2010 HSC NOTES FROM THE MARKING CENTRE
MATHEMATICS EXTENSION 1

Contents

Introduction

This document has been produced for the teachers and candidates of the Stage 6 course in Mathematics Extension 1. It contains comments on candidate responses to the 2010 Higher School Certificate examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

This document should be read along with the relevant syllabus, the 2010 Higher School Certificate examination, the marking guidelines and other support documents which have been developed by the Board of Studies to assist in the teaching and learning of Mathematics Extension 1.

Question 1

(a) Candidates were required to find a standard integral, which most candidates were able to do. The few errors occurred when candidates misread the table of standard integrals. A very small number of candidates were unable to interpret the table and simply wrote down $\sin^{-1}\left(\frac{x}{a}\right)$ with no further working.

(b) Most candidates answered this question correctly. A common error was the exclusion of the endpoints of the domain by stating the answer as $-2 < x < 2$. It was clear that some candidates did not understand the difference between domain and range, giving answers such as $-2 \le f(x) \le 2$ or $-2 \le \cos^{-1}\frac{x}{2} \le 2$.

(c) In their attempts, a significant number of candidates demonstrated a poor understanding of the log laws, producing incorrect statements such as $\ln(x+6) = \ln x + \ln 6$. Generally, if candidates obtained the correct quadratic they proceeded to find the possible solutions $x = -2$ or $x = 3$. Only very rarely, however, did candidates go on to reject the negative solution. Candidates are reminded that they should always check their solutions with the original question. This is especially important when dealing with functions that have a restricted domain.

(d) There were several methods attempted for this part with a varying degree of success. To gain full marks candidates had to acknowledge that $x \neq -2$ and solve the inequality correctly. The most common method was multiplying by the square of the denominator, and candidates using that method were generally successful; however, a significant number of candidates were unable to solve the resulting quadratic inequality. Another common method was to identify important points, with most candidates being able to identify the two critical points. Most who used this method were then able to test their points and come up with the required solution. A small number of candidates attempted a graphical solution and were usually successful in this approach. The most common incorrect method was to simply solve $3 < 4(x+2)$, disregarding the sign of the denominator. Such a response usually failed to gain any marks. Overall, candidates displayed a poor understanding of the distinction between 'and' and 'or', often representing disjoint intervals using a single expression such as $-2 > x > -\frac{5}{4}$.

(e) This part was not very well done. It is recommended that all of the substitution, including the limits, be done in one step, with the working clearly shown to the side to indicate where each part of the substitution has come from. A significant number of candidates evaluated the new limits, but then failed to use them in their solution. A large number of candidates did not correctly find the primitive of $(1-u)u^{\frac{1}{2}}$ with few simply expanding this expression.

(f) About half the candidates successfully completed this part. Of those who recognised the question as binomial probability, a significant number made errors such as using $\binom{6}{2}$ instead of $\binom{5}{2}$, or forgetting the binomial coefficient all together.

Question 2

(a) Many candidates knew that they needed to use a $\cos 2x$ identity in order to integrate $\sin^2 x$. Common errors included mistakes with signs when integrating and arithmetical slips when substituting to find the value of the constant. Many candidates merely quoted a formula. While that formula may lead to the correct answer if quoted correctly, many candidates misquoted this formula.

(b) (i) Many approaches were taken to this part. Some of the attempts led to circular reasoning without the need to differentiate at all. Candidates are encouraged to take note of the mark allocation when devising a proof. Many candidates differentiated correctly then made a substitution to establish the result. Some candidates proved the result by first making t the subject, differentiating with respect to m and then finding the reciprocal. A substantial number misinterpreted the number 35.5 as 35×5.

(ii) Candidates adept at calculator use evaluated the answer showing minimum working. Due to transcription errors, wrong substitution, poor manipulation or change of units, many candidates equated an exponential with a negative number, took logarithms of both sides and fudged the answer. Having calculated the correct answer, 0.079694…, many candidates were unable to round correctly to three decimal places.

(iii) Some candidates substituted increasing values for t to determine the limit, whereas others incorrectly substituted $t = 0$. A number of candidates incorrectly used the sum to infinity of a geometric series.

(c) (i) While many candidates interpreted the information given in the question sufficiently to write a statement like $P(3) = 0$, they did not use this to substitute correctly into the expression for $P(x)$. Some tried to solve convoluted equations.

(ii) A substantial number of candidates did not attempt this part. Of those who attempted it, many substituted their values of a and b from (i) into $P(x)$ but then attempted either to solve or to divide, indicating that they had not really understood the notion of a polynomial remainder. A few candidates correctly identified the remainder.

(d) A large number of candidates achieved full marks on this part. If candidates stated Pythagoras's theorem, correctly differentiated with respect to x and then used the chain rule, they generally obtained the correct solution. A substantial number of candidates did not state Pythagoras's theorem correctly. Candidates who decided to express r in terms of t had greater difficulty differentiating and substituting in order to find $\frac{dr}{dt}$ as a function of x. It was also difficult, in many cases, to determine the meaning of an introduced variable as it was not clearly defined. Differentiation was often executed with respect to the wrong variable. Although x was defined as distance and the car's speed as 100, some candidates converted these to vector quantities and wrote $\frac{dx}{dt} = -100$ giving the justification that the car is travelling away.

Question 3

(a) A significant number of candidates merely stated incorrect answers and could not earn any marks. Candidates who provided a diagram had greater success in obtaining the correct solution. For example, a diagram for part (ii) with the two red doors placed side by side (similar to the arrangement shown below), might prompt candidates to recognise how to proceed with their solution.

(Red Red) Green Blue Orange

(b) (i) A significant number of candidates either did not recognise that the product rule needed to be applied in order to obtain the correct second derivative, or incorrectly applied the product rule. The question stated that there are two points of inflection. This should prompt candidates to check their working if they do not obtain exactly two solutions to $f''(x) = 0$.

(ii) Since this part was only worth one mark, very lengthy explanations were not required. However, many candidates gave explanations which contained insufficient detail. For example, a statement such as 'because it fails the vertical line test' does not provide an adequate explanation to the question, unless it is made clear that it is the proposed inverse which fails this test.

(iii) Most candidates made some progress by correctly taking logarithms of both sides. In the best responses, candidates found the correct expression for the inverse function by taking the positive square root of $-\log x$. Many candidates mistakenly dropped the negative sign inside the square root sign. They did not recognise that since the values of x are between 0 and 1, $\log x$ is negative and so $-\log x$ is positive.

(iv) Most candidates recognised the domain but some had difficulty in expressing this using the correct notation.

(v) Candidates needed to indicate the change in concavity in the curve and asymptote on the y-axis.

(vi) (1) Candidates are reminded to give clear explanations when asked to show a result. Candidates who stated the values of $e^{-0.6^2}$ and $e^{-0.7^2}$ without explaining why the root lies between 0.6 and 0.7 did not earn any marks. In better responses, candidates considered the function $g(x) = x - e^{-x^2}$ and demonstrated that the root lies between 0.6 and 0.7 by showing that the sign of $g(0.6)$ was opposite to the sign of $g(0.7)$.

(2) In better responses, candidates found the correct answer by first showing that the sign of $g(0.65)$ was opposite to the sign of $g(0.7)$.

Question 4

(a) (i) The great majority of candidates found the correct values of x by using $v = 0$ and factorised correctly. Very few errors were noted, those mainly being solving incorrectly after factorising.

(ii) The great majority of candidates gave the correct equation.

(iii) Most candidates gave the correct value of x, but quite a few did not successfully substitute into the formula, or did not take the square root.

(b) (i) A large number of candidates successfully found the correct values for R and α using the subsidiary angle method, and went on to answer the next part correctly. Many candidates simply attempted to find R and α using the stated equation, usually with little success. Many errors in arithmetic were also noted.

(ii) Those candidates who correctly found R and α also made few errors in finding θ, although they did not always check that the answers were within the desired domain.

(c) Although parametric coordinates were involved, this part was a very simple geometry question requiring the candidates to prove that the given shape was a rhombus. Very few candidates showed that they understood the properties of a rhombus. Those who did know the required properties often led themselves astray via weak algebra. Many then fudged their answers. This was particularly noticeable among those trying to prove four sides had equal length. Candidates proved opposite sides equal but did not complete the algebra to show all sides equal. One of the simplest proofs was showing the midpoints of the diagonals equal and that they bisected at right angles. A number of candidates used this method with very few errors. There were other methods attempted with varying results.

Question 5

(a) This part was done well, with most candidates gaining at least one mark. The candidates who showed clear statements involving $\tan 3°$, $\tan 20°$ and $\tan 30°$ were most successful. The most common error was assuming tan 30° was $\sqrt{3}$. Candidates are reminded that if they do not use the names given on a diagram they should clearly define the names that they do use.

(b) (i) This part was generally poorly done, although many candidates made some progress. Most candidates could differentiate $\tan^{-1} x$ but had difficulties with differentiating $\tan^{-1} \frac{1}{x}$, usually by failing to multiply by the derivative of $\frac{1}{x}$. Many candidates who had the correct derivative did not interpret the significance of $f'(x) = 0$ and simply stated the answer rather than demonstrating that it was $\frac{\pi}{2}$ by substituting a suitable value. Candidates using other approaches were less successful as they failed to explain why there was only one possible answer.

(ii) Many candidates failed to see the connection with part (i) and drew an inverse tan graph, or did not attempt this part at all. A significant number of candidates drew $y = \frac{\pi}{2}$, ignoring the fact that they had an odd function.

(c) Many candidates in this question were unfamiliar with basic terminology from the syllabus and did not express reasons in a convincing manner. Candidates should deduce the answers from the information given. A number of candidates assumed that D was the centre of the circle and based their explanations on this basis which made it difficult for them to earn any marks.

(i) Candidates who used an explanation involving exterior angle were more successful than those who used an explanation based on the angle sum of a triangle.

(ii) A number of candidates wrote down $\angle XDB = \angle CAD$ but with an incorrect reason, usually quoting the alternate segment theorem.

(iii) Many candidates did not attempt this part or made little progress. Most candidates appeared to miss the connection between this and the previous parts.

Question 6

(a) (i) Most successful approaches started with the right-hand side of the equation.

(ii) This part was answered well by most candidates, especially if they made the link with (i).

(b) (i) Candidates who were efficient in deriving the cartesian equation by eliminating t were the most successful. Those who took the approach of making v the subject from both x and y and then equating them were less successful, as they had difficulty eliminating t correctly. Some candidates tried to complete this part by making use of the relationship $v^2 = x^2 + y^2$, misunderstanding that v was the velocity at (d, h) and not the initial velocity. There were many careless errors with the algebra. Errors were also caused by the positioning of the vinculum.

(ii) Candidates either deduced that $v \to \infty$ or recognised that velocity was increasing. It was quite common for candidates to state v was decreasing in either (1) or (2) if they found v increasing in the other.

(iii) The most common error in this part was to ignore the fact that α had a fixed value. Many candidates differentiated $\tan\alpha$.

(iv) The more successful candidates were able to make the link with (a)(i). Without this link, it was still possible to arrive at the result through substituting $\theta = \frac{\alpha}{2} + \frac{\pi}{4}$ into $F'(\theta)$, but this required more work.

(v) Some candidates successfully explained that if $F(\theta)$ is a maximum then v^2 is a minimum. It was more common to see candidates conclude that if $F(\theta)$ is a minimum then v^2 is a minimum.

Question 7

(a) Most candidates were able to show that the statement is true for $n = 1$ and this was often the only mark earned in this question. Candidates are reminded to show their substitution of $n = 1$ and not to simply assert that the statement is true in the initial case. A number of candidates confused the correct order of operations by evaluating $(47 + 53) \times 1$ or stating that $147^d = 0$. Although many candidates stated the induction assumption for $n = k$ and stated the case for $n = k + 1$, a large number of candidates had difficulty incorporating the correct assumption into a proof that established the $n = k + 1$ step. The best responses used the substitution $53 \times 147^k = 147\left(100A - 47^k\right)$, as this allowed them to complete the proof more easily. Candidates who did not specify in the assumption step that $47^k + 53 \times 147^{k-1} = 100A$, where A is an integer, usually failed to earn more than one mark for this part. Transcription errors also marred many responses making it difficult for candidates to gain full marks on this part. Candidates are advised not to waste time simply writing out the structure of a mathematical induction proof without any attempt at the proof as this will not gain them any marks.

(b) (i) Many candidates correctly substituted $x = 1$ into the identity to show this result. Weaker responses stated instead that $n = 1$ or substituted $x = 2$ into the identity.

(ii) Candidates who were successful in part (i) were usually able to establish the result in part (ii). A number of candidates attempted unsuccessfully to use formulae for the sum of either an arithmetic or geometric series.

(iii) Candidates were required to correctly differentiate the identity $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + ... + \binom{n}{n}x^n$ and then to substitute $x = 1$ into both sides of the derivative to show the result. Many candidates recognised that differentiation was required but simply stated this and then stated the result. A common error was for candidates to state that $\frac{d}{dx}2^n = n2^{n-1}$. Many candidates used summation notation and correctly differentiated $\sum_{k=0}^{n} k\binom{n}{k}x^k$ but stated their result as $\sum_{k=0}^{n} k\binom{n}{k}x^{k-1}$, failing to demonstrate that the derivative of $\frac{n}{0}$ is 0.

(c) (i) Few candidates were able to articulate that you could select 0 red balls and r blue balls or 1 red ball and $r-1$ blue balls or ... or r red balls and 0 blue balls, giving $r+1$ possible colour combinations. Many candidates used specific examples and attempted to generalise the result from their examples but this was generally not sufficient to earn the mark.

(ii) Some candidates who incorrectly expanded $\binom{n}{k} = \frac{n!}{(n-r)!(n-r-r)!}$ were unable to earn the mark. Only rarely did a candidate use a combinatoric definition of $\binom{n}{n-r}$ as the number of ways of selecting $n - r$ objects is equal to the number of ways of excluding r which is $\binom{n}{r}$.

(iii) Few candidates gained any marks on this part, although there were some who displayed an outstanding understanding of the problem. A number of candidates approached the solution by working backwards from the result $(n - 2)2^{n-1}$ and although they linked this to their results in part (b) they often were unable to connect this to the number of different selections. A number of candidates realised that for a particular selection of r red or blue balls and $n - r$ white balls the number of selections was $(r+1)\binom{n}{r}$ and then correctly wrote a sum from $r = 0$ to n. A few candidates only summed from to $r = 1$ to n. A common incorrect response from candidates was $\frac{(3n)!}{2n!n!}$.