2010 HSC NOTES FROM THE MARKING CENTRE MATHEMATICS

Contents

Introduction

This document has been produced for the teachers and candidates of the Stage 6 course in Mathematics. It contains comments on candidate responses to the 2010 Higher School Certificate examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

This document should be read along with the relevant syllabus, the 2010 Higher School Certificate examination, the marking guidelines and other support documents which have been developed by the Board of Studies to assist in the teaching and learning of Mathematics.

Candidates are advised to read the questions carefully and set out their working clearly. In answering parts of questions, candidates should state the relevant formulas and the information they use to substitute into the formulas. In general, candidates who show working make fewer mistakes. When mistakes are made, marks can be awarded for the working shown. If a question part is worth more than one mark, working is expected to be shown. Any rough working should be included in the answer booklet for the question to which it applies.

Question 1

(a) This part was answered correctly by most candidates. The only common error was to simply divide both sides of the equation in its original form by x resulting in only one solution, namely $x = 4$.

(b) A high percentage of candidates attained full marks for this part. A small but significant number of candidates cross multiplied in the hope of obtaining values for a and b but few if any were successful.

(c) A common error was writing the equation of a circle with the centre at the origin, $x^2 + y^2 = 25$. A significant number of candidates wasted valuable exam time in expanding the equation.

(d) Nearly 90% of candidates were awarded full marks for this part.

(e) Most candidates could differentiate the given function. The common errors were: not using the product rule and simply expressing their answer as $2x\sec^2 x$; and incorrect negative signs.

(f) Most candidates used the correct formula to produce the required result.

(g) Candidates had difficulty with this part. The most common error was stating the domain as $x > 8$. A significant number of candidates correctly used a written statement to answer the question, such as ‘cannot be smaller than 8’.

Question 2

(a) Many candidates were assisted by writing u, u', v and v' first and then proceeding with their working. Mistakes were occasionally made with signs in the numerator and a small number used $\cos 2x$ as the denominator. A significant number did not know the quotient rule. Two areas of concern were the incorrect simplification of $x\sin x$ to $\sin x^2$ and cancelling the x in the numerator with an x in the denominator. Some candidates applied the product rule to the expression $x^{-1}\cos x$, but they were no more successful than those who applied the quotient rule directly.

(b) Most candidates correctly factorised the quadratic but quite a few were then unable to interpret their working correctly. The most common mistake involved having $x < -3$ incorrectly appear as part of the solution. The most efficient solution used was to sketch the concave-up parabola intersecting the x-axis at -3 and 4. This usually led to a correct solution. Candidates who checked several x values often took much more working and time and were sometimes incorrect in one of the substitutions leading to erroneous conclusions.

(c) Most candidates understood that a derivative was required and that it should be evaluated at $x = 2$. Many candidates were not able to correctly differentiate the function. The most common incorrect derivatives were $3\ln 3x$, $\frac{x}{3}$ and $\frac{3}{x}$. Many candidates apparently misread the question and used valuable exam time finding the equation of the tangent.

(d) (i) Most candidates knew that $\sqrt{5x+1} = (5x+1)^{\frac{1}{2}}$. A significant number of candidates multiplied by 5 rather than dividing and some added 5 to $\frac{3}{2}$ in the denominator. Some candidates used some form of differentiation instead of integration and many forgot to include the constant of integration in their final answer.

(ii) Most candidates realised that a logarithm integration was needed. Most incorrect answers had a $\ln(4 + x^2)$ term involved but did not correctly deal with the coefficient or with the numerator. Again, some differentiated while others thought there was an inverse tan integral involved. Of concern was a technique that used the expression $\frac{x}{2x}\int \frac{1}{4+x^2}\,dx$ outside the integral sign as it incorrectly moves the variable outside the integral.

(e) Many different correct solutions were in evidence. Candidates who found incorrect primitives such as $\frac{x^2}{2}+\frac{k^2}{2}$ and $\frac{x^2}{2}+k$ had difficulty gaining any marks because k disappeared when the limits of integration were substituted correctly. Many candidates attempted to cope with this by arbitrarily changing a suitable sign. Another correct approach was to use the primitive $\frac{(x+k)^2}{2}$ but the difficulty of the subsequent working was greatly increased. A novel solution was to draw a sketch of $y = x + k$ from 0 to 6 and equate an expression for the area of the trapezium so formed to 30.

Question 3

(a) (i) Most candidates successfully used the midpoint formula to find the coordinates of M by averaging the x-values and the y-values.

(ii) Most candidates applied the gradient formula between 2 points with success. Some errors occurred when the negative sign was lost.

(iii) Most successful candidates followed the sequence of parts (i) and (ii) and used coordinate geometry to prove that NM is parallel to BC and hence showed that the triangles are similar. Many candidates did not successfully apply the recognised proofs for similar triangles. Many candidates also failed to supply clear and succinct reasons with their statements or clear conclusions to their proofs.

(iv) Most candidates found the appropriate equation in this part, generally using the point-gradient form of a straight line, applying their gradient and using one of the points M or N.

(v) Many candidates applied the distance formula and were explicit with their substitution to achieve the correct distance $d_{NM} = \sqrt{(6-12)^2+(8-6)^2}$.

(vi) A significant number of responses to this part found the equation of BC and then used the perpendicular distance formula to find the required length. This elaborate method was not the most efficient one as this part of the question was only worth one mark. Many algebraic or numeric errors occurred in this process. In better responses, candidates used the area of a triangle formula, $A=\frac{1}{2}b\times h$, and substituted the relevant information of $b=\sqrt{40}$ from part (v), and the area of 44.

(b) (i) Although most candidates sketched a curve, many failed to address very important features of the log function, namely the domain, range, concavity and the asymptote.

(ii) Candidates answered this part by writing $\frac{1}{2}(\ln 1+2\ln 2+\ln 3)$, or $\frac{1}{2}(\ln 1+\ln 2)+\frac{1}{2}(\ln 2+\ln 3)$, or used a table of function values and weightings. Some candidates could not determine the value of h required in the trapezoidal rule and many candidates applied Simpson's rule and so did not obtain many, if any, marks.

(iii) Very few candidates were successful in this part. In better responses, candidates discussed the use of the trapezoidal rule for finding an area. They then said that the approximation was less than the value of the integral since $y = ln\,x$ is concave downwards and so the trapeziums lie beneath the curve. The best attempts came with a justification in words and with two trapeziums indicated on their diagram.

Question 4

(a) This part was generally well done. A number of candidates used km and m in the same calculation or misinterpreted the question and thus gave unrealistic answers like 6000 km being run in the 9th week. Listing all 26 distances was not uncommon and many successfully found all answers this way, but at a large cost in valuable exam time.

(i) Better responses used the formula for the nth term with the substitution clearly shown. Common errors included misquoting the formula, using the formula for the sum of n terms, or using the formula for a geometric series.

(ii) Better responses again used the formula for the nth term with the substitutions clearly made and the resulting equation solved. Many errors were made when solving the equation, including deducing that $n = 11$ from the equation $n - 1 = 12$.

(iii) Many candidates recognised that the arithmetic series stopped at the 13th (or 12th) term and then added the correct multiple of 10 km for the remaining weeks. But many summed 26 terms of a series to reach the common but incorrect result of 269.75 km. Although candidates were well practised in using both formulae, those using $S_n = \frac{n}{2}(a + l)$ tended to make fewer errors. Other candidates quoted the correct formula but did not make the correct substitutions. Some candidates incorrectly identified the number of terms, from the 14th to the 26th.

(b) The majority of candidates knew to use a definite integral involving the difference of the functions. The better responses needed only three or four steps, using a single integral. Where the negative signs were not dealt with efficiently, those candidates were unable to correctly substitute their limits. Many candidates seemed to be well practised in showing the substitution into the expression before evaluation. Common errors included incorrect limits, incorrect primitives, addition (rather than subtraction) of the functions and subtraction of the functions in the wrong order. Many candidates gave a decimal approximation, often incorrect from their initial correct substitutions into the primitive(s).

(c) Better responses used a probability tree, or similar construct, and observed that in part (ii) the probability was 3 times that in part (i), and that in part (iii) the probability was 1 minus that in part (ii). Many candidates responded as if there were replacement of the first selection prior to the second selection being made. Many candidates had difficulty simplifying numerical expressions, further highlighting the need to show working and to avoid giving bald answers.

(i) A common incorrect response was $\frac{4}{12} + \frac{3}{11}$.

(ii) A common error was to multiply the 3 probabilities or cube the answer from part (i).

(iii) The most common error was not recognising that the required answer was the complement of the answer in (ii).

(d) Most candidates correctly used $f(-x) = 1 + e^{-x}$ to prove the identity. Better responses worked from one side, usually the left-hand side, to the other to complete their proof, writing either $e^{-x}e^{x}$ or e^{0} before simplifying to 1. Common errors involved an incorrect expression for $f(-x)$, or an incorrect simplification of $e^{-x}e^{x}$, or not knowing that e^{0} equals 1. Many responses indicated that $2 + e^{x} + e^{-x}$ was the desired result and forced their expansion of $f(x)f(-x)$ to achieve that result. Rewriting e^{-x} with $\frac{1}{e^{x}}$ was a useful step for a number of candidates. Candidates are reminded that substitution of particular values of x does not constitute proof.

Question 5

(a) (i) This part was done in numerous ways including: straight substitution of an expression for h; equating the 2 expressions and ending up with the formula for the volume; and other, more elaborate, substitutions.

Candidates who stated the correct formula $10 = \pi r^2 h$ completed the question efficiently. A significant number of candidates did not know the volume formula and tried to equate the given equations.

(ii) In this part, candidates were required to differentiate and then solve an equation involving a negative index. Those candidates who used the table method to show a minimum existed were the most successful, as many candidates struggled to correctly find the second derivative. The majority of candidates differentiated the result for surface area correctly but there were many errors made when attempting to solve for r. Some candidates also found the minimum value for A which was not required, thus wasting time that could have been used elsewhere in the paper. A small but significant number of candidates found a negative value of r and did not recognise that this must be incorrect.

(b) (i) In this part, most candidates demonstrated their knowledge of trigonometric identities. Most began with the left-hand side but a significant number worked from right to left. Some responses were far too complicated given that this part was only worth one mark.

(ii) A number of candidates equated the required expression with the one in part (i) and then cross multiplied. Many candidates realised that $\cos^2 x = 1 - \sin^2 x$ but did not then factorise the denominator to simplify.

(iii) Far too many candidates went astray by not referring to the table of standard integrals, as suggested in the question. Most candidates who did not use the table did not succeed in integrating correctly. A significant number of candidates who integrated correctly proceeded to confuse signs in their evaluation of the limits of integration and found the solution to be $2 + \sqrt{2}$ instead of just $\sqrt{2}$.

(c) This part was generally well done with many candidates able to earn at least one mark. A common error, however, was to state that the sum of the areas was 1 and not the correct value of 2. It was noticeable that a number of candidates were unable to solve the log equations by simply writing the expressions in exponential form and therefore missed an easy third mark.

Question 6

(a) Most candidates stated correctly that $f'(x) = 0$ is a stationary point. Most candidates successfully showed, using the discriminant or the quadratic formula, that this was not possible for the given function and therefore there were no stationary points. In poorer responses, candidates stated that the equation could not be solved as it was not possible to factorise.

Many candidates did not appreciate the connection between parts (i), (ii) and (iii). Responses from part (ii) often did not correspond with the graph drawn.

Most candidates correctly determined the x and y intercepts. Some did state that $x = \pm 2$ after incorrectly factorising $x^2 + 4$.

Many candidates determined concavity by substituting values into the second derivative. The concept that $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ and the significance of the sign of the second derivative were not well understood.

Candidates are advised to use a ruler and to consider the scale on each axis carefully for all graph work. The better sketches were achieved by candidates who chose appropriate scales on the axes. Candidates are to take care when sketching that curves do not result in turning points at the extremities of the curves.

(b) Many candidates struggled to work in radians. Many attempted to convert the correct answer of 1.8 radians into degrees or radians by multiplying by $\frac{\pi}{180}$ or $\frac{180}{\pi}$. Many candidates were not familiar with radian measure being used in right-angled triangles and so converted back to degrees.

The congruency test of RHS was not well recognised by candidates. Many identified the correct statements and reasons but then declared the test SAS. A number of candidates used circle geometry but reasons for $PT = QT$ were not stated well. Many candidates attempted to use the result in their proof of congruency; for example many used $\angle POT = \angle TOQ$.

Many candidates confused the letters used on the diagram, for example, $PO = QQ$. Many candidates attempted to use a side length of 9 in their calculations. A large number of candidates incorrectly assumed figure $OPTQ$ was a square as it had two right angles and so let $PT = 5$.

It is worth noting that candidates who determined an incorrect value for $\angle POT$ and/or PT were still able to complete the remainder of the question and so earn marks.

Despite candidates being able to state the arc length and area of a sector formulae, the rules were often not used properly or appropriately, for example, the size of the angle was not correct or was used in degrees.

Question 7

(a) (i) Most candidates gained full marks for this part. Virtually all of these integrated the acceleration and used the initial condition to determine the value of the constant of integration. Of the small number of candidates that differentiated the velocity, most failed to show that the velocity function satisfied the given initial condition. The most common error in this part was not explaining why the constant of integration was 1.

(ii) Most candidates realised that the particle is at rest when the velocity is zero. Most errors occurred in solving the resulting trigonometric equation.

Common errors included: working in degrees instead of radians; working with the 4th quadrant answer $2t = -\frac{\pi}{6}$ during their working; trying to convert their answer to a 3rd quadrant solution by adding π to their answer for t rather to their answer for $2t$; and not being able to correctly make t the subject of the equation.

(iii) Better responses showed all of the steps in the derivation of the displacement, including the evaluation of the constant of integration. The most common errors included: not finding the correct primitive; forgetting to integrate the constant 1; finding the primitive of 1 to be x rather than t; not including the constant of integration; and not evaluating the constant of integration.

(b) (i) The most common errors were: using the gradient of AB instead of the gradient of the tangent; incorrectly simplifying the equation $y-1=-2(x+1)$; and finding the gradient of the tangent but not then finding the equation.

(ii) Many candidates could find the midpoint and gradient of AB but the key step in this part was the realisation that the gradient of AB had to be equated to the gradient function of the parabola.

Common errors included: assuming MC is vertical rather than proving it; confusing being vertical with being perpendicular, for example claiming the slope of $MC = -1$ by using $m_1 m_2 = -1$; and not giving any reason at all why MC is vertical.

(iii) The key step in obtaining a better response to this part was knowing that the equation of MC is $x = \frac{1}{2}$.

Question 8

(a) Candidates used a variety of methods to answer this question and a significant number scored full marks. Some of the methods used include:

- using $P = P_0 e^{kt}$, the solution of the differential equation. Common errors made by candidates who used this method included: not being able to write 200 million as a number (writing it as 20000000, 2000000, and so on); writing P_0 as 200 million (in its variety of forms) and P as 102 when solving for k; inability to solve for k correctly, including using $\log_{10}(\ldots)$ not $\log_e(...)$; using $t = 25$ instead of $t = 100$ when evaluating the population in 2035.

- solving for P using a geometric series $= 102r^n$. This method made the solution much easier because logarithms need not be used to solve this equation. However, some candidates who used this method incorrectly used $n = 74$ to determine r.

- solving the differential equation $\frac{dP}{dt} = kP$ by using the relationship $\frac{dP}{dt} = \frac{1}{\frac{dt}{dP}}$ and solving $\int \frac{dt}{dP} dP = \int \frac{1}{kP} dP$. This method was quite lengthy and involved more mathematical processes than the other methods.

This question differs from those in previous years in that the candidates were not asked to verify that $P = P_0 e^{kt}$ is a solution to $\frac{dP}{dt} = kP$. Because of this, many candidates chose inappropriate methods of solution.

(b) This part was not very well answered by most candidates. The most common incorrect answer was $(TT) = 1 - 0.36 = 0.64$. A number of candidates merely wrote down the answer 0.36, reasoning that the required outcome had the same probability as achieving two heads. A significant number of candidates incorrectly determined that $(TT) = 0.64 \times 0.64 = 0.4096$.

(c) (i) Most candidates correctly determined A. A minority incorrectly determined that $A = 3$, possibly referring to the graph that they were required to draw in (c) (iii).

(ii) A significant number of candidates correctly determined b. However, the common error in this part was to simply state the period of the graph and not use it to determine the value of b. A number of candidates wrote the answer for (i) as the answer to (ii) and vice-versa.

(iii) This part was generally well answered, with most candidates scoring at least a mark. Typical errors included: graph drawn with the incorrect period; incorrectly representing the correct period on their graph; lack of symmetry about $y = 1$; and the graph drawn with the incorrect amplitude.

Graphs were often too small, and should be drawn around a third to a half page in size.

(d) Most candidates correctly differentiated the function and realised that $f'(x) > 0$ (for all *x*) for an increasing function. Significantly, many were then unable to (correctly) determine the value of *k* for which this occurred.

A substantial number of candidates incorrectly assumed that increasing meant either $f(x) > 0$ or $f''(x) > 0$.

It is important to note that many candidates could not successfully solve $36 - 12k < 0$, not being able to deal with the required sign change. Some candidates used an alternative approach to the solution by looking at completing the square; others by graphical means.

Question 9

(a) (i) Most candidates were aware that the solution involved compound interest and they used an appropriate formula to correctly find at least one term in the series. However many were unable to continue the pattern to form a geometric series. The most common errors were using $r = 1.5$ or 1.05 instead of the correct 1.005, or not realising that the last term included interest, hence substituting incorrectly into the formula for the sum of a geometric series using 500×1 instead of 500×1.005.

(ii) (1) Some candidates did not recognise this as a time-payment type question. Others had difficulty interpreting the question, with many responses stating incorrectly that $A_1 = (P - 2000)1.005$ rather than $A_1 = P \times 1.005 - 2000$. Other candidates found correct expressions for A_1 and A_2 but were then unable to develop a correct expression for A_n (or even A_3 in some cases) or developed the series for $n + 1$ terms instead of n terms. Incorrect use of brackets also caused the incorrect meaning to be implied in working.

(2) Many candidates attempted this question by stating correctly that $A_n = 0$ and most knew to use the answer provided in the previous part. Most correct solutions involved logarithms to solve the exponential equation and a small percentage of candidates used a trial-and-error approach to complete their solution.

(b) (i) Most candidates attempted this part, presenting a solution involving an inequality. Commonly, errors resulted from the misuse of inequality signs, for example $x > 0$ or $x < 2$ or $0 > x > 2$. A notable percentage of candidates correctly stated the solution in words.

(ii) Many candidates correctly stated that the maximum value was at $x = 2$ but then failed to use the fact that $A_1 = 4$ and $f(0) = 0$ to find the corresponding y value.

(iii) This part was very poorly done and many candidates did not attempt it. Some candidates realised that the solution involved using the area under the curve for $4 \le x \le 6$. Only a small number of candidates interpreted their findings and used $\int_4^6 f'(x)dx = -6$ to arrive at the correct answer. This meant that the most common incorrect answer was $f(6) = 6$.

(iv) Most candidates did not use the connection between this part and parts $(i) - (iii)$. For some, a perfectly drawn graph was their only attempt.

Graphs were generally not well sketched. Candidates are reminded that care should be taken with diagrams and that they should be clearly drawn using a scale on both axes.

Most candidates attempting this part correctly interpreted the concave-down section of the graph, but unfortunately many failed to join this section properly to the rest of their graph at $x = 0$ and $x = 4$. There was limited recognition of the need to sketch a straight line section with a negative gradient for $x \geq 4$. Other incorrect responses included a graph showing more than one turning point, graphs sketched beyond the given domain and failing to indicate $f(0) = 0$ as stated in the question.

Question 10

(a) Many candidates attempted (i); however, in most cases imprecise reasoning and presentation restricted the marks that could be obtained. Candidates are reminded that when dealing with proofs involving multiple overlapping triangles, it is essential to unambiguously identify all angles and objects of interest at each stage of the argument. In many responses, the connection between parts (i) and (ii) was not used, with many candidates incorrectly trying to implement Pythagoras' theorem in part (ii) rather than simply taking ratios of corresponding sides of similar triangles.

Interestingly some candidates achieved full marks in part (ii) by considering instead the areas of the three triangles.

Part (iii) could be answered by applying part (ii) and using the cosine rule on any of the three triangles in the diagram. Although there was some confusion as to the exact nature of the cosine rule, most responses failed to successfully complete the resulting algebra.

Responses in part (iv) were awarded full marks for correctly identifying the amplitude of the cosine function in part (iii). A common error was to claim that $\cos\theta < -1$. Strategies which abandoned the previous results and used a fresh algebraic attack seldom made any progress.

(b) Although most candidates realised the relevance of the formula $V = \pi\int y^2 dx$ in part (i), many could not find the correct limits of integration or an appropriate primitive. Common errors were to integrate r^2 to $\frac{1}{3}r^3$ rather than r^2x or to claim that the lower limit of integration was $r - r\sin\theta$. Once again the crucial connection between parts (i) and (ii) was missed by many candidates.

A common error in part (ii)(1) was to confuse depth with volume, with the resulting equations quickly spiralling out of control. Responses which simply stated a value for θ or measured this angle off the diagram could not be rewarded.

Candidates could still gain full marks in part (ii)(2) by correctly implementing an incorrect angle from part (ii)(1); however, full marks at this stage of the paper were rare, with many candidates clearly running out of time.