A critical analysis of selected Australian and international mathematics syllabuses for the post-compulsory years of secondary schooling.

Prepared for the NSW Board of Studies

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Naturally all responsibility for errors and omissions remains with the author.

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EXECUTIVE SUMMARY

This review analyzes syllabus documents and assessment arrangements for post-compulsory mathematics courses in a selection of Australian states and other countries.

Compared to the mathematics courses offered by other Australian States for university-bound students studying calculus, the current NSW mathematics courses provide an appropriate level of challenge but have not changed in content to reflect current uses of technology, and contemporary applications of mathematics.

Compared to the mathematics courses offered by other Australian States for students not intending to proceed to university, the NSW mathematics courses do not provide the same opportunities that are offered in other states for students needing more time to reach the numeracy levels that most of the cohort have achieved by the end of Year 10.

Awarding and assessing bodies at state and national levels necessarily operate within constraints and policies that are peculiar to their own geographical location and social conditions. For this reason, educational innovations that are successful elsewhere need to be evaluated in the context of local traditions, capabilities, and goals. That being said, studying the structure, content and assessment arrangements of the mathematics courses of other countries, and those of Australian states and Territories, can yield useful ideas for innovations in the NSW situation. For example, the Victorian syllabus structure provides students with much choice and flexibility, and we have an opportunity to observe how teachers and students adapt to a course taught with computer algebra systems. The Queensland courses (among others) offer school students a chance to experience aspects of challenging non-calculus mathematics of both a pure and applied nature. One of the reasons for the success of students from Finland in recent international comparative studies is the commitment to achieving educational goals that reflect equitable standards for as many students as possible.

On the international scene, new developments in Singapore and Hong Kong include radical rearrangements of courses, but are not yet fully implemented. It is interesting to note the inclusion of Statistics and Probability as major components of the new courses, and reduced emphasis on mechanics and applied mathematics. Students in England and in the USA Advanced Placement program are offered separate courses in Statistics, and it is a component of every mathematics syllabus in the International Baccalaureate Diploma Programme. This goes hand in hand with making more use of graphing calculators for teaching, learning, and assessment; and other kinds of technology for graphical display in the classroom.

While considering new ideas from other systems, NSW can also benefit by being aware of less successful innovations such as the AS (Advanced Supplementary) exams in England. The centrally imposed additional set of external examinations to be held one year before the A Level examinations, and courses overloaded with content, turned many students away from mathematics.

The issue of technology for the teaching and learning of mathematics at this level is still a matter for debate. This is considered more deeply in a separate review, where it is shown how enthusiasts are happy to demonstrate that lessons can be enlivened, real world applications can be discussed, and weaker students can be supported through calculations to a stage where they can grasp concepts and solve interesting problems. With changes occurring rapidly in ICT, questions asked must be broadened beyond the issue of "are graphing calculators going to be in the exams?" New technology is changing the way mathematicians work, and is changing the way mathematics is used in the workplace. If mathematics is regarded as something people *do* rather than a disembodied collection of rules and results, the focus changes from the tools to the tasks. In schools the focus could be nudged from the tests towards the solving of interesting problems.

Practical measures to ensure that valuable by-hand skills are still learned have been found in many of the courses considered in this review. Examples have also been found of ways to encourage teachers to embrace the positive features of modern technology - its capacity for engaging and exciting students.

This is a time for evaluating, preserving the best, and incorporating the better of the new.

INTRODUCTION

Three important documents inform this review. *The Review of Senior Secondary Mathematics Curriculum* (Stacey, Dowsey, McCrae and Stephens, 1998) was prepared for the NSW Board of Studies as part of the evaluation process for Mathematics syllabuses in 1998. In the Stacey review, NSW courses were compared with those in selected other Australian states and with five other countries, along with the IB (International Baccalaureate Diploma Program). The current review is structured as an update of the Review of Senior Secondary Mathematics Curriculum. For comparison with New South Wales, the next two largest states, Victoria and Queensland, were chosen for this review for comparison of syllabus content, assessment procedures, and policy concerning technology as an aid to teaching and learning. This provides an opportunity to update the material in Stacey et al (1998). Victoria recently reviewed and reaccredited its mathematics courses for the period 2006-2009 (Victorian Curriculum and Assessment Authority, 2005); and in Queensland, the popular Mathematics A, Mathematics B, Mathematics C suite of subjects has been complemented by Prevocational Mathematics (Queensland Studies Authority, 2004) and Functional Mathematics (Queensland Studies Authority 2006).

The second and third documents that inform the current review are recent publications of the Australian Mathematical Sciences Institute: *Comparison of Year 12 pre-tertiary Mathematics Subjects in Australia 2004-2005* (Barrington and Brown, 2005), and *Participation in Year 12 Mathematics Across Australia 1995-2004* (Barrington, 2006). These documents provide a comprehensive overview of the senior mathematics subjects that are offered by NSW, Victoria, Queensland, South Australia/Northern Territory, Western Australia, Tasmania and the ACT, the different assessment processes across the states, and participation rates in mathematics subjects.

Material for the review was also obtained from websites maintained by state curriculum authorities, and from academic literature such as conference proceedings and journals.

The Big Picture: Enrolments in Year 12 Mathematics in Australian States

Table 1 shows the 2004 enrolments in Year 12 for Australian states. The data are from Barrington (2006), using his classification of senior mathematics subjects as Advanced (studied in NSW by students taking Mathematics Extension 1 and possibly Mathematics Extension 2), Intermediate (studied in NSW by students taking Mathematics 2 Unit), and Elementary (studied in NSW by students taking General Mathematics). Enrolment patterns for NSW are broadly comparable to those in the next most populous state, Victoria.

Table 1. State and Territory Populations and Year 12 Enrolments for 2004.
Data from Barrington (2006), Tables 2A, 2B, 3A, 3B, 4

	Column A: Number of Students in Year 12	Column B: Number of Students in Advanced Mathematics **(In NSW: Mathematics Extension 1 and possibly Extension 2)** (note 1)	Column C: Students in Intermediate Mathematics but not in Advanced Mathematics **(In NSW: Mathematics)** (note 2)	Column D: Students in Elementary Mathematics Subjects **(In NSW: General Mathematics and Mathematics Life Skills)** (note 3)	Column E: Students not in Mathematics (note 4)	
NSW	66279 ***100%***	9959 ***15.0%***	13306 ***20.1%***	30382 ***45.8%***	12632 ***19.1%***	
Vic	49975 ***100%***	6293 ***12.6%***	12090 ***24.2%***	22759 ***45.5%***	8833 ***17.7%***	
Qld	40592 ***100%***	3430 ***8.4%***	12887 ***31.7%***	21246 ***52.3%***	3029 ***7.5%***	(note 5)
WA	19792 ***100%***	1628 ***8.2%***	2655 ***13.4%***	12785 ***64.6%***	2724 ***13.8%***	
SA	13324 ***100%***	1211 ***9.1%***	2134 ***16.0%***	3937 ***29.5%***	6042 ***45.3%***	
Tas	4161 ***100%***	228 ***5.5%***	595 ***14.3%***	1340 ***32.2%***	1998 ***48.0%***	(note 6)
ACT	4098 ***100%***	488 ***11.9%***	1148 ***28.0%***	2156 ***52.6%***	306 ***7.5%***	
NT	1390 ***100%***	45 ***3.2%***	198 ***14.2%***	543 ***39.1%***	604 ***43.5%***	
Totals	199611 ***100%***	23282 ***11.7%***	45013 ***22.6%***	95148 ***47.7%***	36168 ***18.1%***	

Note 1. Barrington (2006, p.1) defines Advanced mathematics subjects as those "generally taken by students who wish to proceed to tertiary studies which require the strongest of mathematical preparations, such as engineering, actuarial studies, mathematics, statistics, physical sciences." In NSW this group of students would be studying Mathematics Extension 1 and also possibly Extension 2. In 2004 there were 3514 students studying Extension 2 in NSW. (Board of Studies, 2006a)

Note 2. Barrington (2006, p.1) defines Intermediate mathematics subjects as those which are "suitable for students who wish to proceed to tertiary studies which require significant but not extensive mathematical preparation, such as science, medicine, economics/commerce, dentistry, agricultural science." In NSW this group of students would be studying Mathematics (often still referred to as "2 Unit Mathematics"). This number does not include the students who study Mathematics *and* Extension 1.

Note 3. Barrington (2006, p.1) defines Elementary mathematics subjects as those which are accredited by State Boards as “suitable for students who wish to study mathematics in their final year at secondary school, but do not intend to enter tertiary courses which require Intermediate or Advanced mathematics subjects.” In NSW this includes students in General Mathematics, and 1006 Special Education students studying the Board Developed Course Mathematics – Life Skills, but does not include 342 students studying Non Board Developed subjects in mathematics such as “Mathematics Applied 2 Unit”.

Note 4. The figures for this column were calculated by subtracting the numbers in columns B, C, and D from column A. The figures in column E may include students who studied mathematics of some kind as a VET subject or a school-based subject – see Notes 3 and 5.

Note 5. For Queensland, column D contains the number enrolled in the subject Mathematics A. In addition, there were many students enrolled in subjects that could be classified as numeracy and work-place related mathematics. Although mathematics is not compulsory in Queensland at this level, participation is nearly 100% when all such subjects are taken into consideration.

Note 6. For Tasmania, column D contains the number enrolled in the subject Mathematics – Applied.

Changes in enrolment patterns over time

Figure 1 was created from data in Barrington (2006), Tables 1A, 2A, and 3A. The national trend in enrolment patterns is for a lower proportion of Year 12 students to study mathematics at an Advanced or Intermediate level, and for an increase in the proportion of students studying mathematics at an elementary level. Barrington notes this with concern, while acknowledging that state differences are tied to local demographic situations and educational conditions that include the influence of tertiary entrance ranking procedures, and students’ perceptions of reward for effort.

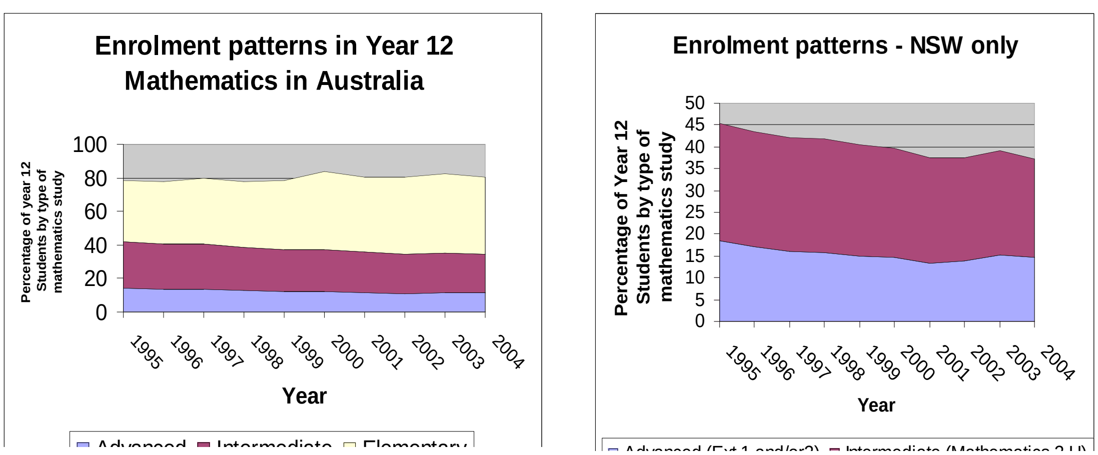

Figure 1: Enrollment patterns in mathematics, Australia and New South Wales

NEW SOUTH WALES

Context for the study of mathematics in Years 11 and 12

The Board of Studies NSW was established in 1990, replacing the Board of Secondary Education which in turn replaced the Secondary Schools Board and the Board of Senior School Studies. The responsibilities of the Board of Studies are listed on its website as follows: (NSW Board of Studies, 2006b)

The Board of Studies:

- sets the core curriculum by developing syllabuses for Kindergarten to Year 12 and provides support materials for teachers and parents;
- manages the NSW School Certificate external Tests (Year 10) and the Higher School Certificate Examinations (Year 12) each year;
- assesses student achievement and awards high quality credentials to meet the needs of the full range of students;
- promotes the provision of quality education by developing, communicating and implementing educational policies and practices;
- provides advice on grading and assessment policy and procedures;
- promotes the provision of quality education through the registration and accreditation of non-government schools, certifying that they may teach students and enter students for the examinations; and
- effectively manages its resources so that educational objectives are met.

A new Mathematics Syllabus has recently been adopted for Years 7 to 10 in New South Wales. The first students to use this syllabus will be in Year 10 in 2006. (Board of Studies NSW, 2003). Although there is only one mathematics subject at this level, three pathways are clearly identified for students to follow, with increases in content and difficulty available at each of pathways 5.1, 5.2 and 5.3.

At Stage 6 (Years 11 and 12), areas of study are called subjects and within subjects there may be several different courses. There are about 150 Board Developed Courses and a large range of Board Endorsed Courses. Both kinds of courses are available in general education subjects as well as in nationally recognised Vocational Education and Training (VET) courses for many industries. (Board of Studies NSW, 2006c). Board Endorsed Courses do not count towards calculation of the Universities Admission Index, which is calculated by the Universities through the Universities Admission Centre (UAC), independently of the Board of Studies; however some VET courses have an optional examination and may be included in the UAI calculation.

Most subjects have a 2 unit Preliminary and a 2 unit Higher School Certificate (HSC) component. Students usually complete their study for the Higher School Certificate over two years, studying Preliminary courses in Year 11 and HSC courses in Year 12. The eligibility requirements for the HSC are set out in the Assessment, Certification and Examination Manual (Board of Studies NSW, 2005). The chief requirements are that at least 12 units must be studied in Year 11 and at least 10 units must be studied in Year 12.

In both years, at least six units need to be Board Developed Courses, at least two units must be in English, at most six units may be in Science, at least three courses should be of two units or greater, and at least four subjects must be studied.

Flexibility is offered by allowing students up to five years to accumulate units for the HSC (to allow for part time study for those with employment or family care commitments, also to allow for repeating of subjects), and also acceleration is possible.

Assessment for most Board Developed Courses consists of 50% determined by the school and 50% by external examinations taken at the end of Year 12.

Rationale and content of subjects

There are four mainstream mathematics courses:

General Mathematics (2 units)	a non-calculus course, 240 hours
Mathematics (2 units)	a calculus based course, 240 hours
Mathematics Extension 1	Requires Mathematics (2 Units), 120 hours
Mathematics Extension 2	Requires Mathematics Extension 1, 120 hours

For students with special educational needs there is also a basic numeracy course: Mathematics – Life Skills. Approximately 1000 students take this course.

Further detail of the Mathematics courses is given in Appendix 1, provided by Margaret Bigelow, Project Officer at the Board of Studies.

Current issues

In the late 1990s, the NSW Government initiated a review of the HSC to address several concerns about the academic standard of the courses being offered. Reforms were put in place in time for students entering Year 11 in 2000. These reforms are described by the Minister for Education and Training of the day in Aquilina, (1997). The structure and content of the calculus courses had been regarded as successful over many years and was unchanged during the reform of the HSC. Consequently, teachers are using syllabus documents substantially unchanged since those approved in 1982 for Mathematics and Mathematics Extension 1. (Board of Senior School Studies, 1982). There were amendments to the Extension 2 course in 1989 when it was known as 4 Unit Mathematics. (Board of Secondary Education, 1989).

As a result of the curriculum reviews that were part of the establishing of the New HSC, the non-calculus subjects *Mathematics in Society* and *Mathematics in Practice* were replaced by *General Mathematics*, examined for the first time in 2001. Enrolments in General Mathematics initially increased, but have declined since 2002 (see Figure 2). Public perception is that the Year 12 component of *General Mathematics* is too difficult for many of the students who might have enrolled in *Mathematics in Practice* (Mitchell, 2004, MANSW 2006) or *Mathematics in Society.*

As noted by McGaw, (1996, p. 136), enrolments in the NSW subject then known as 4 Unit Mathematics (now Extension 2) are very responsive to perceived advantages in the scaling procedures used to calculate the University Admissions Index (UAI), formerly the Tertiary Entrance Rank or TER.

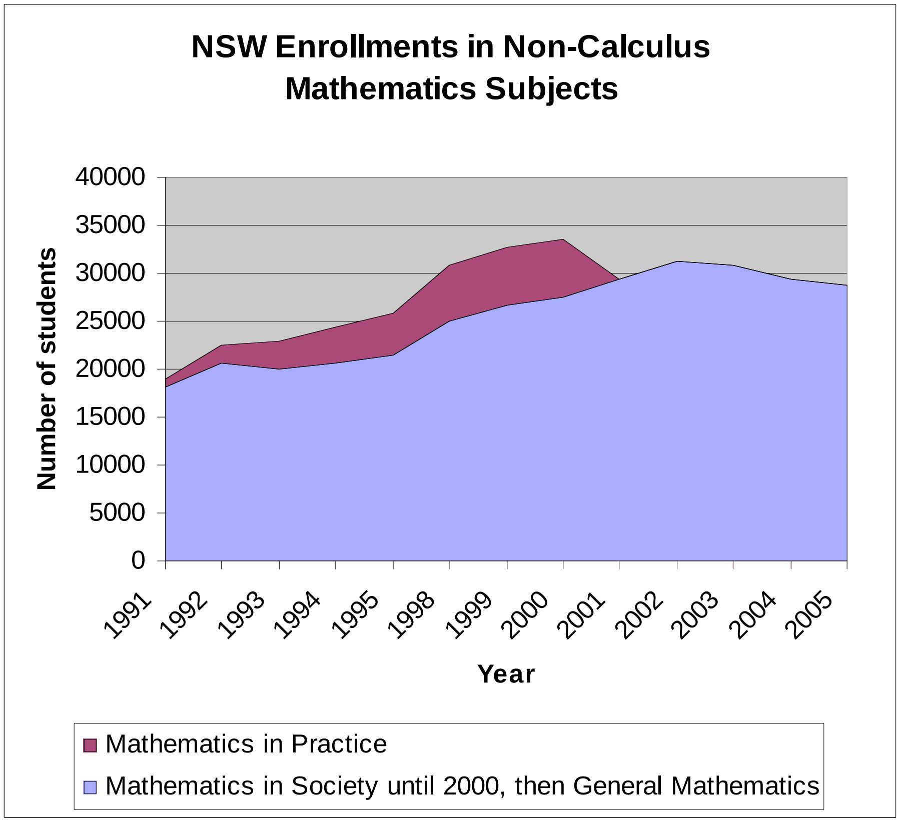

Figure 2. Trends in enrolments in non- calculus mathematics subjects in NSW, 1991-2005. Source of data: NSW Board of Studies Statistics Archives, accessed September 21, 2006 at http://www.boardofstudies.nsw.edu.au/ebos/static/ebos_stats.htm.

VICTORIA

Context for the study of Mathematics in Years 11 and 12

The Victorian Board of Studies is now the Victorian Curriculum and Assessment Authority (VCAA), a statutory authority that reports directly to the Victorian Minister for Education and Training through the VCAA Board. The VCAA was established to

- develop curriculum for all Victorian schools
- assess student learning and monitor student achievement
- conduct research leading to innovative educational programs. (VCAA, 2006a)

The VCAA administers the Victorian Certificate of Education (VCE) and the Victorian Certificate of Applied Learning (VCAL). These are described in the VCAA website as follows:

> The Victorian Certificate of Education (VCE) a certificate that recognises the successful completion of your secondary education. It is an outstanding qualification that is recognised around the world. The VCE provides pathways to further study at university, Technical and Further Education (TAFE) and to the world of work. It is even possible to undertake a school-based apprenticeship or traineeship within your VCE.
>
> The Victorian Certificate of Applied Learning (VCAL) is a senior secondary qualification that sits alongside the VCE. The VCAL is based on hands-on learning and is designed to give students 'employability' skills and the skills to go on to further training – either in the workplace or at TAFE. Students studying the VCAL undertake a work placement where they must learn specific skills. Students can also complete parts of nationally recognised pre-vocational courses as part of their VCAL certificate.

Most students complete their VCE over Years 11 and 12. Many students undertake some VCE units in Year 10. Students choose from 90 studies, or subjects. Of these, 30 are Vocational Education and Training (VET) programs that also provide a nationally recognized industry qualification. Students usually complete Units 1 and 2 of a subject in Year 11 and continue to complete Units 3 and 4 of that subject in Year 12, although it is possible to take only Units 1 and 2 of a subject. Generally, 20 to 24 Units are taken over two years, and at least 16 units must be completed satisfactorily to obtain a VCE. At least three units must be from particular English subjects, including at least one Unit 3. To gain an ENTER (Equivalent National Tertiary Entrance Rank), Units 3 and 4 of an English sequence must be completed, as well as another three sequences of Units 3 and 4 studies.

To provide challenge for very high achievers and a pathway to second year university study in a subject that a student enjoys, Extension Studies are offered by some Victorian universities. The most able 2% of VCE students are considered likely to benefit from this program. These subjects do not count towards the 16 units needed for the VCE, however benefits include completing at least 20% of a first-year university program and additional points in the ENTER calculation. Students apply directly to the universities.

In Victoria, assessment is a mix of school-based assessment and external examinations. More detail is given within each subject description below.

Rationale and Content of Subjects

The document "Mathematics: Victorian Certificate of Education Study Design" (VCAA 2005) details the mathematical content and assessment plans for all the VCE Mathematics subjects accredited for the period 2006-2009. The following information is from that document. A summary of changes for 2006-2009 is available at http://www.vcaa.vic.edu.au/vce/studies/mathematics/summary.print.html.

Foundation Mathematics Units 1 and 2 (Year 11) Victoria

This could be regarded as a Year 11 course only. It provides mathematical development for students entering VCE who need mathematical skills to support other VCE and possibly VET studies. It does not provide a basis for study in mathematics at Units 3 and 4. There are four areas of study: 'Space, shape and design', 'Patterns and number', 'Handling data', and 'Measurement'. Teachers are encouraged to embed the content in contexts which are meaningful to students and sample courses of study include thematic approaches. Although the following list of content (Key Knowledge) may appear to be "light" in terms of mathematical content, there is a longer list of Key Skills in the document. Completing investigations and projects in meaningful contexts makes subjects such as this one more demanding than the list of mathematical content might indicate. In addition, the content listed here is related to Outcome 1 for the subject: "On completion of this unit the student should confidently and competently use mathematical concepts and skills from the areas of study." There are lists of Key Knowledge and Key Skills for Outcome 2 "On completion of this unit the students should be able to apply and discuss mathematical procedures to solve practical problems in familiar and new contexts, and communicate their results" and Outcome 3 "On completion of this unit the student should be able to select and use technology to apply mathematics in a range of practical contexts." Students are expected to be assessed by their schools on achievement in all three Outcomes.

Key Knowledge for Outcome 1 of the Foundation Mathematics Course (Victoria)

1. Space, shape and design
    - Names and properties of common geometric shapes in two dimensions and three dimensions
    - Forms of two-dimensional representations of three-dimensional objects, including nets and perspective diagrams
    - Symbols and conventions for the representation of geometric objects; for example, point, line, ray, angle, diagonal, edge, curve, face and vertex
    - Symbols and conventions for related measurement units
2. Patterns and number
    - Decimal place value basis of number scales
    - Arithmetic operations and what they represent (sum, difference, product, quotient)
    - Equivalent forms of expressing the same quantity; for example, fractions, decimals, percentages

Levels of accuracy required for a particular problem or context
Relationship between quantities and related formulas

3. Handling data
   - Features of graphs, tables, maps and plans
   - Key terminology used in relation to graphs, maps, sketches, plans, charts and tables
   - Purposes for using different forms of data
   - Conventions for correct labeling of graphs, choice of scales and units
   - Data collection in a variety of contexts and for a variety of purposes; for example, keeping personal records for budgeting, keeping records for taxation purposes, gathering opinions through surveys and questionnaires for determining customer/employee satisfaction, monitoring quality control of a production process
   - Key features which pertain to a range of visual presentations
   - Types of data (categorical and numerical) and appropriate forms of representation
   - Use and interpretation of mean and median as 'average', and range as spread
   - Categories into which information can be sorted.
4. Measurement
   - Definitions of common metric units of length, area, volume, and mass
   - Relative scale of metric units, for example mm, cm, m, km
   - Rules for rounding to a specified degree of accuracy
   - Conversion factors for representing common metric quantities in decimal form
   - Procedure for estimation and calculation

Assessment for Foundation Mathematics Course Units 1 and 2 (Year 11) (Victoria)

There is no external assessment for this subject. The Mathematics Study design states that a selection of assessment tasks should be made from investigations and projects, assignments, and tests. Examples are given.

Technology for Foundation Mathematics Course Units 1 and 2 (Year 11) (Victoria)

In the study Design Document the emphasis is on using a range of information and communication technologies where appropriate in applying mathematics in practical contexts. For example, the World Wide Web would be a source of mathematical information and data for investigations and projects. Calculators and computer software would be used to draw graphs and other displays and to organize and present information. Students need to develop skills related to effective use of technology, including for example the selection of viewing windows for graphs.

General Mathematics Units 1 and 2 (Year 11) Victoria

This subject may be implemented in a number of ways. Some students might not proceed to Year 12 studies in mathematics. Others may be intending to take a variety of Year 12 studies. To provide flexibility, schools construct courses for each unit that contain material covering four or more topics selected from at least three different areas of study. The choices must be made with plans for Year 12 study in mind.
The areas of study are

> Arithmetic, Data analysis and simulation, Algebra, Graphs of linear and non-linear relations, Decision and business mathematics, and Geometry and trigonometry.

A summary of topics for each area of study is given here. (See VCAA 2005 for further detail. Some extra detail has been given here selectively where the content is different from NSW subjects or in a different location.)

1. Arithmetic
   - Natural numbers, integers, rational numbers
   - Real and complex numbers (complex numbers to operation with complex numbers of the form $a + b\,i$ where $i^2 = -1$.)
   - Matrices (including definitions, operations, applications to solving linear equations in two variables)
   - Sequences and series (as maps from the natural numbers to the reals, use of technology to create sequences, series and their graphs, recursion, arithmetic and geometric sequences and series, fixed point iteration, practical applications such as financial arithmetic, population modelling, musical scales)
2. Data analysis and simulation
   - Univariate data (including categorical and numerical data, data displays and their interpretation including dot plots, stem and leaf plots, frequency tables and histograms; mean , median, mode, range, variance and standard deviation, boxplot of five-number summary)
   - Bivariate data (including scatterplots and their interpretation, correlation and regression, line of fit)
   - Simulation (including random experiments, event spaces, probability as long run proportion, stages in using simulation in a model, types of simulations such as Bernoulli and Markov trials, simple queueing problems, multi-event problems such as lotto.)
3. Algebra
   - Linear relations and equations
   - Non-linear relations and equations (including construction of tables of values from a formula by use of calculator, computer algebra system or spreadsheet; numerical solution of non-linear equations)
   - Algebra and logic (including propositions, connectives and truth tables; tautologies and proof patterns and application to number patterns, electronic gates and circuits, Boolean algebra)
4. Graphs of linear and non-linear relations
   - Linear graphs and modelling (including linear inequalities)
   - Sketching and interpreting linear and non-linear graphs (includes polar co-ordinates and polar graphs, focus-directrix properties, sketching the graph of reciprocal and square relations from the graph of a simple relation)
   - Variation (includes numerical, graphical and algebraic approaches to direct, inverse and joint variation; transformation of data to establish relationships, modelling using the log function)
   - Kinematics (New topic for this subject – designed to lead in to Specialist Mathematics Units 3 and 4.) (Includes numerical, diagrammatic and graphical representation of rectilinear motion; use of constant acceleration formulas relating velocity, distance, time and acceleration; numerical estimation of instantaneous velocity).

5. Decision and Business Mathematics

Networks (includes Euler's formula, Eulerian and Hamiltonian paths and circuits and applications; trees and spanning trees and applications.)

Linear programming (as far as solving simple optimization problems)

Financial arithmetic (includes interest calculations, simple and compound interest, purchase options and credit, depreciation of assets.)

6. Geometry and trigonometry

Shape and measurement (includes mensuration, Pythagoras, similarity and symmetry)

Geometry in two and three dimensions (includes some straight edge and compass constructions, geometry in art, tessellations, theorems relating to angles in a circle)

Coordinate geometry

Vectors (includes planar representations, addition, subtraction, components, applications to geometric proofs, orienteering, navigation and statics)

Trigonometric ratios and their applications (includes right angled triangles, sine and cosine rules, circle mensuration such as arc length).

Assessment for General Mathematics Course Units 1 and 2 (Year 11) (Victoria)

There is no external assessment for this subject. Students are to demonstrate the achievement of Outcome 1 "On completion of this unit students should be able to define and explain key concepts in relation to the topics from the selected areas of study, and apply a range of related mathematical routines and procedures" through their performance on a selection of assessment tasks, including assignments, tests, summary or review notes. Students are to demonstrate the achievement of Outcome 2 "On completion of this unit students should be able to apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics in at least three areas of study" through their performance on a selection of assessment tasks, including projects, short written responses, problem-solving tasks, and modelling tasks. Achievement of Outcome 3, "On completion of this unit the student should be able to use technology to produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches in at least three areas of study" is to be demonstrated by the student's performance on relevant assessment tasks being undertaken as assessment for Outcomes 1 and 2 which incorporate the effective and appropriate use of technology.

Technology for General Mathematics Course Units 1 and 2 (Year 11) (Victoria)

The appropriate use of technology (such as graphics calculators, spreadsheets, graphing packages, dynamic geometry systems, statistical analysis systems and computer algebra systems) to support and develop the teaching and learning of mathematics is to be incorporated throughout the course.

Mathematical Methods Units 1 and 2 (Year 11) Victoria

The purpose of this subject is to lead on to Mathematical Methods Units 3 and 4 and possibly also Specialist Mathematics Units 3 and 4. A certain amount of knowledge in functions and graphs, and probability is assumed. The Areas of Study are 'Functions and graphs', 'Algebra', 'Rates of change and calculus', and 'Probability'.

A summary of topics for each area of study is given here. (See VCAA 2005 for further detail. Extra detail has been given here selectively but most will be familiar to NSW readers.)

Unit 1

1. Functions and graphs (including co-ordinate geometry, function notation, linear, power, quadratic and cubic polynomials). Sketching by hand.
2. Algebra (including factorizing, expanding, solution of quadratic equations by completing the square and by the formula, graphing and numerical solution in selected cases, simultaneous equations, development of polynomial models, index and logarithm laws). This area of study extends across both units)
3. Rates of change and calculus (including an informal and contextualized approach to average and instantaneous rates of change.)
4. Probability (including events and the representation of event spaces in lists, grids, Venn diagrams, Karnaugh maps, tables and tree diagrams. Impossible, certain, complementary, mutually exclusive, conditional and independent events, compound events.)

Unit 2

1. Functions and graphs (including trig ratios, radian measure, graphs of circular functions in the form $y = af(bx) + c$ where f is the sine, cosine or tangent function; applications, functions involving indices and logs, exponential functions and applications.)
2. Algebra (completing the algebra topics from unit 1)
3. Rates of change and calculus (differentiation by first principles, formal differentiation and anti-differentiation of polynomial functions up to degree 3 and simple power functions and related applications, including graph sketching.)
4. Probability (including counting principles and techniques and their application to probability, total probability law for two events, conditional probability)

Assessment for Mathematical Methods Units 1 and 2 (Year 11)(Victoria)

This very similar to the assessment for General Mathematics Units 1 and 2 outlined above.

Technology for Mathematical Methods Units 1 and 2 (Year 11) (Victoria)

The appropriate use of technology (such as graphics calculators, spreadsheets, graphing packages, statistical software and computer algebra systems) to support and develop the teaching and learning of mathematics is to be incorporated throughout the course.

Mathematical Methods (CAS) Units 1 and 2 (Year 11) (Victoria)

The purpose of this subject is to lead on to Mathematical Methods (CAS) Units 3 and 4, or Mathematical Methods Units 3 and 4, and possibly also Specialist Mathematics Units 3 and 4. This subject was offered as pilot program from 2001-2005, and can be offered by all schools in 2006. The content is very similar to that of Mathematical Methods. Students in Mathematical Methods (CAS) use an approved Computer Algebra System (CAS) calculator or software. The VCAA has foreshadowed that Mathematical Methods and Mathematical Methods (CAS) will be merged into a single CAS enabled (that is, a technology with integrated numerical, graphical and symbolic functionality) study for the next accreditation period from 2010.

Mathematical Methods Units 3 and 4 (Year 12) and Mathematical Methods (CAS) Units 3 and 4 (Year 12) (Victoria)

These subjects have very similar content and will be considered together. As with the Year 11 units, students in the CAS subject use an approved Computer Algebra System (CAS) calculator or software.

There are four areas of study.

1. Functions and Graphs

    Including power, exponential, logarithmic, circular, and modulus functions; graphs of these and transformations. Polynomial functions, sum, difference and composite functions, graphical and numerical solutions of equations, applications.

2. Calculus

    Including limits, continuity, anti-differentiation, applications.

3. Algebra

    Including factorization of polynomials, exponential and log laws, solution of equations involving exponential and trig functions, inverse functions.

4. Probability

    Including random variables, discrete random variables including the binomial distribution, and continuous random variables including the standard normal distribution.

Assessment for Mathematical Methods Units 3 and 4 (Year 12) Mathematical Methods (CAS) Units 3 and 4 (Year 12) (Victoria)

Internal, Unit 3 – 20%, Unit 4 – 14%.
External, Examination 1 (no technology, 1 hour) 22%,
External, Examination 2 (an approved graphics calculator, or CAS calculator for the CAS subject, 2 hours, materials may be brought in) 44%

Teachers are given specific instructions concerning the nature of tasks that contribute marks towards the internal assessment. In Unit 3 a *function and calculus application task* assessing achievement on all three outcomes, and two tests, must be given. In Unit 4 there are to be two *analysis* tasks, one of which must be related to the *probability* area of study. Detailed examples are given in the Study Design document, which show that students are expected to complete modelling and writing tasks which require more than the application of standard procedures and formulas.

Specialist Mathematics Units 3 and 4. (Year 12) (Victoria)

As implied in the name, this subject is designed for students intending to continue to tertiary studies in mathematics itself, or requiring a high level of mathematics. Mathematical structure and proof are highlighted. There are four areas of study. Functions, relations and graphs are usually studied in Unit 3 (Year 11) along with parts of Algebra, Calculus and Vectors. Unit 4 (Year 12) usually would see completed the areas of Algebra, Calculus and Vectors, along with Mechanics.

1. Functions, relations and graphs

    Including sums of simple power functions, reciprocal functions of quadratic functions and circular functions, relations representing circles, ellipses, hyperbolas including parametric forms, analysis of key features of the graphs.

2. Algebra

    Including partial fractions, complex numbers in polar form and Cartesian form, regions and paths in the complex plane, de Moivre's theorem, factoring polynomials over the complex field.

3. Calculus

    Including advanced calculus techniques for analytic and numerical differentiation and integration of a broad range of functions and combinations of functions, applications including curve sketching, evaluation of areas and volumes, solution of differential equations including numerical solutions (Euler's method), kinematics – rectilinear motion, velocity time graphs.

4. Vectors

    Including arithmetic and algebra of vectors, linear dependence and independence of a set of vectors, proof of geometric results using vectors, vector representation of curves in the plane and vector kinematics in one, tow and three dimensions.

5. Mechanics

    Including statics and an introduction to Newtonian mechanics, for both constant and variable acceleration.

Assessment for Specialist Mathematics Units 3 and 4. (Year 12) (Victoria)

Internal, Unit 3 – 14%, Unit 4 – 20%.
External, Examination 1 (no technology, 1 hour) 22%,
External, Examination 2 (an approved graphics calculator, or CAS calculator for the CAS subject, 2 hours, materials may be brought in) 44%

Teachers are given specific instructions concerning the nature of tasks that contribute marks towards the Internal assessment. In Unit 3, two *analysis* tasks must be set, and in Unit 4 a single problem-solving or modelling *application* task must be set, along with two equally weighted tests. Detailed examples are given in the Study Design document, which show that students are expected to complete modelling and writing tasks which require more than the application of standard procedures and formulas.

Further Mathematics Units 3 and 4 (Year 12) (Victoria)

This subject, which has Data Analysis as its focus, may be taken on its own in Year 12 or in conjunction with Mathematical Methods 3 and 4 or Mathematical Methods (CAS) 3 and 4. Unit 3 consists of a Data Analysis area of study which incorporates a statistical application task, and one module selected from six. Unit 4 consists of two more modules selected from the six.
The modules are Number Patterns, Geometry and trigonometry, Graphs and relations, Business-related mathematics, Networks and decision mathematics and Matrices.

Brief description of content – for further detail see VCAA (2005)

Data analysis (Core)

Displaying, summarizing and describing univariate data

- Types of data, review of methods for displaying data, dot plots, stemplots, barcharts, histograms, symmetry and skewness of plots, centre and spread, outliers.
- Review of summary sample statistics: measures of spread and centre
- Modelling bell shaped distributions by the normal distribution, the 68-95-99.7% rule, applications, z scores.
- Random numbers and their use to draw simple random samples from a population; display and appropriately summarise and describe these samples.

Displaying, summarizing and describing relationships in bivariate data

- Identification of dependent (response) and independent (explanatory) variable
- Back to back stem plots, parallel boxplots and their applications
- The relationship between a numerical variable and a categorical variable
- Tables and/or segmented barcharts and their uses
- Scatterplots and their uses
- Approximation of Pearson's product-moment correlation coefficient, r , from a scatterplot and calculation with appropriate technology; use and interpretation of r
- Correlation and causation
- Calculation of r^2 and interpretation in terms of explained variation

Introduction to regression

- Independent and dependent variables; fitting lines of fit by eye, by the three median line, and the least squares methods; interpretation of slope and intercepts; making predictions; residual analysis.
- Using technology to find the least squares regression line.
- Transformation of some forms of non-linear data to linearity.

Displaying, summarizing and describing time series data

- Qualitative analysis, recognition of trend, seasonal, cyclic and random patterns
- Seasonal adjustments
- Median smoothing and smoothing using a moving average
- Trend lines

Modules (three to be studied)

Module 1 Number Patterns

- Arithmetic and geometric sequences, first order linear difference equations, Fibonacci and related sequences, solution of related equations numerically or graphically.

Module 2 Geometry and trigonometry

- Pythagoras' theorem in two and three dimensions, applications of similarity including scale factors for area and volume, surface area and volume of regular and composite solids.
- Solution of right angled triangles by trig ratios, solution of other triangles by sine and cosine rules, areas of non-right angled triangles.
- Applications including location and bearings, angles of elevation and depression, contour maps use to find slope between two points, calculation of unknown angles and distances.

Module 3 Graphs and relations

Construction and interpretation of straight line graphs and step graphs in practical contexts
Graphical and algebraic solution of simultaneous linear equations in two unknowns, applications including break-even points
Non-linear graphs and interpretations of significant points such as turning points in practical applications; constructing such graphs from tables, interpolation and extrapolation
Graphical representation of relations in the form of $y = kx^n$ for n = -2, -1, 1, 2, 3 , obtaining a linear graph by plotting y against x^n and determining the constant of proportionality, practical applications
Linear programming, including modelling in applications such as blending and manufacturing problems.

Module 4 Business-related mathematics
Financial transactions and asset value, including applications such as discounts, capital gains, stamp duty, GST
Inflation (graphical and tabular representation), depreciation by various methods.
Loans and investments including simple and compound interest, annuities, reducing balance loans, effective interest rates. Effect of varying the repayment amount etc on loans.

Module 5 Networks and decision mathematics
Undirected graphs and networks including planar graphs, Euler's formula, Eulerian and Hamiltonian paths and their applications, trees and minimal spanning trees, Prim's algorithm, practical applications
Directed graphs and networks including applications to dominance and reachability; Critical path analysis, network flow, assignment problems, optimal allocation including the Hungarian algorithm.

Module 6 Matrices
Matrix representation of data in a variety of contexts, applications of matrix arithmetic to solving practical problems including simultaneous linear equations. Transition matrices up to 4 by 4 in size. Informal consideration of steady state.

Assessment for Further Mathematics Units 3 and 4 (Year 12) (Victoria)

Internal, Unit 3 – 20%, Unit 4 – 14%.
External, Examination 1 (Multiple choice questions on the core and options; an approved graphics calculator and one bound reference, and notes may be taken into the exam. 1.5 hours) 33%,
External, Examination 2 (Extended answer questions; an approved graphics calculator and one bound reference, and notes may be taken into the exam; 1.5 hours) 33%

Teachers are given specific instructions concerning the nature of tasks that contribute marks towards the Internal assessment. In Unit 3, a data analysis *application task* using contexts for investigation from a suitable data set selected by the teacher must be set, along with an *analysis task* for the module studied in that unit. In Unit 4, two *analysis tasks* must be set.

Detailed examples are given in the Study Design document, which show that students are expected to complete modelling and writing tasks which require more than the application of standard procedures and formulas.

COMPARISON: VICTORIA AND NEW SOUTH WALES

Provision for all students

An obvious contrast is the highly flexible nature of the Victorian arrangement of subjects. Students can study one or two mathematics subjects in each of the Years 11 and 12, and with some restrictions they can proceed to the Unit 3 and 4 study of a different mathematics subject from the one studied for Units 1 and 2. The Study Design (VCAA 2005) lists sixteen examples of such combinations. More flexibility is possible because of the choices available within General Mathematics (Units 1 and 2) and Further Mathematics (Units 3 and 4).

In Victoria, high achievers in mathematics are catered for with the possibility of Extension Studies organized directly with Universities. An advancement strategy makes it possible for selected students to begin their units 1 and 2 mathematics studies in Year 10, their units 3 and 4 in Year 11, then the Extension Studies in Year 12. Other high achievers may opt to take the usual route with school mathematics and do the Extension Studies alongside their Year 12 mathematics.

Advantages of the Victorian arrangement for these students include early contact with university mathematics and mathematicians, at the expense of adjustments to school routine. Extension studies classes are often held at school centres after school for one afternoon a week, and some school teachers are involved in the teaching. Monash University and the University of Melbourne are the main providers, and the latter has around 400 students in its Extension Studies program, about half of them in Mathematics. Additional fees may be as high as $450 per semester. In NSW the Extension 2 (commonly known as 4 Unit) course provides for high achievers within the school system. The NSW arrangement has the advantages of being managed within the school, a long and consistent history of materials available for study, and an additional wider benefit in terms of professional learning for many teachers.

Considering the small numbers involved in the Victorian Extension Studies program, it makes more sense to compare the provision for high achievers by considering the subject Specialist Mathematics alongside NSW Extension 2. The content of Specialist Mathematics (Victoria) is comparable to Mathematics Extension 2 and parts of Extension 1. Obvious differences include Vectors (Victoria only), and Circle Geometry (NSW only).

For students not intending to need mathematics for tertiary study, the possibility in Victoria of making selections from Foundation Mathematics, General Mathematics, and Further Mathematics (within the constraints of school timetabling), allows a range of starting and exiting points for students finishing Year 10 with a range of achievement in mathematics. The possibility of a Year 11 mathematics subject for students with no intention of taking mathematics in Year 12 is provided for, in a subject that is designed to develop mathematical skills within everyday life and work contexts. NSW has lost this capability with the demise of Mathematical Practice, however it appears that many students are using the preliminary year of General Mathematics (NSW) for that purpose.

Content – Calculus based courses (NSW Mathematics and the Extension Courses)

Comparing Victoria's Mathematical Methods (Units 1, 2, 3, 4) with NSW Mathematics (2 Unit) and Mathematics (Extension 1) we see the same emphasis on algebra, functions, and calculus. Where Victoria provides a deeper and broader treatment of probability, NSW instead has plane geometry. The difference in the ages of the syllabus documents is evident in the choice of examples and applications.

Content – NSW General Mathematics and Victorian equivalents

These could be called the "Non-calculus" subjects. Direct comparison is not easy because of the selections possible within the Victorian subjects. Instead, Table 2 lists topics from the subjects General Mathematics (Units 1 and 2,Year 11) followed by Further Mathematics (Units 3 and 4, Year 12, Victoria), and General Mathematics (NSW) that are present in one subject but not the other. It should be borne in mind that in Victoria, not all students study all parts of General Mathematics and Further Mathematics.

Table 2. Comparison of General Mathematics (Units 1 and 2,Year 11) followed by Further Mathematics (Units 3 and 4, Year 12, Victoria), and General Mathematics (NSW) ***by listing content present in one but not the other***.

Content with more emphasis in General Mathematics (Victoria) followed by Further Mathematics, depending on the school's choice of topics	**Content with more emphasis in General Mathematics (NSW)**
Complex numbers (basic introduction) Matrices Sequences including recursion Regression Simulation Logic and Boolean algebra Polar co-ordinates Modelling using the log function Focus-directrix properties of certain graphs Networks Linear programming Kinematics Vectors Geometry in art, tessellations	Wider variety of topics within personal financial mathematics (e.g. shares, taxation, budgeting) Future value, present value calculations Wider applications of area and volume Spherical geometry leading to location on the globe and time differences Financial expectation in games of chance

Making sense of these differences depends upon a familiarity with the purposes of these subjects. Both states aim to provide an alternative mathematical experience, without the demands and abstractions of calculus, for students not intending to study mathematics at tertiary level. These students are not the same as those who need further development in basic numeracy, who are provided for in Victoria by the (Year 11 only) subject Foundation Mathematics, and used to be provided for in NSW by Mathematics in Practice. In Victoria, General Mathematics *taken with* Mathematical Methods (Units 1 and 2) can be used as preparation for Mathematical Methods (Units 3 and 4, with or without CAS), and possibly also Specialist Mathematics. This explains the presence of Kinematics and Vectors in the list above.

Assessment

The assessment structure for each of the Victorian subjects has been described above. Comparisons with NSW are shown in Table 3.

Table 3. Features of assessment in Victoria and in NSW.

	Victoria	NSW
Outcome statements used to structure assessment tasks.	Outcome 1 "On completion of this unit students should be able to define and explain key concepts in relation to the topics from the selected areas of study, and apply a range of related mathematical routines and procedures" Outcome 2 "On completion of this unit students should be able to apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics in at least three areas of study" Outcome 3, "On completion of this unit the student should be able to use technology to produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches in at least three areas of study"	Outcomes are incorporated into the syllabus document for General Mathematics. Course Outcomes (Board of Studies NSW 2000) were written for the calculus-based subjects to distinguish between Preliminary and HSC outcomes. This was part of the change in the New HSC to Standards-referencing procedures. See Appendix 2.
Variety of tasks for internal assessment	As outlined in the subject descriptions, timed pen-and-paper tests are to be balanced by longer investigative and problem solving tasks.	Tasks to be a balance of Component A tasks (largely content based routine procedures) and Component B tasks (more about problem solving, reasoning, unfamiliar contexts), in the weightings Mathematics 80:20, Ext 1 70:30, Ext 2 60:40.
Features of external assessment	Technology free exam as well as technology (and materials) exam.	Hierarchical nature of subjects requires two exams for students in Extension 1 and Extension 2. Scientific calculators expected in all exams, Graphing calculators optional for General Mathematics.

Technology

Graphing calculators are widely used in all Victorian mathematics subjects. In NSW external HSC examinations graphing calculators have been optional for the subject General Mathematics in recent years. The use of scientific calculators is assumed in NSW HSC examinations for other subjects. It is generally agreed that the rules about access to technology in high stakes external assessment have a significant effect on the use of such technology in teaching and learning.

The move towards CAS enabled technology in Victoria for the Calculus-based subjects (described above in the description of Mathematics Methods Year 12, Units 3 and 4), has no equivalent in NSW. In Victoria, examiners were initially directed to assume that students have access to an approved CAS for Examination 2 in Mathematics Methods and Specialist Mathematics examinations for 2009. (VCAA, 2006b). In a later Bulletin, this date was changed to 2010 to allow schools more time to prepare for the changes. (VCAA 2006c).

QUEENSLAND

Context for the study of mathematics in the senior years

The following information is from the website of the Queensland Studies Authority (QSA): http://www.qsa.qld.edu.au/qsa/index.html

> The Queensland Studies Authority is a statutory body responsible for the provision of a range of services and materials relating to syllabuses, testing, assessment, moderation, certification, accreditation, vocational education, tertiary entrance and research.
>
> The Queensland Studies Authority (QSA) was established on 1 July 2002, merging the Queensland School Curriculum Council (QSCC), Queensland Board of Senior Secondary School Studies (BSSSS) and the Tertiary Entrance Procedures Authority (TEPA).

Students receive a Senior Certificate after 12 years of schooling. With some exceptions there are no external examinations: teachers assess their own students using criteria-based assessment and there is a system of moderation conducted by panels of expert teachers who look at samples of student work and compare them across schools. Schools also need to prepare work programs for each subject, outlining a teaching and assessment program. These must be approved by the QSA.

The QSA also administers the Queensland Core Skills Test - a common statewide test for Queensland Year 12 students that is grounded in the Queensland senior curriculum. It has a section on mathematical skills and reasoning that requires no more than Year 10 content. Results on this are used to moderate school assessments to produce Overall Positions and Field Positions that are the basis of Tertiary Entrance. The Queensland Core Skills Test is needed to enable comparison across differing school populations and subject choices.

The QSA is currently conducting a large scale review of syllabuses as part of *Education and Training Reforms for the Future* (ETRF) which introduces a new *Queensland Certificate of Education* (QCE). Students in Year 10 this year (2006) will be the first to receive the QCE in 2008. In the document “Review of Syllabuses for the Senior Phase of Learning Report: Initial Targeted Consultations August – September 2005” (QCA 2006b), the QCE is described as being

> …designed to recognize the diversity of young people and their varied post-school aspirations. While the QCE is clearly an achievement-based qualification that sets higher standards by requiring students to achieve a significant amount of learning to a set standard and achieve a set requirement for literacy and numeracy, it also opens up greater flexibility. The new qualification allows:
> · more learning options
> · more places where learning can occur
> · more time to complete the senior phase of learning.

> The QCE reflects how learning is currently organised. It is therefore timely for the QSA to review its suite of syllabuses for the senior phase of learning and consider whether the current syllabuses:
> · meet the needs of young people and are relevant to their futures in a knowledge economy
> · provide options for flexible delivery in terms of time and location
> · meet the expectations of the community, employers and further education and training institutions and providers.

The QCE will allow students more than two years to accumulate the required 20 credits in a “learning account”. Credits may be gained through a variety of courses including learning in the workplace and the community, such as VET Certificates, apprenticeships, university subjects, and structured workplace projects. There will be a literacy and a numeracy requirement. Legislation has been introduced, requiring all young people in Queensland to complete Year 10 and then participate in further education and/or training for two years. One of the aims of the QCE is to provide a qualification that indicates a significant amount of learning has been achieved for the majority of the Year 12 cohort who do not proceed to university.

Rationale and Content of subjects

Functional Mathematics (QCA 2006c)

This subject will be offered for the first time in 2007 and replaces part of the *Literacy and Numeracy* Subject Area Specification (SAS). It is designed in the main for intellectually impaired students and others who have had difficulty achieving Year 10 numeracy levels. It will not contribute to attaining a QCE. It has an expected enrolment of about 1000 and compares to the NSW subject *Mathematics Life Skills.* There are five topics, which align with the topics in Pre-Vocational Mathematics to allow for articulation between the two subjects.

Pre-Vocational Mathematics (QCA 2004)

This subject has recently been introduced to replace the subject *Trade and Business Mathematics*. Enrolments will be around 4000 to 5000. It will be included in the QCE but will not count for university entrance. Examples of possible courses of study given in the Subject Area Specification indicate that a thematic approach and/or a project based approach and/or an approach based on small units would be suitable for this subject.
The subject does not compare with the NSW subject General Mathematics but appears to be similar to the old NSW Mathematics in Practice syllabus.

The Topic Areas are

1. Mathematics for interpreting society: number
   Whole numbers, fractions, decimals and percentages, ratios, proportions, rates
2. Mathematics for interpreting society: data
   Collect, access and organize data, identify features: statistics content as far as range, bias, mean, median of ungrouped data. Display, present, represent and interpret data: graphs to include pictographs, line, column, bar graphs
3. Mathematics for personal organization: location and time
   Read and use maps to locate points and places: units of measurement, ruler, compass points, maps. Interpret time, clocks and timetables: including 24 hour time, time zones, relationship to longitude.

4. Mathematics for practical purposes: measurement

The metric system and measuring equipment, Two dimensional shapes and regular solids, Representing everyday two-dimensional and regular solids. Includes perimeter, area, volume, nets, scale factors. Applications to be in practical situations.

5. Mathematics for personal organization: finance.

Obtaining and income, including wages, salary, social security payments, pensions, taxation; Spending money, including credit, GST, budgeting; Investing and borrowing money including calculating simple interest using a given rule and compound interest using on-line calculator or tables, shares, real estate.

Mathematics A (Queensland Board of Senior Secondary School Studies, 2000a)

This subject is intended for students who wish to include a mathematics subject in their final two years of high school, but do not intend to study a mathematics or mathematics related course at university. In the rationale for the course is the statement “Mathematics pervades so many aspects of daily life that a sound knowledge is essential for informed citizenship.” (QBSSS, 2000a, p. 1)
In 2005 the enrolment for Year 12 Mathematics A was 21 565, comprising 12 004 females and 9561 males.

Content (Mathematics A, Queensland)
All core topics and one elective topic are to be studied.

Core topics

Managing Money 1

Earnings, taxation including GST, budgeting, spending including discount, profit and loss, foreign exchange.

Managing Money 2

Simple and compound interest for varying compounding periods, inflation, appreciation and depreciation, present value of a lump sum payment, consumer credit, housing loans, investments, real estate, stock market.

Elements of applied geometry

Sine, Cosine, Tangent ratios and applications, Pythagoras’ theorem, area and volume in life-related situations, latitude, longitude and measurement of time.

Linking two and three dimensions

Interpretation and drawing of scale drawings and plans, the geometry of bracing for rigidity, practical tests for squareness, plumbness and levels, estimation of costs in construction

Maps and compasses – either Navigation *or* Land measurement

Navigation: Compass bearings and reverse bearings, magnetic variation, nautical miles and knots, use of maps, charts, dividers and parallel rulers or equivalent, methods of fixing position – bearing fix, dead reckoning, running fix, application to orienteering, air or coastal navigation.

Land measurement: Compass bearings and reverse bearings, perimeters and areas, units, drawing and interpreting site plans, position fix using directions, and vertical and horizontal measurements in relation to a datum, calculation of grades (gradient of the land) interpretation of contour, topographical, detail, cadastral, and land-use maps.

Data collection and presentation

Types of data and variables (continuous/discrete, categorical, ordinal, count…)
Practical aspects of collecting data, data preparation and entry
Data display including pie charts, barcharts, histograms, stemplots, boxplots, scatterplots; compare two or more groups with back-to-back stemplots and adjacent boxplots
Use and misuse of sample means and medians as descriptors of central tendency
Use and misuse of standard deviations and interquartile distances as descriptors of spread
Description of key features of data from graphs and tables

Exploring and understanding data

What a sample represents, sample statistics and estimates of parameters, relative frequencies to estimate probabilities, probability as a measure of chance, contingency (two-way) tables, misuse of probabilities, areas in histograms used to estimate probabilities, modelling data with lines and curves, interpolation, extrapolation.

Electives (one to be studied)

Operations research – linear programming

A working knowledge of the methodology of linear programming, application to solving life-related problems with integer solutions.

Operations research – networks and queueing

Terminology including node, path, branch and tree. Shortest path algorithm, minimum spanning tree algorithm, critical path analysis including slack time, simple and multiple server queues, effects of random arrival and service times.

Introduction to models for data

Probability distributions and expected values for a discrete variable, uniform discrete distribution, random numbers, binomial situations and probabilities, normal distribution model and applications, complements and unions in probability rules, odds as an application of probability.

School elective

Must be appropriate to the course and not a combination of parts of other topics.

Assessment (Mathematics A, Queensland)

Schools make many of their own decisions about assessment.

Extended modelling and problem-solving tasks could be written up as reports of projects or field activities or experiments or investigations. A report could be written as a scientific report, a proposal to a company or organization, a feasibility study.

Assessment tasks other than tests must be included at least twice each year.

Students keep a verification folio of work, which must contain a minimum of 4 and a maximum of 10 pieces of summative work in a range of types of assessment. The verification folios are used to help panels of experienced teachers evaluate the school's judgments about student achievement.

Technology (Mathematics A, Queensland)

It is expected that students use a range of technological tools for learning and assessment, including graphing calculators and computers.

Mathematics B (Queensland Board of Senior Secondary School Studies, 2000b)

This subject is intended for students who intend to study a course at university that requires some mathematics in areas such as health, environmental science, economics and management. It also includes some topics needed for personal participation in society such as applied statistics and applications of algebra and functions to finance. Students intending to study mathematics or mathematics related courses at university would study Mathematics B and Mathematics C. The course is fully prescribed (no option topics).
In 2005 the enrolment for Year 12 Mathematics B was 16 534, comprising 7914 females and 8620 males. This includes the 1057 females and 2260 males who also studied Mathematics C.

Content (Mathematics B, Queensland)

Core topics

Introduction to functions

Focus on the three representations of functions (algebraic, graphical, numerical) and the interrelation of these in modelling situations, from life-related to abstract. Technology use expected. Cncepts of domain and range, representations of functions, properties, practical applications including variation, simple interest as an arithmetic progression, transformations and their graphs, $f(x)$, $f(x)+a$, $af(x)$, $f(x+a)$, $f(ax)$. Absolute value functions, reciprocal functions, polynomials up to fourth degree, inverse and composition of functions, solving simultaneous equations.

Rates of change

A non-rigorous introduction to derivatives. Includes product and chain rule but not the quotient rule. Interpretations and practical applications.

Periodic functions and applications

Basic triangle trigonometry and sine, cosine, tangent of any angle in degrees and radians. Graphs as far as $y = A\sin(Bx + C) + D$ with *A, B, C, D* constants. Applications, trig identities only as far as Pythagorean result, solution of simple trig equations, derivatives of sine and cosine functions and applications of these.

Exponential and log functions and applications

Index laws, log laws and definitions, develop algebraic models for data sets by using logs and/or exponents, applications of geometric progressions to compound interest, and past, present and future values, also to annuities and amortising a loan.

Introduction to integration

Emphasis on applications rather than developing a large repertoire of techniques. Trapezoidal rule.

Applied statistical analysis

Types of data and variables, graphical and tabular displays, summary statistics, describe key features of data, relative frequencies to estimate probabilities, probability distribution and expected value of a discrete variable, binomial probabilities, introduction to hypothesis testing (*not including critical regions, levels, confidence intervals, type 1 and type 2 errors),* concept of probability distribution for a continuous random variable, normal model and standard normal tables.

Assessment and technology (Mathematics B, Queensland)

Similar to Mathematics A. Note the inclusion of extended modelling and problem-solving tasks along with tests. Technology is to be incorporated in assessment. “Although student ownership of graphing calculators is not a requirement, *student access* to appropriate technology *is necessary* to enable students to develop the full range of skills required for successful problem solving during their course of study.” (Queensland Board of Senior Secondary School Studies, 2000b, p.10).

Mathematics C (Queensland Board of Senior Secondary School Studies, 2000c)
This subject is intended as an additional subject to Mathematics B for students who intend to study a courses at university that require a rigorous mathematical preparation, for example the physical sciences and engineering. Because there is some choice in option topics, the subject would be a good preparation for other university courses such as health, management, accounting, business, and life sciences.
In 2005 the enrolment for Year 12 Mathematics C was 3317, comprising 1057 females and 2260 males.

Content (Mathematics C, Queensland)

Core topics

Introduction to groups
    Structures and properties of groups as a basis for identifying common features in systems such as real and complex numbers, matrices and vectors. Closure, associativity, identity, inverse, definition of a group. Suggested applications and contexts include Cayley tables, modular arithmetic and geometry.

Real and complex number systems
    Representations of complex numbers, operations, de Moivre's theorem, proof by mathematical induction.

Matrices and applications
    Definitions, properties and operations, determinant of a matrix, applications in life-related and purely mathematical situations. Use of calculator with matrix operations suggested. Suggested applications include coding, Newtonian mechanics, Leontief matrices in economics, Leslie matrices in ecology, game strategies.

Vectors and applications
    Definitions, properties and operations. Scalar and vector products. Applications to life-related and purely mathematical situations including geometry in three dimensions.

Calculus (builds on Mathematics B calculus)
    Further integration, development and use of Simpson's Rule; simple, linear, first order differential equations with constant coefficients- their solution and applications. (e.g. falling bodies with resisted motion, exponential decay, Newton's law of cooling). Use derivatives to estimate small changes.

Structures and patterns
    Sequences and series including arithmetic and geometric, permutations and combinations, Pascal's Triangle, Fibonacci sequence, method of finite differences, proof by induction. Applications to life-related and purely mathematical situations.

Option topics (two to be studied)

Linear programming (Similar to the option topic for Mathematics A, with some extensions)
    A working knowledge of the methodology of linear programming, application to solving life-related problems with integer solutions. Relationship between algebraic and geometric aspects of problem with constraints in two and three dimensions, use of simplex algorithm in certain cases.

Conics
    Concept of locus, directrix and focal point. Eccentricity. Cartesian, parametric, polar and where appropriate the complex number forms of circles, ellipses, hyperbolas, and parabolas. Applications, including technological and architectural applications.

Dynamics
    Derivatives and integrals of vectors, Newton's laws of motion in vector form applied to objects of constant mass, applications to straight line motion, vertical motion under gravity with and without air resistance, projectile motion without air resistance, simple harmonic motion and circular motion with uniform angular velocity.

Introduction to number theory

Primes, composites, Fundamental Theorem of Arithmetic, divisors, Euclidean algorithm, LCM and GCD, modular arithmetic, congruence including simple simultaneous congruence, simple Diophantine equations. Suggested applications include investigations of ciphers, generation of random numbers, Pythagorean triples, finding information about Goldbach's Conjecture, Fermat's Last Theorem, ancient Egyptian calculations.

Introductory modelling with probability

Includes events and set operations, logical circuits and truth tables, probabilities of events, independence, conditional probability, simple Markov chain models, exponential model. Chaotic behaviour of a simple non-linear system such as the logistic map $x_{n+1} = cx_n(1 - x_n)$.

Advanced periodic and exponential functions

Secant, cosecant, cotangent; expansions e.g. sin(x+y), sin(x-y); shapes of graphs such as $y = e^{ax} \sin bx$, applications of such functions, logistic curve $y = \dfrac{A}{1 + Ce^{-kt}}$. Emphasis on modelling real life situations including radio waves and tuners, superposition of waves, electric circuits, oscillations of springs etc.

School option(s)

Can be developed by the school, must be different from Mathematics B and C material that the student would meet but of similar level of challenge, may be enhanced by use of computer packages but not be the study of computer programming language.

Assessment and technology (Mathematics C, Queensland)

Similar to Mathematics A and B. Note the inclusion of extended modelling and problem-solving tasks along with tests. Technology is to be incorporated in learning experiences and assessment. "The minimum level of higher technology appropriate for the teaching of this course is a graphing calculator." (Queensland Board of Senior Secondary School Studies, 2000c, p. 4).

COMPARISON: QUEENSLAND AND NEW SOUTH WALES

Provision for all students

According to the data in Barrington (2006), Queensland has a very high participation rate in mathematics for Year 12, especially since those figures do not include several thousands of enrolments in more elementary mathematics subjects than are included in the table. The extent to which the conditions of school based assessment rather than external examinations contributes to this high involvement in mathematics can only be conjectured. Other factors are likely to include: the expectations of parents and employers, the need to revise Year 10 mathematics to prepare for the Queensland Core Skills Test which contributes to university entrance, and the variety of mathematics subjects available.

The alignment of the topics in Functional Mathematics and Pre-vocational Mathematics is a strength, and will allow for progression into more demanding work for those who attain numeracy later than others in their age cohort. Some students, from immigrant families, who need to attain English language proficiency before advancing in their mathematics study may be in this category as well.

Content – Calculus based courses (NSW Mathematics and the Extension Courses)

Mathematics B and C (Qld) do not align easily with Mathematics (2 unit), and the Extensions 1 and 2 in NSW. As with Victoria, an obvious difference is the presence of applied statistical analysis in Queensland Mathematics B in the place of Euclidean Geometry in NSW. In terms of calculus topics that students often find difficult, Mathematics B (Qld) is less demanding than Mathematics (2 unit) in NSW, but the breadth of applications that are described in the Suggested Learning Experiences in the syllabus document is wider than in NSW. The main comparison between NSW courses and Mathematics C is the way that the mathematical content in the latter is built upon a foundation of knowledge of groups: this unites the treatment of real and complex numbers, matrices and vectors. Several of the core topics in Mathematics C are not done at all in NSW, which has a heavier emphasis on calculus. The advantage for Queensland students is that they are introduced to a wider and more modern range of mathematical topics. The disadvantage for Queensland students is that their preparation for university level calculus depends on the choice of option topics provided at their school.

Content – NSW General Mathematics and Queensland Mathematics A

The main difference here is the presence of Algebraic Modelling in the NSW General Mathematics Course, and the choice of two operations research electives in the Queensland Mathematics A electives.

Assessment

The internal school-based assessment of all Year 12 subjects in Queensland (and the ACT) provides a completely different assessment environment from the HSC examination system. Strengths include the provision for project work and extended investigations and problem solving tasks in unfamiliar contexts. These are required as part of the NSW internal assessment as Component B, a minor part of the final assessment. Weaknesses include the issue of guaranteeing authenticity and ownership of such work.

Technology

Queensland mathematics syllabuses include statements about expected levels of access to appropriate technology. This varies from not requiring continuous access (Mathematics A) to stating that a minimum requirement is access to a graphing calculator (Mathematics C). This issue is linked to the nature of assessment tasks. Where the teaching and learning of a topic is enhanced by the opportunities for exploration and for investigating real world data that computers and graphing calculators allow, student in Queensland are expected to use this technology.

OVERALL COMPARISON WITH OTHER AUSTRALIAN STATES

Provision for all students

For subjects taken by students aiming for tertiary studies, comparisons with other Australian states are made easier by the work of Frank Barrington and Peter Brown (2005). They refer to these subjects as Advanced and Intermediate. The content of NSW calculus based courses is comparable to the Advanced and Intermediate subjects in most other states. There is considerable variety in what is offered in the different subjects and also in the choice of electives available in the subjects of some other states. Using Table 1 of Barrington and Brown (2005, p. 12) here are the major points of difference:

- No other states have the hierarchical arrangement of calculus subjects that NSW has, with the position of Extension 1 between Mathematics 2 Unit and Mathematics Extension 2 as "half" a subject. No other states have such an advanced course on the "pure" side of mathematics as Extension 2 (4 Unit) in terms of rigour and attention to proof. These arrangements have served NSW well and are highly regarded by respected teachers (for example, see Pender, 2006), and academics (for example, Mack, 2006 p. 2, states that "…what we are able to do at present is reasonably satisfactory for a reasonable number of Stage 6 students.") After a detailed consideration of both examination papers and syllabus documents for all Australian senior mathematics courses, Barrington and Brown (2005, p. 17) conclude that the NSW subject Mathematics Extension 2 is the most "ambitious" of all similar subjects offered in Australia.
- In terms of content, Extension 1 in NSW appears to be almost comparable in its coverage of topics with most other subjects designed for students with the same tertiary destinations. Noting that Extension 1 was originally designed as a "half subject" in terms of teaching time, it is not surprising that there are some topics that these other states include that are not in Extension 1. These include Complex Numbers, Vectors, and some topics from Mechanics. Some other states, but not all, include Matrices and some work on Logic / Proof / Axiomatics in comparable subjects.
- For the NSW subject Mathematics (2 Unit), coverage of topics is again comparable to other states with a few exceptions. These are mainly the *inclusion* of coordinate geometry of the parabola and plane geometry in NSW, and the *lack of* topics on statistics and data analysis. Some other states but not all include work on matrices at this level. Minor topics missing in NSW but present in most other states at this level are binomial theorem, related rates of change problems, and integration by simple substitution, although there is room for some discussion about some of the classifications of sub-topics used by Barrington and Brown (2005).

For subjects taken by students not aiming to progress to tertiary mathematics or not aiming for university studies, the picture is a complex one because of the variety of subjects and course structures on offer, many with options and choices available. Barrington and Brown (2005) do not attempt this comparison of subjects they refer to as

Elementary but it can be seen from the previous sections in this report comparing NSW with Victoria and Queensland that

- the subject General Mathematics is comparable to similar subjects in those states, with less work on data analysis than the Victorian subject Further Mathematics and more work on algebraic modelling than Mathematics A in Queensland.
- Flexibility and a wider coverage of option topics is available at this level in both of the other states compared.

The group of students not aiming to progress to tertiary mathematics or not aiming for university studies also includes students who require more time to achieve the numeracy levels that others achieve by the end of Year 10. Flexibility of offerings appears to be the key to successfully providing for this group so that progression to more demanding subjects can be made. Both Victoria (through Foundation Mathematics and General Mathematics in Year 11) and Queensland (through the new subject Pre-Vocational Mathematics which replaces Trade and Business Mathematics) offer mathematics subjects designed for students not intending to proceed to tertiary studies. Other Australian states provide in different ways for this group of students. In South Australia this is offered through the subject Mathematical Applications, and in Tasmania through the subjects Maths at Work and Maths after College. (Tasmanian Qualifications Authority, 2006). The picture in Western Australia is more complicated as they are in the process of reviewing and rewriting their mathematics subjects as part of a new Western Australian Certificate of Education which will be introduced between 2005 and 2009. There is a subject called Vocational Mathematics (The Curriculum Council of Western Australia 2006) which is designed for one year only and is expected to be integrated with other vocational studies.

Assessment and technology

It is helpful to consider these together when making comparisons with the NSW senior secondary context where the HSC examinations have a major influence on students' experiences of mathematics at school. The issue of allowable access to technology for high stakes external assessment is highly significant for all stakeholders.

In Queensland and the ACT, students are expected to use technology appropriate to the mathematical task being undertaken. All assessment concerning mathematics achievement is conducted internally, with moderation programs in place including local meetings of experienced teachers to compare samples of work across several schools. The Queensland Core Skills Test and the ACT Scaling Test are external tests used to scale internal assessments for the purpose of determining university entrance rankings. (ACT Board of Senior Secondary Studies 2006).

In the remaining states that conduct external exams the use of graphing calculators is widely assumed and in some exams in Victoria, South Australia and Western Australia, students are also permitted to bring prepared notes into the exam room. (Barrington and Brown, 2005).

An interesting exception to this pattern is the decision in Victoria to base the external assessment for the calculus based subjects *Mathematics Methods Units 3 & 4, Mathematics Methods Units 3 & 4 (CAS),* and *Specialist Mathematics* on two exams, one to be a one hour technology free paper as described in the section on Victoria above.

In Victoria, the gradual introduction of CAS technology via a pilot study and parallel subjects provides an opportunity for other states to evaluate the impact these changes are having on the teaching and learning of mathematics.

THE INTERNATIONAL BACCALAUREATE

Context for the study of mathematics in the senior high school years

Australia is one of 124 countries worldwide in which the International Baccalaureate (IB) programme of educational studies is conducted. According to the International Baccalaureate Organization (IBO):

> The International Baccalaureate Organization was founded in Geneva, Switzerland in 1968 as a non-profit educational foundation. Its original purpose was to facilitate the international mobility of students preparing for university by providing schools with a curriculum and diploma recognized by universities around the world. Since then its mission has expanded, and it now seeks to make an IB education available to students of all ages. (IBO, 2006a).

Worldwide there are 1890 schools and 486 000 students participating in the three programmes on offer – the Primary Years Programme (PYP), the Middle Years Program (MYP) and the Diploma Years Programme (DP). The latter is a two year programme for students aged 16-19. According to the IBO, (www.ibo.org/facts/schoolstats/growth.cfm), the increase in the Diploma programme worldwide over the years 1995-2005 was at a compound rate of 10.25%. [1]

Table 4 is from the IBO website (IBO, 2006b) and lists the top ten countries in terms of number of “World Schools”, i.e. schools offering IB Programmes, possibly alongside national curricula.

Table 4. Top ten countries by number of “World Schools”. (IBO, 2006b)

Country	IB World Schools	Programmes		
		Primary Years	Middle Years	Diploma Programme
USA	683	72	168	521
Canada	225	21	118	102
Australia	93	25	44	43
UK	89	4	6	87
Mexico	58	26	13	39
Argentina	43	6	6	42
Spain	36	2	6	35
India	33	5	4	31
Sweden	32	5	7	28
China	31	6	16	25

[1] The interactive capability of this website is a good example of the way mathematics is being built in to everyday tools.

Format of the IB Diploma Programme (IBDP).

This programme is designed to meet the needs of university bound students. It is described in its subject booklets as

> ... a rigorous pre-university course of studies, leading to examinations, that meets the needs of highly-motivated secondary school students between the ages of 16 and 19 years. Designed as a comprehensive two-year curriculum that allows its graduates to fulfill requirements of various national education systems, the DP model is based on the pattern of no single country but incorporates the best elements of many. The DP is available in English, French and Spanish.

There are six subject groups (First language, Second language, Experimental sciences, The Arts, Mathematics and computer sciences, and Individuals and societies.) Students take one subject from each of these groups. At least three and not more than four are taken at higher level (HL), 240 teaching hours, the others at standard level (SL), 150 hours.

There are three other requirements (the core). First, an interdisciplinary Theory of Knowledge course "designed to develop a coherent approach to learning that transcends and unifies the academic area and encourages appreciation of other cultural perspectives" (IBO, 2004a, p. 2) must be studied. Second, students complete an extended essay of around 4000 words requiring independent research and advanced writing skills. Third, there is a requirement that students participate in a range of activities that are (i) creative, (ii) physical and (iii) service oriented.

Provision for all students

Clearly the IBO Diploma Programme is designed for the minority of students in their age cohort who aim for university studies. In the long term this may change as additional programmes such as a vocational diploma are mentioned under Programme Development in the IB Strategic Plan (IBO, 2004c).

Brief descriptions and content of the four mathematics courses in the IB Diploma Programme

Mathematical Studies SL

This course is designed for students who are not likely to continue with mathematics at a tertiary level. Although the list of topics may appear to be more "mathematical" than NSW educators are used to seeing in a course designed for such students, it should be remembered that IB DP students must study subjects in all six areas, and the mathematics in this course is designed to relate as far as possible to other subjects being studied, as well as to common "real world" contexts. The mathematical topics have been chosen so that an approach based on first principles starting with practical investigations can be followed. Less formal, shared learning techniques are encouraged. This course is not a direct equivalent of NSW General Mathematics, although it may be designed for a similar cohort.

Content: IBDP Mathematical Studies SL (IBO 2004a).

The course consists of eight topics and a project, which is to be "an individual piece of work involving the collection of information or the generation of measurements, and the analysis and evaluation of the information or measurements." (IBO, 2004a, p. 8)

Topic 1 Introduction to the graphic display calculator (GDC)
Topic 2 Number and algebra
Topic 3 Sets, logic and probability
Topic 4 Functions
Topic 5 Geometry and trigonometry
Topic 6 Statistics
Topic 7 Introductory differential calculus
Topic 8 Financial mathematics

Technology and Assessment: IBDP Mathematical Studies SL

Students are expected to have access to a GDC at all times during the course.
External assessment; Paper 1, 40% of final mark, GDC required, 15 short answer questions, 1.5 hours. Paper 2, 40% of final mark, GDC required, 5 extended response questions, 1.5 hours. Access to an Information Booklet containing formulas is required for both exams.
Internal assessment: Project, 20%. Considerable guidance is given to students and teachers about managing the project.

Mathematics SL

This course is designed for students preparing for university studies such as chemistry, economics, psychology and business administration.

Content: IBDP Mathematics SL. (IBO 2006c).

The course consists of seven topics and a Portfolio of two pieces of work, based on different areas of the syllabus, representing mathematical investigation and mathematical modelling.

Topic 1 Algebra
Topic 2 Functions and equations
Topic 3 Circular functions and trigonometry
Topic 4 Matrices
Topic 5 Vectors
Topic 6 Statistics and probability
Topic 7 Calculus

Technology and Assessment: IBDP Mathematics SL

Students are expected to have access to a GDC at all times during the course.
External assessment:
Paper 1, 40% of final mark, no calculators allowed, short and extended response questions, 1.5 hours.
Paper 2, 40% of final mark, GDC required, short and extended response questions, 1.5 hours. Access to an Information Booklet containing formulas is required for both exams.

Internal assessment: Portfolio, 20%. Considerable guidance is given about assigning and completing the tasks for the portfolio.

Mathematics HL

This course is designed for students with a good background in mathematics who intend to study it further at university, either in mathematics courses or in physics, engineering and technology courses.
(In terms of content, this is broadly comparable to NSW Mathematics with Extension 1 and elements of Extension 2. In terms of expectations of the students, some experienced teachers who have taught both IBDP and HSC regard this subject as far more demanding.)

Content: IBDP Mathematics HL (IBO 2006d)

Seven core topics and one optional topic are to be studied. The material in the core includes the Mathematics SL material as a subset. A Portfolio of two pieces of work, based on different areas of the syllabus, representing mathematical investigation and mathematical modelling is to be completed.

Topic 1 Algebra
Topic 2 Functions and equations
Topic 3 Circular functions and trigonometry
Topic 4 Matrices
Topic 5 Vectors
Topic 6 Statistics and probability
Topic 7 Calculus
Options (one to be studied)
Topic 8 Statistics and probability
Topic 9 Sets, relations and groups
Topic 10 Series and differential equations
Topic 11 Discrete mathematics

Technology and Assessment: IBDP Mathematics HL

Students are expected to have access to a GDC at all times during the course.
External assessment:
Paper 1, 30% of final mark, no calculator allowed, short and extended response questions, 2 hours.
Paper 2, 30% of final mark, GDC required, short and extended response questions, 2 hours.
Paper 3, 20% of final mark, GDC required, extended response questions mainly on the options, 1 hour. Students answer questions on the option topic they have studied.
Access to an Information Booklet containing formulas is required for all exams.
Internal assessment: Portfolio, 20% of final mark. Considerable guidance is given about assigning and completing the tasks for the portfolio.

Further Mathematics SL

This course caters for students who are very capable in mathematics and interested in studying it at university, either in its own right or in related areas.
(This course is not directly comparable to NSW courses.)

Content: IBDP Further Mathematics SL (IBO 2004b)
The course consists of all the content of Mathematics HL, and in addition, students study the remaining three option topics from that course.

Technology and Assessment: IBDP Further Mathematics SL
Students are expected to have access to a GDC at all times during the course.
External assessment only:
Paper 1, 35% of final mark, GDC required, short-response questions based on the whole syllabus, 1 hour.
Paper 2, 65% of final mark, GDC required, four to six compulsory extended response questions, 2 hours. The emphasis is on problem solving.
Access to an Information Booklet containing formulas is required for both exams.

Why is the IB popular? What are some reasons for its spectacular growth?

The IBO maintains an area of its website headed “Research” and accessible there are its publications called “Research Notes”. The “Research Notes” for January 2004 (IBO 2004d) contained responses to a report commissioned by the IBO and titled *Perceptions of the International Baccalaureate Diploma Programme: A report of an inquiry carried out at UK universities and institutes of Higher Education* (IBO, 2003). As reported in “Research Notes”, the main findings of the report included:

- Just over half of the higher education institutions contacted felt that the IBDP conferred an advantage on applicants to university courses over A levels.
- The IBDP had not suffered “grade inflation”. (Pass rates had been consistent over time).
- The three core features of the IBDP, and its breadth of study requirements, were well received by universities.
- 47000 out of 50000 courses of study available at UK institutions of higher education specified admission requirements in terms of the DP, indicating the wide acceptance of the programme.

Additional points made by university admissions staff and reported in Research Notes (IBO 2004d) included:

- The simple structure of the DP, and the marking scheme, are easy to understand.
- At the University of Glasgow, DP applicants have scored higher than average results for their degrees.
- DP students seem to have more independent study skills, time management skills, and display more analytical and critical thinking skills. They are also more rounded and thoughtful individuals.

- There are high expectations in the DP, for example that everyone studies mathematics and a second language, but students are capable of rising to expectations.

Commentators also raised the point that the positive features shown by IBDP applicants are also shown by some A level applicants, and there were questions about the comparable influence of the DP itself vs the influences of the schools that the applicants attended.

Closer to home, a report was published in the *International Education Journal* by Paul Paris titled "The International Baccalaureate: A Case Study on why Students Choose to do the IB". (Paris, 2003). One government and one non-government school participated in the study, both offering a choice to students between the IBDP and the SACE (South Australian Certificate of Education) programs for Years 11 and 12. In each school, thirty students who had just completed Year 10 participated in the study, 15 who had chosen the IBDP and 15 who had chosen the SACE path for their next two years at school. The students completed written questionnaires and twenty also participated in focus group discussions. In his conclusions, Paris wrote (p. 242):

> The data analysis showed that some Year 10 students in Adelaide chose to do the IB-DP instead of the SACE for a number of reasons.
> - The IB-DP class sizes are smaller.
> - IB-DP teachers are believed to be better, more caring and spend more time towards ensuring the success of their students.
> - The IB-DP offers a superior curriculum to the SACE.
> - The IB-DP secures a higher tertiary entrance ranking score into local universities.
> - The IB-DP is viewed as being only for so called 'smart kids'.
> - The IB-DP provides students with opportunities to study overseas.

Paris notes that some of the students held misconceptions about the comparisons of the IB-DP and the SACE, for example some believed that the SACE would not qualify them for entry into world universities, which is not true.

From the information on the IBO website, there are eleven non-government schools offering the IBDP in NSW, as an alternative to the HSC. Of these, three are international schools, three are co-ed grammar schools, four are schools for females only and one is a school for males only. The wording on their websites about the IBDP emphasizes the following features of the programme – academic integrity, intellectual promise, international recognition. Some emphasize the excellent university entrance scores achieved by recent cohorts of students; others emphasize the global outlook and the CAS (creativity, action and service) aspects of the programme. A highlight is usually the fact that the IBDP marking scheme is straightforward and not scaled, and that it is translated into high UAI rankings.

INTERNATIONAL COMPARATIVE STUDIES IN MATHEMATICS EDUCATION

Introduction

While international comparative studies contain useful information for researchers, policy makers, and curriculum writers, results from such studies are not universally regarded as definitive statements of a country's achievements in mathematics at school level. David Clarke evaluates the impact of international research in mathematics education and makes the point that we study the mathematics classrooms of other countries not to adopt or adapt their practices (which are culturally situated and have been developed for specific purposes and goals), but in order to reflect on our own practice. (Clarke, 2003).

The issues to consider when reviewing international testing programmes include:
(a) What does the test actually test? Are the skills tested by the items located in a narrow band of school mathematics? (b) Is there a (fallacious) assumption behind the test that there is a universal "correct" curriculum that all countries are achieving to a greater or lesser degree? (Keitel & Kilpatrick, 1999). (c) How valid are the claims that the construction of test items has minimized inevitable cultural bias – especially in contextualized mathematics items? (d) To what extent does the reporting of results at a national level conceal local inequities? Clarke claims that "The identification of international differences and similarities in student mathematical performance has limited utility, except as a form of national report card, unless it is accompanied by data that suggest cultural, societal, or instructional differences that might be used to explain such differences and similarities and then to promote improved mathematical learning and associated performance." (Clarke, 2003, p. 158.)

As an example of such considerations, the very high performance of Korean students in the 1994/95 Third International Mathematics and Science Study, when analyzed more deeply, showed that there were large differences in the achievements of students from large and small cities and the largest gender difference of all countries in the study. Furthermore, attitudes towards mathematics showed a very different pattern across the countries. Students in Cyprus reported positive attitudes towards mathematics but achieved at a comparatively low level on average, while the reverse was true for Korea. (Kaur & Gronmo, 2004, p. 366).

In recent years, Australia has participated in two international studies of school mathematics and science: Trends in International Mathematics and Science Study 2002/3 (TIMSS), conducted by the International Association for the Evaluation of Educational Achievement (IEA), and the Programme for International Student Assessment 2003, (PISA), conducted by an international consortium led by the Australian Council for Educational Research (ACER) on behalf of the Organization for Economic Co-operation and Development (OECD). Keeping in mind the cautions expressed above about the meanings to be taken from these studies, the Australian results will be described.

TIMSS: Trends in International Mathematics and Science Study 2002/3
(ACER, n. d. a)

Testing for TIMSS 2002/3 was carried out in 46 countries. In Australia, 210 schools and 5355 Year 8 students as well as 204 schools and 4675 Year 4 students participated in tests of mathematics and science achievement, and completed a short questionnaire about family background, resources in the home, educational aspirations and attitudes toward mathematics and science. The students' teachers and school principals were also surveyed. For present purposes, only the Year 8 mathematics results will be summarized.

TIMSS has a curriculum focus, aiming to find out about the *intended* curriculum, defined as the curriculum specified at national or system level; the *implemented* curriculum, defined as the curriculum as interpreted and delivered by classroom teachers, and the *attained* curriculum, defined as that part of the curriculum that is learned by students, as demonstrated by their attitudes and achievements.

Summary of relevant TIMSS Year 8 mathematics results (ACER, n.d. p. 5)

- Students from Singapore scored significantly higher than students in all other countries.
- In Year 8 mathematics, Australia scored higher than the international average.
- Achievement in the United States, England, Scotland, New Zealand and Malaysia was similar to that of Australian students.

Countries scoring significantly higher than Australia were (in order) Singapore, Korea, Hong Kong, Chinese Taipei, Japan, Belgium, Netherlands, Estonia and Hungary. Countries scoring at no significant difference to Australia were (in order) Malaysia, Latvia, Russian Federation, Slovak Republic, (Australia), USA, Lithuania, Sweden, Scotland, England, Israel, and New Zealand.

Among the Australian states, NSW had the highest proportion of students achieving the "advanced" international benchmark. In terms of content areas, Australia's average score in Year 8 was significantly higher than the international average in all mathematics content areas (Number, Algebra, Measurement, Geometry, and Data). The content area Data was Australia's highest score and Geometry was the lowest.

There was no significant gender difference in overall mathematics achievement at year 8, although males significantly outperformed females in Number and Measurement.

It should be noted that the average age of the Australian participants was 13.9 years, which is less than the average age of all the groups from countries whose students scored more highly than Australia. In the other international comparison study, PISA, students are selected by age rather than by year group, and the focus is on applying knowledge to real-life problems and situations (Reading Literacy, Scientific Literacy and Mathematical Literacy) rather than on curriculum.

PISA: Programme for International Student Assessment (ACER n. d. b)

Testing for PISA 2003 was carried out in 41 countries, including all OECD countries and eleven non-OECD countries. In Australia, 321 schools and 12 551 students participated in tests of the four domains - Reading Literacy, Scientific Literacy, Problem Solving, and Mathematical Literacy, and completed a 30 minute questionnaire about family background, their attitudes to school and learning strategies they used. The students' school principals were also surveyed about the atmosphere and resources at school for learning and the kinds of programs the students were studying. The organizers of PISA claim that the programme assesses young people's ability to apply their knowledge and skills to real-life problems and situations rather than how well they had learned a specific curriculum. (ACER, n. d. b, p. 2).

In Mathematical Literacy, countries scoring significantly higher than Australia were (in order) Hong Kong, Finland, Korea, Netherlands, and Singapore. Countries scoring at no significant difference to Australia were (in order) Liechtenstein, Japan, Canada, Belgium, Macao-China, Switzerland, (Australia), New Zealand, Czech Republic, and Denmark. The response rate from the UK was too low to allow comparisons to be made. (OECD Directorate of Education, 2004).

In Problem Solving, countries scoring significantly higher than Australia were (in order) Korea, Hong Kong, Finland, and Japan. Countries scoring at no significant difference to Australia were (in order) New Zealand, Macao-China, (Australia), Liechtenstein, Canada, Belgium, Switzerland, Netherlands and France. It is interesting to note that Finland achieved the highest scores in Reading Literacy and Scientific Literacy as well as in Mathematical Literacy.

There were few significant differences among the Australian States and Territories. The Australian States and Territories all performed at or better than the OECD average in all four domains.

The main policy messages from PISA 2003 for Australia are summarized on page 13 of ACER (n. d. b). In brief these are:

- Students with higher socio-economic backgrounds score more highly on average than students with lower socio-economic backgrounds. Assuming that social equity in education is a goal of the school system, more effective ways to counter the disadvantage experienced by groups in our society need to be found.
- Indigenous students need support to improve their overall low performance.
- There were no overall gender differences in mathematics performance, but females appear to be less engaged, more anxious, and less confident in mathematics than males.

CHOICE OF COUNTRIES FOR INTERNATIONAL COMPARISON OF SYLLABUSES WITH AUSTRALIA

Based on a number of factors, including relative standings on the TIMSS and PISA indicators, similar cultures and/or colonial history, and shared language, the following countries were chosen for comparison of Senior Mathematics Syllabuses:

Singapore
Hong Kong (Hong Kong Special Administrative Region of China)
Finland
UK
USA

Important features of the populations of these countries are illustrated in Table 5:

Table 5: Population data for selected countries

Source: The World Factbook https:www.cia.gov/cia/publications/factbook/index.html
Adaptations and collations have been made.
The data is approximate only as some figures are for 2004, others for 2005 or 2006.

	Australia	***Singapore***	***Hong Kong***	***Finland***	***UK***	***USA***
Population July 2006 estimate in millions	20.264	4.492	6.94	5.231	60.609	298.444
Pop. Growth rate	0.85%	1.42%	0.59%	0.14%	0.28%	0.91%
Labour Force percentages						
Primary production	3.60%	0%	0%	8%	1.50%	0.70%
Manufacturing, construction, transportation and communications	21.20%	35%	17.50%	36%	19.10%	22.90%
Services including financial services, community and social services	75.20%	65%	82.30%	56%	79.50%	76.40%
Unemployment	5.10%	3.10%	5.50%	8.40%	4.70%	5.10%
Mobile phones per person, estimated for 2004 or 2005 data as available	0.83	0.96	1.20	0.95	1.01	0.66
Internet users as % of population, 2005 data	71%	55%	71%	63%	63%	69%
Gini index	35.2	42.5	43.4	26.9	36.8	45

SINGAPORE

Context for the study of mathematics in the senior secondary school

This report begins with overall descriptions from O'Donnell (2004), especially pages 23 and 38 from which this summary was constructed:

> Education is centrally controlled (by the Ministry of Education). There are six years of compulsory primary education. Classes are streamed from the end of Primary Year 4 (age 10+) according to children's performance in a school-based examination in the mother tongue (Chinese, Malay or Tamil), English, and mathematics, and children may repeat Year 5 before taking the Primary School Leaving Examination (PSLE) in Year 6. On the basis of their PSLE results, students are placed in secondary education in 'special' or 'express' streams (leading to the Singapore GCE O Level certificate in four years, age 16), or in 'normal' streams (which lead to the Singapore GCE 'N' level qualification in four years and enable some students to take the Singapore GCE 'O' level in a fifth year, age 17). Singapore-Cambridge GCE Advanced Level examinations are available at age 18+.

The information in the following paragraphs was obtained from the website of the Singapore Ministry of Education. (MOE, 2006). There are several pathways following the Singapore GCE 'O' level examinations. Students admitted to Junior Colleges (about 25% of the age cohort) are on an academic track and complete the Singapore-Cambridge A level exams after two years. Students admitted to the Centralised Institute (the Millennia Institute) may sit for the Singapore-Cambridge A level exams after three years, and may also take commercial stream subjects such as accounting. About 40% of the age cohort are admitted to Polytechnics for applied and practice oriented training. The Polytechnics offer three year courses in areas such as accounting, nursing, optometry, and applied engineering. Students graduating from Polytechnics with high grades may apply for university entrance. Finally there are ten Institutes of Technical Education which provide two year vocational courses. Students doing well enough here may go on to Polytechnics for Diploma courses.

Recent innovations have resulted from a Review of Junior College/Upper Secondary Education in 2002 which recommended a broader and more flexible Junior College curriculum. An Integrated Programme for the top 10% of the age cohort has been introduced. Particular schools have been identified as suitable for this programme and their students may complete the secondary school years without the burden of preparing for the GCE O Level examinations. In addition, one school has been given permission to offer the International Baccalaureate Programme, and a new school, the National University of Singapore High School for Mathematics and Science has been established. This school will prepare students for the NUS High School Diploma and will also offer preparation for the Scholastic Aptitude Test (SAT) and for the Advanced Placement exams conducted by the USA College Board.

Increasing breadth in the final years is to be achieved by replacing the scheme under which students took 3 or 4 A Level subjects to allow for various programs. Subjects are being re-written as H1, H2 or H3 level subjects, with H1 being half the content but similar in difficulty to H2 subjects, which are similar to the old A Levels. Students will take three H2 subjects and one H1 content based subject. They must include one contrasting subject (so that, for example, those studying mainly humanities must take one subject from the science area).The H3 subjects are to be of a higher level again and allow for acceleration to University level study. A new subject, called KI (Knowledge and Inquiry), is to be introduced and may be used as the contrast subject. It appears to be similar to the IB Core subject Theory of Knowledge.

Mathematics is offered as H1, H2, and H3 subjects.

Content – lower secondary years

It is appropriate to start with an overview of the Singapore lower secondary mathematics syllabus (Curriculum Planning and Development Division, 2006) as it is quite advanced by Australian standards.

> In Secondary 1, the broad topics are Numbers and Algebra, Geometry and Measurement, Statistics and Probability. Functions are introduced in Numbers and Algebra.
> In Secondary 2, the same areas are studied, with the usual progression through content topics. Functions to simple quadratics are studied. A section on set language and notation is included.
> In Secondary 3/ 4, Numbers and Algebra includes sketching quadratics given in factor form and establishing gradients of curves by drawing tangents. Matrices are introduced. Geometry and measurement includes circle properties, and trigonometry includes the Sine and Cosine rules, arc length, radian measure. Vectors in 2 dimensions are introduced. Statistics and probability is also continued.
>
> There is a simpler T program available in Secondary 1 to 4.
>
> An O Level Additional Mathematics for Secondary 3/4 consists of Algebra (including quadratic theory, polynomials with the factor and remainder theorems, solving cubic equations, partial fractions, binomial expansion, exponential, log, and modulus functions; Geometry and Trigonometry includes expansions and identities and solving simple trig equations, coordinate geometry and proof; Calculus, somewhat equivalent to a NSW 2 unit treatment, is also studied here.

Content for A level (New H1, H2, H3 courses) (SEAB, 2006).

> *Mathematics Higher 1 (HI)*
> This subject is designed for students intending to study business, economics, and social sciences and similar courses at university.
> Topic areas are Functions and graphs, Calculus, Probability.
> (In Calculus, the product and quotient rules are not included. Definite integrals are introduced as areas under curves, not limiting values of sums. Numerical integration on the Graphing Calculator is included. The Probability section includes binomial and normal distributions, hypothesis testing, correlation and regression.)
>
> The content for H1 is examined by one 3-hour paper: Section A - 40 marks on Pure Mathematics, Section B - 60 marks on Statistics).
>
> *Mathematics Higher 2 (H2)*
> This subject is designed for students intending to study mathematics, physics, and engineering.

Topic areas are Functions and graphs, Sequences and series (includes proof by mathematical induction), Vectors (includes scalar and vector products and 3 dimensional geometry), Calculus (includes McLaurin Series, integration techniques, definite integrals are introduced as limiting values of sums), Permutations combinations and probability, Binomial Poisson and normal distributions, Sampling and hypothesis testing, Correlation and regression (including transformation to achieve linearity).

The content of H2 is examined by two 3-hour papers. Paper 1 is on Pure mathematics. Paper 2 has two sections: Section A (40 marks on Pure Mathematics) and Section B (60 marks on Statistics).

Mathematics Higher 3 (H3)
This is for students who have a strong aptitude for mathematics and are passionate about it. Students taking H3 must also take H2.
The topics are Differential Equations and a choice of two more topics from Plane Geometry, Graph Theory, and Combinatorics.

The H3 syllabus is examined by one 3-hour paper with a compulsory Section A (40 marks – 4 questions) based on Differential Equations, and a Section B (56 marks – any 4 questions from six that will be offered, two on each of the three options). The remaining 4 marks are allocated for style and clarity.

Students are expected to use a Graphing Calculator with no CAS in all 3 syllabuses – note that Graphing Calculators were not used in the earlier A-level courses. Mechanics, which was an essential feature of the earlier syllabus, is now removed from the A-level syllabus.

Comments on a comparison of the Singapore Mathematics Curriculum with the current NSW HSC courses

- Singapore provides in a structural way for the diverse needs of its students in mathematics. Performance on the Year 6 PSLE determines whether students will be expected to complete the lower stage of secondary school in four years or five: allowing time for most students to reach the highest standard they can. Performance on the O Level then determines the kind of upper secondary school experience the student will experience – but again, the pathways are not blocked and some late developers seeking a University education can find a route there via the Polytechnics or the Centralised Institute after three years instead of the two years taken at Junior Colleges.
- The new H1, H2, H3 subjects are for students intending to study at university. Statistics features in all three subjects while mechanics is missing.
- All assessment at this level is external in Singapore.
- In Singapore, graphing calculators are expected for classes and in the external examinations.

HONG KONG

Context for the study of mathematics in the senior secondary school

Hong Kong is at a major turning point in its educational history, planning for a change from seven years of secondary school and three year university degrees to a system of six years of secondary school and four years of university[2]. The catchphrase chosen for this change is "3 – 3 – 4". Wide consultation is being undertaken on a series of documents and plans. At the time of writing, the source of information is the website of the Education and Manpower Bureau of the Government of the Hong Kong Special Administrative Region, China. On this site is a series of articles by Mrs Fanny Law, Permanent Secretary for Education and Manpower. The most recent one explains:

> Reforming the academic structure is a massive undertaking. In view of the extensive impact of the reform, which includes senior secondary and university curricula, new ways of learning, public examination and university admission criteria, and a wider set of post-school pathways for our young people, the first year of implementation of the new senior secondary curriculum will be 2009/10 allowing ample time for all to be fully prepared. (Law, 2006a)

New Curriculum and Assessment Frameworks for all subjects are being developed. A major change is the decision to increase the amount of School Based Assessment (SBA) as a component of final marks. Reasons given for this include (a) improving the validity of assessment, as some important learning outcomes cannot be assessed within the context of a written examination, (b) to improve the reliability of assessment by using teacher assessments over a period of time rather than relying on "one-off" exams, and (c) to promote a positive effect on teaching and learning by engaging students in activities that lead to meaningful learning. (An overview of School-based Assessment in the New Senior Secondary Curriculum, www.emb.gov.hk).

Rationale for changes to Mathematics – mathematics to be compulsory.

This is from another article by Law (2006b)

> The rapid advancement of modern technology, coupled with the fast development of the Internet, has not only transformed the way in which information is disseminated, but also brought about tremendous changes to mathematics education. For instance, while basic understanding of number, measurement and data handling will always be a necessary foundation, the need for drilling computation skills has reduced significantly due to the availability of calculators and the emergence of computer algebra systems in recent years. With the help of information technology, many learning topics and meaningless drilling have become obsolete. Mathematics education is no longer confined to the rote learning of formulae, principles and algorithms. In light of societal changes, secondary students should not only possess foundation mathematical skills and concepts; more importantly, they should build up confidence in learning mathematics and be well-trained in logical deduction, abstract thinking, analysis and problem solving, and communication through symbols and graphs. They should also cultivate in themselves a spirit of perseverance in finding solutions and a habit of independent rational thinking. This is why Mathematics is included as one of the core subjects to be taken by all students under the New Senior Secondary (NSS) academic structure. The curriculum emphasizes mathematical exploration, research and application.

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[2] This is a change from a system adapted from the British to the one currently used by China and by the USA.

The pathway to the making of these decisions can be traced through various consultation reports including that of a team which worked from April 1998 to June 1999 to survey the views of students, parents, teachers, university lecturers, curriculum planners and human resources personnel in the commercial sector. (Wong, Lam, Leung, Mok and Wong, 1999). Points made in that report include

- Mathematical knowledge, concepts, problem solving skills as well as opportunities to discover and to invent should be encompassed.
- Interest in learning mathematics needs to be cultivated.
- Although Hong Kong students performed well in international comparisons, they lacked confidence in solving mathematical problems.
- In Hong Kong, on the average, topics were taught one or two years earlier than in other countries.
- Students regarded the curriculum as "too packed".
- Students saw mathematics as a set of rules, hated tedious calculations, hoped for "liveliness" and real life applications both in teaching and in textbooks.
- Students see a good mathematics teacher as one "who is nice, lively, provides a variety of activities and offers clear, step-by-step explanation, who allows time for students to think, checks frequently to see if students understand from time to time, explains how to approach problems and would not penalize weaker students." (Wong et al, 1999, p. 3).
- Teachers found the curriculum too bulky, lacking in flexibility, boring and unrelated to real life, and unable to cater for individual differences and to provoke thinking.
- Teachers found that students were often passive, lacking in initiative, and not serious about learning. Teachers needed more time to prepare teaching materials and smaller class sizes.

In any 'trimming down' of content, the desire expressed was to make room for deeper understanding of the material and not a watered-down curriculum. The report concludes with a strong statement about the need for upgrading teacher professionalism.

It should be noted that entry to the final two years of secondary schooling is not automatic, but is based upon the results of the Hong Kong Certificate of Education Examination (HKCEE), similar to the GCSE "O" Level examinations in the UK. Students must reach a required level and entry is competitive.

Table 6 compares the old and new structures.

Table 6. Existsing and new Senior Secondary curriculum structures for Hong Kong.

Existing	*For introduction in 2009*
Six years primary education (P1 – P6)	Six years primary education (P1 – P6)
Three years lower secondary (S1 – S3) Two years middle secondary (S4 – S5) Examination for the HKCEE Two years senior secondary (S6 – S7) Three year university courses	Three years lower secondary (S1 – S3) Three years senior secondary (S4 – S6) Four year university courses (the new " 3 – 3 – 4" system)
Usual pattern for S6 and S7: 2 Languages (Chinese and English) 2 Advanced level (A level) subjects Liberal Studies (AS level, Advanced Supplementary Level) 1 other AS level subject. (A level subject: 8x40 mins per week) (AS level subject: 4x40 mins per week) Some students may take only one A level. Mathematics subjects: A level Applied Mathematics A level Pure Mathematics AS level Applied Mathematics AS level Mathematics and Statistics	Students will study four core subjects and up to three electives. Core subjects: Chinese language English Language Mathematics Liberal Studies Electives may be in the areas of Applied Science, Business, management and law, Creative studies, engineering and production, Media and communication, Services. Mathematics will consist of a Compulsory Part and an Extended Part which consists of two modules. Students may only take one module. Module 1 – Calculus and Statistics Module 2 – Algebra and Calculus

Further description of mathematics under the new arrangement. (CDC and HKEAA, 2006).

Mathematics will consist of a Compulsory Part, and an Extended Part which consists of two modules. Students may only take one module. Module 1 is called "Calculus and Statistics" and Module 2 is called "Algebra and Calculus." The Compulsory Part includes sub-topics identified as Foundation and Non-Foundation Topics. Schools may teach some classes all, part, or none of the Non-Foundation Topics, depending on student preparation, needs and interests. Teaching time is estimated at 270 – 338 hours. The compulsory part with a module is estimated at 405 hours, or 15% of the total lesson time in the senior secondary curriculum.

Compulsory Part <u>Note that Non-Foundation topics are underlined in this list.</u>

Number and Algebra

1. Quadratic equations in one unknown. Includes factor method, formula, discriminant. <u>Add, subtract, multiply and divide complex numbers.</u>
2. Functions and graphs.
3. <u>Exponential and logarithm functions. Includes laws and graphs.</u>
4. More about polynomials. Includes division, remainder theorem and factor theorem.
5. <u>More about equations. Includes solving equations which can be transformed into quadratic equations, simultaneous equations.</u>

6. Variations. Includes direct and inverse variation and applications to real-life problems, as well as partial and joint variation.
7. Arithmetic and geometric sequences and their summations. Includes applications to interest, growth and depreciation. Sum to infinity of certain geometric series.
8. Inequalities and linear programming
9. More about graphs of functions. Includes curve sketching. Transformations such as $f(x + k)$.

Measure, Shape and Space

10. Basic properties of circles. Includes chord and arc properties, tests for concyclic points, angles in alternate segments.
11. Locus. Includes describing and sketching the locus of points satisfying given conditions, describing with algebraic equations.
12. Equations of straight lines and circles
13. More about trigonometry. Includes sine, cosine and tangent functions in degrees, solving trig equations, Sine and Cosine rules, Heron's formula, 2 and 3 dimensional problems.

Data Handling

14. Permutation and combination
15. More about probability. Includes set language, mutually exclusive events, complementary events, independent events, conditional probability.
16. Measures of dispersion. Includes range and interquartile range, box and whisker diagrams, standard deviation from the formula, comparing data sets. Applications to real life problems including standard scores and normal distribution. Effect on dispersion of data when items are added etc.
17. Uses and abuses of statistics. Recognize different techniques in survey sampling and the basic principles of questionnaire design, discuss and recognize the uses and abuses of statistical methods in various daily-life activities or investigations, assess statistical investigations presented in different sources such as news media, research reports etc.

Further Learning Unit. (Notional 20 hours of class time for each of 18 and 19)

18. Further applications. Explore and solve more sophisticated real-life problems, appreciate the connections between different areas of mathematics. Examples are given from problem solving in finance, taxation, transformation of data, Fibonacci sequence, cryptography, as well as pure mathematics and history of mathematics.
19. Inquiry and investigation. Further improve the ability to inquire, communicate, reason and conceptualize mathematical concepts. This time could be used for School Based Assessment.

Module 1 Calculus and Statistics

This module is designed for students who will be involved in study and work which demands a wider knowledge and deeper understanding of the application of mathematics, in particular, statistics.

1. Foundation Knowledge

   1.1 Binomial expansion, 1.2 Exponential function and logarithmic function.

2. Introduction to differentiation with applications

   2.1 Derivative of a function (a conceptual approach rather than using limits – first principles not required), 2.2 Differentiation of a function (includes quotient and product rules etc), 2.3 Second derivative, 2.4 Applications.

3. Introduction to integration with applications

   3.1 Indefinite integrals, 3.2 Definite integrals and applications, 3.3 Approximation by trapezoidal rule.

4. Further Probability.
   4.1 Conditional probability and independence, 4.2 Bayes' theorem
5. Binomial, Geometric, and Poisson Distributions. (Proofs are not required)
   5.1 Discrete random variables, 5.2 Probability distribution, expectation and variance. 5.3 Binomial distribution, 5.4 Geometric distribution, 5.5 Poisson distribution, 5.6 Applications.
6. Normal Distribution
   6.1 Basic definition and properties, 6.2 Standardization of a normal variable and use of the standard normal table, 6.3 Applications of the normal distribution.
7. Point and Interval estimation
   7.1 Sampling distribution and point estimates, 7.2 Confidence interval for a population mean, 7.3 Confidence interval for a population proportion.
8. Further Learning Unit (notional time 10 hours).
   Inquiry and investigation. May be used for School Based Assessment.

Module 2 Algebra and Calculus

This module is designed for students who will be involved in mathematics related disciplines and careers.

1. Foundation Knowledge
   1.1 Surds, 1.2 Mathematical induction, 1.3 Binomial theorem. 1.4 More about the trigonometric functions (includes radian measure, cosecant etc, identities including sum-to-product), 1.5 Introduction to the number *e*.
2. Limits and differentiation
   2.1 Limits, graphical approach to continuity, 2.2 Differentiation including first principles, standard rules and results, implicit differentiation, 2.3 Applications of differentiation.
3. Integration
   3.1 Indefinite integration including substitution and by parts, 3.2 Definite integration, 3.3 Applications of definite integration, including areas, and volumes by disc and shell methods.
4. Matrices and systems of linear equations
   4.1 Determinants and properties for up to 3 by 3 size, 4.2 Matrices, standard properties and operations, with inverses, up to 3 by 3 size. 4.3 Systems of linear equations including Cramer's Rule and Gaussian elimination.
5. Vectors
   5.1 Introduction including operations and properties, 5.2 Scalar product and vector product, 5.3 applications.
6. Further Learning Unit. (Notional time 10 hours).
   Emphasis on problem solving and investigations. May be used for school based assessment.

Technology and assessment

The curriculum document for mathematics (CDC and HKEAA, 2006) refers several times to the impact that new technologies have made on the teaching and learning of mathematics, and encourages teachers to use software such as geometric sketching software and computer graphing software to make the learning of mathematics more interesting for their students. However there is no prescription about the kinds of calculators to which students may require access in lessons, and for public exams only "electronic calculators" (not graphing calculators) are mentioned.

The assessment design includes for the Compulsory Part, 15% School-based assessment, (to be phased in over the years 2012 - 2015) and Public Examination:

Paper 1 Conventional questions weighted 55%, 2.25 hours long.
Paper 2 Multiple choice questions weighted 30%, 1.25 hours long.

For each of the modules there is to be a public examination of conventional questions, 2.5 hours in length.

Comments on a comparison of the new Hong Kong Mathematics Curriculum with the current NSW HSC courses

- Unlike the NSW situation, mathematics will be compulsory in Hong Kong to Year 12 under the new arrangements.
- Hong Kong has in the past had a lower retention rate to year 12 than in NSW, because of the HKCEE. A typical school would reduce from 4 classes of 45 students in S5 to 2 classes of 35 in S6.
- The level of the compulsory mathematics in Hong Kong is more advanced and algebra based than in NSW – more a “pure mathematics” flavour.
- NSW HSC General Mathematics has more emphasis on practical measurement, areas and volumes etc. In Hong Kong, students study these topics in Primary and lower Secondary classes and the topics are not revisited.
- Hong Kong Module 1 with its heavy emphasis on data handling has no equivalent in the NSW HSC courses.
- Topics in Applied Mathematics such as kinematics are not in the new Hong Kong courses.
- There is a thorough treatment of matrices and vectors in Module 2, unlike the situation in some Australian states where a minor introduction plus applications give a taste without much mathematical background.
- There is concern in Hong Kong about repetitive drill of lower level skills. Teachers are encouraged to consider direct instruction as one of several teaching styles, alongside an enquiry approach and a “co-construction” approach that engage students more and involve students in the process of making and testing conjectures rather than being passive recipients.
- There is to be a gradual introduction of School Based Assessment in Hong Kong but only in the compulsory part, not the extension parts.

FINLAND

Context[3] for the study of mathematics in the senior secondary years

There are nine years of basic education in Finland, from age 7 to age 16. At the end of this compulsory schooling there is no examination: students may choose to attend an Upper Secondary School (academic) or a Vocational Institute and about 50% opt for each of these paths. After a further three years students may sit for a matriculation examination for selection to universities and polytechnics. Some students may sit this exam after two years, others may take four. This exam comprises papers in the mother tongue, (Finnish, Swedish or Sami), the second national language (Finnish or Swedish), the first foreign language, mathematics, and general studies. There are two exam periods each year and students may sit for tests in one exam period, or in parts over three consecutive exam periods. In the general upper secondary schools, students are not grouped into classes by year.

Content of secondary mathematics curriculum

At the time of writing, a copy of the relevant syllabus in English has not been located. A further paper will be prepared when this has been achieved.

Finland and PISA

In the PISA surveys of 2000 and 2003, the average scores of students from Finland was either the highest or in the top four on all scales – reading literacy, mathematical literacy, scientific literacy and problem solving. After the PISA 2000 results were released, a group of Finnish academics reported on the features of the educational environment that they believed led Finnish students to their successful performance. (Välijärvi, Linnakylä, Kupari, Reinikainen & Arffman, 2002). Among the points that they made are:

- The excellent results in reading literacy, scientific literacy and mathematical literacy featured a low spread; a measure of high equality in educational achievement.
- Finnish students did exceptionally well in interpreting graphs and diagrams, but not so well in algebra; a considerable proportion of Finnish students left unanswered a great number of tasks requiring generalization or explanations.
- In reading literacy, the factors *engagement in reading* and *interest in reading* were more important for Finnish students than family background, which proved more important in other countries. This suggests that the kind of schooling had evened out the impact of socio-economic background.
- A high *self-concept in mathematics* was strongly and positively associated with performance in mathematics for all countries, however this did not explain variations across countries. For example, high scoring Korea was one of the countries where students displayed a low self concept in mathematics.

[3] General information on Finland is from the websites http://www.edu.fi/english/ and http://virtual.finland.fi/Education_Research/

- The tasks used in PISA matched the Finnish curriculum, with an emphasis on use and application of knowledge, and problem solving.
- Finland has had in place since 1996 a program aiming to develop knowledge and skills in mathematics and science at all levels of schooling (The LUMA program).
- In mathematical literacy, high achieving Finnish boys were close to other high achieving boys in the sample as a whole, but Finnish high achieving girls outscored in a significant way other high achieving girls in the sample as a whole.
- Differences between schools in Finland was very low, a feature of all Nordic countries where education is non-selective. This reflects the Finnish philosophy of equality in education. One reason for this is the small size of the population and a conviction that no student should be left out of high quality education, both for the student's sake and for the good of the country.
- Finnish schools and teachers provide a supportive, caring environment. Warm meals and school health services, as well as counselling, are provided free.
- Finnish teachers are highly qualified and the teaching profession is highly valued. Only highly motivated and multi-talented students are accepted into teacher education programmes, and once qualified they appear to be willing to engage in further education and training.
- There are no national tests or examinations during the years of compulsory education: the teachers' assessments are relied upon and the teachers are considered expert at this.
- The national curriculum is flexible, decentralized, and does not contain a high level of detail.
- There is a high degree of school autonomy in Finland, which contributes to the professional role of teachers.
- The cultural homogeneity of the Finnish population makes it easier than other places to reach mutual understanding on education policy.

Many of these points are reminders of the fact that the reasons for relative placement in the international tests such as PISA and TIMSS are complex and culturally based. Curriculum content is only one feature among many.

UNITED KINGDOM
England

Context for the study of mathematics in the senior secondary school

Within the United Kingdom, responsibilities for education are devolved in varying ways to Wales and Northern Ireland, and Scotland has a different system for post-compulsory curriculum and examinations. This report focuses on England. It begins with overall descriptions from O'Donnell (2004), especially pages 29 and 27 from which this summary was constructed:

> Overall responsibility for all aspects of publicly-funded education in England lies with the national Government, but many responsibilities are delegated to local education authorities and school governing bodies. At the completion of compulsory education (age 16), assessment generally involves the General Certificate of Secondary Education (GCSE). Although not compulsory, these qualifications, traditionally taken in a range of academic subjects, and increasingly available in a range of vocational subject areas, are usually required for access to further study. Students who attend the final two years of schooling generally study for GCE (General Certificate of Education) 'AS' (Advanced Subsidiary) qualifications or GCE 'A' Levels (Advanced levels). GCE 'A' levels (sometimes known as A2 examinations) are single subject examinations which usually take two years to complete and comprise six modules or units. The three modules/units taken in the first year of a GCE 'A' Level course comprise the GCE 'AS' qualification, taken at age 17. This is a 'stand-alone' qualification. Consequently, students may choose not to complete a further three modules to gain the GCE 'A' Level in the second year.

The usual pattern is to take four GCE (academic subjects) or VCE (vocational subjects) AS Levels in Year 12 (Lower sixth form) and then three A2 Levels in Year 13 (Upper Sixth Form). Vocational awards are being phased out and replaced by "Applied GCEs". Recently a high failure rate in AS exams and a review of curriculum offerings has resulted in a change to these requirements and now two modules (units), comprising half of the usual A Level course are sufficient for AS study in a subject (QCA, 2006a). This arrangement was in place for the academic year beginning September 2004.

The Qualifications and Curriculum Authority (QCA), a public body sponsored by the Department for Education and Skills (DfES), maintains and develops the national curriculum and associated assessments, tests, and examinations. At the senior secondary level QCA publishes Subject Criteria from which the three independent awarding bodies, AQA, Edexcel, and OCR, formerly called examining boards, construct syllabuses and assessment regimes (called specifications). The awarding bodies write and conduct their own exams and award the appropriate qualifications. Schools choose with which awarding bodies they wish to work, and these sponsor networks of support for teachers, often including arrangements with textbook publishers.

The amalgamation of many examining boards into the three awarding bodies has happened over recent years. The Joint Council for Qualifications (JCQ), established in 2004, is an umbrella body for the awarding bodies and among other things maintains statistics about participation in subjects and levels of award. Around 9% of total candidates for A level awards in 2006 took Mathematics, (32719 males and 20178 females) and less than 1% of total candidates for A level awards in 2006 took Further Mathematics A level awards (4238 males and 1695 females), (JCQ 2006). More detailed statistics are maintained by the Department for Education and Skills. (DfES, 2006).

Content

For Mathematics, the QCA Subject Criteria (QCA 2002) outlines Core Content, which is to be developed into four units, and gives guidance about other units. The Core Content contains the headings Proof (including understanding and using mathematical language and grammar), Algebra and functions, Coordinate geometry in the (x,y) plane, Sequences and Series, Trigonometry, Exponentials and logarithms, Differentiation, Integration, Numerical methods (including location of roots by sign change, iterative methods, numerical integration), and Vectors. The Subject Criteria document includes Assessment Objectives and weightings, and the condition that one AS unit of Core Content must be assessed without any “calculating aid”. All other units are to permit the use of graphic calculators. Specifications are to “encourage the appropriate use of graphic calculators and computers as tools by which the teaching and learning of mathematics may be enhanced.” (QCA, 2002, Section 6.4 g). A typical pattern of study would include two core modules and one application module in each of Years 12 and 13.

The OCR Specification lists the contents of its modules as follows: (OCR, 2003)

C1	Core Mathematics 1	AS	Indices and surds, polynomials, coordinate geometry and graphs, differentiation.
C2	Core Mathematics 2	AS	Trigonometry, Sequences and series, Algebra, Integration
C3	Core Mathematics 3	A2	Algebra and functions, Trigonometry, Differentiation and integration, Numerical methods
C4	Core Mathematics 4	A2	Algebra and graphs, Differentiation and integration, Differential equations, Vectors

FP1	Further Pure Mathematics 1	AS	Summation of series, Mathematical induction, Roots of polynomial equations, Complex Numbers, Matrices
FP2	Further Pure Mathematics 2	A2	Rational functions and graphs, Polar coordinates, Hyperbolic functions, Differentiation and integration, Numerical methods
FP3	Further Pure Mathematics 1	A2	Differential equations, Vectors, Complex numbers, Groups

M1	Mechanics 1	AS	Force as a vector, Equilibrium of a particle, Kinematics of motion in a straight line, Newton’s laws of motion, Linear momentum
M2	Mechanics 2	A2	Centre of mass, Equilibrium of a rigid body, Motion of a projectile, Uniform motion in a circle, Coefficient of restitution and impulse, Energy, work and power.
M3	Mechanics 3	A2	Equilibrium of rigid bodies in contact, elastic strings and springs, Impulse and momentum in two dimensions, Motion in

			a vertical circle, Linear motion under a variable force, Simple harmonic motion.
M4	Mechanics 4	A2	Relative motion, Centre of mass, Moment of inertia, Rotation of a rigid body, Stability and oscillations

S1	Probability and Statistics 1	AS	Representation of data, Probability, Discrete random variables, Bivariate data
S2	Probability and Statistics 2	A2	Continuous random variables, The normal distribution, The Poisson distribution, Sampling and hypothesis tests
S3	Probability and Statistics 3	A2	Continuous random variables, Linear combinations of random variables, Confidence intervals and the *t* distribution, Difference in population means and proportions, Chi-square tests
S4	Probability and Statistics 4	A2	Probability, Non-parametric tests, Probability generating functions, Moment generating functions, Estimators, Discrete bivariate distributions

D1	Decision mathematics	AS	Algorithms, Graph Theory, networks, Linear programming
D2	Decision mathematics	A2	Game theory, Flows in a network, Matching and allocation problems, Critical path analysis, Dynamic programming

All of these units are externally assessed, that is, there is no coursework or school based assessment requirement. This is actually listed as a feature of this particular scheme. The examinations are typically 1 hour 30 minutes for each module.

The modules are combined in various ways to construct awards. For example, to gain an Advanced GCE AS Level in Mathematics awarded by OCR students would pass modules C1, C2, and one Application module. To gain an Advanced GCE A Level in Mathematics (mixed Statistics and Mechanics), awarded by OCR, a student would pass the modules C1, C2, and S1 in Year 12 and C3, C4, and M1 in Year 13. To gain an Advance GCE A Level in Mathematics and an Advanced GCE in Further Mathematics (Balanced Option), a student would pass C1, C2, S1, M1, D1, and FP1 in year 12, followed by C3, C4, S2, M2, D2, and FP2 or 3 in Year 13.

OCR also administers a set of modules developed by the Mathematics in Education and Industry Schools Project (MEI) in which some coursework (school based assessment) is included in the form of projects for appropriate modules. The MEI group has a reputation for innovation (being the first to include probability at A level); and for strong support of teachers through programmes designed, for example, to enable their skills and confidence in the teaching of Advanced Mathematics.

Several MEI modules have no exact equivalent in the previously described OCR scheme: for example Decision Mathematics Computation, for which candidates require access to a computer with a spreadsheet program and a linear programming package, and suitable printing facilities, throughout the 2.5 hour examination. In Australia, the material studied in that module would be found in a university course in Operations Research.

Provision for all students

In England, the specialization of A Level study has led to the situation where mathematics is studied at a high level by a small proportion of students. An even smaller proportion of the most able students may choose to study for an Advanced Extension Award in Mathematics. This requires no additional content but is designed to assess candidate's abilities to apply and communicate their knowledge of mathematics effectively, to use the skills of critical analysis, evaluation and synthesis, to reach clear, logical and coherent judgments. Assessment consists of a single 3 hour written exam without the use of any scientific or graphic calculators, or computer algebra systems. The emphasis is on solving unfamiliar problems. Approximately 1000 students sit for this exam. (Smith, 2004, p. 64).

Concern about large numbers of students not studying mathematics of any kind after the compulsory years of schooling has led to the introduction of Free-standing mathematics qualifications (FSMQs). These are described on the QCA website as follows:

> Free-standing Mathematics Qualifications have been designed to meet a range of needs. They can be used in a variety of ways by a variety of students. In particular, the following student groups have been identified:
>
> - Students who wish to gain some mathematics equivalent in demand to GCSE Mathematics, either at the lower grades (G to D) or at the higher grades (C-A*).
> - Students who already have GCSE mathematics at grade C-A*, who can use the units to reinforce or extend particular areas of mathematics to help with their other studies.
> - A Level or GNVQ students and others who have enjoyed mathematics and been successful at it, but who have chosen non-mathematical options; the units will allow the possibility of continuing some study of the subject in discrete units.
> - Advanced GNVQ or A Level students in need of mathematical back-up in their main studies.
> - Students applying for Initial Teacher Training courses (or trainees on such courses) – two units have been specifically developed for this target group.
> - Students on Access or Foundation courses to Higher Education.
> - Students in Higher Education who may use them as revision units.
> - Modern Apprentices and National Trainees.
> - Employees (NVQ students and others).

The Foundation FSMQs are "Working in 2 and 3 dimensions", "Making sense of data", "Managing money". The Intermediate ones are "Solving problems in shape and space", "handling and interpreting data", "Making connections in mathematics", (designed for students who may apply for Primary teacher training), "Using algebra, functions and graphs", "Calculating finances". The Advanced units are "Working with algebraic and graphical techniques", "Modelling with calculus", and "Using and applying statistics". The foundation units may be combined into an AS award: a combination of "Working with algebraic and graphical techniques" with a choice of either "Modelling with calculus", or "Using and applying statistics", along with a terminal unit "Applying Mathematics" can be put together for a new AS Level award called "AS use of mathematics". The content of this Applying Mathematics Course is selected to show the application of mathematics to a wide spread of topics. For example, the QCA specification (QCA 2006b) lists

The design of household appliances
The spread of an epidemic
Planetary orbits
How historians and archaeologists recreate human faces
How literature scholars decide if a manuscript was written by Shakespeare or another author
The growth of queues
How Speed cameras work
Economic growth.

In the unit Applying Mathematics, students are expected to use a graphic calculator in class and in exams. All assessment is by external exams, consisting of a comprehension paper lasting one hour (30%) and a further paper lasting 1.5 hours (60%) assessing the application of mathematical techniques.

In addition, there are Application of Number qualifications, largely taught in technical colleges. These are designed to increase the numeracy of students studying for GNVQ vocational qualifications.

Comments on a comparison of the English Mathematics Curriculum with the current NSW HSC courses

- Because of the extra year[4] of secondary schooling in England and the specialization in the final two years it is not appropriate to compare the content of A level mathematics subjects with Australian school mathematics subjects. Many of the topics studied would be found in university courses in NSW.
- England is struggling with the issue of participation rates in mathematics. Innovations such as the AS subjects and FSMQs have not been universally successful to date.
- For AS and A2 Level subjects in England, the modular approach with shorter exams (1.5 hours) provides flexibility for students in choosing aspects of mathematics that suit their interests and further study ambitions.
- The issue of retaining essential by-hand techniques while allowing students to use contemporary mathematical tools is the subject of considerable debate. In England this has been resolved by having one technology free exam, for the first module of the core in AS Level mathematics. Graphing calculators are expected for all other exams.
- England has a history of innovations in mathematics education. Detailed consideration of the topics and applications studied within traditional algebra and calculus units would be useful.

[4] Some would say this is only an extra six months because of the way the academic year runs September to June in the northern hemisphere.

- The ongoing debate about the place of statistics in school mathematics is relevant to the NSW situation.
- A major report on Post-14 Mathematics Education in the UK (Smith, 2004) has relevance to the NSW situation because of shared cultural backgrounds and some shared issues such as declining participation in the more demanding mathematics courses, and a shortage of mathematics teachers.

THE UNITED STATES OF AMERICA - USA

Context for the study of mathematics in the senior secondary school

This report begins with overall descriptions from O'Donnell (2004), especially page 41 from which this summary was constructed:

> School education is the responsibility of individual states. The United States Congress has enacted legislation which affects States, communities and schools such as the "No Child Left Behind" national education reform strategy. Education is usually structured as primary (ages 5+ to 8), Intermediate (ages 8-12), junior high (ages 12-15) and senior high (ages 16-18). There are voluntary national tests during senior high school education including Scholastic Aptitude Tests (SATs), American College Testing (ACT) and Advanced Placement (AP) examinations, which all assess suitability for higher education.

Content of mathematics curriculum at senior high level

In the Stacey Review, on which this current review is modeled, the point was made that Calculus was not a mainstream subject in most high schools in the USA. However, to make comparisons with senior high school mathematics courses in Australia, those authors reviewed instead the Advanced Placement (AP) courses Calculus AB, Calculus BC and AP Statistics. These are courses offered by The College Board and taught in High Schools across the USA and also in other countries.

The following descriptions of the AP programs were obtained from the website of The College Board.
http://www.collegeboard.com/student/testing/ap/calculus_ab/topic.html?calcab

Calculus AB

This is designed as a full year course for students who have completed four years of secondary mathematics designed for college-bound students. In those courses topics studied should be algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include those that are linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise defined. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions of the numbers 0, pi/6, pi/4, pi/3, pi/2, and their multiples.

Course Goals (Calculus AB)

Students should be able to:

- work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- understand the meaning of the derivative in terms of a rate of change and local linear approximation and they should be able to use derivatives to solve a variety of problems.
- understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and should be able to use integrals to solve a variety of problems.

- understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.

- communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.

- model a written description of a physical situation with a function, a differential equation, or an integral.

- use technology to help solve problems, experiment, interpret results, and verify conclusions.

- determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.

- develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

List of topics (Calculus AB)

I. Functions, Graphs, and Limits

A. Analysis of Graphs

With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

B. Limits of Functions (incl. one-sided limits)

An intuitive understanding of the limiting process.
Calculating limits using algebra.
Estimating limits from graphs or tables of data.

C. Asymptotic and Unbounded Behavior

Understanding asymptotes in terms of graphical behavior.
Describing asymptotic behavior in terms of limits involving infinity.
Comparing relative magnitudes of functions and their rates of change. (For example, contrasting exponential growth, polynomial growth, and logarithmic growth.)

D. Continuity as a Property of Functions

An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
Understanding continuity in terms of limits.
Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

II. Derivatives

A. Concept of the Derivative

Derivative presented graphically, numerically, and analytically.
Derivative interpreted as an instantaneous rate of change.
Derivative defined as the limit of the difference quotient.
Relationship between differentiability and continuity.

B. Derivative at a Point

Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
Tangent line to a curve at a point and local linear approximation.
Instantaneous rate of change as the limit of average rate of change.

Approximate rate of change from graphs and tables of values.

C. Derivative as a Function

Corresponding characteristics of graphs of f and f '.
Relationship between the increasing and decreasing behavior of f and the sign of f '.
The Mean Value Theorem and its geometric consequences.
Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

D. Second Derivatives

Corresponding characteristics of the graphs of f, f ', and f ".
Relationship between the concavity of f and the sign of f ".
Points of inflection as places where concavity changes.

E. Applications of Derivatives

Analysis of curves, including the notions of monotonicity and concavity.
Optimization, both absolute (global) and relative (local) extrema.
Modeling rates of change, including related rates problems.
Use of implicit differentiation to find the derivative of an inverse function.
Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

F. Computation of Derivatives

Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
Basic rules for the derivative of sums, products, and quotients of functions.
Chain rule and implicit differentiation.

III. Integrals

A. Interpretations and Properties of Definite Integrals

Definite integral as a limit of Riemann sums.
Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f'(x)dx = f(b) - f(a)$

Basic properties of definite integrals. (Examples include additivity and linearity.)

B. Applications of Integrals

Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to give accumulated change or using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, and the distance traveled by a particle along a line.

C. Fundamental Theorem of Calculus

Use of the Fundamental Theorem to evaluate definite integrals.
Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

D. Techniques of Antidifferentiation

Antiderivatives following directly from derivatives of basic functions.
Antiderivatives by substitution of variables (including change of limits for definite integrals).

E. Applications of Antidifferentiation

Finding specific antiderivatives using initial conditions, including applications to motion along a line.
Solving separable differential equations and using them in modeling. In particular, studying the equation y' = ky and exponential growth.

F. Numerical Approximations to Definite Integrals

Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

External exam Calculus AB

There is a three hour 15 minute exam.

Multiple Choice 105 minutes	Free Response 90 minutes
Part A 55 minutes 28 questions No calculator	Part A 45 minutes 3 questions with calculator
Part B 50 Minutes 17 questions Graphing Calculator	Part B 45 minutes 3 questions No calculator
50% of exam mark	50% of exam mark

Calculus BC

This course has the same prerequisites and goals as Calculus AB.
Calculus BC is a full-year course in the calculus of functions of a single variable. It includes all topics covered in Calculus AB plus additional topics. Both courses represent college-level mathematics for which most colleges grant advanced placement and credit. The content of Calculus BC is designed to qualify the student for placement and credit in a course that is one course beyond that granted for Calculus AB.

Additional topics in Calculus BC

Parametric, Polar, and Vector Functions
The analysis of planar curves includes those given in parametric form, polar form, and vector form.
Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration.

Numerical solution of differential equations using Euler's method.
L'Hôpital's Rule, including its use in determining limits and convergence of improper integrals and series.

Derivatives of parametric, polar, and vector functions.

Applications of Integrals

Appropriate integrals are used in a variety of applications to model physical, social, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and for BC only the length of a curve (including a curve given in parametric form).

Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only).

Improper integrals (as limits of definite integrals).

Solving logistic differential equations and using them in modeling.

Polynomial Approximations and Series

Concept of Series

A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence or divergence.

Series of constants

Motivating examples, including decimal expansion.

Geometric series with applications.

The harmonic series.

Alternating series with error bound.

Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series.

The ratio test for convergence and divergence.

Comparing series to test for convergence or divergence.

Taylor Series

Taylor polynomial approximation with graphical demonstration of convergence. (For example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve.)

Maclaurin series and the general Taylor series centered at x = a.

Maclaurin series for the functions ex, sin x, cos x, and 1/(1-x).

Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series.

Functions defined by power series.

Radius and interval of convergence of power series.

Lagrange error bound for Taylor polynomials.

External exam Calculus BC

There is a three hour 15 minute exam with the same structure as the exam for Calculus AB.

AP Statistics

The purpose of this course is to introduce students to the major concepts and tools for collecting, analyzing, and drawing conclusions from data. Students are exposed to four broad conceptual themes:
Exploring Data: Describing patterns and departures from patterns
Sampling and Experimentation: Planning and conducting a study
Anticipating Patterns: Exploring random phenomena using probability and simulation
Statistical Inference: Estimating population parameters and testing hypotheses

Students who successfully complete the course and examination may receive credit and/or advanced placement for a one-semester introductory college statistics course. This does not necessarily imply that the high school course should be one semester long. Each high school will need to determine the length of time for its AP Statistics course to best serve the needs of its students. Statistics, like some other AP courses, could be effectively studied in a one-semester, a two-trimester, or a one-year course. Most schools, however, offer it as a two-semester course.

Content (AP Statistics)

I. Exploring Data: Describing patterns and departures from patterns (20%-30%)

Exploratory analysis of data makes use of graphical and numerical techniques to study patterns and departures from patterns. Emphasis should be placed on interpreting information from graphical and numerical displays and summaries.
Constructing and interpreting graphical displays of distributions of univariate data (dotplot, stemplot, histogram, cumulative frequency plot)
Center and spread
Clusters and gaps
Outliers and other unusual features
Shape
Summarizing distributions of univariate data
Measuring center: median, mean
Measuring spread: range, interquartile range, standard deviation
Measuring position: quartiles, percentiles, standardized scores (z-scores)
Using boxplots
The effect of changing units on summary measures
Comparing distributions of univariate data (dotplots, back-to-back stemplots, parallel boxplots)
Comparing center and spread: within group, between group variation
Comparing clusters and gaps
Comparing outliers and other unusual features
Comparing shapes
Exploring bivariate data
Analyzing patterns in scatterplots
Correlation and linearity
Least-squares regression line
Residual plots, outliers, and influential points
Transformations to achieve linearity: logarithmic and power transformations
Exploring categorical data
Frequency tables and bar charts
Marginal and joint frequencies for two-way tables
Conditional relative frequencies and association
Comparing distributions using bar charts

II. Sampling and Experimentation: Planning and conducting a study (10%-15%)
Data must be collected according to a well-developed plan if valid information on a conjecture is to be obtained. This plan includes clarifying the question and deciding upon a method of data collection and analysis.

Overview of methods of data collection
Census
Sample survey
Experiment
Observational study
Planning and conducting surveys
Characteristics of a well-designed and well-conducted survey
Populations, samples, and random selection
Sources of bias in sampling and surveys
Sampling methods, including simple random sampling, stratified random sampling, and cluster sampling
Planning and conducting experiments
Characteristics of a well-designed and well-conducted experiment
Treatments, control groups, experimental units, random assignments, and replication
Sources of bias and confounding, including placebo effect and blinding
Completely randomized design
Randomized block design, including matched pairs design
Generalizability of results and types of conclusions that can be drawn from observational studies, experiments, and surveys

III. Anticipating Patterns: Exploring random phenomena using probability and simulation (20%-30%)

Probability is the tool used for anticipating what the distribution of data should look like under a given model.
Probability
Interpreting probability, including long-run relative frequency interpretation
'Law of Large Numbers' concept
Addition rule, multiplication rule, conditional probability, and independence
Discrete random variables and their probability distributions, including binomial and geometric
Simulation of random behavior and probability distributions
Mean (expected value) and standard deviation of a random variable, and linear transformation of a random variable
Combining independent random variables
Notion of independence versus dependence
Mean and standard deviation for sums and differences of independent random variables
The normal distribution
Properties of the normal distribution
Using tables of the normal distribution
The normal distribution as a model for measurements
Sampling distributions
Sampling distribution of a sample proportion
Sampling distribution of a sample mean
Central Limit Theorem
Sampling distribution of a difference between two independent sample proportions
Sampling distribution of a difference between two independent sample means
Simulation of sampling distributions

t-distribution
Chi-square distribution

IV. Statistical Inference: Estimating population parameters and testing hypotheses (30%-40%)

Statistical inference guides the selection of appropriate models.
Estimation (point estimators and confidence intervals)
Estimating population parameters and margins of error
Properties of point estimators, including unbiasedness and variability
Logic of confidence intervals, meaning of confidence level and confidence intervals, and properties of confidence intervals
Large sample confidence interval for a proportion
Large sample confidence interval for a difference between two proportions
Confidence interval for a mean
Confidence interval for a difference between two means (unpaired and paired)
Confidence interval for the slope of a least-squares regression line

Exam for AP Statistics.

The three-hour test includes a 90-minute multiple-choice section and a 90-minute free-response section. These sections count equally to the final grade. A graphing calculator with statistical capabilities is required.

Section I: Multiple-Choice
The multiple-choice section tests proficiency on a wide variety of topics.
Section II: Free-Response
The free-response section tests analytical, organizational, and communication skills through five questions and an investigative task.

Points of comparison with NSW courses

- In terms of structure, there are few points of comparison since the traditional four-year high school mathematics curriculum in the USA is a differentiated curriculum – one year spent on each of the areas Algebra, Geometry, Algebra and Trigonometry, then Pre-Calculus. Various integrated schemes are available but pressure of time prevents a full discussion.
- The use of graphing calculators has been approved for the AP courses and exams since the mid 1990s

CONCLUSION

In preparing this analysis, I found that the voice of students was not often heard through the lists of content and descriptions of assessment regimes, nor was there much description of teachers and their work.

In the section on Hong Kong there are a few lines that I will repeat here to bring the report to a close:

- Students saw mathematics as a set of rules, hated tedious calculations, hoped for "liveliness" and real life applications both in teaching and in textbooks.
- Students see a good mathematics teacher as one "who is nice, lively, provides a variety of activities and offers clear, step-by-step explanation, who allows time for students to think, checks frequently to see if students understand from time to time, explains how to approach problems and would not penalize weaker students." (Wong et al, 1999, p. 3).

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Appendices

Appendix 1 Course descriptions for NSW subjects

Course: General Mathematics **Course No:** 15230

2 units for each of Preliminary and HSC
Board Developed Course

Prerequisites: The course is constructed on the assumption that students have achieved the outcomes in the core of the Standard Mathematics course for the School Certificate, together with the recommended options *Trigonometry* and *Further Algebra*.

Exclusions: Students may **not** study any other Stage 6 Mathematics course in conjunction with General Mathematics.

Course Description

General Mathematics focuses on mathematical skills and techniques which have direct application to everyday activity. The course content is written in five areas of study, with an emphasis on application of specific skills and on tasks that involve integrating mathematical skills and techniques across a range of familiar and unfamiliar situations. These tasks may draw from more than one area of study, and encourage transfer of knowledge across the entire course, as well as linking with study in other Stage 6 subjects.

The course is fully prescribed, and is designed to support TAFE and other vocational courses. It provides an appropriate mathematical background for students who do not wish to pursue the formal study of mathematics at tertiary level, while giving a strong foundation for university study in the areas of business, humanities, nursing and paramedical sciences.

Main Topics Covered

Preliminary Course

- Financial Mathematics
- Data Analysis
- Measurement
- Probability
- Algebraic Modelling

HSC Course

- Financial Mathematics
- Data Analysis
- Measurement
- Probability
- Algebraic Modelling

External Assessment	Weighting	Internal Assessment	Weighting
A single HSC examination of two and one half hours' duration. No more than 30% of the examination will be based on the Preliminary course. Questions based on the Preliminary course can also be asked when they lead in to questions based on the HSC course. Marks from these lead-in questions will not be counted in the 30% Preliminary allowance. Calculators, including graphics calculators, that meet Board requirements (as advised through the Official Notices section of the *Board Bulletin*) may be used. Geometrical instruments and approved geometrical templates may be used.	100	A variety of assessment tasks across all of the content of the course. Once the assessment of the HSC course has commenced, some Preliminary course work can be included in assessment tasks for General Mathematics. No more than 30% of the assessment is to be based on the Preliminary course.	100
	100		100

Course: Mathematics	**Course No:** 15240

2 units for each of Preliminary and HSC
Board Developed Course
Prerequisites: The course is constructed on the assumption that students have achieved the outcomes in the core of the Intermediate Mathematics course for the School Certificate, along with the recommended options.
Exclusions: General Mathematics

Course Description
The course is intended to give students who have demonstrated general competence in the skills of Stage 5 Mathematics an understanding of and competence in some further aspects of mathematics which are applicable to the real world. It has general educational merit and is also useful for concurrent studies in science and commerce. The course is a sufficient basis for further studies in mathematics as a minor discipline at tertiary level in support of courses such as the life sciences or commerce. Students who require substantial mathematics at a tertiary level, supporting the physical sciences, computer science or engineering, should undertake the Mathematics Extension 1 course or both the Mathematics Extension 1 and Mathematics Extension 2 courses.

Main Topics Covered

Preliminary Course
- Basic arithmetic and algebra
- Real functions
- Trigonometric ratios
- Linear functions
- The quadratic polynomial and the parabola
- Plane geometry – geometrical properties
- Tangent to a curve and derivative of a function

HSC Course
- Coordinate methods in geometry
- Applications of geometrical properties
- Geometrical applications of differentiation
- Integration
- Trigonometric functions
- Logarithmic and exponential functions
- Applications of calculus to the physical world
- Probability
- Series and series applications

External Assessment

A single written examination paper of three hours duration, consisting of ten questions of equal value.

No more than the equivalent of two questions will be based on the Preliminary course. Questions from the Preliminary course will be short and represent a minor part of a total question. Marks can be awarded for demonstration of knowledge and skills from the Preliminary course (or earlier) when required for questions on the HSC course. That is, questions based on the Preliminary course can be asked when they lead in to questions based on topics from the HSC course. Marks from these lead-in questions will not be counted in the two-question allowance from the Preliminary course.

Board-approved calculators, geometrical instruments and approved geometrical templates may be used.

Internal Assessment

The objectives of the course are grouped into two components, Component A and Component B, for assessment purposes. Component A (80%) is primarily concerned with the student's knowledge, understanding and skills developed in each Content Area listed in the syllabus. Component B (20%) is primarily concerned with the student's reasoning, interpretative, explanatory and communicative abilities. A number of tasks will be used to determine a student's school-based assessment and any one task may contribute to measuring attainment in both components.

Once the assessment of the HSC course has commenced, some Preliminary course work can be included in assessment tasks for Mathematics. No more than 20% of the assessment is to be based on the Preliminary course.

Course: Mathematics Extension 1	**Course No:** 15250

1 unit in each of Preliminary *(Preliminary Mathematics Extension)* and HSC
Board Developed Course

Prerequisites: The course is constructed on the assumption that students have achieved the outcomes in the core of the Advanced Mathematics course for the School Certificate, along with the recommended options.

Exclusions: General Mathematics

Course Description

The content of this course and its depth of treatment indicate that it is intended for students who have demonstrated a mastery of the skills of Stage 5 Mathematics and are interested in the study of further skills and ideas in mathematics. The course is intended to give these students a thorough understanding of and competence in aspects of mathematics, including many which are applicable to the real world. It has general educational merit and is also useful for concurrent studies of science, industrial arts and commerce. The course is a recommended minimum basis for further studies in mathematics as a major discipline at a tertiary level and for the study of mathematics in support of the physical and engineering sciences. Although the course is sufficient for these purposes, students of outstanding mathematical ability should consider undertaking the Mathematics Extension 2 course.

Main Topics Covered

Preliminary Course

- Other inequalities
- Further geometry
- Further trigonometry
- Angles between two lines
- Internal and external division of lines into given ratios
- Parametric representation
- Permutations and combinations
- Polynomials
- Harder applications of the Mathematics Preliminary course topics

HSC Course

- Methods of integration
- Primitive of $\sin^2 x$ and $\cos^2 x$
- Equation $\frac{dN}{dt} = k(N - P)$
- Velocity and acceleration as a function of x
- Projectile motion
- Simple harmonic motion
- Inverse functions and inverse trigonometric functions
- Induction
- Binomial theorem
- Further probability
- Iterative methods for numerical estimation of the roots of a polynomial equation
- Harder applications of Mathematics HSC course topics

External Assessment	Internal Assessment
Two written examination papers. One paper is the Mathematics course paper and is of three hours duration. The other paper, of two hours duration, is based on the Mathematics Extension 1 course and consists of seven questions of equal value. No more than the equivalent of two questions will be based on the Preliminary course. Questions from the Preliminary course will be short and represent a minor part of a total question. Marks can be awarded for demonstration of knowledge and skills from the Preliminary course (or earlier) when required for questions on the HSC course. That is, questions based on the Preliminary course can be asked when they lead in to questions based on topics from the HSC course. Marks from these lead-in questions will not be counted in the two-question allowance from the Preliminary course. Board-approved calculators, geometrical instruments	The objectives of the course are grouped into two components, Component A and Component B, for assessment purposes. Component A (70%) is primarily concerned with the student's knowledge, understanding and skills developed in each Content Area listed in the syllabus. Component B (30%) is primarily concerned with the student's reasoning, interpretative, explanatory and communicative abilities. A number of tasks will be used to determine a student's school-based assessment and any one task may contribute to measuring attainment in both components. School assessment for the Mathematics Extension 1 HSC course can be based on the whole of the Mathematics Extension 1 course (Preliminary and HSC courses). Assessment for this course should not begin until the school program of HSC assessments for other subjects begins (this is

and approved geometrical templates may be used.	usually no earlier than Term 4 of Year 11).

Course: Mathematics Extension 2	**Course No:** 15260
1 unit for the HSC Board Developed Course The course is designed for students with a special interest in mathematics who have shown that they possess special aptitude for the subject. **Exclusions:** General Mathematics	
Course Description The course offers a suitable preparation for study of mathematics at tertiary level, as well as a deeper and more extensive treatment of certain topics than is offered in other mathematics courses. It represents a distinctly high level in school mathematics involving the development of considerable manipulative skill and a high degree of understanding of the fundamental ideas of algebra and calculus. These topics are treated in some depth. Thus, the course provides a sufficient basis for a wide range of useful applications of mathematics as well as an adequate foundation for the further study of the subject.	
Main Topics Covered ▪ Graphs ▪ Complex Numbers ▪ Conics ▪ Integration ▪ Volumes ▪ Mechanics ▪ Polynomials ▪ Harder Mathematics Extension 1 topics	
External Assessment	**Internal Assessment**
Two written examination papers. One paper is the Mathematics Extension 1 course paper and is of two hours duration. The other paper, of three hours duration, is based on the Mathematics Extension 2 course and consists of eight questions of equal value. Board-approved calculators, geometrical instruments and approved geometrical templates may be used.	The objectives of the course are grouped into two components, Component A and Component B, for assessment purposes. Component A (60%) is primarily concerned with the student's knowledge, understanding and skills developed in each Content Area listed in the syllabus. Component B (40%) is primarily concerned with the student's reasoning, interpretative, explanatory and communicative abilities. A number of tasks will be used to determine a student's school-based assessment and any one task may contribute to measuring attainment in both components.

Appendix 2 Outcomes statement for Mathematics, Mathematics Extension 1 and Mathematics Extension 2. (See following pages).

Mathematics, Mathematics Extension 1 and Mathematics Extension 2

Course Outcomes

The current calculus-based mathematics courses will remain unchanged in the introductory years of the New Higher School Certificate. During this time the course content and internal assessment arrangements of *2 Unit*, *3 Unit* and *4 Unit Mathematics* will be maintained. (See *Mathematics 2/3 Unit Syllabus* and *Mathematics 4 Unit Syllabus*.) However, under the new HSC structure, the courses will be called *Mathematics, Mathematics Extension 1* and *Mathematics Extension 2,* respectively.

The HSC results of students studying these courses will be reported using the standards-referencing procedures in place for all Board developed courses under the new structure.

Following are the outcomes developed for the *Mathematics, Mathematics Extension 1* and *Mathematics Extension 2* courses. The outcomes have been derived from the content of the courses, and together with the content, determine the breadth and depth of study to be undertaken by students.

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Mathematics, Mathematics Extension 1 and Mathematics Extension 2 Outcomes

	Mathematics		*Mathematics Extension 1*		**Mathematics Extension 2**
Objectives	**Preliminary Outcomes**	**HSC Outcomes**	**Preliminary Outcomes**	**HSC Outcomes**	**HSC Outcomes**
Students will develop:	**A student:**	**A student:**	A student:	**A student:**	**A student:**
• **appreciation of the scope, usefulness, beauty and elegance of mathematics**	**P1** • **demonstrates confidence in using mathematics to obtain realistic solutions to problems**	**H1** • **seeks to apply mathematical techniques to problems in a wide range of practical contexts**	**PE1** • **appreciates the role of mathematics in the solution of practical problems**	**HE1** • **appreciates interrelationships between ideas drawn from different areas of mathematics**	**E1** • **appreciates the creativity, power and usefulness of mathematics to solve a broad range of problems**
• **the ability to reason in a broad range of mathematical contexts**	P2 • **provides reasoning to support conclusions which are appropriate to the context**	**H2** • **constructs arguments to prove and justify results**	**PE2** • **uses multi-step deductive reasoning in a variety of contexts**	**HE2** • **uses inductive reasoning in the construction of proofs**	**E2** • **chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings**

Objectives	Preliminary Outcomes	HSC Outcomes	Preliminary Outcomes	HSC Outcomes	HSC Outcomes
Students will develop:	A student:	A student:	A student:	A student:	A student:
• **skills in applying mathematical techniques to the solution of practical problems**	**P3** • **performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities** P4 • chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques	H3 • **manipulates algebraic expressions involving logarithmic and exponential functions** H4 • **expresses practical problems in mathematical terms based on simple given models** H5 • applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems	**PE3** • **solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations**	**HE3** • **uses a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion, or exponential growth and decay**	**E3** • **uses the relationship between algebraic and geometric representations of complex numbers and of conic sections** E4 • uses efficient techniques for the algebraic manipulation required in dealing with questions such as those involving conic sections and polynomials E5 • uses ideas and techniques from calculus to solve problems in mechanics involving resolution of forces, resisted motion and circular motion

Objectives	Preliminary Outcomes	HSC Outcomes	Preliminary Outcomes	HSC Outcomes	HSC Outcomes
Students will develop:	A student:	A student:	A student:	A student:	A student:
• **understanding of the key concepts of calculus and the ability to differentiate and integrate a range of functions**	**P5** • **understands the concept of a function and the relationship between a function and its graph** P6 • relates the derivative of a function to the slope of its graph P7 • determines the derivative of a function through routine application of the rules of differentiation	**H6** • **uses the derivative to determine the features of the graph of a function** H7 • uses the features of a graph to deduce information about the derivative H8 • uses techniques of integration to calculate areas and volumes	**PE4** • uses the parametric representation together with differentiation to identify geometric properties of parabolas PE5 • determines derivatives which require the application of more than one rule of differentiation	HE4 • uses the relationship between functions, inverse functions and their derivatives HE5 • applies the chain rule to problems including those involving velocity and acceleration as functions of displacement HE6 • determines integrals by reduction to a standard form through a given substitution	E6 • combines the ideas of algebra and calculus to determine the important features of the graphs of a wide variety of functions E7 • uses the techniques of slicing and cylindrical shells to determine volumes E8 • applies further techniques of integration, including partial fractions, integration by parts and recurrence formulae, to problems

Objectives	Preliminary Outcomes	HSC Outcomes	Preliminary Outcomes	HSC Outcomes	HSC Outcomes
Students will develop:	A student:	A student:	A student:	A student:	A student:
• the ability to interpret and communicate mathematics in a variety of forms	**P8** • **understands and uses the language and notation of calculus**	**H9** • **communicates using mathematical language, notation, diagrams and graphs**	**PE6** • **makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations**	**HE7** • **evaluates mathematical solutions to problems and communicates them in an appropriate form**	**E9** • communicates abstract ideas and relationships using appropriate notation and logical argument