Literature Review and Annotated Bibliography to Inform the Stage 6 Mathematics Review

Prepared for the NSW Board of Studies

Dr Mary Coupland
Department of Mathematical Sciences
University of Technology, Sydney.

October 2006

LITERATURE REVIEW

This literature review focuses on selected current issues relevant to the teaching and learning of mathematics at the post-compulsory level, and is structured in four parts.

Part 1 provides a conceptual basis by outlining the theoretical bases of recent relevant research in mathematics education. Part 2 addresses the role of new technologies in the teaching and learning of mathematics in the post-compulsory years.[1] Part 3 investigates the place of statistics in mathematics courses at this level. Part 4 identifies the issues in mathematics education that are relevant to the current discussions about a proposed Australian Certificate of Education.

PART 1: RELEVANT THEORIES OF LEARNING

Research in education seldom "proves" that one approach (to teaching, to curriculum construction…) is to be favoured over another. An obvious reason for this is the complex nature of the phenomena that are studied in educational research. People disagree about how best to research complex social systems. In mathematics education research, there is no single agreed paradigm – researchers have adopted paradigms from more fundamental disciplines, often psychology and more recently sociology and anthropology.

To evaluate research in mathematics education, the paradigms within which the researchers choose to frame their research questions and construct their methodologies should be taken into account.

Recent empirical research in mathematics education, including the work reviewed here, can be regarded as being situated in one of two contrasting views of learning, labelled by Anna Sfard (1996) as "acquisitive" and "participatory" models. The "acquisitive" models include those based on information processing, which treat knowledge as an entity that can be acquired as concepts and skills through learning. These concepts and skills are assumed to be transferable across contexts. On the other hand, "participatory" models regard knowledge as a social construct, with the context being of paramount importance. Context here includes physical and social classroom conditions, the influence of the teacher, and the broader social setting of the learning situation, including what the student knows about reward for effort. It is assumed that much of the motivation for engaging with learning arises from these contexts.

Within the acquisitive model, research seeks to identify and describe the concepts and schema (Davis, 1984; Evans, 1991; Skemp, 1986), that individuals are assumed to construct as they seek to make personal meaning from their experiences. Skills are regarded as kinds of schemas that encode "knowing how" or procedural knowledge. (Bell, Costello, & Küchemann, 1983; Hiebert & Lefevre, 1986). The mechanism of metacognition provides direction to the process of concept acquisition. Metacognition is described by Kilpatrick (1986, p. 13) as "knowledge about how one thinks, knowledge of

[1] The material in Part 2 is developed from Chapter 2 of Coupland (2004).

how one is thinking at the moment (monitoring), and control over one's thinking". Schoenfeld (1987) expands this definition of metacognition for the purposes of mathematics education to include the beliefs and intuitions about mathematics that a person brings to their study, and how those beliefs shape the way that mathematical work is undertaken (p. 190). Models of metacognitive control (Zimmerman & Schunk, 1989) include various theories of self-regulated learning that attempt to explain student motivation as well as learning.

In the acquisitive model of learning, the emphasis is on the individual learner and on the individual control that the learner can exercise over their actions: deciding whether to engage with a task and how much persistent effort will be devoted to it. The social context of the learner, and issues around who chooses the tasks and why they are chosen, are largely unquestioned. It is also assumed that learning is accessible to the research methods of the "hard sciences", which bring with them the positivist view that it is possible to describe events and systems unambiguously through the correct choice of variables and appropriate data collection and analysis.

Keeves and Stacey (1999) note that in recent years there has been "an emerging awareness that mathematics education is influenced by the social and cultural context in which it takes place and that the social and cultural characteristics of teachers and students affect outcomes" (p. 209). Stephen Lerman (1994) points out that a theory of learning that takes the cognizing individual as the central element cannot account for the communications between people that are necessary to assess the viability of one's personal constructions of meaning.

Related to this new awareness of the importance of social and cultural contexts is the challenge to positivism from the alternative paradigms offered by the interpretive view and by critical theory (Kanes & Atweh 1993), so that views of mathematics education are changing along with views of how research in the field should be conducted.

PART 2: RESEARCH CONCERNING TEACHING AND LEARNING WITH NEW TECHNOLOGIES, INCLUDING COMPUTER ALGEBRA SYSTEMS (CAS) AND GRAPHING DISPLAY CALCULATORS (GDCs)

This part will be structured as follows:
(1) Brief descriptions of the relevant hardware and software.
(2) How new learning environments can be created using the dynamic visual features of new technology.
(3) Experimental and quasi-experimental studies of learning in technology rich environments.
(4) Research within the acquisitive view of learning that aims to build theory.
(5) Research within the participatory view of learning.

A brief description (Graphing display calculators - GDC)

Graphing display calculators or graphing or graphics calculators are battery operated machines with a multi-line text display that can also display graphic objects such as statistical plots and graphs on Cartesian axes. Some are also capable of handling matrix algebra and numerical equation solving. Most have the capability of handling tables of data and some have a spreadsheet capability. Figure 1 shows the Casio fx 9860G AU - graphics calculator

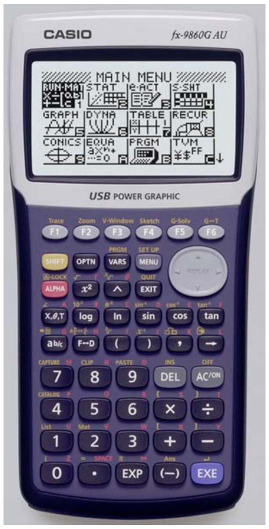


Figure 1. Casio fx 9860G graphing calculator.

A brief description (CAS)

In general, computer algebra systems (CAS) are tools built by and for mathematicians. The software Derive was released on October 1988 to run on PC compatible computers and was a rewrite of the software muMATH, itself released in 1979 for 8080 and Z80 computers, and Radio Shack TRS-80 computers. Before that, CAS were only available on large mainframe computers (Kaineder, 2003). Texas Instruments produced the TI-92, (see Figures 2, 3), a hand-held calculator with built-in Geometry, graphing, and CAS software for the school and college market in the USA. Texas Instruments and the authors of the Derive program jointly developed the CAS software on the TI-92. A main feature is the QWERTY keyboard, which makes this machine more like a mini-computer than a calculator.

Figure 2. TI-92 Calculator

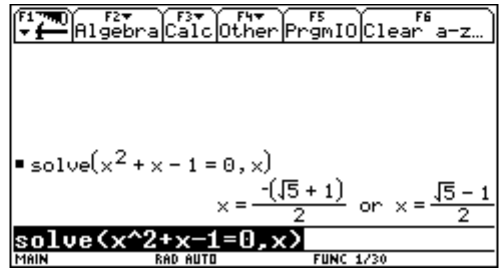


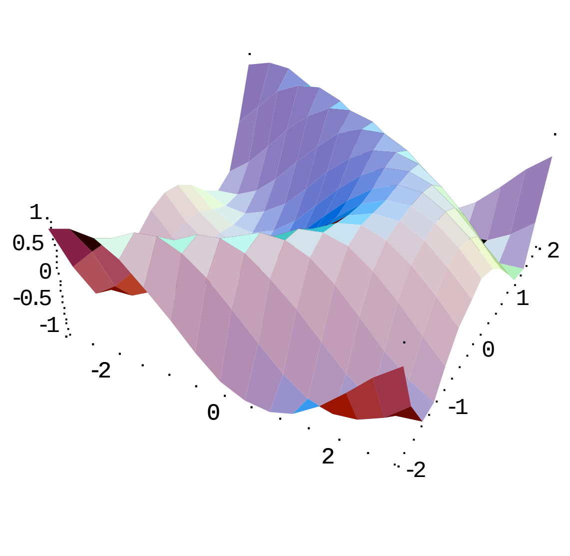
Figure 3. Screen of TI-92 running Derive software

The compact size and low price of these machines have contributed to their widespread introduction in some countries and in some states within other countries. More recent developments include the ClassPad 300 described on the Casio website as follows:

Figure 4. Casio ClassPad 300.

The ClassPad 300 is the natural evolution of the graphics calculator. It comes packed with an impressive collection of applications that support self-study, like 3D Graph, Geometry, eActivity, and lots more. A big 160 x 240-dot LCD touch screen enables easy and intuitive stylus-based operation. The ClassPad 300 lets you input expressions and displays expressions as they appear in a real life. Factorization of expressions, calculation of limit values of functions, and other operations can be performed quickly and easily while viewing the results on a large LCD screen. (http://www.casioed.net.au/products/classpad_product.php)

The CAS software *Mathematica* was released as Version 1.0 in June 1988, and as at 2006 version 5.2 available. The system handles numerical, algebraic, graphical and programming tasks required by engineers, mathematicians, scientists, and students. See Figures 5 and 6 for samples of *Mathematica* output. Other CAS includes Maple and Matlab.



$\int_0^1 \frac{\ }{1+x^2}\, dx$

$\frac{\text{p}}{4}$

Expand[… $\frac{\ }{b}$ … ^4]

$a^4 + \frac{1}{b^4} + \frac{4\,a}{b^3} + \frac{6\,a^2}{b^2} + \frac{4\,a^3}{b}$

Figures 5 and 6: Samples of *Mathematica* input and output

A brief description (interactive, dynamic software)

This classification includes geometric software such as Cabri-geometre (Laborde, 1995), and the Geometric Supposer (Yerushalmy & Houde, 1986). This software is built on the ability of computer software to capture and replay a student's actions. For example, a triangle can be drawn on screen, the angles measured and the sum displayed, then the triangle can be deformed into other triangles: what the student sees is that the size of the triangle’s angles change but their sum remains the same – 180 degrees.

How new learning environments can be created using the dynamic visual features of new technology

The dynamic nature of computer-generated images is an essential part of demonstrating variance. This is well explained by Kaput (1992, p. 525, original emphasis):

> One very important aspect of mathematical thinking is the abstraction of invariance. But, of course, to recognize invariance - to see what stays the same - one must have variation. *Dynamic media inherently make variation easier to achieve.*

With *Mathematica*, students can construct for themselves short animations that show how a graph changes as one or more parameters changes – a series of pictures is produced and then animated at the touch of a button. Other possibilities include the incorporation of “slidergraphs” into screens. As the student moves with the mouse the position of a slider on a scale, the value of a parameter changes and a graph takes a new shape. The idea of using the power of CAS software to create new representations and even complete microworlds is very appealing to mathematicians and to mathematics educators. Using a graphing calculator or spreadsheet software, students can be encouraged to work with multiple representations of functions: moving from a formula to a table to a graph. A simple home-made example that I prepared for my own students with the spreadsheet program Excel is provided in Figure 7. My intention was to provide students with the opportunity to experiment with the parameters in an equation and see the corresponding change in the shape of the graph. There are two functions so that students can vary one variable at a time if they wish, and compare the results.

LOGISTIC FUNCTION GRAPHER

The general format is y = K / (1 + b*EXP(- k*x)).

Change the values for K, b, and k to see how the graph changes.

BLUE GRAPH

K= 90 b= 10 k= 0.3

PINK GRAPH

K= 60 b= 4 k= 0.3

x	y	
0	8.18	12.00
1	10.70	15.14
2	13.87	18.78
3	17.77	22.85
4	22.43	27.21
5	27.85	31.70
6	33.92	36.12
7	40.46	40.27
8	47.19	44.02
9	53.83	47.29
10	60.09	50.04
11	65.75	52.29
12	70.69	54.09
13	74.85	55.51
14	78.26	56.60
15	81.00	57.45
16	83.16	58.09
17	84.83	58.57
18	86.11	58.94
19	87.09	59.21
20	87.82	59.41

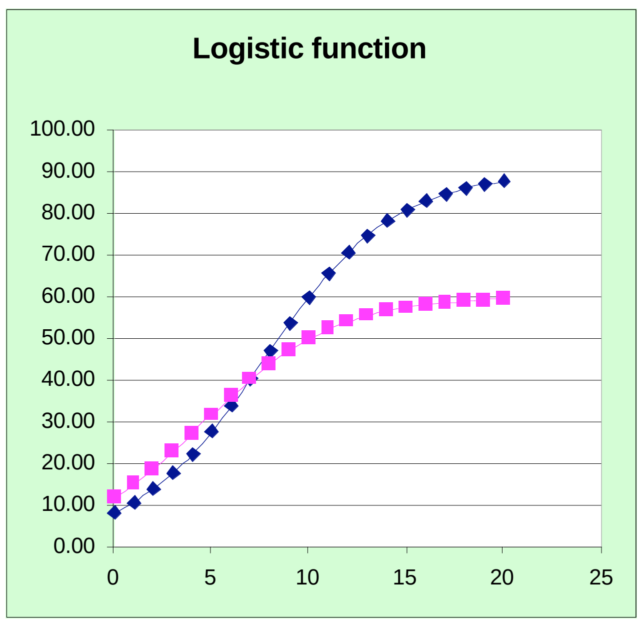


Figure 7. “Logistic function grapher” prepared by the author.

John Mason (1995) sounds a cautionary note about the place of screen images in his paper *Less may be more on a screen*. Mason points out “Just as manipulation of physical apparatus guarantees neither construal nor abstraction of mathematical ideas from that experience, so experience of screen images guarantees neither construal of nor abstraction from those images” (p. 119). Mason appeals for images that challenge or surprise, as they invoke the sense-making powers of the student, and for tasks that allow time and space for students to construe their own understandings of the ideas represented by screen images. This method promotes the use of images as starting points for mathematical investigations.

Images can also be new tools, replacing algebraic manipulations in many cases where numerical solutions are the desired result. For example, a CAS can solve differential equations numerically and graph the results for different parameters without the need for finding exact solutions in algebraic form (Bulmer, 2001). The questions to be asked here concern the place of those exact solutions in the standard curriculum. One of Bulmer’s examples is the solution of the initial value problem $\frac{dy}{dx} = -ky, \quad y(0) = 1$.
As he says, the solution can be found numerically and plotted for different values of the parameter *k*, without reference to the exponential function: but it is often the case that this initial value problem is used to *introduce* the exponential function, a standard component of calculus courses. This is a case where a mathematician sees a use for a tool from the point of view of an expert who already possesses knowledge of a kind that students lack. Eric Love (1995, p. 114) reminds us that “Expert users … project their previous experiences of paper-and-pencil mathematics on to the situations in the computer software and use these tools as surrogates for their previous manual techniques. Learners, of course, do not have this previous experience and thus have the double handicap of knowing neither under what circumstances they might use the tool, nor how it works”.

The work of the French researchers Guin, Trouche, and Lagrange (e.g. Guin & Trouche, 1998; Lagrange, 1999a, 1999b), considered later in this review, emphasises the need for teachers to draw students’ attention to the techniques of both pen and paper *and* the CAS to encourage the reflection that is necessary for learners to achieve the construction of schemes and concepts. It can be argued that both pen and paper and CAS are authentic tools of practising mathematicians, and induction is essential into the appropriate use of both, and appropriate use of combinations of both.

Innovators in the area of using new technologies in mathematics regularly claim that a significant advantage is the ability to broaden the scope of the applications of mathematics that can be considered. Data from the “real world” can be incorporated into learning tasks, and students can be freed from repetitive routine calculations in order to concentrate on higher level skills such as planning an investigation, and making and checking conjectures. Evidence that this can occur in mainstream mathematics subjects is provided by Forster, Mueller, Haimes, and Malone (2003). This team analysed the content of assessment items known as “extended pieces of work” designed at eight schools for the Western Australian subject *Applicable Mathematics*. In this subject, graphing calculators have been mandatory equipment since 1998. The conclusions were that

> …the main influences of the availability of graphics calculators on the design of the 'extended pieces of work' that we collected were requirements for conjecture, calling for extensive calculation and inclusion of practical applications that would not be feasible to solve without the technology. Also, a range of models in regression analysis and the random number generator for simulation were utilized. This, mandatory use of technology in Applicable Mathematics in Western Australia has widened the scope of approaches in 'extended pieces of work' in ways we see as potentially valuable, and these mainly related to possibilities for mathematical inquiry." (Forster et al, 2003, p. 358).

According to André Heck (2001), the pioneering work on the innovations around the introduction of CAS in schools has resulted in a consensus on the most important advantage of using computer algebra in mathematics education:

> Computer algebra has the potential of making mathematics more enjoyable for both teachers and pupils because it turns mathematical entities into concrete objects, which can be directly investigated, validated, manipulated, illustrated, and otherwise explored. … abstraction, exact reasoning, and careful use of symbolism are not solely a hobby of the mathematics teacher anymore, but are immediately rewarded when using computer algebra (p. 210).

This statement captures many of the achievements of innovators in this field. It also signals a shift in the ownership of the process of authentication that can happen with CAS use. Zehavi and Mann (1999) found that students using a CAS to solve word problems were able to check quickly whether or not their model made sense, and then reflect on their models: an advantage over the usual way of teaching "where pupils do not have any immediate control over their work and therefore have to wait until the teacher reacts to their solution before they start to reflect, if at all" (p. 66). *Of importance here is the fact that it is not the CAS use alone that carries the possibility of change, but the structure of learning tasks and the framing in the social context of the classroom of the activities that the students undertake.*

In the next section, a sample of experimental research is reported. Again, changes in the teaching and learning contexts often include more than the use of CAS. The availability of the CAS allowed for such changes, and this was exploited by innovative teachers.

Experimental and quasi-experimental studies of learning in technology rich environments

One of the most frequently cited reports of an experimental study comparing the learning of students taught with CAS to students taught in a traditional way is by Kathleen Heid (1988). The setting was a college in the USA, the students were majoring in business, architecture, and life science. In Heid's two experimental classes in introductory calculus (which she taught herself, 18 students in one class and 17 in another), concepts and applications were emphasised, with computers used for computing derivatives, graphing, and so on until the final three weeks of 15-week course. During those last three weeks the students learned the pen and paper algorithms that a comparison class had been learning all semester. The comparison class was larger (100 students) and was taught by a different instructor. The primary sources of data were transcripts of interviews with students, conceptual questions in a quiz taken before the three weeks on traditional pen and paper skills, and final examination results. The main conclusions were:

> Students from the experimental classes spoke about the concepts of calculus in more detail, with greater clarity, and with more flexibility than did students from the comparison group. They applied calculus concepts more appropriately and freely. As a group, the students in the experimental classes were better able than the students in the comparison class to answer conceptually oriented questions – an indication of a more refined ability to translate a mathematical concept from one representation to another. They performed almost as well on the final examination as the comparison class. Their performance was remarkably suggestive that compressed and minimal attention to skill development was not necessarily harmful, even on a skills test. (Heid, 1988, p. 21)

Students in Heid's experimental classes "felt the computer aided in their conceptual understanding by refocusing their attention" (p. 22), by relieving them of some manipulation tasks, by giving them results that they could rely on in when drawing conclusions, and by helping them to focus attention on more global aspects of problem solving.

Kenneth Ruthven (2002) has challenged the contrast of concepts and techniques that Heid emphasized in her report. He points out that the students in Heid's experimental group *were* engaged in practising techniques, but of a different kind from the conventional class. They worked on application problems. They also had more small group discussions. He claims that, when viewed from the French theory developed by (among others), Artigue and Lagrange (e.g. Artigue, 2001, 2002; Lagrange 1999a, 1999b) in which techniques have a broader scope than routine algorithms, the conceptual development of Heid's experimental group "grew out of new techniques constituted in response to this broader range of tasks, and from greater opportunities for the theoretical elaboration of these technique" (Ruthven, 2002, p. 284). This raises the issue that concepts in mathematics need not be regarded as "pure" and "abstract" mental abstractions.

Jeanette Palmiter (1991) also investigated the teaching of an early calculus course, this time on integration, a "second quarter" course. Unlike Heid's study in which allocation to the experimental classes was not random, 120 volunteers were randomly assigned to an experimental or control group. Other differences were that the students were in engineering courses, and those in the experimental group used the CAS for their homework and their final examination, which was identical to the exam taken by the control group. Both classes also sat for a pen and paper conceptual examination. For the control group, both exams were taken at the end of the ten week course. The experimental group took the exams after five weeks, so that the final five weeks could be spent on learning the pen and paper algorithms that the control group had studied. This was thought important since those skills would be assumed in later courses.

Palmiter (1991) found that the experimental class outscored the control group in both the conceptual and the computational exams. Students proceeding to later calculus courses from the experimental class fared "at least as well" as those from the control group (p. 155). Both Heid and Palmiter point out that the use of the computer algebra systems in their studies (MuMath in Heid, MACSYMA in Palmiter) was not the only difference in the experiences of their groups of students. The classes had different teachers, although in Palmiter's study some attempt was made to reduce the impact of this by having the classes taught by lecturers and teaching assistants of the same gender (female) with similar teaching reputations (highly regarded). The teachers met weekly to ensure that the same material with the same focus was being presented.

The work just described by Heid (1988) and Palmiter (1991) are examples of many studies conducted prior to 1995 using an experimental research design to investigate the effects of using CAS on the conceptual understanding of students in early calculus courses. More recently and in an Australian context, McCrae, Asp, and Kendal (1999) found that access to a CAS enabled students in year 11 to perform differentiations with the same level of success as year 12 students, and they obtained a better conceptual understanding than a comparison non-CAS year 11 class. Other studies are reviewed by David Meel (1998) who concludes that this body of research indicates that "CAS-integrated calculus curricula either minimally or significantly impacted student achievement on conceptually-oriented items in comparison to the achievement evidenced from performances by traditional calculus students on instruments designed specifically to gather this information" (p. 166).

Meel (1998) also notes the change in focus in research in this area, away from studying responses to common final examinations and attitude surveys, and towards "examining conceptual, procedural, and problem-solving differences between students of CAS-integrated calculus curricula with respect to students of traditional calculus curricula" (Meel, 1998, p. 165). Park and Travers (1996) do indeed include achievement tests and attitude surveys in their data collection, but an innovation is their use of student generated concept maps to assess conceptual understanding. Their "experimental" group were students in the *Calculus&Mathematica* [*sic*] course at the University of Illinois, in which lectures are replaced by laboratory work on prepared CAS files designed to introduce calculus ideas through experimentation in a largely visual and interactive environment (Ohio State University, 1998 - 2003). The concept maps prepared by these students scored more highly on measures of detail, number of concepts and cross-links, and congruence with instructors' concept maps than those of students in a comparison traditional calculus class.

Research within the acquisitive view of learning that aims to build theory

Guin and Trouche and Instrumental Genesis

Since 1980 all types of calculators have been freely used in French secondary school examinations and a corresponding interest is shown by the Ministry of Education and Technology in promoting their use in classrooms (Guin & Trouche, 1998). Dominique Guin and Luc Trouche (1998) describe the resistance of many teachers to this new technology, in spite of its official approval. In this context, their research suggests that without guidance from teachers about reading the images on a graphing display calculator screen students tend to confuse mathematical objects with their screen images, and can construct incorrect ideas about mathematical objects that the tools cannot show, for example the behaviour of functions for large values of the independent variable. Research by Guin and Trouche emphasises the distinction between a *tool* and an *instrument*, the latter being a psychological construct achieved when a person has obtained sufficient knowledge of the potentialities and constraints of an artefact in order to use the tool effectively in the process of an activity: in effect the person has appropriated the tool.

As said so clearly by Eric Love (1995, p. 114): “It is a mistake to think, especially in the teaching-learning situation, of tools as having some kind of existence outside of the contexts in which they are used” In Guin and Trouche’s terms, an instrument is more than a physical tool. They give an analysis of the constraints and potentialities of the graphic calculator with Derive and then show how a reorganisation of classroom dynamics can help students to make the necessary integrations to achieve what they call an *instrumental genesis.* They set out to develop situations that aim:

> to foster experimental work (investigation and anticipation) with interactions between graphic observations and theoretical calculus, and to encourage students to compare various results of different registers in order to tackle the distortion between the paper and machine environments, precisely because it is not a natural behaviour. This reflection is needed in order to seek mathematical consistency in various results and will motivate students to improve the mathematical knowledge required to overcome these contradictions (such as the distinction between approximate and exact calculation, control of numerical approximations, reflection on the unavoidable discretization of the screen and the nature of representatives and calculation algorithms). (Guin & Trouche, 1998, p. 208)

In these rearranged classrooms, students worked in pairs or groups of three on problems specially chosen to provide challenges and to bring out difficulties in the representations on the machines. The students were obliged to prepare written reports at this stage. Later the teacher, assisted by a “sherpa student” who operated a graphic calculator attached to a screen, orchestrated a class analysis of the problem. “The teacher’s role was to compare different strategies, pointing out the contribution of each group, and suggesting questions designed to make students discuss the various results found” (Guin & Trouche, 1998, p. 211). The authors developed a typology of five kinds of student behaviours with the graphic calculators, and found that students who used a *rational*, *resourceful* or *theoretical* work method were likely to make more progress over time than students using a *random* or *mechanical* work method. The authors make a case for changing the arrangement of classes to encourage student research, and conclude with practical points for teachers concerning technological issues.

Guin and Trouche (1998) have made significant progress towards finding arrangements for working in the classroom that encourage students to achieve *instrumental genesis*, that is the appropriation of the CAS as personal tools. Notably they achieve this through an arrangement of social and intellectual tasks that is not at all like the traditional didactic teaching model and also not a “one person one machine” model. A more complex role for the teacher is indicated.

The work of Guin and Trouche (1998) reported above has been used to introduce this section as it features the difficulties that many writers have observed in the introduction of CAS into classrooms. It is simply not the case that having access to the machines and the software leads *inevitably* to increased exploration and improved mathematical thinking on the part of students. Other researchers have reported these findings in their own research both with CAS and with graphing calculators. These difficulties include (i) technical difficulties, (ii) student attitudes, (iii) individual differences, (iv) role of the teacher, and (v) significance of the learning tasks.

(i) Technical difficulties in using CAS and GDCs.

Jean-Baptiste Lagrange has been investigating the use of CAS in French schools since 1994. In summarising this work he states “technical difficulties in the use of CAS replaced the usual difficulties that pupils encountered in paper/pencil calculations. Easier calculation did not automatically enhance students’ reflection and understanding” (Lagrange, 1999a, p. 6). Lagrange makes the point that conceptual reflection on techniques, not on tasks, is necessary for concept building, and that without the step of reflection, students know that their own understanding has not been enhanced: “so it appeared that many students did not consider problem solving using computer algebra as a convincing support of their understanding of mathematics, even when they liked it. They felt that their understanding developed from the techniques that they built in the ordinary context, and solving problem with CAS seemed to them very apart from these techniques.” (Lagrange, 1999a, p. 6). Lagrange proposes that the use of a CAS needs to be taught with an emphasis on its own techniques, to foster student reflection.

In the Netherlands, Paul Drijvers (1999, 2000, 2001) set out to explore the nature of obstacles that students encountered in using a CAS. Obstacles encountered by students using CAS include (Drijvers, 2000, p. 205):

- The difference between the algebraic representations provided by the CAS and those students expect and conceive as ‘simple’.
- The difference between numerical and algebraic calculations and the implicit way the CAS deals with this difference.
- The limitations of the CAS and the difficulty in providing algebraic strategies to help the CAS to overcome these limitations.
- The inability to decide when and how computer algebra can be useful.
- The flexible conception of variables and parameters that using a CAS requires.

The first of these is similar to the kind of technical difficulty described by Lagrange (1999a). As Drijvers notes, these obstacles have a technological component, but dealing adequately with them requires mathematical insight. This is where teachers have a special role, as other researchers have also concluded. Drijvers (2001, p. 226) agrees with Guin and Trouche (1998) about the role of "student interaction, classroom discussions and demonstrations" as an issue for teaching. It is in these discussions that students will find out how to "help the machine" when its limitations prevent it from giving an output when expected.

Lynda Ball (2001) also describes this aspect of the interplay between "by hand" manipulations guided and prompted by algebraic insight, and using the CAS to do what it *can* do. In order to use CAS successfully, students will need to have an algebraic sense of the various forms that equations may take, and know how to transform a given equation into one of those standard forms. Robyn Pierce and Kay Stacey (2001) give a framework for algebraic insight, which they are developing to guide the construction of CAS experiences for first year university students in an 'introduction to calculus' course. They define *algebraic insight* as "the algebraic knowledge and understanding which allows a student to correctly enter expressions into a CAS, efficiently scan the working and results for possible errors, and interpret the output as conventional mathematics" (Pierce & Stacey, 2001, p. 418). McRae, Asp and Kendal (1999) also conclude that "indications are that, paradoxically, a strong algebra facility is needed to get the best out of a CAS" (p. 364). Those authors had investigated the progress of three year 11 classes using CAS calculators in an introductory calculus course and used a year 11 (non-CAS) class, and a year 12 class for comparison. Lagrange (1999b), in describing teaching experiments with the TI-92 in year 11 French classrooms, makes the same point: "once more we see how good algebraic schemes are essential to be able to make sense of computer algebra output" (p. 75).

In responding to Drijvers' 1999 paper, Kaye Stacey (1999) states that research that she has been involved with in Melbourne has also found the obstacles listed by Drijvers. She notes a parallel with earlier work in the 1970s concerning the introduction of electronic calculators into schools (Etlinger, 1974, cited in Stacey, 1999), in that machines using CAS have enormous power "but the pedagogical opportunity often arises at the edge of this power" (Stacey, 1999, p. 55). This is the opportunity to improve students' mathematical understanding by working at things the CAS cannot do alone. However, the limitations can also be frustrations (p. 56).

The obstacles 2, 3, and 4 found by Drijvers have consequences for the use of CAS enabled calculators in high stakes assessment such as examinations. Ute Mueller and Patricia Forster (1999) analysed the way that students used graphing calculators with some CAS functions in the Western Australian Tertiary Entrance Examinations in 1998, the first year that the technology was allowed in the examinations. Although they concluded that calculator-based answers were not associated, in general, with higher (or lower) marks than traditional alternatives they identified several problem areas arising from the use of such technology: "foremost are the interpretation of graphical information and students' apparent uncertainty as to when use of graphics calculators is appropriate" (Mueller & Forster, 1999, p. 402). Examples included the misinterpreting of graphs when asymptotes were involved, reporting a definite integral as "35.9999" directly from a CAS display rather than giving the exact value of 36, not including sufficient reasoning in questions about limits, (tables of values showing a trend would have been accepted but students only gave two or three values), and failing to recognise errors that were caused by the limitations of storing very large or very small numbers in the calculator.

Misconceptions about function values due to the pixel-bound nature of graphical displays have been found in student learning with graphing display calculators (GDCs). Mitchelmore and Cavanagh (2000) report on a study designed to identify misconceptions (and causes for those misconceptions) that arise when students use GDCs to graph straight lines and parabolas. Twenty-five students with some experience of GDCs were interviewed as they performed a series of tasks designed to create problematic situations at the limits of the GDC display capabilities. For example, in the first of seven tasks the students were asked to draw a sketch of a quadratic function. The function was chosen so that in the default window of the GDC the portion of graph that was shown appeared as a straight line. In many of the tasks the students resorted to zooming in and reading the traced co-ordinate values displayed on the screen, without demonstrating an appreciation of scale. Summarising, Mitchelmore and Cavanagh (2000, p. 254) attribute student errors to four main causes: "…a tendency to accept the graphic image uncritically, without attempting to relate it to other symbolic or numerical information; a poor understanding of the concept of scale; an inadequate grasp of accuracy and approximation; and a limited grasp of the processes used by the calculator to display graphs". In a follow up study the authors noted that teachers participating in in-service instruction in the use of graphing calculators did acquire confidence and proficiency in structuring lessons that initially avoided problems with the graphing displays, and later, when students were more comfortable with the technology, used those problems to initiate discussion and provide opportunities for students to increase their mathematical understanding.

(ii) Student attitudes

A report by Galbraith, Pemberton, and Cretchley (2001) describes the learning experiences of students in two different institutions: the University of Queensland, (UQ), and the University of Southern Queensland, (USQ). The report compares results obtained from using the Galbraith and Haines scales (at UQ) with results from a different set of questions designed to measure Mathematics Confidence, Computer Confidence, Math-Tech Attitudes (attitudes to technology use in the learning of mathematics), and Math-Tech Experience (views on experience with software in learning mathematics), at USQ. Similarities in the structure of correlations were observed, with a very weak correlation between Mathematics Confidence and Computer Confidence, and Math-Tech attitude and Math-Tech Experience correlating more highly with Computer Confidence than with Mathematics Confidence. The pattern of pre-post scores was quite different between the two institutions, with students at UQ reporting a decline in both Mathematics and Computer Confidence *and* Motivation that was not evident at USQ. In attempting to explain these results, the descriptions of what the students actually did, and how the assessment experiences were structured, became essential. Here are the most telling sentences:

> For the UQ students the *Maple* environment was an effective gatekeeper to success in mathematics because of the central role it played in the program. Feelings about computing would likely be integrated with concern with success, even among the supremely competent, and it is most unlikely that such high stakes featured in their earlier computer experiences. … For the USQ students MATLAB was provided as a support, indeed a powerful support but not a gatekeeper to success because of the continuing priority accorded parallel approaches such as hand calculations. This meant that the computer power on offer had an element of choice, with students able to access it as they saw the opportunity and value in doing so. The students were in control. (Galbraith, Pemberton & Cretchley, 2001, p. 239)

Here we see the importance of context, both the assessment context, and the positioning of the CAS. In the different contexts, the CAS was seen by the students as a high-stakes hurdle on the one hand (UQ), and as a support in the development of pen and paper skills, which the students still saw as “real” mathematics, on the other hand (USQ).

(iii) Individual differences

As noted above, Guin and Trouche (1998) identified five kinds of student behaviours with graphic calculators. Their contrast of rational, resourceful and theoretical working styles with random and mechanical working styles could be applied to the descriptions in Weigand and Weller (2001), who used a program running in the background to capture the keystrokes and windows used by the students in their study. Working styles, of course, are not independent of the task, nor of the social environment of the classroom. Weigand and Weller found examples of students who were influenced by the requirements of the task, and the use of the computer, to think about mathematical functions, precisely because the software needed them to declare functions in order to draw objects on the screen. (One of the tasks required students to replicate a “ski-jump” by finding the equations of two parabolas.) Some students in their study created many graphs one after another, with little pause for reflection. Those authors also make the point that “It requires a teacher – or well-constructed materials – to help the student reflect upon or think over the computer results” (Weigand & Weller, 2001, p. 106).

(iv) The role of the teacher

Kendal and Stacey (2001) investigated the role of the teacher in influencing the use that students made of computer algebra systems and consequently the mix of conceptual and procedural knowledge acquired by students. They used Wertsch’s notion of “privileging” to describe a teacher’s individual way of teaching, including decisions about what is taught and how it is taught. Two teachers of parallel year 11 classes in introductory calculus were observed and interviewed, and their classes also took tests designed to assess competencies in numerical, graphical and symbolic aspects of differentiation. “Students of the teacher who privileged conceptual understanding and student construction of meaning were more able to interpret derivatives. Students of the teacher who privileged performance of routines made better use of the CAS for solving routine problems” (Kendal & Stacey, 2001, p. 143). This is entirely consistent with Ruthven’s (2002) interpretation of Heid’s 1988 research, as outlined earlier. From the acquisitive / constructivist theory of learning perspective, students acquire transferable schema from reflection on techniques, and from the participatory / socio-cultural viewpoint, students increasingly appropriate the practices of the classroom in which they are situated.

The 1990s saw an increased focus on the teacher in all areas of education research, and John Monaghan (2001) points out that more research is needed to assess how mainstream teachers (rather than innovators) are likely to regard the integration of CAS into their teaching. Some may see it as a threat to their role as the only expert in the classroom. Monaghan argues for a link to be made with the existing tools of the algebra teacher: the textbook and their command of algebraic techniques: “curriculum change must start by recognising (and valuing) teachers’ practices” (Monaghan, 2001, p. 465).

(v) Significance of the learning tasks and assessment considerations

The implications for mathematics assessment of the introduction of CAS on computers or on hand-held calculators such as the TI-89 or the TI-92 are tied up with implications for the mathematics curriculum (Meagher, 2001). On the one hand, with little change in curriculum it is possible to construct assessment items that do not excessively reward students who use such software, as described by Mueller and Forster (1999). There are negative consequences to a course of action that allows CAS for teaching purposes but not for assessment, as predicted by Kemp, Kissane, and Bradley (1995). These include the possibility that many students would choose *not* to use the technology at all so that they are prepared for a non-technology assessment, and this has been empirically verified by research studies. (For example, see Macintyre & Forbes, 2002). Other negative consequences are that many students may be denied access to technology that is not seen as essential. On the other hand, unrestricted access to technology may result in inappropriate uses, and an avoidance of learning to do anything by hand. Mathematicians such as Eisenberg (1999) have voiced these concerns. A way forward is suggested by Bernhard Kutzler (2002) among others, who proposes the assessing in separate ways of essential by-hand techniques and technology-assisted problem-solving. This structure of examinations in two parts – "technology free" and "technology required" is used for selected exams in several systems including Victoria, the AP Calculus subjects in the USA, and some A Level exams in the UK.

It is interesting to note that David Tall (Tall 1991, 2000a; Tall, Blokland, & Kok 1990), one of the major researchers in the cognitive / constructivist paradigm in the field of computer supported mathematics learning, and designer of the successful microworld *A Graphical Approach to the Calculus* (Tall et al., 1990), acknowledges in his more recent work the role of the social dynamics of the classroom:

> The moral of the story is that it is possible, with a well-designed microworld, to build an environment for exploring highly subtle theories in an informal way. However, I do not see the computer microworld as the sole agent in facilitating student exploration and peer discussion. The role of the teacher as mentor is vital – to draw out ideas from students and to encourage them to express verbally what they see occurring visually. (Tall, 2000b, p. 229).

A stronger challenge, however, comes from Kate Crawford (1994) who calls for a review of the social organisation of educational contexts, since technological advances now require us to place a higher priority on creative and adaptive human capabilities. "Implicit prerequisites to the creative use of knowledge as a basis for objective organisation are more independent and creative learning experiences for students" (p. 103). Crawford underlines Pratt's claim (cited in Crawford, 1994), that the *creation* of microworlds by students rather than the interaction with microworlds developed by others can be the source of educational power in the new technologies.

Research within the participatory view of learning

In this section, several reports concerning research with graphing calculators or CAS will be considered, in which socio-cultural aspects of learning theory are made explicit.

Jean-Baptiste Lagrange (1999b) has found a need to go beyond constructivism to incorporate explicitly the Vygotskian notion of mediation by tools. He points out that:

> Mediation changes the nature of the action of human over objects…the idea of mediation is useful in our project because a purely constructivist view of the use of computers is insufficient to analyse the interaction between the user, his/her instrument and the objects in the settings. A constructivist view assumes that the computer settings will provide the means for a predictable and meaningful interaction. What actually happened when we observed the use of DERIVE was different: interaction situations of the students and DERIVE were often less productive than teachers' expectations. (Lagrange, 1999b, p. 56.)

Lagrange's (1999b) paper provides an analysis of what Lagrange sees as the main issue in the teaching and learning of mathematics with CAS in French high schools. (The students in the study all had TI-92 machines, with DERIVE built in.) The main issue to Lagrange appears to be the finding of a new balance between concepts and techniques. He points out that students develop concepts not by reflection on tasks, but by reflection on the techniques used to accomplish the tasks.

> Every topic, mathematical or not, has a set of tasks and methods to perform these tasks. Newcomers in the topic see the tasks as problems. Progressively they acquire the means to achieve them and they become skilled. That is how they acquire techniques in a topic. Furthermore, in teaching and learning situations the students and the teachers are not interested in simply acquiring and applying a set of techniques. They want to talk about them, and therefore they develop a specific language. Then, they can use this language to question the consistency and the limits of the techniques. In this way they reach a theoretical understanding of a topic. (Lagrange, 1999b, p. 63)

Lagrange (1999b) emphasises the need to help students to develop their own schemes ("internal adaptive constructs of a person", p. 63), and makes the distinction between schemes and techniques: "techniques are official means of achieving a task but, in facing the task, a person doesn't 'follow' a technique, especially when the task is new or more complex or more problematic than usual. When knowledge is requested a person acts through schemes" (p. 63). In his observations in classrooms, Lagrange notes that it is the role of the teacher, by choosing tasks and encouraging discussion, to promote the reflection on techniques that results in learning.

Hilary Povey and Myka Ransom (2000) explicitly set out to report university students' experiences with a CAS from the students' perspective, using a methodology based in grounded theory and focussing on the sense that the students made for themselves of their experiences. They do not claim to report opinions that are representative of all students in their study, and indeed they state that most of the students were positive about their experiences at least some of the time. However, they describe two themes that emerged in the report writing of students from various cohorts (engineering, mathematics, and mathematics education), which they interpret as indicating a resistance to the adoption of CAS techniques. The themes are: a strong desire for understanding, often linked to pen and paper mastery; and issues of control. Students expressed concern that "technology, through its speed and opacity, can hinder reflection" (Povey & Ransom, 2000, p. 52). Students were also concerned about becoming lazy and unmotivated if machines did all the work for them, and wanted to know about how the computer "did" the mathematics. An interesting point is that some of the students felt more comfortable and more in control after checking the computer's work by hand, even though they used calculators as part of that "manual" checking: "an example of someone using a familiar technology to support learning with a less familiar technology" (p. 57). As pointed out by Crawford (1986, p. 5), "when one uses a calculator to solve a mathematical problem the action is not broken by this 'extra-cerebral link'. Rather, the arithmetic computation has become a 'technicalized' operation: the function of a machine." A question that arises for proponents of the use of CAS in teaching is whether the use of CAS as an "extra-cerebral link" in the way that calculators are used is firstly an achievable and secondly a desirable goal.

A feature of the research reported by Povey and Ransom (2000) is their concern with more than the technical skills of the students. They say that they "are not arguing *against* the use of computers in the teaching and learning of mathematics; rather that we need to heed and work with the students' concerns in ways which respect their roots in issues of personal worth and identity" (Povey & Ransom, 2000, p. 61).

One way to use technology to enhance student control is to plan lessons in which the computer or graphic display calculator is explicitly assigned a 'servant' role, as described by Galbraith, Renshaw, Goos, and Geiger (1999). These researchers set out to build and research a classroom culture that supports learning as a "collective process of enculturation into practices of mathematical communities" (p. 223), with solving real life problems being the focus and incorporating student presentations to the whole class with a large screen allowing the presenter to demonstrate how they used their graphing calculator. They theorise other 'voices' in the classroom besides the one that regards technology as a servant, describing situations where the technology is a "partner" in explorations of mathematical problems, and also situations where the technology has become "an extension of self" – the highest level of functioning. This resonates with Crawford's (1986) description of the calculator as sometimes being an "extra-cerebral link" and the way that the students in Povey and Ransom's (2000) study regarded their calculators as extensions of themselves – to be used without hesitation to check the work on the less familiar computers. Galbraith et al. (1999) describe a successful integration of technology into a classroom that uses it as a support for the main purpose of the class – learning to *be* mathematicians.

Summary of Part 1.

This literature review began with a description of a dichotomy in the theoretical background of mathematics education research. This dichotomy was described as a map of, on the one hand, *acquisitive* models of learning based on constructivism and on the other hand *participatory* models of learning incorporating a more expansive and situated view of cognition. With this background, research into the introduction of CAS to mathematics teaching was considered. Most of the reviewed research was based in the constructivist paradigm, concerned with the acquiring of concepts and skills by individual students, however there are strong indications that the role of the teacher in encouraging discussion and reflection about techniques is essential to ensure that learning (as in the *acquiring* of concepts) occurred in technology-rich classroom environments.

Early research within the acquisitive model was concerned with demonstrating that there were advantages to learning mathematics with the new tools (GDCs, CAS). Due to the complex nature and many variables in classroom settings, this was (and always will be) difficult to establish conclusively and research next focused on describing the ways that students worked with the new tools. Unforeseen difficulties with the technology were identified. Much effort has been expended in producing innovative teaching activities and assessment tasks to take advantage of the new ways of working with mathematical ideas that are afforded by GDCs and CAS. More recently there has been recognition of the essential role played by the teacher in guiding students' attention when there is much to be considered in a technology rich environment.

From the socio-cultural view, it is precisely that classroom discussion and reflection, motivated by a genuine social need to engage in sense-making, that constitutes participation in authentic mathematical activities and this *participation* is the key to learning. As described by Goos, Galbraith, Renshaw and Geiger (2000, p. 306), a socio-cultural perspective on learning is manifested in classrooms where students "are expected to defend and critique ideas by proposing justifications, explanations, and alternatives". In these classrooms students learn to use the tools of GDCs and computers as resources to support the mathematical discussion and problem solving that are the real focus of the lessons.

The role of tools is a complex one in educational theory and this is appropriate in considering the role of computers in mathematics education. As Eric Love (1995) has described the situation:

> There is a dynamic interaction between the work that people do, computer software to carry out operations hitherto taught as part of a mathematics curriculum, and of what 'mathematics' is thought to consist. Trying to pin down parts of this dynamic by speaking of "the mathematics embedded in software", of "producing software for teaching mathematics" or "tools for doing mathematics" will inevitably fail adequately to characterise this continually changing interaction. (p. 117)

The purposes for the CAS tool in "doing mathematics" in a professional context are quite different from the purposes of using a CAS when "doing mathematics" in a mathematics classroom. Mathematics can be regarded as *a social practice* as well as *a collection of knowledge and disembodied skills.* CAS are one of many mathematical artefacts that change the nature of mathematics and mathematics education, not just because they allow mathematics to be done in new ways, but because they change the nature of mathematical practice – what mathematicians *do*.

Concluding comments on the place of new technologies in the post-compulsory mathematics curriculum.

Mathematics education in the post-compulsory years has more than one goal. First, students need to be equipped for participation in the world of work and to participate in society as responsible citizens. As Kaye Stacey points out, improved methods of mathematical computation are unproblematic in the world of work, where tools that make computation easier, more accurate and faster have always been important to progress. (Stacey, 2005, p. 8.) One kind of technology readily used in the workplace and also in the home, and perhaps too often overlooked in school mathematics is spreadsheet software.

Second, students intending to study mathematics at university or to use mathematics to support other university studies need to learn the language of advanced mathematics, based as it is on algebra and functions. While these topics have traditionally been taught with pen and paper methods, the availability of graphics calculators and CAS supports a widening of approaches to include more exploration and reduce time spent perfecting routines. As outlined in this review, this does not happen automatically and requires expert teaching and carefully prepared materials.

Barry Kissane (2000) raises the possibility of broadening the curriculum to include topics that were not possible without graphics calculators (or CAS), for example simulation as a means of tackling some situations involving uncertainty, iterative techniques to study some of the mathematics of chaos, and dealing with real world data collected electronically and transmitted to a calculator.

For both of the goals of post-compulsory mathematics, participation in work and society *and* further learning, the new technologies and internet access support a much enriched approach to the study of statistics. This will be considered in the next part of this review.

PART 2: THE PLACE OF STATISTICS IN POST-COMPULSORY MATHEMATICS CURRICULA

The current situation in New South Wales compared to other school systems

Compared to other Australian states and territories, NSW has much less content described as statistics and probability in its mathematics course for non-specialists, Mathematics 2 Unit. Barrington and Brown call this kind of course "Intermediate": a course designed for students intending to study subjects at university that need some mathematics, but not as much content nor depth as required by the physical sciences, engineering, or mathematics itself. The following extract from Table 2 in Barrington and Brown, 2005 shows the comparison.

State:		NSW	VIC	QLD	SA	WA	TAS	ACT
	Highest level subject studied:	(2 Unit) Maths	Math'l Methods	Maths B	Math'l Studies	Applicable Maths	Maths - Methods	Math'l Methods (major)
Topic	Sub-topic							
5. Statistics/Probability								
	events, sample spaces, ...	YES	YES	YES	YES*	YES	YES	YES
	mutually exclusive events	YES	YES	YES	YES*	YES	YES	YES
	P(AUB)=P(A)+P(B)-P(AB)	YES	YES	YES	YES*	YES	YES	YES
	tree diagrams	YES	YES*	YES*	YES*	YES*	YES*	YES*
	independent events	YES	YES	YES	YES*	YES	YES	YES
	conditional probability	NO	YES	YES	YES*	YES	YES	YES
	discrete prob distributions	NO	YES	YES	YES*	YES	YES	YES
	expected value	NO	YES	YES	YES*	YES	YES	YES
	standard deviation	NO	YES	YES	YES*	YES	YES	YES
	central limit theorem	NO	NO	NO	YES	NO	NO	YES
	confidence intervals	NO	NO	NO	YES	NO	NO	YES
	null hypothesis	NO	NO	NO	YES	NO	NO	NO
	binomial distribution	NO	YES	YES	YES	YES	YES	YES
	hypergeometric distribution	NO	YES	NO	NO	YES	YES	OPT
	normal distribution	NO	YES	YES	YES	YES	YES	YES
	Poisson distribution	NO	NO	NO	NO	YES	NO	NO
	boxplots	NO	NO	YES	NO	YES	NO	NO
	correlation coefficient	NO	NO	OPT	NO	YES	NO	YES
	least squares regression	NO	NO	OPT	NO	YES	NO	YES

Figure 8. Extract from Table 2: Australian I-Type students' mathematics coverage, in Barrington and Brown, 2005, p. 16.

As shown in the accompanying report prepared for the Stage 6 review, *A critical analysis of selected Australian and international mathematics syllabuses for the post-compulsory years of secondary schooling*, the NSW subject General Mathematics has much less content described as statistics and probability than the comparable subject in Victoria, *Further Mathematics.*

On the international scene, new developments in Singapore and Hong Kong include radical rearrangements of courses, but are not yet fully implemented. It is interesting to note the inclusion of Statistics and Probability as major components of the new courses, and reduced emphasis on mechanics and applied mathematics. Students in England and in the USA Advanced Placement program are offered separate courses in Statistics, and it is a component of every mathematics syllabus in the International Baccalaureate Diploma Programme. This goes hand in hand with making more use of graphing calculators for teaching, learning, and assessment; and other kinds of technology for graphical display in the classroom.

Arguments for the inclusion of statistics and/or data handling refer to the notion of statistical literacy.

Definitions of Statistical Literacy

"Statistical Literacy is the ability to understand and critically evaluate statistical results that permeate our daily lives – coupled with the ability to appreciate the contributions that statistical thinking can make in public and private, professional and personal decisions". (Katherine Wallman, 1992 Presidential Address to the American Statistical Association. Quoted in Watson, 2006, p. 10.)

"Statistical literacy is essential in our personal lives as consumers, citizens and professionals. Statistics plays a role in out health and happiness. … A statistically literate high school graduate will know how to interpret the data in the morning newspaper and will ask the right questions about statistical claims. He or she will be comfortable handling quantitative decisions that occur on the job, and will be able to make informed decisions about the quality of life issues." (Kader and Perry, 2006, p. 1).

"Our view is that this [Statistical literacy] means the ability to understand how quantitative data are generated, and how they can be summarized, modeled and interpreted in ways that allow substantively useful conclusions to be drawn about the functioning of the world from which they are derived. It is also, and most importantly, an understanding of uncertainty and how the measurement of uncertainty can be put to constructive uses, for example in decision making and in handling risk, and especially in the formulation of evidence based policy. It involves the ability critically to evaluate the use of statistical data by others, in the media and elsewhere. This refers especially to the use of official statistics, both in providing 'snapshots' of current situations and in showing important changes over time.
It is also our view that statistical literacy has to be acquired through the experience of handling data themselves, but within the context of an appropriate mathematical framework. The process of learning statistics is iterative, moving continually between theory and practice." (Royal Statistical Society, 2005).

Arguments for the inclusion of statistics and/or data handling in the mathematics curriculum USA (adapted from Franklin, Kader, Mewborn, Moreno, Peck, Perry and Schaeffer, 2005).

- Statistical literacy is essential for citizenship. Students who are reaching the age of voting need to be able to understand the principles behind opinion polling and the data gathering conducted for and by government agencies.
- Personal choices concerning health, safety, investments, and purchases are all informed by statistical information. Interpreting this requires statistical literacy.
- Statistical literacy is required for productive work in many jobs and careers, contributing to individual advancement and the good of a nation's economy.
- Quality control and accountability in the workplace rely on statistical procedures.
- Science contributes to the health of modern societies and the communication of scientific findings relies heavily on statistics.

Additional arguments especially relevant to the NSW situation

- Statistics can be used to motivate topics in the mathematics curriculum. For example, the equation of a straight line is used to describe a line of best fit when finding a trend in data; basic equation solving is required to convert to and from standardized scores.
- For students needing further development of numeracy, statistics provides realistic examples where the calculation of percentages, ratios, and rates is required, along with units of measurement. Newspaper stories illustrated with data and charts provide numbers over a large range, reinforcing number concepts.
- Considering the choice of broad field of study, for many students proceeding to university in NSW, statistics is a more useful application of mathematics than some of the current topics in the calculus courses. This is seen in the data shown in Table 1 and Figure 9.

Broad field of study	***2002***	***2003***	***2004***	***2005***	***2005***
Creative Arts	5490	5509	5441	5849	7%
Society and Culture	23709	22276	20715	24566	29%
Management and Commerce	18812	19126	17690	18482	22%
Education	11086	10091	9472	10688	12%
Health	9039	9090	9331	10237	12%
Agriculture, Environmental and Related Studies	1955	2112	2062	1686	2%
Architecture and Building	1461	1639	1657	1951	2%
Engineering and Related Technologies	3896	3832	3729	3738	4%
Information Technology	3908	3726	2973	2746	3%
Natural and Physical Sciences	5909	5755	5909	5768	7%
				85711	100 %

Table 1. NSW Domestic students commencing university by broad field of study. Data assembled from yearly tables accessed 20th October at http://www.dest.gov.au/

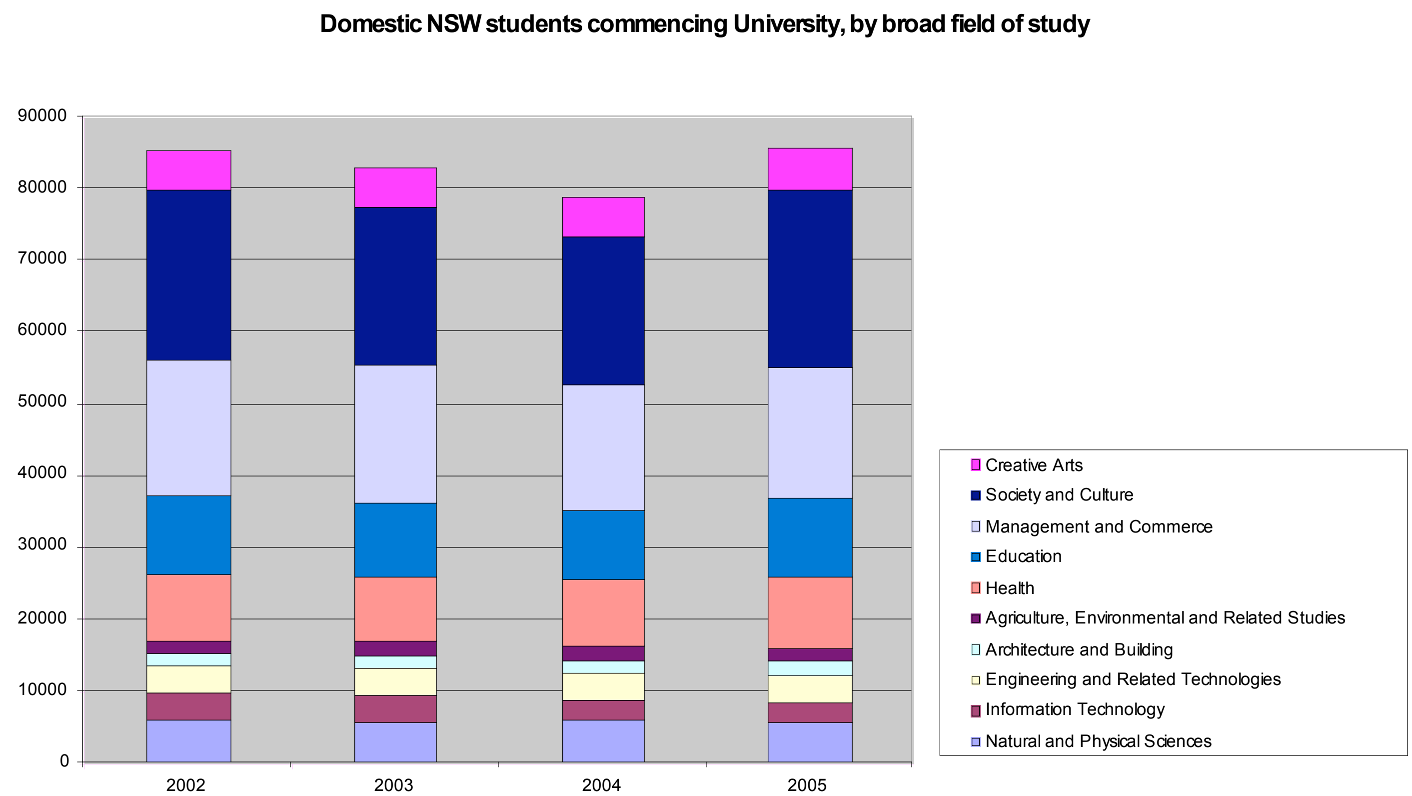


Figure 9. NSW Domestic students commencing university by broad field of study. Data assembled from yearly tables accessed 20th October at http://www.dest.gov.au/

PART 3: IMPLICATIONS FOR THE NSW STAGE 6 MATHEMATICS REVIEW IN THE LIGHT OF CURRENT PROPOSALS FOR AN AUSTRALIAN CERTIFICATE OF EDUCATION

A search of the websites of the Australian Council for Educational Research (ACER), the Department of Education, Science and Training (DEST), and Federal and State Ministers of Education yielded one public document (ACER 2006), and several media releases.

ACER was commissioned by the Australian government to investigate and report on models and implementation arrangements for an Australian Certificate of Education (ACE) for the final years of secondary school, in the light of the diverse arrangements that currently exist at state and territory level for curriculum, assessment, and accreditation at that level. The first outcome is the report *Australian Certificate of Education: Exploring a way forward* (ACER, 2006). The abstract of the report is taken from the DEST website, accessed 24th October, 2006: http://www.dest.gov.au/sectors/school_education/publications_resources/profiles/australian_certificate_education.htm

> The report *Australian Certificate of Education: exploring a way forward:*
>
> - notes that at present there are significant inconsistencies in senior secondary school across Australia in such matters as:
>
> - terminology relating to curricula, assessment and reporting;
> - requirements for the award of the senior certificate;
> - what is taught in particular subjects;
> - how vocational learning is incorporated;
> - how student achievement is assessed; and
> - how student results are reported.
>
> - analyses the following four models
>
> - A national certificate as an alternative to the existing state-based final year certificates;
> - A national certificate which evolves from the existing state-based certificates;
> - A national certificate which is a general aptitude test similar to that used in the United States;
> - A national certificate modelled on the International Baccalaureate.
>
> The report sets out a clear proposal for an Australian Certificate of Education that would be available to all senior secondary school students, regardless of where they live in Australia.

ACER conducted national consultations to seek the views of stakeholders in this enterprise. An Options Paper was distributed to all invitees prior to each meeting, providing an overview of the four options listed in the abstract above as models for the national certificate. As a result of the consultations it appeared that there was a strong preference for the third of the options listed. (This is actually the second of the options in the list as it appears in the abstract above.) "The perceived advantages of Option 3 were that it recognized and built on to the strengths of existing arrangements, including attempts to increase student participation and to provide learning opportunities for the broad range of students now staying on to senior secondary education." (ACER, 2006, p. 23)

The report made six recommendations, which are summarised here:

1. Curriculum essentials in at least some nominated mathematics, English, science and social science/humanities subjects should be identified by national subject panels.
2. Achievement standards in at least some nominated mathematics, English, science and social science/humanities subjects should be benchmarked internationally and perhaps take the form of A to E grades in a subject.
3. As part of the ACE, all students undertake a national Key Capabilities Assessment (KCA) part way through year 12. (These Key Skills are listed as reading literacy/verbal reasoning, mathematical literacy/quantitative reasoning, written English, and ICT literacy.) The purpose of this assessment is to provide information about a number of capabilities important to life and work beyond school. Another purpose is to assist with the equating of university entrance scores across the different systems, and this test might replace existing generic skills tests in some states.
4. An ACE Award of Excellence be introduced for students who meet international standards of excellence in their school subjects and on the KCA.
5. A national standards body be established to identify essential curriculum content, develop achievement standards and manage the KCA. (This would not be an awarding body.)
6. All students in the final years of secondary school be given access to the ACE, which may be awarded by the existing state and territory authorities following agreement to incorporate essential curriculum content in nominated subjects, to report against common achievement standards, and to incorporate the KCA. Each of the existing senior secondary certificates would be eligible to become the ACE. A positive outcome of this arrangement is the opportunity to share syllabus and assessment materials across states.

Throughout the report there are implications for the Stage 6 review of NSW Mathematics. These will now be listed and discussed.

- The imposition of awarding grades A to E across the subject area would be made easier if a common core of material in mathematics courses could be decided. Given the wide range of both content and difficulty level addressed in the existing subjects, General Mathematics through to Extension 2, this seems an impossible task, however. A way forward could be to identify two parallel streams, calculus and non-calculus, with challenging mathematics in both, so that students not taking the full calculus stream are not "locked out" of the higher grades. On the other hand, if the NSW mathematics education community regards the plan to identify essential core material in all subjects (p. 26) as undesirable or impossible, given the hierarchical nature of mathematical studies, then that position needs to be put as soon as possible to the bodies responsible for the construction of the ACE.

- Given the fact that no other states use the “half a subject” model of the NSW mathematics plus Extension 1, the viability of that arrangement needs to be considered. It is flagged on page 64 that among the many questions to be considered is whether the general framework for a subject might include number of hours of study.
- The possibility of sharing some syllabus and assessment materials with other states is to be welcomed at this time in particular. In NSW we have had limited experience with teaching and assessing topics in statistics and data handling.
- In the report, the ACE is conceptualised as a broad certificate, not just an academic one. (ACER, 2006, p. 6 and p. 20). This has implications for considering a numeracy-based mathematics subject in the revised suite of mathematics courses in NSW.
- The report acknowledges the existence of state-based arrangements that have been negotiated in detail with local communities. (p. 22). In NSW there is currently an excellent rapport between officers of the Board of Studies and members of the mathematics teaching profession. This is a strength of the current arrangements and should be supported.
- The report states that it does not recommend the introduction of a single examination or a single set of assessment processes for all states. (p. 72). In the short term, including the life of the current Stage 6 Review for Mathematics in NSW, this indicates that there is no essential need to “maintain the status quo” just because the ACE may be around the corner.
- The mathematical literacy test proposed as part of the KCA is described on page 82 as “a test of the ability to use mathematical principles and concepts in the solution of problems”. The role of problem solving needs to be considered in the design of the new syllabuses for Stage 6 mathematics in NSW.

REFERENCES

ACER. (2006). Australian Certificate of Education: Exploring a way forward. Canberra: Commonwealth of Australia.

Artigue, M. (2001). The teaching and learning of mathematics a the university level: Critical questions for contemporary research in education. *Notices of the American Mathematical Society, 46*(11), 1377-1385.

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*, 245-274.

Ball, L. (2001). Solving equations: Will a more general approach be possible with CAS? In H. Chick, K. Stacey, J. Vincent, & J. Vincent (Eds.), *Proceedings of the 12th study conference of the International Commission on Mathematics Instruction: The future of the teaching and learning of algebra* (pp. 48-52). Melbourne, Australia: University of Melbourne.

Barrington, F. & Brown, P. (2005). *Comparison of Year 12 pre-tertiary Mathematics Subjects in Australia 2004-2005.* Melbourne, Australian Mathematical Sciences Institute. Retrieved 18th August 2006 from www.amsi.org.au.

Bell, A. W., Costello, J., & Küchemann, D. E. (1983). *A review of research in mathematical education, prepared for the committee of inquiry into the teaching of mathematics in school: Part A, Research on learning and teaching.* Windsor, U.K.: NFER-Nelson.

Bulmer, M. (2001). Algebra in an age of numerical mathematics. In H. Chick, K. Stacey, J. Vincent, & J. Vincent (Eds.), *Proceedings of the 12th study conference of the International Commission on Mathematics Instruction: The future of the teaching and learning of algebra* (pp. 136-139). Melbourne, Australia: University of Melbourne.

Chick, H., Stacey, K., Vincent, J., & Vincent, J. (Eds.). (2001). *Proceedings of the 12th study conference of the International Commission on Mathematics Instruction: The future of the teaching and learning of algebra*. Melbourne, Australia: University of Melbourne.

Coupland, M. (2004). *Learning with new tools.* Unpublished PhD thesis, University of Wollongong, Australia.

Crawford, K. (1986). *Simultaneous and successive processing, executive control and social experience: Individual differences in educational achievement and problem solving in mathematics.* Unpublished doctoral dissertation, University of New England, Australia.

Crawford, K. (1994). The context of cognition: The challenge of technology. In P. Ernest (Ed.), *Constructing mathematical knowledge: Epistemology and mathematics education* (pp. 92-106). London: Falmer Press.

Davis, R. B. (1984). *Learning mathematics: The cognitive science approach to mathematics education.* London: Croom Helm.

Drijvers, P. (1999). Students encountering obstacles using CAS: A developmental-research pilot study. In P. Kent, J. Monaghan, & N. Zehavi (Eds.), *Papers (Presentations, Reactions and Keynotes) of the CAME (Computer Algebra in Mathematics Education) meeting at the Weizmann Institute of Science, Rehovot, Israel, August 1-2, 1999* (pp. 34-49). Retrieved May 19, 2001, from http://metric.ma.ic.ac.uk/came/events/weizmann

Drijvers, P. (2000). Students encountering obstacles using a CAS. *International Journal of Computers for Mathematical Learning, 5*(3), 189-209.

Drijvers, P. (2001). The concept of parameter in a computer algebra environment. In H. Chick, K. Stacey, J. Vincent, & J. Vincent (Eds.), *Proceedings of the 12th study conference of the International Commission on Mathematics Instruction: The future of the teaching and learning of algebra* (pp. 221-227). Melbourne, Australia: University of Melbourne.

Eisenberg, T. (1999). A skeptic replies to Edith Schneider's "On using CAS in teaching mathematics". In P. Kent, J. Monaghan, & N. Zehavi (Eds.), *Papers (Presentations, Reactions and Keynotes) of the CAME (Computer Algebra in Mathematics Education) meeting at the Weizmann Institute of Science, Rehovot, Israel, August 1-2, 1999* (pp. 30-33). Retrieved May 19, 2001, from http://metric.ma.ic.ac.uk/came/events/weizmann

Evans, G. (1991). Student control over learning. In J. Biggs (Ed.), *Teaching for learning: The view from cognitive psychology* (pp. 51-70). Hawthorn, Australia: Australian Council for Educational Research.

Forster, P., Mueller, U., Haimes, D., and Malone, J. (2003). Impact on school assessment where use of graphics calculators is mandatory in a related public examination. *International Journal of Mathematical Education in Science and Technology. 34* (3), 343-359.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., and Schaeffer, R. (2005). *A curriculum framework for preK-12 statistics education.* A report endorsed by the Board of Directors of the American Statistical Association. Retrieved 23rd October from http://www.amstat.org/education/GAISEPreK-12.htm

Galbraith, P., Pemberton M., & Cretchley, P. (2001). Computers, mathematics, and undergraduates: What is going on? In J. Bobis, B. Perry, & M. Mitchelmore (Eds.), *Numeracy and beyond: Proceedings of the 24th annual conference of the Mathematics Education Research Group of Australasia Inc, University of Sydney, June 2001* (pp. 233-240). Sydney, Australia: MERGA.

Galbraith, P., Renshaw, P., Goos, M., & Geiger, V. (1999). Technology, mathematics, and people: Interactions in a community of practice. In J. Truran & K. Truran (Eds.), *Proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australasia, Adelaide* (pp. 223-230). Adelaide, Australia: MERGA.

Goos, M., Galbraith, P., Renshaw, P. & Geiger, V. (2000). Reshaping teacher and student roles in technology-enriched classrooms. *Mathematics Education Research Journal. 12*(3), 303 – 320.

Guin, D., & Trouche, L. (1998). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematical Learning, 3*(3), 195 – 227.

Heck, A. (2001). Variables in computer algebra, mathematics and science. *The International Journal of Computer Algebra in Mathematics Education, 8*(3), 195-221.

Heid, M. K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. *Journal for Research in Mathematics Education, 19*(3), 3-25.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In Hiebert, J. & Lefevre, P. (Eds.), *Conceptual and procedural knowledge: The case of mathematics.* Hillsdale, NJ: Erlbaum.

Kader, G. & Perry, M. A framework for teaching statistics within the K-12 mathematics curriculum. Paper presented at the International Conference for Teaching Statistics ICOTS – 7, Salvador, Brazil. Accessed October 23rd, 2006 at http://www.stat.auckland.ac.nz/~iase/publications/17/2B3_KADE.pdf

Kaineder, B. (2003). *A brief history of the muMATH / Derive TM CASs.* Retrieved February 14, 2003, from http://www.derive-europe.com/theroots.asp?history

Kanes, C., & Atweh, B. (1993). A comparative analysis of research questions, methods and methodologies in mathematics education: 1979 and 1993. In B. Atweh, C.

Kanes, C. Carss, & G. Booker (Eds.), Contexts in mathematics education (pp. 369-374). Brisbane, Australia: Mathematics Education Research Group of Australasia.

Keeves, J., & Stacey, K. (1999). Research into mathematics education. In J. Keeves & K. Marjoribanks (Eds.), Australian Education: Review of Research 1965-1998 (pp. 205-241). Melbourne, Australia: Australian Council for Educational Research.

Kemp, M., Kissane, B., & Bradley, J. (1995). Assessment and the graphics calculator. In A. Richards (Ed.), *FLAIR- Forging links and integrating resources: Proceedings of the 15th biennial conference of the Australian Association of Mathematics Teachers* (pp. 235-241). Adelaide, Australia: Australian Association of Mathematics

Kendal, M., & Stacey, K. (2001). The impact of teacher privileging on learning differentiation with technology. *International Journal of Computers for Mathematical Learning, 6*(2), 143-165.

Kilpatrick, J. (1986). Reflection and recursion. In M. Carss (Ed.), *Proceedings of the 5th international congress on mathematical education*. Boston: Birkhauser.

Kissane, B. (2000). Technology and the curriculum: the case of the graphics calculator. In M.O.J. Thomas (Ed.) *Proceedings of TIME 2000: An international Conference on Technology in Mathematics Education.* (pp. 60 – 71). Auckland, New Zealand: University of Auckland.

Kutzler, B. (2002). *Two-tier exams as a way to let technology in.* Retrieved on August 27, 2002, from http://www.ketzler.com/article/art_exam/exams.htm (Originally in the proceedings of Portoroz 2000, bk-teachware, Hagenberg)

Laborde, C. (1995). Designing tasks for learning geometry in a computer-based environment. In L. Burton & B. Jaworski (Eds.), Technology in mathematics teaching - A bridge between teaching and learning (pp. 35-67). Bromley, U.K: Chartwell-Bratt.

Lagrange, J. (1999a). A didactic approach of the use of Computer Algebra Systems to learn mathematics. In P. Kent, J. Monaghan, & N. Zehavi (Eds.), *Papers (Presentations, Reactions and Keynotes) of the CAME (Computer Algebra in Mathematics Education) meeting at the Weizmann Institute of Science, Rehovot, Israel, August 1-2, 1999* (pp. 4-15). Retrieved May 19, 2001, from http://metric.ma.ic.ac.uk/came/events/weizmann

Lagrange, J. (1999b). Complex calculators in the classroom: Theoretical and practical reflections on teaching pre-calculus. *International Journal of Computers for Mathematical Learning, 4*(1), 51-81.

Lerman, S. (1994). Articulating theories of mathematics learning. In P. Ernest (Ed.), *Constructing mathematical knowledge: Epistemology and mathematics education* (pp. 41-49). London: Falmer Press.

Lerman, S. (1999). Response to Jean-Baptiste Lagrange: A didactic approach of the use of CAS to learn mathematics. In P. Kent, J. Monaghan, & N. Zehavi (Eds.), *Papers (Presentations, Reactions and Keynotes) of the CAME (Computer Algebra in Mathematics Education) meeting at the Weizmann Institute of Science, Rehovot, Israel, August 1-2, 1999* (pp. 16-18). Retrieved May 19, 2001, from http://metric.ma.ic.ac.uk/came/events/weizmann

Love, E. (1995). Software for mathematics education. In L. Burton & B. Jaworski (Eds.), Technology in mathematics teaching - A bridge between teaching and learning (pp. 109-118). Bromley, U.K: Chartwell-Bratt.

Macintyre, T., & Forbes, I. (2002). Algebraic skills and CAS – Could assessment sabotage the potential? *The International Journal of Computer Algebra in Mathematics Education, 9,* (1) 29-56.

Mason, J. (1995). Less may be more on a screen. In L. Burton & B. Jaworski (Eds.), *Technology in mathematics teaching - A bridge between teaching and learning* (pp. 119-134). Bromley, U.K.: Chartwell-Bratt.

McCrae, B., Asp, G., & Kendal, M. (1999). Learning calculus with supercalculators. In J. Truran & K. Truran (Eds.), *Proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australasia, Adelaide* (pp. 359-364). Adelaide, Australia: MERGA.

Meagher, M. (2001). Curriculum and assessment in the age of computer algebra systems. *International Journal of Computer Algebra in Mathematics Education, 8*(1), 89-96.

Meel, D. (1998). Honors students'calculus understanding: Comparing *Calculus & Mathematica* and traditional calculus students. In A. Schoenfeld, J. Kaput, & E Dubinsky (Eds.), *Research in collegiate mathematics education III* (pp. 163-215). Providence, RI: American Mathematical Society.

Monaghan, J. (2001). Teachers and computer algebra systems. In H. Chick, K. Stacey, J. Vincent, & J. Vincent (Eds.), *Proceedings of the 12th study conference of the International Commission on Mathematics Instruction: The future of the teaching and learning of algebra* (pp. 462-467). Melbourne, Australia: University of Melbourne.

Mueller, U., & Forster, P. (1999). Graphics calculators in the public examination of calculus: Misuses and misconceptions. *Proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australasia, Adelaide* (pp. 396-403). Adelaide, Australia: MERGA.

Palmiter, J. (1991). Effects of computer algebra systems on concept and skill acquisition in calculus. *Journal for Research in Mathematics Education, 22*, 151-156.

Park, K., & Travers, K. (1996). A comparative study of a computer-based and a standard college first-year calculus course. In A. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.), *Research in collegiate mathematics education II* (pp. 155-176). Providence, RI: American Mathematical Society.

Pierce, R., & Stacey, K. (2001). A framework for algebraic insight. In J. Bobis, B. Perry, & M. Mitchelmore (Eds.), *Numeracy and beyond: Proceedings of the 24th annual conference of the Mathematics Education Research Group of Australasia, University of Sydney, June 2001* (pp. 418-425). Sydney, Australia: MERGA.

Povey, H., & Ransom, M. (2000). Some undergraduate students' perceptions of using technology for mathematics: Tales of resistance. *International Journal of Computers for Mathematical Learning, 5*(1), 47-63.

Royal Statistical Society. (2005). Teaching statistics across the 14-19 curriculum. Accessed 23rd October, 2006, at http://www.rss.org.uk/main.asp?page=2638

Ruthven, K. (2002). Instrumenting mathematical activity: reflections on key studies of the educational use of computer algebra systems. *International Journal of Computers for Mathematical Learning, 7,* 275-291.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189-215). Hillsdale, N.J.: Erlbaum.

Sfard, A. (1996). On acquisition metaphor and participation metaphor for mathematics learning. In C. Alsina, J. Alverez, B. Hodgson, C. Laborde, & A. Pérez (Eds.), *Selected lectures from the 8th international congress on Mathematical Education,* Seville, 1996 (pp. 397-411). Seville: S. A. E. M. "THALES".

Skemp, R. (1986). *The psychology of learning mathematics* (2nd ed.). Harmondsworth, U.K.: Penguin Books.

Stacey, K. (1999). Understanding algebra and using CAS: A reaction to "Students encountering obstacles using CAS: A developmental research study". In P. Kent, J. Monaghan, & N. Zehavi (Eds.), *Papers (Presentations, Reactions and Keynotes) of the CAME (Computer Algebra in Mathematics Education) meeting at the Weizmann Institute of Science, Rehovot, Israel, August 1-2, 1999* (pp. 50-56). Retrieved May 19, 2001, from http://metric.ma.ic.ac.uk/came/events/weizmann

Stacey, K. (2005). Accessing results from research on technology in mathematics education. *Australian Senior Mathematics Journal. 19*(1), 8 – 15.

Tall, D. O. (1991). The psychology of advanced mathematical thinking. In D. O. Tall (Ed.), *Advanced mathematical thinking* (pp. 3-21). Dordrecht: Kluwer.

Tall, D. O. (2000a). *Biological brain, mathematical mind & computational computers: How the computer can support mathematical thinking and learning.* Plenary presentation for ATCM conference, Chang Mai Thailand, December 2000. Retrieved April 14, 2003, from http://www.warwick.ac.uk/staff/David.Tall/downloads.html

Tall, D. O., Blokland, P., & Kok, D. (1990). *A graphical approach to the calculus.* [Computer Software]. New York: Sunburst.

Watson, J. (2006). *Statistical literacy at school: growth and goals.* Mahwah, New Jersey: Lawrence Erlbaum Associates.

Weigand, H., & Weller, H. (2001). Changes of working styles in a computer algebra environment – The case of functions. *International Journal of Computers for Mathematical Learning, 6*(1), 87-111.

Wolfram Research. (2003). *The History of Mathematica*. Retrieved April 19, 2003, from Wolfram Research Website: http://www.wolfram.com/company/history

Yerushalmy, M., & Houde, R.A. (1986). The geometric supposer: Promoting thinking and learning. *Mathematics Teacher, 79,* 418-422.

Zbiek, R., & Heid, M. K. (2001). Dynamic aspects of function representations. In H. Chick, K. Stacey, J. Vincent, & J. Vincent (Eds.), *Proceedings of the 12th study conference of the International Commission on Mathematics Instruction: The future of the teaching and learning of algebra* (pp. 682-689). Melbourne: University of Melbourne.

Zehavi, N., & Mann, G. (1999). Didactical use of CAS in story problems. In P. Kent, J. Monaghan, & N. Zehavi (Eds.), *Papers (Presentations, Reactions and Keynotes) of the CAME (Computer Algebra in Mathematics Education) meeting at the Weizmann Institute of Science, Rehovot, Israel, August 1-2, 1999* (pp. 57-67). Retrieved May 19, 2001, from http://metric.ma.ic.ac.uk/came/events/weizmann

Zimmerman, B., & Schunk, D. (Eds.). (1989). *Self-regulated learning and academic achievement: Theory, research, and practice*. New York: Springer-Verlag.

ANNOTATED BIBLIOGRAPHY OF SIGNIFICANT LITERATURE

ACER. (2006). Australian Certificate of Education: Exploring a way forward. Canberra: Commonwealth of Australia.

Major report of 208 pages commissioned by the Australian Government. It contains background to the project and a description of the consultation process that was undertaken. Four models for the ACE are considered in detail against desirable features of an ACE – inclusivity, high standards for curriculum and achievement, greater national consistency, increased comparability, reduced duplication, recognition of general capabilities, and clear reporting. The model considered most desirable is that of a national curriculum framework with delivery, assessment, and awarding still being undertaken at state and territory level. A new Key Capabilities Assessment test for mid-year 12 is proposed. Useful appendices include descriptions of the International Baccalaureate Diploma Programme, Scholastic Aptitude Tests, summaries of existing state and territory certificates.

Barrington, F. & Brown, P. (2005). *Comparison of Year 12 pre-tertiary Mathematics Subjects in Australia 2004-2005.* Melbourne, Australian Mathematical Sciences Institute. Retrieved 18th August 2006 from www.amsi.org.au.

Describes and compares the content and assessment details of post-compulsory mathematics courses and subjects across Australia, with an emphasis on calculus based courses.

Chick, H., Stacey, K., Vincent, J., & Vincent, J. (Eds.). (2001). *Proceedings of the 12th study conference of the International Commission on Mathematics Instruction: The future of the teaching and learning of algebra*. Melbourne, Australia: University of Melbourne.

A wide ranging, international perspective on the issue of teaching and learning algebra in schools and in universities. Comprehensive summaries of mathematical education research in that area. Chapters include reports from Working Groups on Early algebra, Approaches to algebra, Technological environments, CAS and algebra, Algebra history in mathematics education, Symbols and language, Teachers' knowledge and the teaching of algebra, Teaching and learning tertiary algebra, Goals and content of an algebra curriculum for the compulsory years.

Forster, P., Mueller, U., Haimes, D., and Malone, J. (2003). Impact on school assessment where use of graphics calculators is mandatory in a related public examination. *International Journal of Mathematical Education in Science and Technology. 34* (3), 343-359.

Chosen here as an example of the comprehensive research that is undertaken in WA in the context of the introduction of graphing calculators at the Year 12 high stakes public examinations.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., and Schaeffer, R. (2005). *A curriculum framework for preK-12 statistics education.* A report endorsed by the Board of Directors of the American Statistical Association. Retrieved 23rd October from http://www.amstat.org/education/GAISEPreK-12.htm

An excellent resource for statistics in the mathematics curriculum. Includes a framework and many practical examples of statistical investigations of real data.

Goos, M., Galbraith, P., Renshaw, P. & Geiger, V. (2000). Reshaping teacher and student roles in technology-enriched classrooms. *Mathematics Education Research Journal. 12*(3), 303 – 320.

Included here as an excellent example of case study research in mathematics education with a focus on teacher-student interactions in the participatory paradigm for learning.

Guin, D., & Trouche, L. (1998). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematical Learning, 3*(3), 195 – 227.

A much quoted research paper from France. Shows how classroom dynamics can be changed to help students become more capable users of modern technology when they need to discuss mathematical situations and their representations.

Heid, M. K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. *Journal for Research in Mathematics Education, 19*(3), 3-25.

A classic piece of research, often quoted and also criticised, but one of the first papers to claim to show advantages for conceptual learning when students use computer algebra for routine calculations and spend more time on learning applications and principles.

Kendal, M., & Stacey, K. (2001). The impact of teacher privileging on learning differentiation with technology. *International Journal of Computers for Mathematical Learning, 6*(2), 143-165.

This paper is often quoted as an example of the importance of teacher interactions in classrooms where technology is being used. Two teachers of parallel year 11 classes in introductory calculus were observed and interviewed, and their classes also took tests designed to assess competencies in numerical, graphical and symbolic aspects of differentiation. Students of the teacher who privileged conceptual understanding and student construction of meaning were more able to interpret derivatives. Students of the teacher who privileged performance of routines made better use of the CAS for solving routine problems.

Stacey, K. (2005). Accessing results from research on technology in mathematics education. *Australian Senior Mathematics Journal. 19*(1), 8 – 15.

A useful summary of research in the Melbourne CAS CAT project.

Watson, J. (2006). *Statistical literacy at school: growth and goals.* Mahwah, New Jersey: Lawrence Erlbaum Associates.

Recently published summary of many years of research into the teaching and learning of statistics, with clear applications to and guidelines for constructing syllabuses. Mostly relevant to the compulsory years of schooling.