# 2008 HSC Notes from the Marking Centre Mathematics Extension 2

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# Contents

# 2008 HSC NOTES FROM THE MARKING CENTRE
# MATHEMATICS EXTENSION 2

## Introduction

This document has been produced for the teachers and candidates of the Stage 6 Mathematics Extension 2 course. It contains comments on candidate responses to the 2008 Higher School Certificate examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

This document should be read along with the relevant syllabus, the 2008 Higher School Certificate examination, the marking guidelines and other support documents which have been developed by the Board of Studies to assist in the teaching and learning of Mathematics Extension 2.

Many parts in the Extension 2 paper require candidates to prove, show or deduce a result. Candidates are reminded of the need to give clear, concise reasons in their answers.

## Question 1

(a) Most responses made use of a substitution such as $u = x^3$ or $u = 5 + x^3$. Where a substitution approach was used there was a surprising number that contained basic errors such as $\frac{d}{dx}\left(x^3\right) = 2x^2$, $\frac{1}{3}\int \frac{3x^2}{(5+x^3)}dx = -\frac{1}{9(5+x^3)}$ or $\frac{-1}{3(5+x^3)^3}$ and by having incorrect signs. Some candidates failed to recognise the expression as being one to which the reverse chain rule idea could be applied. Such candidates generally approached the question by expanding $\left(5 + x^3\right)^2$ or by rewriting the integral as $\int x^2(5+x^3)^{-2}\,dx$ and treating it as a product.

(b) While the majority of candidates recognised the expression as being one to which a standard integral could be applied, many made basic errors in using the results. Common errors produced answers with an incorrect coefficient of the logarithmic term, or an incorrect handling of the $\sqrt{x^2 + a^2}$ term, or an attempt to rewrite the expression so as to obtain an inverse trigonometric function as the primitive. Others who attempted a substitution used various approaches such as $u = 2x$ or $x = \frac{1}{2}\tan\theta$. These experienced varying degrees of success.

(c) Typical responses recognised that the integral could be found through integration by parts. As an alternative some candidates sought to evaluate the integral in the form

$\int_0^1 \tan^{-1}x\,dx = \left(1 \times \frac{\pi}{4}\right) - \int_0^{\frac{\pi}{4}} \tan y\,dy$, being the difference between the area of the rectangle formed by the ordinate $x = 1$ on the curve $y = \tan^{-1} x$ and the area between that curve and the $y$-axis. The main error made in this approach was in finding the primitive of the tangent function.

Other responses used the substitution $\theta = \tan^{-1} x$ to attempt to evaluate the integral. A minority of these responses produced $\int_0^{\frac{\pi}{4}} \theta \sec^2\theta \, d\theta$ as a correct version of the original integral and fewer still proceeded to a correct solution.

(d) Those candidates who recognised the substitution $u^2 = 2x - 1$ were generally successful, especially if implicit differentiation was used. More substantial difficulties were encountered by those candidates who used the substitutions $u = 2x - 1$ or $u = \sqrt{2x - 1}$. Other substitution methods were tried with varying degrees of success. For example, many candidates successfully used the substitution $x = \frac{1}{2}\sec^2\theta$.

The common errors in this part were a failure to correctly convert the limits of the integral and a failure to correctly simplify the integrand once the substitution had been made. In the latter case an incorrect differentiation of the substitution expression was often the reason. The substitution $u = 2x - 1$ produced difficulties for many responses that used this approach. This substitution produces the integral $\int \frac{du}{u^{\frac{3}{2}} + u^{\frac{1}{2}}}$, which created problems as a further substitution is required. Common errors in trying to deal with this expression included rewriting the integral as $\int \left\{ u^{\frac{-3}{2}} + u^{\frac{-1}{2}} \right\} du$, or attempting to use a partial fractions approach.

There were candidates who approached this part of the question using integration by parts. This approach produced limited success.

(e) Most candidates started this part of the question well, recognising that the primitive of $\frac{2x}{2 - x^2}$ involved a logarithm and also that evaluating $\int \frac{4 - 2x}{2 - 2x + x^2} dx$ required splitting the numerator.

Minor errors prevented many candidates from achieving full marks on this part. Errors included an incorrect sign on the $\log\left|2 - x^2\right|$ term and incorrect coefficients associated with the $\log\left|x^2 - 2x + 2\right|$ and $\tan^{-1}(x - 1)$ terms. Others obtained correct primitive functions and were then unable to obtain a correct expression or value after substituting for the limits of the integral.

**Question 2**

(a) Successful candidates demonstrated a correct expansion of a binomial product in association with the knowledge that $i^2 = -1$.

(b) (i) Successful candidates indicated multiplication of the numerator and denominator by $1 - i$, followed by an appropriate simplification of the expression.

(ii) In correct responses, candidates expressed $1+i\sqrt{3}$ and $1+i$ as $2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ and $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$, respectively. This was then followed by finding the quotient of the moduli and the difference of the arguments to write the final expression as $\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$. Common errors included calculating the argument of $1+i\sqrt{3}$ to be $\frac{\pi}{6}$ and/or incorrectly finding the quotient of 2 and $\sqrt{2}$.

(iii) Successful candidates recognised the equality of the real part of the expressions in parts (i) and (ii).

(iv) Successful candidates demonstrated a successful application of De Moivre's Theorem to the final expression from part (ii).

(c) In better responses, candidates interpreted the locus as $z^2+\left(\bar{z}\right)^2=8$ and then $\left(x+iy\right)^2+\left(x-iy\right)^2=8$. They then simplified down to the final equation of the locus, $x^2-y^2=4$, or equivalent. Most then recognised this as a hyperbola. Common errors included incorrect expansion and simplification of $\left(x+iy\right)^2+\left(x-iy\right)^2$ or labelling $x^2-y^2=4$ as a conic other than a hyperbola, for example, a circle.

(d)(i) The simplest correct responses presented $OM=\frac{1}{2}\left(OQ+OR\right)$ which led to the complex number $\frac{1}{2}z\left(\omega+\bar{\omega}\right)=-\frac{z}{2}$ representing $M$. Other correct responses involved letting $z=x+iy$ or $r\left(\cos\theta+i\sin\theta\right)$ and working to the correct answer. Incorrect responses included finding $QM=\frac{1}{2}QR=\left(OR-OQ\right)$ or attempting to rotate $OP$ about $O$.

(ii) Similarly to part (i), candidates who applied a midpoint principle $\frac{S+P}{2}=M$ were generally successful. Such responses recognised that $M$, and not $O$, was the centre of the parallelogram. Other correct responses included noting that as $PQ=RS$ we have $\omega z-z=S-\bar{\omega}z$ and progressed from there. However, such responses were vulnerable to getting the sign of the vector incorrect. Incorrect responses also included treating $O$ as the centre of the diagonal $PS$.

**Question 3**

(a) Candidates are reminded to present large, clear sketches. Sometimes it is not obvious which are preliminary working sketches and which are final answers. Candidates should clearly mark axes and label any intercepts and any asymptotes. A large number of candidates did not sketch $y = g(2 - x)$ for $x < 1$ correctly and many of those who made a good attempt assumed the final domain was $x > 0$.

(b) There were quite a large variety of successful methods of solving these parts of the question.

(i) Some candidates showed that the sum of the squares of the roots was negative and thought that this meant that the roots must all be non-real. Some thought that showing that $p(1) \neq 0$ and $p(-1) \neq 0$ was sufficient.

(ii) and (iii)
It was not necessary to solve $p(z) = 0$ to answer the question and those who did had the more onerous task of proving $\alpha^6 = 1$ and $p(\alpha^2) = 0$ are true for each value of $\alpha$. Also some responses contained simple index law errors such as $(\alpha^2)^4 = \alpha^6$.

(c) (i) This question was answered more successfully with responses starting with $I_n = \int_0^{\frac{\pi}{4}} \tan^{2n-2}\theta \tan^2\theta \, d\theta = \int_0^{\frac{\pi}{4}} \tan^{2n-2}\theta (\sec^2\theta - 1) \, d\theta$, an approach that leads directly to the reduction formula. Quite a few responses indicated a misconception that a reduction formula question must involve integration by parts, an approach that made the question much more complicated. A significant number of candidates incorrectly assumed that this part required a proof using mathematical induction.

(ii) Some candidates made the task more difficult by evaluating $I_1$ instead of $I_0$. A very large number made calculation errors such as $1 - \frac{\pi}{4} = \frac{3\pi}{4}$. Most of the responses that were unsuccessful with part (i) still attempted this part.

(d) This question was very well done, but a few candidates confused forces and distances.

**Question 4**

(a) (i) Most candidates succeeded in finding an expression for the area.

(ii) A substantial proportion of candidates recognised the need to apply a similar result to the one used for part (i) and achieved the required proof.

(iii) Responses that contained an attempt at this part usually achieved some credit. However careless algebra when solving achieved many varied results. Some candidates used the geometry result that tangents from an external point are equal, but failed to acknowledge this theorem in any form.

When completing parts (iii) and (iv) a diagram was a useful tool in some responses.

(iv) Responses that demonstrated an understanding of the result $A = \frac{1}{2}Pr$ in part (iii) usually achieved the required result in part (iv). Errors in this part occurred when candidates calculated the perimeter or area of an incorrect triangle.

(b) (i) Better responses successfully found the gradient by implicit differentiation. However some candidates failed to substitute the point $P(x_1, y_1)$ into the equation of the tangent.

(ii) In better responses, candidates substituted one equation into the other or subtracted the two equations and achieved the required result very quickly. Others who attempted to solve the equations of the tangents to find the coordinates of T became lost in the algebra and generally did not complete the required working.

(iii) Better responses included substituting the origin and midpoint into the equation from part (b)(ii) to show that $O$, $M$ and $T$ were collinear. Many candidates who attempted to find gradients tried to manipulate their answers in order to prove the points were collinear. Some candidates had difficulty finding the midpoint.

**Question 5**

(a) (i) The most common approach to this question was to apply the chain rule to $P(t)$ and then use substitution to arrive at the differential equation in terms of $P$. Candidates are advised to show all steps in their working. Multiple cancelling of terms in a single line is also undesirable as it often leads to errors. Alternative approaches were to make $t$ the subject and then differentiate, to reciprocate the given differential equation and use partial fractions to integrate to find $t = f(P)$; to reciprocate the given $P(t)$ and then use implicit differentiation.

(ii) Most responses substituted $t = 0$. A few chose $t = 2008$.

(iv) Common errors were to find $\frac{dP}{dt} \times 100\%$ or to use the gradient between the two points on the graph of $P$ where $t = 0$ and $t = 1$.

(b) (i) Mid-range responses only established the condition $p'(1) = 0$. The very best responses excluded the possibility of a zero of order greater than 2.

(ii) While candidates may have realised that $(1,0)$ was a minimum turning point, many did not communicate it clearly. In the better responses, a graph was drawn. Candidates often failed to acknowledge that the minimum value was 0 even if they correctly determined the nature of the stationary point at $x = 1$. Many successfully showed $p''(x) > 0$ for $x > 0$ but then focused on the $y$-intercept.

(iii) Efficient responses used part (i) to find the second quadratic factor, namely $x^2 + 2x + 3$. In many responses, this quadratic was factorised incorrectly.

(c) (i) Many candidates expanded the given quadratic and then applied the quadratic formula rather than take the square root of both sides first. Some did not use the given information that $x_2 > x_1$ to assign the correct signs to the square roots in the expressions for $x_1$ and $x_2$.

(ii) In weaker responses, candidates did not recognise the cross-section was an annulus but instead used the rectangle given in the diagram.

(iii) In better responses, candidates recognised the use of a semicircle to calculate the definite integral. Those who chose a trigonometric substitution were often successful but at the expense of time.

**Question 6**

(a) Most candidates who were successful in answering the question found the value of $\omega$ and proceeded from there. Those who worked solely from the relationships between roots and coefficients were less likely to have had success.

(b) (ii) Very few responses justified the removal of the absolute value sign in this part.

(iii) In better responses, candidates completed this part using the formula $b^2 = a^2\left(e^2 - 1\right)$ or an equivalent relationship.

(c) (i) This part required candidates to have good algebraic skills, particularly in adding fractions involving factorials. Those candidates who had this skill generally did well.

(ii) This question was generally well answered by candidates who had familiarity with handling telescoping sums.

**Question 7**

(a) Most candidates attempted this part by writing down an expression for the probability involving either combinations or permutations, with relatively few using the simpler tree diagram approach. It is important for candidates to explain their computations.

(i) In better responses, candidates first computed the probability that all three balls were red, and then multiplied that answer by three. This is a good approach to this question, but not if it is set out as

$$P = \frac{\binom{n}{3}}{\binom{3n}{3}} = \frac{n(n-1)(n-2)}{3n(3n-1)(3n-2)} = \frac{(n-1)(n-2)}{(3n-1)(3n-2)}$$

While it is possible such a response is following the argument outlined above, examiners must, in the absence of additional explanation, view this as an incorrect expression for the probability with an error in the last step of the simplification.

(b) (i) A substantial number of candidates appeared to have incorrectly named one or more angles in their answer, and some did not provide adequate justification for the assertions in their working. Candidates who copied the diagram into their writing booklet were much more likely to name the angles correctly, and many avoided the angle-naming problem entirely by labelling the relevant angles on their diagram with unique numbers or letters.

(ii) Most candidates knew that this part had something to do with the product of the secants, although a common error was to write something like $PT^2 = RQ \cdot QT$. A significant number of responses essentially proved the result on the product of the secants in their answer, choosing to begin by showing that triangles $QPT$ and $PRT$ are similar. Others tried to use the sine rule, almost invariably without success.

(c) (ii) Candidates who referred to the terminal or final velocity, or gave an answer that was related to the speed of the current or some similar physical phenomenon that would cause the boat to eventually move at a constant velocity, were awarded the mark. A very common incorrect response was the single-word response 'resistance'.

(iii) In better responses, candidates correctly found the primitive of $\frac{dx}{dt}$. Weaker responses did not do this, and, instead, found the distance as a function of $v$ by integrating $v\frac{dv}{dx}$.

(iv) Most candidates correctly substituted into the expression given in part (c) (iii).

**Question 8**

Many candidates used the sum/difference to product trigonometrical identities but they were mostly unsuccessful in applying them correctly in all three parts.

(a) There were three main approaches used by candidates. The first was the simplest where the candidates used the given identity to complete the proof. The other two involved compound angle expansions, rearranging and applying Mathematics Extension 1 results to produce the RHS of the proof for $n = k + 1$.
It was noticeable that many candidates did not earn full marks due to poor application of algebraic skills in trigonometry.

(c) (i) In typical responses, candidates differentiated each term of $f(t) = \sin(a + nt)\sin b - \sin a \sin(b - nt)$, for example differentiating $\sin(a + nt)\sin b$ they would write $n\cos(a + nt)\sin b + \sin(a + nt)\cos b$ and so on. They would then go on to produce four terms for $f'(t)$ and then 8 or more terms for $f''(t)$.
Showing that $f(0) = 0$ was particularly well done.

(ii) Many candidates were successful in this part if they used part (c) (i) to expand then simplify to produce the desired result. Many did not realise/recognise that $f(t)$ was of the form $A\sin nt$ due to the information in part (c) (i). Those who did follow this approach had problems in showing that $A = \sin(a + b)$.

(iii) There was a mixture of responses. Carelessness resulted in many candidates losing marks when finding all the solutions to $\sin(nt) = 0$.

# Mathematics Extension 2

## 2008 HSC Examination Mapping Grid

| Question | Marks | Content | Syllabus outcomes |
|---|---|---|---|
| 1 (a) | 2 | 4.1 | E8 |
| 1 (b) | 2 | 4.1 | E8 |
| 1 (c) | 3 | 4.1 | E8 |
| 1 (d) | 4 | 4.1 | E8 |
| 1 (e) | 4 | 4.1 | E8 |
| 2 (a) | 2 | 2.1 | E3 |
| 2 (b) (i) | 2 | 2.1 | E3 |
| 2 (b) (ii) | 3 | 2.2 | E3 |
| 2 (b) (iii) | 1 | 2.2 | E3 |
| 2 (b) (iv) | 1 | 2.4 | E3 |
| 2 (c) | 2 | 2.1, 2.2, 2.5, 3.2 | E3 |
| 2 (d) (i) | 2 | 2.1, 2.2, 8.0 | E3 |
| 2 (d) (ii) | 2 | 2.1, 8.0 | E2, E9 |
| 3 (a) (i) | 1 | 1.3 | E6 |
| 3 (a) (ii) | 2 | 1.5 | E6 |
| 3 (a) (iii) | 2 | 1.3 | E6 |
| 3 (b) (i) | 1 | 2.1, 7.1 | E4 |
| 3 (b) (ii) | 1 | 2.1, 7.1 | E4 |
| 3 (b) (iii) | 1 | 2.1, 7.1 | E4 |
| 3 (c) (i) | 2 | 4.1 | E8 |
| 3 (c) (ii) | 2 | 4.1 | E8 |
| 3 (d) | 3 | 6.3.1 | E5 |
| 4 (a) (i) | 1 | 8.0 | E2 |
| 4 (a) (ii) | 1 | 8.0 | E2 |
| 4 (a) (iii) | 3 | 8.0 | E2, E9 |
| 4 (a) (iv) | 3 | 8.0 | E2, E9 |
| 4 (b) (i) | 2 | 3.1 | E3, E4 |
| 4 (b) (ii) | 2 | 3.1 | E3, E4 |
| 4 (b) (iii) | 3 | 3.1 | E3, E4 |

| Question | Marks | Content | Syllabus outcomes |
|---|---|---|---|
| 5 (a) (i) | 2 | 8.0 | E2 |
| 5 (a) (ii) | 1 | 8.0 | E2 |
| 5 (a) (iii) | 1 | 8.0 | E2 |
| 5 (a) (iv) | 1 | 8.0 | E2 |
| 5 (b) (i) | 2 | 7.2 | E2, E4 |
| 5 (b) (ii) | 1 | 8.0 | E2, E4 |
| 5 (b) (iii) | 2 | 7.2, 8.0 | E2, E4 |
| 5 (c) (i) | 1 | 5.1 | E7 |
| 5 (c) (ii) | 2 | 5.1 | E7 |
| 5 (c) (iii) | 2 | 5.1 | E7 |
| 6 (a) | 3 | 2.1, 7.0 | E3, E4 |
| 6 (b) (i) | 2 | 3.2 | E3, E4, E6 |
| 6 (b) (ii) | 1 | 3.2 | E3, E4 |
| 6 (b) (iii) | 3 | 3.2 | E3, E4 |
| 6 (c) (i) | 3 | 8.0 | E2, E9 |
| 6 (c) (ii) | 2 | 8.0 | E2, E9 |
| 6 (c) (iii) | 1 | 8.0 | E2, E9 |
| 7 (a) (i) | 2 | 8.0 | E2 |
| 7 (a) (ii) | 1 | 8.0 | E2 |
| 7 (a) (iii) | 1 | 8.0 | E2 |
| 7 (a) (iv) | 1 | 8.0 | E2 |
| 7 (b) (i) | 2 | 8.1 | E2, E9 |
| 7 (b) (ii) | 2 | 8.1 | E2, E9 |
| 7 (c) (i) | 1 | 6.2.1 | E2, E5 |
| 7 (c) (ii) | 1 | 6.1.2 | E2, E5 |
| 7 (c) (iii) | 3 | 6.2.1 | E2, E5, E8 |
| 7 (c) (iv) | 1 | 6.1.2 | E2, E5 |
| 8 (a) | 3 | 8.2 | E2, E4 |
| 8 (b) (i) | 4 | 8.0 | E2, E9 |
| 8 (b) (ii) | 1 | 8.0 | E2, E9 |
| 8 (c) (i) | 3 | 8.0 | E2, E9 |
| 8 (c) (ii) | 2 | 8.0 | E2, E9 |
| 8 (c) (iii) | 2 | 8.0 | E2, E9 |

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# 2008 HSC Mathematics Extension 2
# Marking Guidelines

The following marking guidelines were developed by the examination committee for the 2008 HSC examination in Mathematics Extension 2, and were used at the marking centre in marking student responses. For each question the marking guidelines are contained in a table showing the criteria associated with each mark or mark range.

The information in the marking guidelines is further supplemented as required by the Supervisor of Marking and the senior markers at the marking centre.

A range of different organisations produce booklets of sample answers for HSC examinations, and other notes for students and teachers. The Board of Studies does not attest to the correctness or suitability of the answers, sample responses or explanations provided. Nevertheless, many students and teachers have found such publications to be useful in their preparation for the HSC examinations.

A copy of the Mapping Grid, which maps each question in the examination to course outcomes and content as detailed in the syllabus, is also included.

**Question 1 (a)**

*Outcomes assessed: E8*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct primitive | 2 |
| • Attempts substitution $a = 5 + x^3$ or equivalent merit | 1 |

**Question 1 (b)**

*Outcomes assessed: E8*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct primitive | 2 |
| • Attempts to apply the appropriate standard integral from the table | 1 |

**Question 1 (c)**

*Outcomes assessed: E8*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 3 |
| • Correct primitive | 2 |
| • Applies integration by parts correctly | 1 |

**Question 1 (d)**

*Outcomes assessed: E8*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 4 |
| • Obtains $\int_1^{\sqrt{3}} \frac{2}{u^2+1}\,dx$ or equivalent merit | 3 |
| • Obtains $\int \frac{2}{u^2+1}\,dx$ or equivalent merit | 2 |
| • Attempts to substitute $u = \sqrt{2x-1}$ or equivalent merit | 1 |

**Question 1 (e)**

*Outcomes assessed: E8*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 4 |
| • Finds correct primitive for $\dfrac{4-2x}{2-2x+x^2}$ | 3 |
| • Finds correct primitive for $\dfrac{-2x}{2-x^2}$ | 2 |
| • Uses the given result | 1 |

**Question 2 (a)**

*Outcomes assessed: E3*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 2 |
| • Finds $a$ or $b$ | 1 |

**Question 2 (b) (i)**

*Outcomes assessed: E3*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 2 |
| • Attempts to realise the denominator | 1 |

**Question 2 (b) (ii)**

*Outcomes assessed: E3*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 3 |
| • Writes both in modulus-argument form | 2 |
| • Writes $1+i\sqrt{3}$ or $1+i$ in modulus-argument form | 1 |

**Question 2 (b) (iii)**

*Outcomes assessed: E3*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 1 |

**Question 2 (b) (iv)**

*Outcomes assessed: E3*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 1 |

**Question 2 (c)**

*Outcomes assessed: E3*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 2 |
| • Finds the equation of the locus in terms of $x$ and $y$ | 1 |

**Question 2 (d) (i)**

*Outcomes assessed: E3*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 2 |
| • Shows that $M$ corresponds to $\frac{z}{2}(\omega + \overline{\omega})$ | 1 |

**Question 2 (d) (ii)**

*Outcomes assessed: E2, E9*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 2 |
| • Recognises that $M$ is the midpoint of $PS$ or equivalent merit | 1 |

**Question 3 (a) (i)**

*Outcomes assessed: E6*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct sketch | 1 |

**Question 3 (a) (ii)**

*Outcomes assessed: E6*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct sketch | 2 |
| • Shows asymptote at $x = 1$ | 1 |

**Question 3 (a) (iii)**

*Outcomes assessed: E6*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct sketch | 2 |
| • Correct sketch for $x \geq 1$ | 1 |

**Question 3 (b) (i)**

*Outcomes assessed: E4*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 1 |

**Question 3 (b) (ii)**

*Outcomes assessed: E4*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 1 |

### Question 3 (b) (iii)

*Outcomes assessed: E4*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 1 |

### Question 3 (c) (i)

*Outcomes assessed: E8*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 2 |
| • Substitutes $\tan^2\theta = \sec^2\theta - 1$ | 1 |

### Question 3 (c) (ii)

*Outcomes assessed: E8*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 2 |
| • Shows $I_3 = \frac{1}{5} - I_2$ | 1 |

### Question 3 (d)

*Outcomes assessed: E5*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 3 |
| • Shows $T\sin\alpha = (m\ell\sin\alpha)\omega^2$ and $T\cos\alpha = mg$ | 2 |
| • Resolves the tension in the horizontal and vertical directions | 1 |

**Question 4 (a) (i)**

*Outcomes assessed: E2*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 1 |

**Question 4 (a) (ii)**

*Outcomes assessed: E2*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 1 |

**Question 4 (a) (iii)**

*Outcomes assessed: E2, E9*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 3 |
| • Shows $(3x-8)^2 = 64 + x^2$ | 2 |
| • Shows $4x = 8 + x + \sqrt{64 + x^2}$ | 1 |

**Question 4 (a) (iv)**

*Outcomes assessed: E2, E9*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 3 |
| • Finds perimeter of second triangle | 2 |
| • Finds the length of one board | 1 |

**Question 4 (b) (i)**

*Outcomes assessed: E3, E4*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 2 |
| • Shows gradient of $PT$ is $\dfrac{-b^2x_1}{a^2y_1}$ | 1 |

**Question 4 (b) (ii)**

*Outcomes assessed: E3, E4*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 2 |
| • Writes down equation of $QT$ | 1 |

**Question 4 (b) (iii)**

*Outcomes assessed: E3, E4*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 3 |
| • Shows $M$ is on the line given in part (ii) | 2 |
| • Observes that $O$ lies on the line given in part (ii) | 1 |

**Question 5 (a) (i)**

*Outcomes assessed: E2*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 2 |
| • Differentiates the expression for *P* correctly | 1 |

**Question 5 (a) (ii)**

*Outcomes assessed: E2*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 1 |

**Question 5 (a) (iii)**

*Outcomes assessed: E2*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 1 |

**Question 5 (a) (iv)**

*Outcomes assessed: E2*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 1 |

**Question 5 (b) (i)**

*Outcomes assessed: E2, E4*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 2 |
| • Shows $p(x)$ has a zero at $x = 1$ | 1 |

**Question 5 (b) (ii)**

*Outcomes assessed: E2, E4*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 1 |

**Question 5 (b) (iii)**

*Outcomes assessed: E2, E4*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 2 |
| • Attempts to divide by $(x-1)^2$ or equivalent merit | 1 |

**Question 5 (c) (i)**

*Outcomes assessed: E7*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 1 |

**Question 5 (c) (ii)**

*Outcomes assessed: E7*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 2 |
| • Writes $A = \pi\left(x_2^2 - x_1^2\right)$ or equivalent merit | 1 |

**Question 5 (c) (iii)**

*Outcomes assessed: E7*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 2 |
| • Obtains $V = \int_{-b}^{b} 4\pi a\sqrt{b^2 - h^2}\,dh$ or equivalent merit | 1 |

**Question 6 (a)**

*Outcomes assessed: E3, E4*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 3 |
| • Uses sum and product of roots to find *a* and *c* or equivalent merit | 2 |
| • Writes $p(z)=(z-1)(z+\omega)(z+\overline{\omega})$ | 1 |

**Question 6 (b) (i)**

*Outcomes assessed: E3, E4, E6*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 2 |
| • Shows that the gradient is $\dfrac{b\sec\theta}{a\tan\theta}$ | 1 |

**Question 6 (b) (ii)**

*Outcomes assessed: E3, E4*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 1 |

**Question 6 (b) (iii)**

*Outcomes assessed: E3, E4*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 3 |
| • Makes substantial progress | 2 |
| • Correct expression for $S'R'$ | 1 |

**Question 6 (c) (i)**

*Outcomes assessed: E2, E9*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 3 |
| • Makes substantial progress | 2 |
| • Writes $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$ or equivalent merit | 1 |

**Question 6 (c) (ii)**

*Outcomes assessed: E2, E9*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 2 |
| • Recognises the telescoping sum | 1 |

**Question 6 (c) (iii)**

*Outcomes assessed: E2, E9*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 1 |

### Question 7 (a) (i)

*Outcomes assessed: E2*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 2 |
| • Shows understanding of without replacement | 1 |

### Question 7 (a) (ii)

*Outcomes assessed: E2*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 1 |

### Question 7 (a) (iii)

*Outcomes assessed: E2*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 1 |

### Question 7 (a) (iv)

*Outcomes assessed: E2*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 1 |

### Question 7 (b) (i)

*Outcomes assessed: E2, E9*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution. Justifications (abbreviated or otherwise) which indicate the appropriate geometric fact are acceptable | 2 |
| • Proof without sufficient justification | 1 |

**Question 7 (b) (ii)**

*Outcomes assessed: E2, E9*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 2 |
| • Applies $PT^2 = QT \times RT$ | 1 |

**Question 7 (c) (i)**

*Outcomes assessed: E2, E5*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 1 |

**Question 7 (c) (ii)**

*Outcomes assessed: E2, E5*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 1 |

**Question 7 (c) (iii)**

*Outcomes assessed: E2, E5, E8*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 3 |
| • Evaluates constant and attempts to eliminate $t$ | 2 |
| • Correct primitive | 1 |

**Question 7 (c) (iv)**

*Outcomes assessed: E2, E5*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 1 |

**Question 8 (a)**

*Outcomes assessed: E2, E4*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 3 |
| • Establishes the induction step | 2 |
| • Verifies the result for $n = 1$ | 1 |

**Question 8 (b) (i)**

*Outcomes assessed: E2, E9*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 4 |
| • Obtains correct expression in terms of $R$, $n$ and $\delta$ | 3 |
| • Writes $A = A_1 + \cdots + A_n$ and attempts to apply the result in part (a) | 2 |
| • Writes $A = A_1 + \cdots + A_n$ | 1 |

**Question 8 (b) (ii)**

*Outcomes assessed: E2, E9*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct answer | 1 |

**Question 8 (c) (i)**

*Outcomes assessed: E2, E9*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 3 |
| • Shows $f''(t) = -n^2 f(t)$ | 2 |
| • Shows $f(0) = 0$ | 1 |

**Question 8 (c) (ii)**

*Outcomes assessed: E2, E9*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 2 |
| • Observes that $f''(t) = -n^2 f(t)$ | 1 |

**Question 8 (c) (iii)**

*Outcomes assessed: E2, E9*

**MARKING GUIDELINES**

| Criteria | Marks |
|---|---|
| • Correct solution | 2 |
| • Observes that solution occurs when $f(t) = 0$ | 1 |