2011 HSC Notes from the Marking Centre
Mathematics Extension 1

Contents

Introduction

This document has been produced for the teachers and candidates of the Stage 6 course in Mathematics Extension 1. It contains comments on candidate responses to the 2011 Higher School Certificate examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

This document should be read along with the relevant syllabus, the 2011 Higher School Certificate examination, the marking guidelines and other support documents developed by the Board of Studies to assist in the teaching and learning of Mathematics Extension 1.

Question 1

(b) Generally the quotient rule was applied correctly. In some cases the product rule was also applied correctly.

(c) Many candidates recognised the domain and wrote $x \neq 0$. The most common method used to establish the critical points was to multiply both sides by the square of the denominator. A few candidates found it difficult to correctly interpret their working to reach the final solution and gave responses such as $0 > x > 2$ or $x > 0,2$.

(d) Most candidates correctly found the derivative for the given substitution and/or changed the limits.

(e) Commonly the responses were $\frac{2\pi}{3}$ or 120°.

(f) Approximately half of the candidates gave the correct response of $y \geq 1$. Successful responses recognised some feature of the function and investigated appropriate domain values, which led to establishing the range. For example, many noted that it was an even function, or some found a minimum value. However, in some poor responses candidates simply identified a log function and mistakenly wrote the natural domain as a range, or incorrectly assumed that if $x > 0$ then $y > 0$.

Question 2

(a) While in some responses candidates wrote a statement, such as $P(3)=12$, they did not substitute correctly into $P(x)$. Where long division was used, the correct values of a or $P(-1)$ were generally not found. Some wrote that $P(-1)=-4$ then concluded that the remainder was 4, suggesting a lack of understanding of the remainder theorem.

(b) This proved to be the most challenging part in Question 2. In many responses, candidates did not apply the Table of Standard Integrals to differentiate correctly, nor quote the correct formula for Newton's Method, nor substitute into the formula and evaluate correctly nor remember to use radian mode.

(c) Most candidates were able to either quote the correct general term in the required expansion or determine that the term involving $x^5 \times \frac{1}{x^3}$ yielded the coefficient of x^2.

Some candidates evaluated $\binom{8}{3} 3^5 (-4)^3$ incorrectly. Others omitted the negative sign or the brackets in their expansion or in their evaluation of $\binom{8}{3}(3x)^5\left(-\frac{4}{x}\right)^3$.

Some candidates correctly found the value of r in the general term, but confusion with the subscripts led to the coefficient of a different term. A substantial number of candidates stated the term rather than the coefficient.

(d) In a substantial number of responses, there were dotted lines at $x = \pm 1$ or arrowheads on the end of the graph, indicating confusion about the restrictions or definitions of the inverse trigonometric functions. In some cases the y-intercept was omitted and many graphs incorrectly included a horizontal point of inflexion.

(e) (ii) The most common misinterpretation was to group the three favourite songs then play them randomly among the other 37 songs.

Question 3

(a) (i) Some approaches made the question harder than intended. For example using the auxiliary angle method to convert the given expression into a single trigonometric expression, integrating the equation of motion twice, which required the use of later parts to establish some initial conditions to resolve the constants of integration and using $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ met with little success.

(ii) The common errors resulted from poor algebra skills in substituting $t = 0$ into nt and finding a value for n, or deducing $2n = 0 + Bn$ and concluding that $B = 0$ or $B = n$. Some candidates who were unsuccessful at, or did not attempt, part (i), differentiated the displacement correctly in order to solve (ii). However, they still did not see the significance of what they were doing in terms of part (i).

(iii) Many misinterpreted this and found where the particle was at its greatest distance rather than the required ‘when’. Some candidates showed a poor understanding of the use and importance of radians by providing an answer of $t = \frac{90}{n}$.

(iv) Many candidates did not distinguish between distance and displacement and incorrectly concluded that the total distance the particle travelled was 0. Better responses recognised that $t = \frac{2\pi}{n}$ represented the end of one period, thus the particle must have travelled four times the amplitude (one complete oscillation). A common error was to conclude that it had travelled twice the amplitude.

(b) (i) Candidates should be aware that simply substituting the coordinates into the equation establishes that the point is on the line, but does not show the line is a tangent. Some responses included attempts using the memorised parametric equation $xx_1 = 2a(y + y_1)$.

(ii) Many candidates derived the equation of the tangent, rather than substituting the new parameter into the given equation from part (i). This indicates that some candidates may not understand the benefits of the parametric approach.

(iii) An appropriate approach was to substitute the coordinates of the point R into both tangents to show that it lies on both and is the point of intersection. Most candidates attempted to find the coordinates of R. Some only found the x coordinate and simply quoted the given value for y.

(iv) This part was challenging. A common response was to identify the locus as $y = t - t^2$, rather than the required $x = \frac{1}{2}$, and thus claiming that the locus was another parabola. Very few candidates were able to state the restriction on the y coordinate. Some noted the value $y = \frac{1}{4}$, although unaware of its significance.

Question 4

(a) Candidates are advised to sketch large graphs (about a 1/3 of a page), clearly showing all required features. There were a number of very small graphs drawn, without clear labels.

(i) The majority of candidates were able to correctly differentiate the given function. Some candidates confused $f^{-1}(x)$ with $f'(x)$.

(ii) Some candidates, in solving the equation $f'(x) = 0$, misused basic logarithmic laws. Some candidates found the x coordinate of the maximum, but could not simplify the expression involving $e^{-\log 4} - 2e^{-2\log 4}$ or equivalent to find the y coordinate, or did not realise that this was required.

(iii) Some candidates entered the values into a calculator to gain the correct answer. Some did not realise that this value represented the x-intercept, even after getting the value of 0 at $x = \log 2$.

(iv) Some candidates thought the function was equal to 0 as opposed to approaching 0. This indicated a lack of understanding of the concept of a limit and the requirement to describe the behaviour. Some candidates entered very large numbers in their calculators and instead of extrapolating the limit of zero from the digits on the screen, they wrote their calculator display as the limit.

(v) This part was generally done well, although some candidates tried to find the x-intercept or left the part out, but indicated the correct intercept on the graph.

(vi) Candidates who were successful in parts (ii) to (v) were generally successful in sketching the graph, as the x-intercept, the y-intercept, the asymptote and the single maximum turning point were known.

(b) (i) This was generally done well. A number of candidates did not use the correct terminology, confusing circumference and radius.

(ii) In better responses part (i) and angle CDA were used to establish the result. Many candidates proved a cyclic quadrilateral by showing opposite angles supplementary, spending more time on this part than required.

(iii) This was a challenging question. Candidates had difficulty in expressing themselves clearly and often made inappropriate assumptions. The better responses showed a sophisticated approach, such as showing that the size of angle MPO is 180º. Some candidates attempted to use other theorems, often unsuccessfully relating to two circles intersecting and the line joining their centres.

Question 5

(a) (i) Candidates should be aware that when the question asks for a particular fact to be used, in this case similar triangles, then the most efficient solution is most likely to be the one that uses that fact.

(ii) This part of the question was generally done well with many different methods used. A successful approach was to multiply the left-hand side by $\dfrac{1+\sin\theta}{1+\sin\theta}$.

(iii) This part was poorly answered. Those few responses that used circle geometry were generally successful. The candidates who recognised the isosceles triangle and used this knowledge, usually did not fully justify their arguments. As the answer was again given, many candidates were blindly stating facts. Candidates are reminded that correct reasons must support their statements.

(iv) Many candidates successfully completed this part, despite weak setting out.

(v) This part was generally done well, with most candidates noting the relationship from part (iv).

(b) (i) Some candidates, after writing $k=\log\left(\dfrac{3}{5}\right)$, then simply stated that $k=\log\left(\dfrac{5}{3}\right)$, as was required by the question.

(ii) Many candidates did not correctly interpret $t=-1.23$ as a time before 10am.

Question 6

(a) In most responses, candidates showed the statement true for $n=1$ by showing their substitution. Many responses did not demonstrate an understanding that the induction process depends on assuming the statement true for $n=k$ and using this assumption to show the statement also true for $n=k+1$. Good practice is to write the statement that is to be proved. Common errors in the proof of the statement for $n=k+1$ included transcription errors and poor or inefficient algebraic approaches. Many candidates expanded both sides of the statement to show they were equal, often factorising a cubic into the required linear factors without any justification. The better responses involved factorisation rather than expansion. In some responses, candidates simply wrote out the structure of a mathematical induction proof without any attempt at the proof.

(b) (i) Most candidates correctly substituted $y=0$ into the vertical displacement equation to show this result. Some candidates correctly verified the result by substituting $t=\sqrt{\frac{2h}{g}}$ into the vertical displacement equation to show that $y=0$. There were some inefficient algebraic approaches with candidates first finding the cartesian equation then substituting $x=vt$ to show the result. Weaker responses stated that $y=-h$ and incorrectly obtained the result. Candidates need to be careful with notation, making sure the radical sign includes all required terms, $t=\sqrt{\frac{2h}{g}}$ not $t=\frac{\sqrt{2h}}{g}$.

(ii) In many responses, candidates stated $d=v\sqrt{\frac{2h}{g}}$ but had difficulties linking this to the velocity components and/or dealing with the negative velocity when the ball strikes the ground. Many responses were marred by fudging of negative signs when tan 45° was given as $\frac{\dot{y}}{\dot{x}}$ rather than $\frac{|\dot{y}|}{\dot{x}}$.

(c) (i) Responses that made use of a tree diagram were often successful, as were those which considered $1-(1-p)^2$. In weaker responses, $\binom{2}{1}p(1-p)$ was often given as the solution. In some responses, there were insufficient or inconsistent steps in the working.

(ii) In better responses, candidates used tree diagrams or found $\binom{3}{2}p^2(1-p)+\binom{3}{3}p^3$. Some who tried to expand $1-\left[(1-p)^3+3p(1-p)^2\right]$ struggled with the algebra.

(iii) In better responses, candidates found the difference between the probabilities of winning Game 1 and Game 2 and were able to show that this difference was positive, or started with $(1-p)^2 > 0$ and were able to elegantly establish the inequality, sometimes reversing their steps after starting with the inequality to be proved. Some candidates attempted a proof by contradiction, but a number commenced with $2p - p^2 < 3p^2 - 2p^3$ rather than $2p - p^2 \le 3p^2 - 2p^3$. Many candidates stated the inequality and proceeded until they found an expression that was positive, but sometimes were unable to justify that. Most responses did not successfully use the domain $0 < p < 1$. Weaker responses showed the inequality true for a specific value of *p* rather than in general.

(iv) Many candidates wrote an equation that incorrectly interpreted the problem, often having Darcy being twice as likely to win Game 2 as Game 1. In many responses where candidates did correctly interpret the wording, they were unable to solve the equation or interpret their solutions over the domain.

Question 7

(a) (i) Responses using the relationship $\tan 45° = \frac{r}{l}$ leading to $r = l$ were generally successful.

(ii) Most responses included the result $\frac{dV}{dt} = \frac{dV}{dl} \times \frac{dl}{dt}$ leading to the correct expression for $\frac{dV}{dt}$. In some responses the confusion between the variable *l* and the constant *h* led to an incorrect derivative.

(iii) In many responses candidates were unable to establish the relationship between the volumes involving *l* and *h*. Most unsuccessful attempts started by finding an alternate version of $\frac{dV}{dt}$, even though part (iii) followed directly from the result in part (ii).

(b) (i) Better responses included expressions such as 'find the derivative' and 'multiply by *x*'. Responses in which candidates multiplied by *x* first and then differentiated were largely unsuccessful.

(ii) Most candidates were able to differentiate $\sum_{r=1}^{n} \binom{n}{r} r x^r$ correctly, although careless errors were common. Most candidates who attempted this part were able to recognise that the next step involved substituting $x = 1$.

(iii) Responses in which candidates attempted to prove the result using mathematical induction were generally unsuccessful.