2011 HSC Notes from the Marking Centre
Mathematics Extension 2

Contents

Introduction

This document has been produced for the teachers and candidates of the Stage 6 Mathematics Extension 2 course. It contains comments on candidate responses to the 2011 Higher School Certificate examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

This document should be read along with the relevant syllabus, the 2011 Higher School Certificate examination, the marking guidelines and other support documents developed by the Board of Studies to assist in the teaching and learning of Mathematics Extension 2.

Many parts in the examination require candidates to prove, show or deduce a result. Candidates are reminded of the need to give clear, concise reasons in their answers.

Question 1

(a) Candidates needed to demonstrate knowledge of the method of integration by parts, either by stating the values of $\frac{du}{dx}$, u, $\frac{dv}{dx}$ and v or quoting the relevant formula. Many candidates did neither, so it was not always clear what they were doing. A significant number wrote $\frac{du}{dx} = x$ and $u = 1$.

(b) This part was generally done well by a variety of methods. Substitution using $u^2 = x + 1$ or integration by parts were the most common methods used. The integration was simplified by replacing $x\sqrt{x+1}$ with $(x+1)\sqrt{x+1} - \sqrt{x+1}$ leading to $(x+1)^{\frac{3}{2}} - (x+1)^{\frac{1}{2}}$. Some candidates tried to use the substitution $x = \tan^2 \theta$, which can lead to a solution, but few candidates achieved the correct answer because it is a difficult choice.

(c) (i) This part was generally done well. A common error occurred when candidates, in multiplying both sides by $x^3(x-1)$, wrote $ax^2(x-1)+bx(x-1)+cx^3=1$ rather than x on the right hand side.

(ii) This part was done very well but, in many responses, candidates did not accurately transcribe their values of a, b and c from part (i).

(d) This part was generally done very well by the method of replacing $\cos^2\theta$ with $1-\sin^2\theta$, or by using integration by parts, or by using $\cos^3\theta = \frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta$. Some candidates used a substitution of $u=\sin\theta$ leading to $u-\frac{u^3}{3}$, but did not give their final answer in terms of θ.

(e) The most common errors were to not set the calculator to radians mode or to not consider the negative sign, giving the integration of $\frac{1}{4+(1-t)^2}$ as $\frac{1}{2}\tan^{-1}\left(\frac{1-t}{2}\right)$.

Question 2

(b) Most candidates realised that z was the diagonal of a rhombus found by the addition of two complex numbers. In part (ii) those who worked with angles were much more successful than those who worked with vectors. A significant number successfully found the required angle using trigonometry. However, many who tried this method experienced difficulty calculating or working with $|z|$. Better responses included labelled angles on a diagram.

(c) Many successful responses used a diagram stating $z=2$ and used the idea that the solutions were equally spaced at angles of $\frac{2\pi}{3}$ around a circle of radius 2.

(d) (i) The expansion in this part was done well but, in some responses, the signs were incorrect when raising i to various powers.

(ii) This part was also done well as candidates used de Moivre's theorem and their result from part (i) to produce the given result.

(iii) Candidates used the result from part (ii), with most responses arriving at $\cos 3\theta = 1$. In better responses, candidates listed several answers before choosing the one that answered the question. The smallest positive solution was often incorrectly given to be 0.

Question 3

(a) (i) Some candidates did not label or number the x- and y-axes. Some did not find the period correctly and so did not see the connection with the restricted domain given in the question.

(iii) The better responses clearly connected the graph and part (ii). Many responses with the correct limit in part (ii) did not have this value on the graph, but had a graph with a limit of zero or approaching infinity as x approached zero. Some graphs were hard to read, as the final graph was drawn directly over the original graph or included several working attempts all drawn in the same pen. The use of a second colour, pencil or highlighter would help make the intention clear.

(b) Responses in which candidates calculated the height of the cross section, $\sin x$, then found the area, were generally more successful than those that attempted to start with an integral. Errors in the use of Pythagoras' theorem made this part more difficult to solve. Difficulties in determining the area led to incomplete solutions. Minor errors occurred in both evaluating the integral and substituting the limits of integration. Some candidates incorrectly treated the question as a volume of revolution.

(c) There were many different methods used to answer this question. Some unusual approaches were attempted. The better responses worked with only one side of the inequality, rather than with both sides at once. Checking that the statement was true for $n=1$ was done well. Some did not simplify the answers to actually demonstrate equality, some checked for 0, some for 2. A common error was to state $(2k)! = 2!k!$. In the better responses, candidates simplified the left hand side, $[2(k+1)]! = (2k+2)(2k+1)(2k)!$, then used the induction hypothesis to simplify. The most common error was to not use the induction hypothesis to allow comparison of both sides of the inequality.

(d) (i) Some candidates did not realise that $e \geq 1$.

(iii) In some responses asymptotes were confused with directrices.

(iv) Most graphs were neat and well labelled.

(v) In many responses, candidates did not describe the effect as was required. Words such as wider, bigger, steeper were not conclusive. Some responses included a series of chronological diagrams to illustrate the effect.

Question 4

(a) (i) In better responses, candidates recognised that $|x+iy-a|^2 - |x+iy-b|^2 = 1$ was $(x-a)^2 + y^2 - \left[(x-b)^2 + y^2\right] = 1$. Responses that recognised the correct usage of $|z|$ were generally successful, although careless errors were common in algebraic manipulation. A few candidates misinterpreted the modulus symbol as brackets and proceeded to expand.

(ii) In successful responses candidates recognised that $x = \dfrac{a+b}{2} + \dfrac{1}{2(b-a)}$ is real from part (i) and hence justified that the locus of z is a vertical line. A variety of incorrect responses were given including ellipse, hyperbola, parabola and circle.

(b) Candidates are reminded that a copy of the diagram is a useful tool, particularly when labelling extra points to use in subsequent proofs.

(i) A variety of methods were used to prove this result. Candidates are reminded that they need to display enough information to fully justify the proof.

(ii) Most candidates successfully completed this part. However, candidates are reminded that alternate angles are only equal if the two lines are parallel.

(iii) Candidates who attempted this part recognised the need to use part (ii), in conjunction with the property related to angles standing on the same arc, to complete the required proof.

(c) (i) In weaker responses, candidates tried to use the given facts about $f(t)$ and $g(t)$ as their starting point, instead of starting with $y = Af(t) + Bg(t)$.

(ii) Weaker responses used the result from part (i) as the solution to the differential equation rather than $y = e^{kt}$.

(iii) In better responses, candidates used $t = 0$ and $y = 0$ to obtain a linear equation in A and B. A common error was to not use $t = 0$ after having found $\frac{dy}{dt}$, leading to a non-linear equation. Those who found two linear equations did not always complete this part.

Question 5

(a) (i) Better responses included a diagram to show the resolution of forces vertically and horizontally, including the direction of these components.

(ii) Most candidates were able to solve the equations simultaneously to obtain the required result for this part.

(iii) In better responses, candidates stated that the bead stayed in contact for $N \geq 0$ and then solved the resulting inequality using the result $\tan\theta = \frac{r}{h}$, or they recognised that $\sec\theta = \frac{R}{h}$ and that $\operatorname{cosec}\theta = \frac{R}{r}$ and substituted into the appropriate inequality. The least successful approach was to substitute $\frac{g}{h}$ for ω to show that $N = 0$ and to then explain why $\omega \leq \sqrt{\frac{g}{h}}$.

(b) Using a common denominator or multiplying both sides by $(1+p)(1+q)(1+r)$ then correctly simplifying the denominator led to the result $p + q - r + 2pq + pqr$. In better responses the information given in the question was then used to complete the proof.

(c) (i) Many candidates knew the reflection property, as stated in the syllabus, but there were a large number of candidates who stated results, which are a consequence of the reflection property, rather than the property itself.

(ii) The relationship $SP + S'P = 2a$ was known and the majority of candidates were able to combine this with the fact that $SP = PR$ to obtain the appropriate result.

(iii) This part of the question was challenging. The most common successful approach was to join O to Q and use similar triangles, or to use the fact that OQ was the join of the midpoints of two sides of a triangle and hence OQ was parallel to, and half the length, of the third side $S'R$.

Question 6

(a) (i) Most candidates could explain the result, arguing that the acceleration $\frac{dv}{dt} \to 0$ for $v \to v_T$.

(ii) Most candidates could separate variables to give $t = \int \frac{m}{mg - kv^2} dv$. Those who realised that partial fractions should be used generally did well. Those who quoted a formula often did not deal with the coefficient of v^2 in the denominator. A very common and serious error was to assume that $\int \frac{m}{mg - kv^2} dv = m(?)\log\left(mg - kv^2\right)$ with '?' being some constant or $-\frac{1}{2kv}$.

(iii) In better responses, candidates substituted twice for v and v_0 in terms of v_T straight away.

(b) (i) Many candidates appeared to be confused about how to prove what was required. The direction of their argument was often unclear, with confusion also about the use of 'or' and 'and'.

(ii) Few candidates were able to explain clearly why the stationary point was a horizontal point of inflection if $f(a) = 0$ and $f'(a) \neq 0$.

(iii) In many responses, the sketch of $y = (f(x))^3$ was not carefully drawn compared to that of $y = f(x)$.

(c) Relatively few simplified $\left|1 + \frac{1}{z}\right|$ to $\frac{|z+1|}{|z|}$ which led to a quick solution. The algebra was generally problematic in cases where $z = x + iy$ was immediately substituted.

Question 7

(a) In better responses, candidates included an integral for the volume of the solid of revolution in the form $2\pi\int_0^1 \left(shell \quad radius\right)\left(shell \quad height\right)dx$ or $2\pi\int_0^1 \frac{(1-x)x}{1+x^2}dx$. The integrand needed to be converted to a usable form using polynomial division or algebraic manipulation. Common errors included incorrectly identifying the shell radius or incorrectly evaluating the definite integral.

(b) (i) In better responses, candidates substituted correctly before showing that $\cos^2\left(\frac{\pi}{8}(4-u)\right)=\cos^2\left(\frac{\pi}{2}-\frac{\pi}{8}u\right)=\sin^2\left(\frac{\pi}{8}u\right)$. Many candidates did not make productive use of basic trigonometric identities. Some candidates made only partial substitutions into the integral, for example neglecting to change the limits of integration and/or dx.

(ii) The responses that recognised the need to add the two different integrals for I to obtain $2I=\int_1^3 \frac{1}{x(x-4)}dx$ were mostly successful. Errors were made in obtaining the correct partial fraction decomposition of the integrand. Weaker responses included attempts to directly evaluate the integral (for example by parts) or by making the incorrect assumption that $\frac{\sin^2\left(\frac{\pi}{8}u\right)}{u(u-4)}$ can be written in the form $\frac{A}{u}+\frac{B}{u-4}$.

(c) (i) While most candidates followed the directions to substitute the equation for l into the equation for the ellipse, a significant number either did not set the discriminant of the resulting quadratic to zero or made algebraic errors.

(ii) This part was generally done well. However, some responses lacked sufficient detail.

(iii) Better responses included a correct expression for $QS\times Q'S'$ with clear working proving this to be equal to b^2. Weaker responses did not make the necessary use of the result in part (i). Errors occurred with the absolute value function, for example, $\left|1-m^2\right|$ was replaced with $\left|1+m^2\right|$.

Question 8

(a) This part was done reasonably well with several methods used. Many gave the incorrect primitive, $\int\left(x^2-1\right)^5 dx=\frac{\left(x^2-1\right)^6}{12x}$.

(b) (ii) Many responses involved the subtraction of the answer to part (i) from 1.

(iii) This part was challenging. Many candidates interpreted it as a question on binomial probability.

(c) (i) There was evidence of problems due to a weak understanding of the properties of modulus and the extended triangle inequality.

(ii) In better responses candidates were able to obtain the desired result with justification.

(d) This part was challenging. It involved a proof by contradiction. In only a few responses were candidates able to link part (d) with parts (c) (i) and (ii).