates appeared unable to simplify the expression $(\frac{1}{q} - \frac{1}{p}) \div (q - p)$, some successfully obtaining the expression $\frac{p-q}{pq(q-p)}$ but not being able to simplify further until they took the negative reciprocal, to find the gradient of the perpendicular, and had a minus sign to help them.

(ii) A large minority of candidates answered this part as asked and simply wrote down the correct answer. Many responses treated this as a repeat of part (i) and derived the equation of the line. Some candidates correctly answered part (ii) but did not then go back and correct mistakes in part (i).

(iii) Most candidates recognised that they needed to solve a system of equations, but some took a very long time to do it. Some candidates did not recognise that $pqr - rqp = 0$, while others, as in part (i), could not simplify expressions like $\dfrac{p-r}{pqr^2 - p^2qr}$. A few spent many pages of calculations solving the equations, some successfully. Once they had obtained the coordinates of the point T most, but not all, then checked that T lay on the hyperbola, either by calculating xy for their coordinates or by calculating $\frac{1}{x}$ and obtaining y.

(d) This part was done poorly, if at all. Many candidates did not attempt any part or only attempted part (ii).

(i) Of the few candidates who attempted this part, most gave unnecessarily lengthy arguments involving similar or congruent triangles. A few obtained the answer quickly by recognising the intercept property of lines through the mid-points of two sides of a triangle.

The most common mistake, among those who attempted this part, was to argue only that one pair of sides was parallel, or that one pair of sides was equal. Of course, many responses correctly observed that one pair of sides was both parallel and equal.

(ii) This part was mostly done well. Some responses merely noted that, in a cyclic quadrilateral, the exterior angle is equal to the interior opposite (or remote) angle, while others gave a two-line answer using supplementary angles in a cyclic quadrilateral and a straight line.

(iii) Only a very small minority of candidates obtained full marks for this part. Candidates who did not first show that ΔKBP was isosceles rarely made any progress at all. A small but significant number of responses assumed that ΔKBP was isosceles and so could not obtain full marks.

Once ΔKBP was known, or assumed, to be isosceles most deduced that $KA=KB=KP$. From here responses either continued using a second isosceles triangle and a calculation via angles, or noted that this made AB the diameter of the circle through A, B and P and hence $\angle APB$ was a right-angle. The third common argument showed that the two parts of ΔAKP created by the line KM were congruent.

Question 5

(a) In the better responses, candidates drew a correct graph and a typical shell. This made clear the correct limits of integration. They were then able to expand the perfect square and integrate. Careless errors were common in the integration step.

(b) (i) Candidates were successful in expanding the two trigonometric expressions.

(ii) The most efficient approach was to use the rule in part (i) three times, leading to the equation $4\cos\theta\cos\frac{\theta}{2}\cos\frac{5\theta}{2} = 0$. A common response was to apply part (i) only twice, arriving at the equation $2\cos\theta\,(\cos 2\theta + \cos 3\theta) = 0$, then solving $\cos 2\theta + \cos 3\theta = 0$. To do this, care was needed in several points. The 'otherwise' method relied on the expansion of trigonometric expressions leading to a quartic equation in $\cos\theta$. This then required use of the factor theorem and long division of a polynomial followed by application of the quadratic formula.

(c) (i) Candidates were able to draw force diagrams, but are reminded to then write out the resulting force equations.

(ii) Successful candidates were able to write $r = l\sin\alpha$ and divide the simplified equations of part (i). Those using Pythagoras' theorem rarely arrived at a solution in terms of g, l and α, as required by the question.

(d) (i) Successful candidates were able to see 3^4. Unsuccessful candidates tried to list the sample space or use factorial notation.

(ii) This part was done well.

(iii) An efficient approach was to recognise $p(\text{win}) = (1 - p(\text{draw}))\div 2$. This required noting the three cases (DDDD, WLDD, WWLL) to arrive at a draw in four boards, as well as calculating the number of arrangements and probabilities of each case. A more common approach was to list the six cases (WWWW, WWWD, WWWL, WWDD, WWDL,WDDD) for winning directly. Candidates frequently omitted the case WDDD. Many candidates assumed the occurrence of the cases were equally likely, ignoring the number of arrangements of each case and/or the probabilities of each case.

Question 6

Question 6 consisted of two parts. The first part involved trigonometry and the second part involved the motion of a particle satisfying given conditions.

(a) (i) This question was done well by most candidates. Many candidates failed to mention that they were using the 'sine rule'.

(ii) This question was done well by most candidates. Some candidates confused ratios. For example $\frac{\sin(\alpha-\theta)}{\sin\theta}$ was sometimes given as $\frac{OA}{OC}$ or $\frac{OB}{OC}$ instead of $\frac{OC}{OA}$.

(iii) This question was done well by most candidates.

(iv) This question was done well by many candidates. Quite a few incorrectly wrote $\cot\theta - \cot\alpha = \frac{\sin(\theta-\alpha)}{\sin\theta\sin\alpha}$.

(v) Most candidates were unable to find the angles of the given triangle. Very few calculated all of $\cot\frac{\pi}{2}$, $\cot\frac{\pi}{4}$, $\mathrm{cosec}\frac{\pi}{2}$, $\mathrm{cosec}\frac{\pi}{4}$ correctly. Of the few who found that $\cot\theta(\cot\theta - 1)^2 = 2$, most did not solve it.

(b) (i) Many candidates did not state that $\ddot{x} = -g$ when $x = R$.

(ii) This question was done well by most candidates. Quite a few tried to manipulate their incorrect answer into the given equation.

(iii) Many candidates tried to show that as $t \to \infty, x \to \infty$. This may be true but it does not prove that $x > R$ for $t > 0$. Most did not realise the significance of the fact that $-(gR - u^2) \geq 0$.

(iv) (1) Most candidates who realised that $x = D$ when $v = 0$ in part (ii) were successful.

(2) Most candidates who realised that $x = R$ in part (iii) were successful. A significant number of candidates used $x = D$ in part (iii), which created a lot of work to reach the required solution. Quite a few candidates mistakenly substituted $x = 0$ into part (iii).

Question 7

(a) (i) Some candidates demonstrated good understanding of this part. Their approaches varied widely once they had calculated the gradients, $-\sin\alpha$ and $\sec^2\alpha$. Many were not able to see that a simple manipulation of $\cos\alpha = \tan\alpha$ at *P*, was necessary. Others thought they had to solve a quadratic equation, $\sin^2\alpha + \sin\alpha - 1 = 0$, first. However, their understanding of the solutions, $\frac{-1 \pm \sqrt{5}}{2}$, and applying them to the product was not well demonstrated. Those who used the angle between two lines fared worst in trying to produce 90°.

(ii) Many candidates did not attempt this part. Those who attempted it were mostly successful. There were many approaches used to obtain $\sec^2\alpha = \frac{1+\sqrt{5}}{2}$ correctly from $\cos\alpha = \tan\alpha$.

(b) (i) In the better responses, many candidates demonstrated they had the necessary knowledge and skill, once they organized I_n into $\sec^{n-2} t \times \sec^2 t$. However, many became lost when they chose $\sec^n t \times 1$, or $\sec^{n-1} t \times \sec t$, to integrate by parts. A few converted I_n into $I_{n-2} + \int \sec^{n-2} t.\tan^2 t\, dt$, but became confused. Few were successfully able to continue with this approach, by using $I_{n-2} + \int \tan t.\sec^{n-3} t \sec t.\tan t\, dt$. Some tried using $\cos^{2-n} t.\sec^2 t$.

(ii) Candidates demonstrated they could complete this part by using the expression in part (i). Many stated that $\sec^2 \frac{\pi}{3}$ was $\frac{1}{4}, \frac{3}{4}, \frac{4}{3}, 2,$ or 8. It was noticeable that some thought $\int_0^{\frac{\pi}{3}} \sec^4 t\, dt$ meant $\int_0^{\frac{\pi}{3}} \frac{\sec^2 t . \tan t}{3}\, dt + \frac{2}{3}\int_0^{\frac{\pi}{3}} \sec^2 t\, dt$.

(c) (i) Candidates demonstrated their skills fairly well, testing $n = 1$ (or $n = 2$) correctly. Many were able to apply the inductive hypothesis, but their algebraic skills became a problem. The many candidates who started with the right-hand side instead of starting with the left-hand side became confused or lost. It was noticeable that many incorrectly thought $1+\alpha^k = 1-\left(\frac{1}{3}\right)^k$.

(ii) Many candidates correctly answered 2.

Question 8

Overall, candidates' responses to this question were of a high standard, with a significant number of candidates obtaining full marks, and most candidates making significant progress on many parts of the question. It was particularly pleasing to see that most candidates were able to see the connections between the various parts and apply the results stated in the question to subsequent parts, even if they had not been able to establish those results themselves.

(a) (i) The inequality that candidates were asked to prove prompted a number of them to attempt to apply the substitution $t = \tan\frac{\theta}{2}$, which was never successful. Candidates who observed that $1-t^2 \geq \frac{1}{2}$ were usually able to earn full marks. It is important to remember that in proving that an inequality holds, it is essential that candidates work from established inequalities towards the desired result, or provide a careful argument which runs through a chain of equivalent statements, with appropriate justification. Many candidates' responses, read literally, simply showed that it was possible to deduce some true statement by assuming that the desired result is true. This does not establish anything.

(ii) Many candidates recognised that $\frac{1}{1+t} + \frac{1}{1-t} - 2 = \frac{2t^2}{1-t^2}$ and were able to obtain the mark in this part very easily. However, many attempted to apply partial fractions to $\frac{2t^2}{1-t^2}$ and often lost the –2 in the process.

(iii) and (iv) These parts were generally done very well, with most candidates who attempted them scoring the available marks.

(b) (i) Most candidates scored at least one mark in this part by computing the first derivative. Errors in the second derivative were frequent, but it was pleasing to see the number of candidates who deduced that *a* and *b* were zeros of a quadratic polynomial. However, the examiners were surprised by the large number of candidates who attempted to answer this question by applying the technique of integration by parts to compute $\int x^n e^{-x} dx$, which was of no relevance.

(ii) Candidates who had found two distinct real values for a and b in part (i) were able to earn at least one mark by making the appropriate substitution in part (ii). There were many mistakes in substitution, and many candidates had difficulty in recognising the need to divide the numerator and denominator by the highest power of n to get the result in the desired form.

(iii) A substantial proportion of the candidature recognised the need to apply the result in (a) part (iv) with $x = \square\frac{1}{\sqrt{n}}$, although very few mentioned that this value was in the domain for which (a) part (iv) was valid. Many of those who substituted correctly into this result were then able to obtain the desired bounds on $\frac{f(b)}{f(a)}$ by taking nth powers.

(iv) A pleasing proportion of the candidature recognised the connection between (b) part (iii) and (b) part (iv). However, many thought that $e^{\frac{4}{3\sqrt{n}}}$ approaches 0 as $n \to \infty$, and so were unable to gain this mark.

Mathematics Extension 2

2006 HSC Examination Mapping Grid

Question	Marks	Content	Syllabus outcomes
1 (a)	2	4.1	E8, HE6
1 (b)	2	4.1	E8, HE6
1 (c) (i)	3	7.6	E8
1 (c) (ii)	2	4.1	E8
1 (d)	3	4.1	E8
1 (e)	3	4.1	E8, HE6
2 (a) (i)	1	2.1	E3
2 (a) (ii)	1	2.1	E3
2 (a) (iii)	1	2.1	E3
2 (b) (i)	2	2.2	E3
2 (b) (ii)	2	2.2, 2.4	E3
2 (b) (iii)	1	2.4	E3
2 (c)	2	2.4	E3
2 (d) (i)	1	2.5	E2, E3, E9
2 (d) (ii)	3	3.1	E3
2 (d) (iii)	1	2.2	E3, E9
3 (a) (i)	2	1.6	E6
3 (a) (ii)	2	1.5	E6
3 (a) (iii)	2	1.4	E6
3 (b) (i)	1	3.2	E3
3 (b) (ii)	2	3.2	E3
3 (b) (iii)	2	3.2	E3
3 (c) (i)	3	7.4	E4, E9
3 (c) (ii)	1	7.4	E4
4 (a)	3	7.3	PE3, E4
4 (b)	3	5.1	E7
4 (c) (i)	2	3.3	E4
4 (c) (ii)	1	3.3	E4
4 (c) (iii)	2	3.3	E4
4 (d) (i)	1	8.1	PE3
4 (d) (ii)	1	8.1	PE3
4 (d) (iii)	2	8.1	PE3, E9
5 (a)	3	5.1	E7

Question	Marks	Content	Syllabus outcomes
5 (b) (i)	1	8.0, 5.7E1	E3, H5
5 (b) (ii)	3	8.0, 5.9E1	E3, H5
5 (c) (i)	2	6.3.1	E5
5 (c) (ii)	1	6.3.2	E5
5 (d) (i)	1	8.0	PE3
5 (d) (ii)	1	8.0	PE3, H5
5 (d) (iii)	3	8.0	PE3, H5, E9
6 (a) (i)	1	8.0	H5
6 (a) (ii)	2	8.0	H5
6 (a) (iii)	1	8.0	H5
6 (a) (iv)	1	8.0	H5
6 (a) (v)	2	8.0, 7.4	PE3, H5
6 (b) (i)	1	6.1.2	E5
6 (b) (ii)	3	6.1.2	E5
6 (b) (iii)	2	6.1.2	E5
6 (b) (iv) (1)	1	6.1.2	E5
6 (b) (iv) (2)	1	6.1.2	E5
7 (a) (i)	3	8.0	H6, E2
7 (a) (ii)	2	8.0	H5, E2
7 (b) (i)	3	4.1	E8
7 (b) (ii)	2	4.1	E8
7 (c) (i)	4	8.2	HE2, E2, E4, E9
7 (c) (ii)	1	8.0	E2, E9
8 (a) (i)	2	8.3	PE3, E4
8 (a) (ii)	1	8.3	PE3
8 (a) (iii)	2	8.3	E4, E8
8 (a) (iv)	1	8.3	HE4
8 (b) (i)	4	8.0	H6, E2
8 (b) (ii)	2	8.0	E2, E4, E9
8 (b) (iii)	2	8.3	E2, E4, E9
8 (b) (iv)	1	8.0	E2, E4, E9

BOARD OF STUDIES
NEW SOUTH WALES

2006 HSC Mathematics Extension 2
Marking Guidelines

Question 1 (a)

Outcomes assessed: E8, HE6

MARKING GUIDELINES

Criteria	Marks
• Correct primitive	2
• Makes an appropriate substitution or equivalent merit	1

Question 1 (b)

Outcomes assessed: E8, HE6

MARKING GUIDELINES

Criteria	Marks
• Correct primitive	2
• Attempts to complete the square or equivalent merit	1

Question 1 (c) (i)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Makes substantial progress	2
• Demonstrates some understanding of partial fractions	1

Question 1 (c) (ii)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct primitive	2
• Correctly integrates one of the terms in part (i)	1

Question 1 (d)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Correct primitive or equivalent merit	2
• Applies the method of integration by parts	1

Question 1 (e)

Outcomes assessed: E8, HE6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Makes substantial progress	2
• Demonstrates some understanding of the t substitution	1

Question 2 (a) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (a) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (a) (iii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (b) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Correct modulus or argument	1

Question 2 (b) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Shows some understanding of de Moivre’s theorem	1

Question 2 (b) (iii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (c)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Finds all three solutions	2
• Attempts to apply de Moivre’s theorem or equivalent merit	1

Question 2 (d) (i)

Outcomes assessed: E2, E3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (d) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Finds the lengths of both axes or equivalent merit	2
• Finds the length of the major axis or equivalent merit	1

Question 2 (d) (iii)

Outcomes assessed: E3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 3 (a) (i)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct graph	2
• A non-negative function with zeros at 0 and 3 or equivalent merit	1

Question 3 (a) (ii)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct graph	2
• A graph with the correct behaviour at 0 and 3 or equivalent merit	1

Question 3 (a) (iii)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct graph	2
• A graph with a maximum at the origin or equivalent merit	1

Question 3 (b) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 3 (b) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Finds the eccentricity	1

Question 3 (b) (iii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Finds the equations of the asymptotes or equivalent merit	1

Question 3 (c) (i)

Outcomes assessed: E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	3
• Finds the value of a or equivalent merit	2
• Recognises that the zeros are $a \pm ib$ and $a \pm 2ib$ or equivalent merit	1

Question 3 (c) (ii)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 4 (a)

Outcomes assessed: PE3, E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Makes substantial progress	2
• Shows some understanding of the meaning of multiple zero or equivalent merit	1

Question 4 (b)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Shows that the cross-sectional area is $4y$	2
• Attempts a computation of the form $\int_0^1 A(y)\,dy$ or equivalent merit	1

Question 4 (c) (i)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Calculates the gradient or equivalent merit	1

Question 4 (c) (ii)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 4 (c) (iii)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Finds one of the coordinates of the point T	1

Question 4 (d) (i)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Correct proof	1

Question 4 (d) (ii)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Correct proof	1

Question 4 (d) (iii)

Outcomes assessed: PE3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct proof	2
• Shows ΔKPB is isosceles or equivalent merit	1

Question 5 (a)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Correct integral expression for the volume or equivalent merit	2
• Recognises that the region lies between $x = 0$ and $x = 1$ or equivalent merit	1

Question 5 (b) (i)

Outcomes assessed: E3, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 5 (b) (ii)

Outcomes assessed: E3, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Makes substantial progress	2
• Applies the result of part (i) or equivalent merit	1

Question 5 (c) (i)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Resolves correctly in one direction or equivalent merit	1

Question 5 (c) (ii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 5 (d) (i)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 5 (d) (ii)

Outcomes assessed: PE3, H5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 5 (d) (iii)

Outcomes assessed: PE3, H5, E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	3
• Finds the probability of drawing the match or equivalent merit	2
• Attempts to calculate the probability of drawing the match or equivalent merit	1

Question 6 (a) (i)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (a) (ii)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Makes some progress	1

Question 6 (a) (iii)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (a) (iv)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (a) (v)

Outcomes assessed: PE3, H5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Finds the cubic equation satisfied by $\cot\theta$ in this case	1

Question 6 (b) (i)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (b) (ii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Applies $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ and attempts to use the initial conditions or equivalent merit	2
• Applies $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ or equivalent merit	1

Question 6 (b) (iii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Makes some progress	1

Question 6 (b) (iv) (1)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 6 (b) (iv) (2)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 7 (a) (i)

Outcomes assessed: H6, E2

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Computes the product of the gradients or equivalent merit	2
• Differentiates both functions or equivalent merit	1

Question 7 (a) (ii)

Outcomes assessed: H5, E2

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Shows sin α is a root of $x^2 + x - 1 \quad 0$ or equivalent merit	1

Question 7 (b) (i)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Makes substantial progress	2
• Applies the technique of integration by parts or equivalent merit	1

Question 7 (b) (ii)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Applies the result of (b) (i)	1

Question 7 (c) (i)

Outcomes assessed: HE2, E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	4
• Proves the inductive step or equivalent merit	3
• Shows the case for $n = 1$ and attempts to prove the inductive step or equivalent merit	2
• Shows the case for $n = 1$ or equivalent merit	1

Question 7 (c) (ii)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 8 (a) (i)

Outcomes assessed: PE3, E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Shows that $\frac{1}{2} \le 1 - t^2 \le 1$ or equivalent merit	1

Question 8 (a) (ii)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 8 (a) (iii)

Outcomes assessed: E4, E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to integrate the expression in part (ii)	1

Question 8 (a) (iv)

Outcomes assessed: HE4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 8 (b) (i)

Outcomes assessed: H6, E2

MARKING GUIDELINES

Criteria	Marks
• Correct solution	4
• Deduces that a and b satisfy $x^2 - 2nx + n(n-1) \quad 0$ or equivalent merit	3
• Evaluates $''(x)$ or equivalent merit	2
• Evaluates $'(x)$ f	1

Question 8 (b) (ii)

Outcomes assessed: E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Evaluates (a) or (b) f	1

Question 8 (b) (iii)

Outcomes assessed: E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to apply a (iv)	1

Question 8 (b) (iv)

Outcomes assessed: E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1