7.6 Content for Stage 4

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Mathematics • Stage 4

Number and Algebra
Computation with Integers

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.3 recognises and explains mathematical relationships using reasoning
4.4 operates efficiently with different representations of numbers and numerical relationships, including financial calculations

Students:

Apply the associative, commutative and distributive laws to aid mental and written computation

- use an appropriate non-calculator method to divide two-and three-digit numbers by a two-digit number
  - compare initial estimates with answers obtained by written methods and check with a calculator (Understanding, Problem Solving) [N] [CCT]
- apply a practical understanding of commutativity to aid mental computation [N] [CCT],
  eg $3+9=9+3=12$, $3\times 9=9\times 3=27$
- apply a practical understanding of associativity to aid mental computation [N] [CCT],
  eg $3+8+2=3+(8+2)=13$, $2\times 7\times 5=7\times (2\times 5)=70$
- apply a practical understanding of the distributive law to aid mental computation [N] [CCT],
  eg to multiply any number by 13, first multiply by 10 and then add 3 times the number
- use factors of a number to aid mental computation of multiplication and division [N],
  eg to multiply a number by 12, first multiply the number by 6 and then multiply by 2

Compare, order, add and subtract integers

- place directed numbers on a number line [N]
- order directed numbers [N]
- add and subtract directed numbers
  - determine, by developing patterns or using a calculator, that subtracting a negative number is the same as adding a positive number (Reasoning) [N] [CCT]
  - apply directed numbers to problems involving money and temperature (Problem Solving) [N] [CCT]

Carry out the four operations with integers, using efficient mental and written strategies and appropriate digital technologies

- multiply and divide directed numbers

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Number and Algebra
Computation with Integers

- investigate, by developing patterns or using a calculator, the rules associated with multiplying and dividing directed numbers (Reasoning, Understanding) [N] [CCT]

- use the $\boxed{+/-}$ key to enter directed numbers into a calculator
- use a calculator to perform operations with integers
  - evaluate whether it is more appropriate to use mental strategies or a calculator when performing certain operations with integers (Fluency) [CCT]
- use grouping symbols as an operator with integers
- apply the order of operations to mentally evaluate expressions involving integers, including where fractions contain an operator within their numerator or denominator, eg $\frac{15+9}{6}$, $\frac{15+9}{15-3}$, $5+\frac{18-12}{6}$, $5+\frac{18}{6}-12$, $5\times(2-8)$
  - investigate whether other calculators, such as those found in computer software and on mobile phones, use order of operations (Understanding, Fluency) [N] [CCT]

Background Information:

To divide two- and three-digit numbers by a two-digit number, students may be taught the long division algorithm or, alternatively, to transform the division into a multiplication.

eg (i) $88 \div 44 = 2$ because $2 \times 44 = 88$;
(ii) $356 \div 52 = \square$ becomes $52 \times \square = 356$. Knowing that there are two fifties in each 100, students may try 7 so that $52 \times 7 = 364$
which is too large. Try 6 so that $52 \times 6 = 312$. Answer is $6\frac{44}{52} = 6\frac{11}{13}$.

Students also need to be able to express a division in the following form in order to relate multiplication and division:
$356 = 6 \times 52 + 44$; and then division by 52 gives: $\frac{356}{52} = 6 + \frac{44}{52} = 6\frac{11}{13}$

Complex recording formats for directed numbers such as raised signs can be confusing. The following formats are recommended:

$$-2-3=-5$$
$$-7+(-4)=-7-4=-11$$
$$-2--3=-2+3=1$$

Brahmagupta, an Indian mathematician and astronomer (c.598 – c.665 CE) is noted for the introduction of zero and negative numbers in arithmetic.

Language:

Note the distinction between the use of fewer/fewest for number of items and less/least for quantities, eg ‘There are fewer students in this class; there is less milk today.’

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Number and Algebra
Fractions, Decimals and Percentages

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.3 recognises and explains mathematical relationships using reasoning
4.4 operates efficiently with different representations of numbers and numerical relationships, including financial calculations

Students:

Compare fractions using equivalence. Locate and represent fractions and mixed numerals on a number line

- determine highest common factors and lowest common multiples
- generate equivalent fractions
- reduce a fraction to its simplest form
- express improper fractions as mixed numerals and vice versa
- place fractions and mixed numerals on a number line [N]
  - interpret a given scale to determine fractional values represented on a number line (Understanding) [N]
  - choose an appropriate scale to display given fractional values on a number line, eg when plotting thirds or sixths, choose a scale of 3 cm for every one unit value (Fluency) [N]

Solve problems involving addition and subtraction of fractions, including those with unrelated denominators

- add and subtract fractions, including mixed numerals, using written and calculator methods
  - recognise and explain incorrect operations with fractions eg explain why $\frac{2}{3}+\frac{1}{4}\neq\frac{3}{7}$ (Reasoning) [L] [N] [CCT]
  - interpret fractions and mixed numerals on a calculator display (Understanding) [N] [CCT]
- subtract a fraction from a whole number, eg $3-\frac{2}{3}=2+1-\frac{2}{3}=2\frac{1}{3}$

Multiply and divide fractions and decimals using efficient written strategies and digital technologies

- determine the effect of multiplying or dividing by a number less than one [N]
- multiply and divide decimals using written methods, limiting operators to two digits
  - compare initial estimates with answers obtained by written methods and check with a calculator (Understanding, Problem Solving) [CCT]
- multiply and divide fractions and mixed numerals
  - demonstrate multiplication of a fraction by another fraction using a diagram to illustrate the process (Understanding, Reasoning) [L] [N]

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Number and Algebra
Fractions, Decimals and Percentages

- explain why division by a fraction is equivalent to multiplication by its reciprocal using a numerical example (Understanding, Reasoning) [L] [N] [CCT]

- calculate fractions and decimals of quantities [N]
  - choose the appropriate equivalent form for mental computation [N] [CCT], eg 0.25 of $\$60$ is equivalent to $\frac{1}{4}$ of $\$60$ which is equivalent to $\$60 \div 4$ (Understanding, Fluency)

Express one quantity as a fraction of another, with and without the use of digital technologies

- express one quantity as a fraction of another [N]
  - choose appropriate units to compare two quantities as a fraction [N] [CCT], eg 15 minutes is $\frac{15}{60} = \frac{1}{4}$ of an hour (Understanding, Fluency)

Round decimals to a specified number of decimal places

- round decimals to a given number of places
- use symbols for approximation, eg $\approx$ [L]

Investigate terminating and recurring decimals

- use the notation for recurring (repeating) decimals [L], eg $0.33333\ldots = 0.\dot{3}$, $0.345345345\ldots = 0.\dot{3}4\dot{5}$, $0.266666\ldots = 0.2\dot{6}$
- convert fractions to terminating or recurring decimals as appropriate
  - recognise that calculators may show approximations to recurring decimals, and explain why, eg $\frac{2}{3}$ displayed as 0.666666667 (Understanding, Reasoning) [CCT]
  - verify on a calculator that $0.\dot{9} = 1$ and explain why this is true (Reasoning) [CCT]

Connect fractions, decimals and percentages and carry out simple conversions

- convert fractions to decimals (terminating and recurring) and percentages
- convert terminating decimals to fractions and percentages
- convert percentages to fractions and decimals
  - evaluate the reasonableness of statements in the media that quote fractions, decimals or percentages, eg 'the number of children in the average family is 2.3' (Understanding, Problem Solving) [N] [CCT]
- order fractions, decimals and percentages

Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies

- calculate percentages of quantities [N]
  - choose an appropriate equivalent form for mental computation [N] [CCT], eg 20% of $\$40$ is $\frac{1}{5} \times \$40$ (Understanding)

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Number and Algebra
Fractions, Decimals and Percentages

- express one quantity as a percentage of another, eg 45 minutes is 75% of an hour [N]

Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies

- increase and decrease a quantity by a given percentage [N]
  - recognise equivalences when calculating [N], eg multiplication by 1.05 will increase a number/quantity by 5%; multiplication by 0.87 will decrease a number/quantity by 13% (Understanding)
- interpret and calculate percentages greater than one hundred [N], eg an increase from $2 to $3 is an increase of 150%
- solve a variety of real-life problems involving percentages, including percentage composition (Problem Solving) [N]
  - interpret calculator displays in formulating solutions to problems by appropriately rounding decimals (Fluency, Understanding)
  - use the unitary method to solve problems involving percentages, eg find the original value, given the value after an increase of 20% (Fluency) [N]
  - interpret nutritional information panels on products (Understanding) [L] [N]
  - interpret media and sport reports involving percentages (Understanding) [N] [CCT]

Background Information:

The earliest evidence of fractions can be traced to the Egyptian papyrus of Ahmes (about 1650 BCE). In the seventh century CE the method of writing fractions as we write them now was invented in India, but without the fraction bar (vinculum), which was introduced by the Arabs. Fractions were widely in use by the 12th century.

One cent and two cent coins were withdrawn by the Australian Government in 1990. Prices can still be expressed in one-cent increments but the final bill is rounded to the nearest five cents (except for electronic transactions). In this context, rounding is different to normal conventions in that totals ending in 3, 4, 6, and 7 are rounded to the nearest 5 cents, and totals ending in 8, 9, 1, and 2 are rounded to the nearest 0 cents.

Language:

The word fraction comes from the Latin word ‘frangere’ meaning ‘to break’.

The word ‘cent’ comes from the Latin word ‘centum’ meaning ‘one hundred’. Percent means ‘out of one hundred’ or ‘hundredths’.

Students may need assistance with the subtleties of the English language when solving word problems, eg ‘$\frac{1}{10}$ **of** $50’ is not the same as ‘$\frac{1}{10}$ **off** $50’.

Students may wrongly interpret words giving a mathematical instruction (eg estimate, multiply, simplify) to just mean ‘get the answer’.

Mathematics • Stage 4

Number and Algebra
Financial Mathematics

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.3 recognises and explains mathematical relationships using reasoning
4.4 operates efficiently with different representations of numbers and numerical relationships, including financial calculations

Students:

Investigate and calculate Goods and Services Tax (GST), with and without digital technologies

- calculate GST and GST-inclusive prices for goods purchased in Australia [N] [PSC]
  - interpret GST information contained on receipts (Understanding) [N]
  - investigate efficient methods of computing GST and GST-inclusive prices (Problem Solving) [N]
  - explain why the GST is not equivalent to 10% of the GST-inclusive price (Reasoning) [N]
- determine pre-GST prices for goods given the GST-inclusive price [N]
  - explain why the pre-GST price is not equivalent to 10% off the GST-inclusive price (Reasoning) [N]

Investigate and calculate 'best buys', with and without digital technologies

- solve problems involving discounts, including calculating the percentage discount (Problem Solving) [N]
  - evaluate special offers such as percentage discounts, buy-two-get-one-free, buy-one-get-another-at-half-price, etc to determine how much is saved (Problem Solving) [N] [PSC]
- calculate 'best buys', eg 500 grams for $4.50 compared with 300 grams for $2.75 [N] [PSC]
  - investigate 'unit pricing' used by retailers and use this to determine the best buy (Understanding, Fluency) [N] [PSC]
  - recognise that in practical situations there are other considerations than just the 'best buy', eg the amount required, waste due to spoilage (Reasoning) [N] [CCT] [PSC]
  - use price comparison websites to make informed decisions related to purchases under given conditions (Problem Solving) [N] [ICT] [CCT] [PSC]

Solve problems involving profit and loss, with and without digital technologies

- calculate the selling price given the percentage profit/loss on the cost price [N]
- express profit/loss as a percentage of the cost price [N]
- calculate the cost price given the selling price and percentage profit/loss [N]

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Number and Algebra
Financial Mathematics

Background Information:

The Goods and Services Tax (GST) in Australia is a value added tax on the supply of goods and services. It was introduced by the Federal Government and took effect from 1 July 2000. Prior to the GST, Australia operated a wholesale sales tax implemented in the 1930s when Australia had an economy dominated by goods. The GST is levied at a flat rate of 10% on most goods and services, apart from GST exempt items (usually basic necessities like milk and bread).

Language:

GST stands for 'Goods and Services Tax'.

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Number and Algebra
Proportion

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.3 recognises and explains mathematical relationships using reasoning
4.4 operates efficiently with different representations of numbers and numerical relationships, including financial calculations

Students:

Recognise and solve problems involving simple ratios

- use ratio to compare quantities measured in the same units [N]
- write ratios using the symbol ':', eg $4:7$ [L]
  - express one part of a ratio as a fraction of the whole, eg in the ratio $4:7$ the first part is $\frac{4}{11}$ of the whole (Understanding)
- simplify ratios, eg $4:6=2:3$, $\frac{1}{2}:2=1:4$, $0.3:1=3:10$
- divide a quantity in a given ratio [N]

Solve a range of problems involving rates and ratios, with and without digital technologies

- interpret and calculate ratios that involve more than two numbers
- solve a variety of real-life problems involving ratios (Problem Solving) [N]
  - interpret and use scales on maps, plans and enlargement diagrams (Understanding, Fluency) [N]
  - apply the unitary method to ratio problems (Fluency) [N]
- distinguish between ratios, where quantities are measured in the same units, and rates, where quantities are measured in different units [N]
- convert given information into rates, eg 150 kilometres travelled in 2 hours = 75 km/h [N]
- solve a variety of real-life problems involving rates (Problem Solving) [N] [CCT]

Investigate, interpret and analyse graphs from authentic data

- interpret distance/time graphs (travel graphs) made up of straight line segments [N]
  - write or tell a story which matches a given travel graph (Understanding) [L] [N]
  - match a travel graph to a description of a particular event and explain reasons for the choice (Understanding, Reasoning) [L] [N]
  - compare travel graphs of the same situation, decide which one is the most appropriate and explain why (Understanding, Fluency, Reasoning) [L] [N] [CCT]
- recognise concepts such as change of speed and direction in distance/time graphs [N]
  - describe the meaning of different slopes for the graph of a particular event (Understanding) [N]

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Number and Algebra
Proportion

  - calculate speeds for straight line segments of given graphs (Fluency) [N]

- recognise the significance of horizontal lines in distance/time graphs [N]
- determine which variable should be placed on the horizontal axis in distance/time graphs [N]
- draw distance/time graphs made up of straight line segments [N]
- sketch informal graphs to model familiar events, eg noise level during the lesson [N]
  - record the distance of a moving object from a fixed point at equal time intervals and draw a graph to represent the situation,
    eg move along a measuring tape for 30 seconds using a variety of activities that involve a constant rate such as walking forwards or backwards slowly, and walking or stopping for 10-second increments (Understanding, Problem Solving) [N]
- use the relative positions of two points on a graph, rather than a detailed scale, to interpret information [N]

Background Information:

Work with ratio may be linked with the Golden Rectangle. Many windows are Golden Rectangles, as are some of the buildings in Athens such as the Parthenon. The ratio of the dimensions of the Golden Rectangle was known to the ancient Greeks: $\frac{\text{length}}{\text{width}} = \frac{\text{length} + \text{width}}{\text{length}}$

In this Stage, the focus is on examining situations where the data yields a constant rate of change. It is possible that some practical situations may yield a variable rate of change. This is the focus in the 5.3 pathway.

It is the usual practice in Mathematics to place the independent variable on the horizontal axis and the dependent variable on the vertical axis. This is not always the case in other subjects, eg Economics.

Language:

The word *ratio* derives from the Greek logos originally meaning ‘reason’ but which developed into a term for the principle of order and knowledge.

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Number and Algebra
Algebraic Techniques 1

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.3 recognises and explains mathematical relationships using reasoning
4.5 generalises number properties to operate with algebraic expressions and solves linear equations

Students:

Introduce the concept of variables as a way of representing numbers using letters

- use letters to represent numbers and develop the concept that a letter is used to represent a variable [N]
- use concrete materials to model the following:
  - expressions that involve a variable, and a variable plus a constant, eg a, $a+1$ [N]
  - expressions that involve a variable multiplied by a constant, eg $2a$, $3a$ [N]
  - sums and products, eg $2a+1$, $2(a+1)$ [N]
  - equivalent expressions, such as $x+x+y+y+y=2x+2y+y=2(x+y)+y$ [N]
  - and to assist with simplifying expressions, such as
    $(a+2)+(2a+3)=(a+2a)+(2+3)$ [N]
    $=3a+5$
- recognise and use equivalent algebraic expressions [L],
  eg $y+y+y+y=4y$

  $w\times w=w^2$

  $a\times b=ab$

  $a\div b=\dfrac{a}{b}$
- use algebraic symbols to represent mathematical operations written in words and vice versa, eg the product of x and y is xy; $x+y$ is the sum of x and y [L] [N]
  - generate a variety of equivalent expressions that represent a particular situation (Understanding, Fluency) [N] [CCT]

Extend and apply the laws and properties of arithmetic to algebraic terms and expressions

- recognise like terms and add and subtract them to simplify algebraic expressions, eg $2n+4m+n=4m+3n$
  - determine equivalence of algebraic expressions by substituting a given number for the letter (Understanding, Reasoning) [N]
  - connect algebra with the commutative and associative properties of arithmetic to determine that $a+b=b+a$ and $a+b+c=a+(b+c)$ (Understanding) [CCT]
- recognise the role of grouping symbols and the different meanings of expressions, such as $2a+1$ and $2(a+1)$ [CCT]

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Number and Algebra
Algebraic Techniques 1

- translate from everyday language to algebraic language and vice versa [L] [N]
  - use algebraic symbols to represent simple situations described in words [L] [N], eg write an expression for the number of cents in x dollars (Understanding)
  - interpret statements involving algebraic symbols in other contexts [L] [N] [ICT], eg creating and formatting spreadsheets (Understanding)
- simplify algebraic expressions that involve multiplication and division, eg $12a \div 3,\ 4x \times 3,\ 2ab \times 3a,\ \frac{8a}{2},\ \frac{2a}{8},\ \frac{12a}{9}$
  - connect algebra with the commutative and associative properties of arithmetic to determine that $a \times b = b \times a$ and $a \times b \times c = a \times (b \times c)$ (Understanding) [CCT]

Simplify algebraic expressions involving the four operations

- simplify a range of algebraic expressions including those involving mixed operations
  - apply the order of operations to simplify algebraic expressions (Fluency) [N]

Background Information:

It is important to develop an understanding of the use of letters as algebraic symbols for variable numbers.

The recommended approach is to spend time over the conventions for the use of algebraic symbols for first-degree expressions and to situate the translation of generalisations from words to symbols as an application of students' knowledge of the symbol system rather than as an introduction to the symbol system.

The recommended steps for moving into symbolic algebra are:

- the variable notion, associating letters with a variety of variable numbers
- symbolism for a variable plus a constant
- symbolism for a variable times a constant
- symbolism for sums and products.

Thus if $a = 6, a + a = 6 + 6$ but $2a = 2 \times 6$ and **not** 26.

To gain an understanding of algebra, students must be introduced to the concepts of variables, expressions, unknowns, equations, patterns, relationships and graphs in a wide variety of contexts. For each successive context, these ideas need to be redeveloped. Students need gradual exposure to abstract ideas as they begin to relate algebraic terms to real situations.

It is suggested that the introduction of the symbol system precede Linear Relationships Stage 4, since this topic presumes students are able to manipulate algebraic symbols and will use them to generalise patterns.

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Number and Algebra
Algebraic Techniques 2

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.3 recognises and explains mathematical relationships using reasoning
4.5 generalises number properties to operate with algebraic expressions and solves linear equations

Create algebraic expressions and evaluate them by substituting a given value for each variable

- substitute into and evaluate algebraic expressions and evaluate the result
  - calculate and compare the value of x^2 for values of x with the same magnitude but opposite sign (Fluency, Understanding) [CCT]
  - determine and justify whether a simplified expression is correct by substituting numbers for variables (Problem Solving, Reasoning) [CCT]
- generate a number pattern from an algebraic expression [N]

eg

x	1	2	3	4	5	6	10	100
$x+3$	4	5	6					

- replace written statements describing patterns with equations written in algebraic symbols, eg 'you add five to the first number to get the second number' could be replaced with '$y = x + 5$' [L] [N]
  - determine whether a particular pattern can be described using algebraic symbols (Problem Solving, Understanding) [CCT]

Extend and apply the distributive law to the expansion of algebraic expressions

- expand algebraic expressions by removing grouping symbols, eg $3(a+2)=3a+6$, $-5(x+2)=-5x-10$, $a(a+b)=a^2+ab$
  - connect algebra with the distributive property of arithmetic to determine that $a(b+c)=ab+ac$ (Understanding) [CCT]

Factorise algebraic expressions by identifying numerical factors

- factorise a single algebraic term, eg $6ab = 3\times 2\times a\times b$
- factorise algebraic expressions by finding a common factor, eg $6a+12=6(a+2)$, $-4t-12=-4(t+3)$
  - check expansions and factorisations by performing the reverse process (Fluency, Reasoning) [CCT]

Factorise algebraic expressions by identifying algebraic factors

- factorise algebraic expressions by finding a common algebraic factor, eg $x^2-5x=x(x-5)$, $5ab+10a=5a(b+2)$

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Number and Algebra
Algebraic Techniques 2

Background Information:

When evaluating expressions, there must be an explicit direction to replace the letter by a number to ensure full understanding of notation occurs.

Mathematics • Stage 4

Number and Algebra
Indices

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.3 recognises and explains mathematical relationships using reasoning
4.4 operates efficiently with different representations of numbers and numerical relationships, including financial calculations

Students:

Investigate index notation and represent whole numbers as products of powers of prime numbers

- evaluate numbers expressed as powers of positive whole numbers, eg $2^3 = 8$
- describe numbers written in index form using terms such as base, power, index, exponent [L]
- use index notation to express powers of numbers (positive indices only), eg $8 = 2^3$ [L]
- determine and apply tests of divisibility [CCT]
  - verify the various tests of divisibility using a calculator (Problem Solving)
  - apply tests of divisibility mentally as an aid to calculation (Fluency) [CCT]
- express a number as a product of its prime factors, using index notation where applicable
  - find the highest common factor of large numbers by first expressing the numbers as products of prime factors (Problem Solving, Fluency) [CCT]

Investigate and use square roots of perfect square numbers

- use the notation for square root $\left(\sqrt{\ }\right)$ and cube root $\left(\sqrt[3]{\ }\right)$ [L]
- recognise the link between squares and square roots and cubes and cube roots [CCT], eg $2^3 = 8$ and $\sqrt[3]{8} = 2$
- determine through numerical examples that:
  $(ab)^2 = a^2b^2$, eg $(2\times3)^2 = 2^2\times3^2$
  $\sqrt{ab} = \sqrt{a}\times\sqrt{b}$, eg $\sqrt{9\times4} = \sqrt{9}\times\sqrt{4}$
- express a number as a product of its prime factors to determine whether it has an integer square and/or cube root [N]
- find square roots and cube roots of any whole number using a calculator, after first estimating [N]
  - determine whether it is more appropriate to use mental strategies or a calculator to find the square root of a given number (Understanding, Fluency) [CCT]

Use index notation with numbers to establish the index laws with positive integral indices and the zero index

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Number and Algebra
Indices

- develop index laws arithmetically by expressing each term in expanded form [L],
  eg $3^2 \times 3^4 = (3\times3)\times(3\times3\times3\times3) = 3^{2+4} = 3^6$
  $3^5 \div 3^2 = \dfrac{3\times3\times3\times3\times3}{3\times3} = 3^{5-2} = 3^3$
  $(3^2)^4 = (3\times3)\times(3\times3)\times(3\times3)\times(3\times3) = 3^{2\times4} = 3^8$
  - verify the index laws using a calculator,
    eg use a calculator to compare the values of $(3^4)^2$ and 3^8 (Fluency, Understanding)
  - explain the incorrect use of index laws, eg why $3^2 \times 3^4 \neq 9^6$ (Reasoning) [L] [CCT]
- establish the meaning of the zero index, eg by patterns [CCT]

3^5	3^4	3^3	3^2	3^1	3^0
243	81	27	9	3	1

  - verify the zero index law using a calculator (Fluency, Understanding)
- use index laws to simplify expressions with numerical bases, eg $5^2 \times 5^4 \times 5 = 5^7$

Background Information:

The square root sign signifies a positive number (or zero). Thus $\sqrt{9} = 3$ (only). However, the two numbers whose square is 9 are $\sqrt{9}$ and $-\sqrt{9}$ ie 3 and –3.

Language:

Words such as 'square' have more than one mathematical context, eg draw a square; square three; find the square root of 9. Students may need to have these differences explained.

Words such as 'product', 'prime', 'power', 'base' and 'index' have different meanings in mathematics from their everyday usage. This may be confusing for some students.

Mathematics • Stage 4

Number and Algebra
Equations

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.3 recognises and explains mathematical relationships using reasoning
4.5 generalises number properties to operate with algebraic expressions and solves linear equations

Students:

Solve simple linear equations

- distinguish between algebraic expressions where letters are used as variables, and equations where letters are used as unknowns [CCT]
- solve simple linear equations using concrete materials, such as the balance model or cups and counters, stressing the notion of doing the same thing to both sides of an equation [N]
- solve linear equations using algebraic methods that involve one or two steps in the solution process and may have non-integer solutions,

  eg $x-7=15$ $\quad \frac{x}{7}=5$
  $2x-7=15$ $\quad \frac{2x}{7}=5$
  $7-2x=15$

  – compare and contrast strategies to solve a range of linear equations, such as guess-check-improve, one-to-one matching and backtracking (Fluency, Reasoning) [CCT]
  – generate equations with a given solution, eg find equations that have the solution $x=5$ (Fluency, Problem Solving) [N]

Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution

- solve linear equations using algebraic methods that involve at least two steps in the solution process and may have non-integer solutions,

  eg $3x+4=13$ $\quad 3x+4=x-8$ $\quad \frac{x}{3}+5=10$ $\quad \frac{x+5}{3}=10$
  $3(x+4)=13$ $\quad 3x+4=8-x$

- check solutions to equations by substituting [CCT]

Mathematics • Stage 4

Number and Algebra
Equations

Background Information:

Simple equations can usually be solved using arithmetic methods. Students need to solve equations where the solutions are not whole numbers and that require the use of algebraic methods.

Five models have been proposed to assist students with the solving of simple equations.

Model 1 uses a two-pan balance and objects such as coins or centicubes. A light paper wrapping can hide a 'mystery number' of objects without distorting the balance's message of equality.

Model 2 uses small objects (all the same) with some hidden in containers to produce the 'unknowns' or 'mystery numbers'.

eg place the same number of small objects in a number of paper cups and cover them with another cup. Form an equation using the cups and then remove objects in equal amounts from each side of a marked equals sign.

Model 3 uses one-to-one matching of terms on each side of the equation.

$3x+1=2x+3$
eg $x+x+x+1=x+x+2+1$ giving $x=2$ through one-to-one matching.

Model 4 uses a substitution approach. By trial and error a value is found for the unknown that produces equality for the values of the two expressions on either side of the equation (this highlights the variable concept).

Model 5 uses backtracking or a reverse flow chart to unpack the operations and find the solution. This model only works for equations with all letters on the same side.

eg $3d+5=17$
$17-5\rightarrow \square \div 3\rightarrow \Delta$
$17-5\rightarrow 12\div 3\rightarrow 4$
$\therefore d=4$

Mathematics • Stage 4

Number and Algebra
Linear Relationships

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.3 recognises and explains mathematical relationships using reasoning
4.6 graphs and interprets linear relationships on the number plane

Students:

Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point

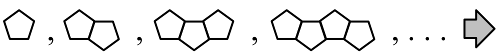
- read, plot and name ordered pairs on the number plane including those with values that are not whole numbers [N]

Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries

- use the notation P' to name the 'image' resulting from a transformation of a point P on the Cartesian plane [L]
- plot and name the coordinates for P' resulting from translating P one or more times
- plot and name the coordinates for P' resulting from reflecting P in either the x- or y-axis
  – investigate and describe the relationship between the coordinates of P and P' following a reflection in the x- or y-axis,
    eg if P is reflected in the x-axis, P' has the same x-coordinate, and its y-coordinate is the same but opposite in sign (Understanding) [CCT]
  – recognise that a translation can produce the same result as a single reflection and vice versa (Understanding) [CCT]
- plot and name the coordinates for P' resulting from rotating P by a multiple of $90°$ about the origin
  – investigate and describe the relationship between the coordinates of P and P' following a rotation of $180°$ about the origin,
    eg if P is rotated $180°$ about the origin, P' has the same x- and y-coordinates but both are opposite in sign (Understanding) [CCT]
  – recognise that a combination of translations and/or reflections can produce the same result as a single rotation and that a combination of rotations can produce the same result as a single translation and/or reflection (Understanding) [CCT]

Plot linear relationships on the Cartesian plane with and without the use of digital technologies

- use objects to build a geometric pattern, record the results in a table of values, describe the pattern in words and algebraic symbols and represent the relationship on a number grid

eg 

number of pentagons (p)	1	2	3	4	…
number of matches (m)	5	9	13	17	…

[N]

  – check pattern descriptions by substituting further values (Reasoning, Fluency) [CCT]
  – describe the pattern formed by plotting points from a table and suggest another set of points that might form the same pattern (Understanding, Reasoning) [CCT]

Mathematics • Stage 4

Number and Algebra
Linear Relationships

  - describe what has been learnt from creating patterns, making connections with number facts and number properties (Understanding) [CCT]
- recognise a given number pattern (including decreasing patterns), complete a table of values, describe the pattern in words and algebraic symbols, and represent the relationship on a number grid [L]
  - generate a variety of number patterns that increase or decrease and record them in more than one way (Fluency, Understanding) [N]
  - explain why a particular relationship or rule for a given pattern is better than another (Reasoning, Understanding) [CCT]
  - distinguish between graphs that represent an increasing number pattern and those that represent a decreasing number pattern (Understanding) [CCT]
  - determine whether a particular number pattern can be described using algebraic symbols (Understanding, Fluency) [CCT]
- use a rule generated from a pattern to calculate the corresponding value for a larger number [N]
- form a table of values for a linear relationship by substituting a set of appropriate values for either of the letters and graph the number pairs on the number plane, eg given $y = 3x + 1$, form a table of values using $x = 0$, 1 and 2 and then graph the number pairs on a number plane with appropriate scale [N]
  - explain why 0, 1 and 2 are frequently chosen as x-values in a table of values (Understanding, Reasoning) [CCT]
- extend the line joining a set of points to show that there is an infinite number of ordered pairs that satisfy a given linear relationship [N]
  - interpret the meaning of the continuous line joining the points that satisfy a given number pattern (Understanding) [CCT]
  - read values from the graph of a linear relationship to demonstrate that there are many points on the line (Understanding) [CCT]
- derive a rule for a set of points that has been graphed on a number plane [N]
- graph more than one line on the same set of axes using ICT and compare the graphs to determine similarities and differences, eg parallel, pass through the same point [N] [ICT] [CCT]
  - identify similarities and differences between sets of linear relationships (Understanding) [CCT],
    eg $y = 3x$, $y = 3x + 2$, $y = 3x - 2$
    $y = x$, $y = 2x$, $y = 3x$
    $y = -x$, $y = x$
  - determine which term affects the slope of a graph, making it increase or decrease (Reasoning) [CCT]
- use ICT to graph linear and simple non-linear relationships such as $y = x^2$ [ICT]

Mathematics • Stage 4

Number and Algebra
Linear Relationships

  - recognise and explain that not all patterns form a linear relationship (Understanding) [CCT]
  - determine and explain differences between equations that represent linear relationships and those that represent non-linear relationships (Fluency, Understanding) [L] [CCT]

Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution

- recognise that each point on the graph of a linear relationship represents a solution to a particular linear equation [N]
- use graphs of linear relationships to solve a corresponding linear equation with or without ICT,
  eg use the graph of $y = 2x + 3$ to find the solution of the equation $2x + 3 = 11$ [N] [ICT]
- graph two intersecting lines on the same set of axes and read off the point of intersection [N]
  - explain the significance of the point of intersection of two lines in relation to it being the only solution of both equations (Fluency, Reasoning) [L] [CCT]

Background Information:

When describing number patterns algebraically, it is important that students develop an understanding of the use of letters as algebraic symbols for variable numbers of objects rather than for the objects themselves. The practice of using the first letter of the name of an object as a symbol for the number of such objects (or worse still as a symbol for the object) can lead to misconceptions and should be avoided.

In 'Linear Relationships', linear refers to straight lines.

While alternative grid systems may be used in early experiences, it is intended that the standard rectangular grid system be established.

Descartes and Fermat used coordinates to identify points in terms of positive or zero distances from axes. Isaac Newton introduced negative values.

Language:

Students will need to become familiar with and be able to use new terms including coefficient, constant term, and intercept.

Mathematics • Stage 4

Measurement and Geometry
Length

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.7 calculates time durations, lengths and areas of plane shapes, and volumes of prisms and cylinders

Students:

Find perimeters and areas of parallelograms, rhombuses and kites

- find the perimeter of a range of plane shapes including parallelograms, rhombuses, kites and simple composite figures [N]
  - compare perimeters of rectangles with the same area (Understanding) [CCT]
- solve problems involving the perimeter of plane shapes,
  eg find the dimensions of a rectangle given its perimeter and the length of one other side (Problem Solving) [N] [CCT]

Investigate the concept of irrational numbers, including π

- identify and name parts of a circle and related lines, including arc, tangent, chord, sector and segment [L]
- demonstrate by practical means that the ratio of the circumference to the diameter of a circle is constant,
  eg measure and compare the diameter and circumference of various cylinders or use dynamic geometry software to measure circumferences and diameters [N] [ICT] [CCT]
- define the number π as the ratio of the circumference to the diameter of any circle [L]
  - compare the various approximations for π used throughout the ages and the concept of irrational numbers (Understanding) [CCT]

Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area

- develop and use the formulae to calculate the circumference of circles in terms of the radius r or diameter d
  $\text{Circumference of circle} = \pi d$ and $\text{Circumference of circle} = 2\pi r$ [N] [CCT]
  - use mental strategies to estimate the circumference of circles, using an approximate value of π such as 3 (Fluency) [N]
  - find diameter and/or radii of circles given their circumference (Fluency) [N]
- find the perimeter of quadrants and semi-circles [N]
- calculate the perimeter of simple composite figures consisting of two shapes including quadrants and semicircles [N]
- calculate the perimeter of sectors [N]

Mathematics • Stage 4

Measurement and Geometry
Length

- solve problems involving circles and parts of circles, giving an exact answer in terms of π and an approximate answer using the calculator's approximation for π [N]

Background Information:

Graphing of the relationship between the length of a rectangle with a constant perimeter and possible areas of the rectangle links to non-linear graphs in Algebra.

Students should develop a sense of the levels of accuracy that are appropriate to a particular situation, eg the length of a bridge may be measured in metres to estimate a quantity of paint needed but would need to be measured far more accurately for engineering work.

The number π is known to be irrational (not a fraction) and also transcendental (not the solution of any polynomial equation with integer coefficients). At this Stage, students only need to know that the digits in its decimal expansion do not repeat (all this means is that it is not a fraction), and in fact have no known pattern.

Pi (π) is the Greek letter equivalent to 'p', and is the first letter of the Greek word 'perimetron' meaning perimeter. In 1737, Euler used the symbol for pi for the ratio of the circumference to the diameter of a circle.

Language:

Perimeter comes from the Greek word 'perimetron' meaning perimeter.

Mathematics • Stage 4

Measurement and Geometry
Area

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.7 calculates time durations, lengths and areas of plane shapes, and volumes of prisms and cylinders

Students:

Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving

- develop and use the formulae for the area of squares and rectangles
  Area of rectangle $= lb$ where l is the length and b is the breadth of the rectangle
  Area of square $= s^2$ where s is the side length of the square [N] [CCT]
  - explain the relationship that multiplying, dividing, squaring and factoring have with the areas of squares and rectangles with integer side lengths (Understanding, Fluency) [N] [CCT]
  - explain the relationship between the formulae for the area of squares and rectangles (Understanding, Fluency) [CCT]
  - compare areas of rectangles with the same perimeter (Understanding, Fluency) [N] [CCT]
- develop, with or without ICT, and use the formulae for the area of parallelograms and triangles
  Area of parallelogram $= bh$ where b is the length of the base and h is the perpendicular height
  Area of triangle $= \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height [N] [ICT] [CCT]
  - identify the perpendicular height of triangles and parallelograms in different orientations (Understanding) [CCT]
- find the areas of simple composite figures that may be dissected into squares, rectangles, parallelograms and triangles [N]
- solve problems relating to area (Problem Solving) [N]

Choose appropriate units of measurement for area and volume and convert from one unit to another

- choose an appropriate unit to measure area of different shapes and surfaces, eg floor space, fields [CCT]
  - use the areas of familiar surfaces to assist with estimation of larger areas (Fluency) [N] [CCT]
- convert between metric units of area
  $1\text{ cm}^2 = 100\text{ mm}^2$, $1\text{ m}^2 = 1000000\text{ mm}^2$, $1\text{ ha} = 10000\text{ m}^2$, $1\text{ km}^2 = 1000000\text{ m}^2 = 100\text{ ha}$

Find perimeters and areas of parallelograms, rhombuses and kites

Mathematics • Stage 4

Measurement and Geometry
Area

- develop, with or without ICT, and use the formulae for the area of a kite or rhombus,
  Area of rhombus/kite $= \frac{1}{2}xy$ where x and y are the lengths of the diagonals [N] [ICT] [CCT]
- select and use the appropriate formula to calculate the area of a quadrilateral [N] [CCT]

Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area

- develop, with or without ICT, the formula to calculate the area of circles,
  Area of circle $= \pi r^2$ where r is the length of the radius [N] [ICT] [CCT]
  - find radii of circles given their circumference or area (Fluency) [N]
- find the area of quadrants and semi-circles [N]
- calculate the area of sectors [N]
- solve problems involving circles or parts of circles, giving an exact answer in terms of π and an approximate answer using the calculator's approximation for π (Problem Solving) [N]

Background Information:

Finding the areas of rectangles and squares with integer side lengths is an important link between geometry and multiplication, division, factoring and squares. Factoring a number into the product of two numbers is equivalent to forming a rectangle with these side lengths, and squaring is equivalent to forming a square.

Area formulae for the triangle and parallelogram need to be developed by practical means and related to the area of a rectangle. The rhombus is treated as a parallelogram and the area found using the formula $A = bh$.

The formula for finding the area of a rhombus or kite depends upon the fact that the diagonals are perpendicular, and so is linked with the geometry of special quadrilaterals. The formula applies to any quadrilateral in which the diagonals are perpendicular.

The formula for finding the area of a circle may be established by using one or both of the following dissections:

- cut the circle into a large number of sectors, and arrange them alternately point-up and point-down to form a rectangle with height r and base length πr
- inscribe a number of congruent triangles in a circle, all with vertex at the centre and show that the area of the inscribed polygon is half the length of perimeter times the perpendicular height
- dissect the circle into a large number of concentric rings, cut the circle along a radius, and open it out to form a triangle with height r and base $2\pi r$.

The Greek writer, Heron, is best known for his formula for the area of a triangle:
$A = \sqrt{s(s-a)(s-b)(s-c)}$ where a, b and c are the lengths of the sides of the triangle and s is half the perimeter of the triangle.

Mathematics • Stage 4

Measurement and Geometry
Area

One of the three famous problems left unsolved by the ancient Greek mathematicians was the problem of ‘squaring the circle’, ie using straight edge and compasses to construct a square of area equal to a given circle.

Language:

The abbreviation m^2 is read ‘square metre(s)’ and not ‘metre squared’ or ‘metre square’.

The abbreviation cm^2 is read ‘square centimetre(s)’ and not ‘centimetre squared’ or ‘centimetre square’.

Mathematics • Stage 4

Measurement and Geometry
Volume

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.7 calculates time durations, lengths and areas of plane shapes, and volumes of prisms and cylinders

Students:

Draw different views of prisms and solids formed from combinations of prisms

- represent three-dimensional objects in two dimensions from different views [CCT]
- identify and draw the cross-section of a prism [L] [CCT]
- visualise, construct and draw various prisms from a given cross-sectional diagram [CCT]
- determine if a solid has a uniform cross-section [CCT]
  - recognise solids with uniform and non-uniform cross-sections (Understanding) [CCT]

Choose appropriate units of measurement for area and volume and convert from one unit to another

- choose an appropriate unit to measure volume or capacity of different objects, eg swimming pools, household containers [L] [N]
  - use the capacity of familiar solids to assist with estimation of larger capacities (Fluency, Reasoning) [N] [CCT]
- convert between units of volume and capacity
  $1\text{ cm}^3 = 1000\text{ mm}^3$, $1\text{ L} = 1000\text{ mL} = 1000\text{ cm}^3$, $1\text{ m}^3 = 1000\text{ L} = 1\text{ kL}$

Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume

- develop the formula for volume of prisms by considering the number and volume of layers of identical shape; Volume of prism $=$ base area $\times$ height leading to $V = Ah$ [N] [CCT]
- calculate the volume of a prism given its perpendicular height and the area of its cross-section [N]
- calculate the volume of prisms with cross-sections that are rectangular and triangular [N]
- solve practical problems involving volume and capacity of right prisms (Problem Solving) [N]

Calculate the surface area and volume of cylinders and solve related problems

- develop and use the formula to find the volume of cylinders,
  Volume of cylinder $= \pi r^2 h$
  where r is the length of the radius of the base and h is the perpendicular height [N] [CCT]
  - recognise and explain the relationship between the volume formulae for cylinders and prisms (Understanding) [N] [CCT]
- solve problems involving volume and capacity of right prisms and cylinders,
  eg calculate the capacity of a cylindrical can of drink or water tank (Problem Solving) [N]

Mathematics • Stage 4

Measurement and Geometry

Volume

Background Information:

The volumes of rectangular prisms and cubes are linked with multiplication, division, factorisation and powers. Factoring a number into the product of three numbers is equivalent to forming a rectangular prism with these side lengths, and to forming a cube if the numbers are all equal. Some students may be interested in knowing what fourth and higher powers, and the product of four or more numbers, correspond to.

When developing the volume formula students require an understanding of the idea of cross-section and can visualise, for example, stacking unit cubes layer by layer into a rectangular prism, or stacking planks into a pile.

The focus here is on right prisms and cylinders, although the formulae for volume also apply to oblique prisms and cylinders provided the perpendicular height is used. In a right prism, the base and top are perpendicular to the other faces. In a right pyramid or cone, the base has a centre of rotation, and the interval joining that centre to the apex is perpendicular to the base (and thus is its axis of rotation).

Oblique prisms, cylinders, pyramids and cones are those that are not right.

Language:

The abbreviation m^3 is read 'cubic metre(s)' and not 'metre cubed' or 'metre cube'.

The abbreviation cm^3 is read 'cubic centimetre(s)' and not 'centimetres cubed' or 'centimetre cube'.

Mathematics • Stage 4

Measurement and Geometry
Time

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.7 calculates time durations, lengths and areas of plane shapes, and volumes of prisms and cylinders

Students:

Solve problems involving duration, including using 12- and 24-hour time within a single time zone

- adding and subtracting time mentally using bridging strategies, eg from $2:45$ to $3:00$ is 15 minutes and from $3:00$ to $5:00$ is 2 hours, so the time from $2:45$ until $5:00$ is 15 minutes + 2 hours = 2 hours 15 minutes [N] [CCT]
- adding and subtracting time with a calculator using the 'degrees, minutes, seconds' button
- rounding calculator answers to the nearest minute or hour [N]
- interpreting calculator displays for time calculations, eg 2.25 on a calculator display for time means $2\frac{1}{4}$ hours [N]
- solve problems involving calculations with mixed time units, eg 'How old is a person today if he/she was born on 30 / 6 / 1999 ?' (Problem Solving) [N]

Solve problems involving international time zones

- compare times and calculate time differences between major cities of the world, eg 'Given that London is 10 hours behind Sydney, what time is it in London when it is 6 : 00 pm in Sydney?' [N] [CCT]
  - interpret and use information related to international time zones from maps (Understanding, Fluency) [N] [CCT]
  - solve problems about international time relating to everyday life, eg determine whether a particular soccer game can be watched live on television during normal waking hours (Problem Solving) [N]

Background Information:

The calculation of time can be done on a scientific calculator and links with fractions and decimals.

The Babylonians thought that the Earth took 360 days to travel around the Sun (last centuries BCE). This is why there are 360° in one revolution and hence 90° in one right angle. There are 60 minutes (60') in one hour and 60 minutes in one degree. The word 'minute' (meaning 'small') and minute (time measure), although pronounced differently, are really the same word. A minute (time) is a minute (small) part of one hour. A minute (angle) is a minute (small) part of a right angle.

Mathematics • Stage 4

Measurement and Geometry

Right-Angled Triangles (Pythagoras)

Outcomes

A student:

4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism

4.2 applies appropriate mathematical techniques to solve problems

4.7 calculates time durations, lengths and areas of plane shapes, and volumes of prisms and cylinders

Students:

Investigate Pythagoras' Theorem and its application to solving simple problems involving right angled triangles

- identify the hypotenuse as the longest side in any right-angled triangle and also as the side opposite the right angle [L]
- establish the relationship between the lengths of the sides of a right-angled triangle in practical ways, including using ICT [N] [ICT] [CCT]
  - describe the relationship between the sides of a right-angled triangle (Understanding) [CCT]
- use Pythagoras' theorem to find the length of sides in right-angled triangles [N]
- write answers to a specified or sensible level of accuracy, using the 'approximately equals' sign [L] [N] [CCT]
- solve practical problems involving Pythagoras' theorem, approximating the answer as a decimal (Problem Solving) [N]
  - apply Pythagoras' theorem to solve problems involving perimeter and area (Problem Solving) [N]
- identify a Pythagorean triad as a set of three numbers such that the sum of the squares of the first two equals the square of the third [L] [N]
- use the converse of Pythagoras' theorem to establish whether a triangle has a right angle [N] [CCT]

Investigate the concept of irrational numbers, including π

- use technology to explore decimal approximations of surds
  - recognise that surds can be represented by decimals that are neither terminating or have a repeating pattern (Understanding)
- solve practical problems involving Pythagoras' theorem, giving an exact answer as a surd, eg $\sqrt{5}$ (Problem Solving) [N]

Mathematics • Stage 4

Measurement and Geometry

Right-Angled Triangles (Pythagoras)

Background Information:

Students should gain an understanding of Pythagoras' theorem, rather than just being able to recite the formula in words. By dissecting and rearranging the squares, they will appreciate that the theorem is a statement of a relationship amongst the areas of squares.

Pythagoras' theorem becomes, in Stage 5, the formula for the circle in the coordinate plane. These links can be developed later in the context of circle geometry and the trigonometry of the general angle.

Pythagoras' theorem was probably known many centuries before Pythagoras (c.580-c.500 BCE), to at least the Babylonians.

In the 1990s, Wiles finally proved a famous conjecture of Fermat (1601-1665), known as 'Fermat's last theorem', that says that if n is an integer greater than 2, then $a^n + b^n = c^n$ has no integer solution.

Mathematics • Stage 4

Measurement and Geometry
Angle Relationships

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.3 recognises and explains mathematical relationships using reasoning
4.8 identifies and uses angle relationships and properties of plane shapes, including transformations and congruent figures

Students:

Use the language, notation and conventions of geometry

- label and name points, lines and intervals using capital letters [L]
- label the vertex and arms of an angle with capital letters [L]
- label and name angles using $\angle B$ and $\angle NPW$ notation [L]
- use the common conventions to indicate right angles and equal angles on diagrams [L]

Recognise the geometric properties of angles at a point

- use the words 'complementary' and 'supplementary' for angles adding to 90° and 180° respectively, and the associated terms 'complement' and 'supplement' [L] [N]
- identify and name adjacent angles (two angles with a common vertex and a common arm), vertically opposite angles, straight angles and angles of complete revolution, embedded in a diagram [L] [CCT]
  - recognise that adjacent angles can form right angles, straight angles and angles of complete revolution (Understanding) [CCT]

Identify corresponding, alternate and co-interior angles when two parallel straight lines are crossed by a transversal

- use the common conventions to indicate parallel lines on diagrams [L]
- identify and name a pair of parallel lines and a transversal [L]
  - identify parallel lines in the environment (Understanding) [N]
- use common symbols for 'is parallel to' ($||$) and 'is perpendicular to' ($\perp$)
- identify, name and measure alternate angle pairs, corresponding angle pairs and co-interior angle pairs for two lines cut by a transversal [CCT]
  - use dynamic geometry software to investigate angle relationships formed by parallel lines and a transversal (Fluency, Understanding) [N] [ICT]
- recognise the equal and supplementary angles formed when a pair of parallel lines are cut by a transversal [CCT]

Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning

Mathematics • Stage 4

Measurement and Geometry
Angle Relationships

- use angle properties to identify parallel lines [CCT]
  - explain why a pair of given lines is or is not parallel, giving a reason (Reasoning) [CCT]
- find the unknown angle in a diagram using angle relationships, including angles at a point and angles associated with parallel lines, giving a reason (Problem Solving) [N]

Background Information:

Students should give reasons when finding unknown angles. For some students formal setting out could be introduced. For example,

$\angle PQR = 70°$ (corresponding angles, $PQ \parallel SR$)

Eratosthenes' calculation of the circumference of the earth used parallel line results.

Dynamic geometry software or prepared applets are useful tools for investigating angle relationships; angles and lines can be dragged to new positions while angle measurements automatically update.

Students could explore the results about angles associated with parallel lines cut by a transversal by starting with corresponding angles – move one vertex and all four angles to the other vertex by a translation. The other two results then follow using vertically opposite angles and angles on a straight line. Alternatively, the equality of the alternate angles can be seen by rotation about the midpoint of the transversal.

Mathematics • Stage 4

Measurement and Geometry
Properties of Geometrical Figures 1

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.3 recognises and explains mathematical relationships using reasoning
4.8 identifies and uses angle relationships and properties of plane shapes, including transformations and congruent figures

Students:

Classify triangles according to their side and angle properties and describe quadrilaterals

- label and name triangles (eg ΔABC) and quadrilaterals (eg $ABCD$) in text and on diagrams [L]
- use the common conventions to mark equal intervals on diagrams [L]
- recognise and classify types of triangles on the basis of their properties (acute-angled triangles, right-angled triangles, obtuse-angled triangles, scalene triangles, isosceles triangles and equilateral triangles) [L] [CCT]
  - recognise that a given triangle may belong to more than one class (Understanding) [CCT]
  - explain why the longest side of a triangle is always opposite the largest angle (Reasoning) [CCT]
  - explain why two sides of a triangle must together be longer than the third side (Reasoning) [CCT]
  - sketch and label triangles from a worded or verbal description (Understanding, Fluency) [N]
- distinguish between convex and non-convex quadrilaterals (the diagonals of a convex quadrilateral lie inside the figure) [L] [CCT]
- investigate the properties of special quadrilaterals (trapeziums, kites, parallelograms, rectangles, squares and rhombuses) [L] [CCT]
  Properties to be considered include:
    - opposite sides parallel
    - opposite sides equal
    - adjacent sides perpendicular
    - opposite angles equal
    - diagonals equal in length
    - diagonals bisect each other
    - diagonals bisect each other at right angles
    - diagonals bisect the angles of the quadrilateral
  - use techniques such as paper folding, measurement or dynamic geometry software to investigate the properties of quadrilaterals (Fluency, Understanding) [ICT] [CCT]
  - sketch and label quadrilaterals from a worded or verbal description (Understanding, Fluency) [N]
- classify special quadrilaterals on the basis of their properties [L] [CCT]

Mathematics • Stage 4

Measurement and Geometry
Properties of Geometrical Figures 1

- describe a quadrilateral in sufficient detail for it to be sketched (Understanding) [N] [CCT]

Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries

- investigate the line symmetries and the order of rotational symmetry of polygons, including the special quadrilaterals [CCT]
  - determine if particular triangles and quadrilaterals have line and/or rotational symmetry (Problem Solving) [CCT]
- investigate the line symmetries and the rotational symmetry of circles and of diagrams involving circles, such as a sector and a circle with a marked chord or tangent [CCT]

Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral

- justify informally that the interior angle sum of a triangle is $180°$, and that any exterior angle equals the sum of the two interior opposite angles [L] [CCT]
  - use dynamic geometry software or otherwise to investigate the interior angle sum of a triangle, and the relationship between any exterior angle and the sum of the two interior opposite angles (Understanding, Reasoning) [ICT] [CCT]
- use the angle sum of a triangle to establish that the angle sum of a quadrilateral is $360°$ [L] [CCT]
- find the unknown angle in a triangle and/or a quadrilateral, giving a reason (Problem Solving) [N] [CCT]

Use angle properties and relationships to solve problems with appropriate reasoning

- find the unknown side and/or angle in a diagram, using angle relationships and the properties of special triangles and quadrilaterals, giving a reason (Problem Solving) [N] [CCT]
  - recognise special types of triangles and quadrilaterals embedded in composite figures or drawn in various orientations (Understanding) [CCT]

Mathematics • Stage 4

Measurement and Geometry

Properties of Geometrical Figures 1

Background Information:

The properties of special quadrilaterals are important in Measurement. For example, the perpendicularity of the diagonals of a rhombus and a kite allow a rectangle of twice the size to be constructed around them, leading to formulae for finding area.

At this Stage, the treatment of triangles and quadrilaterals is still informal, with students consolidating their understandings of different triangles and quadrilaterals and being able to identify them from their properties.

Students who recognise class inclusivity and minimum requirements for definitions may address this Stage 4 content concurrently with content in Stage 5 Properties of Geometrical Figures, where properties of triangles and quadrilaterals are deduced from formal definitions.

Students should give reasons orally and in written form for their findings and answers. For some students formal setting out could be introduced.

A range of examples of the various triangles and quadrilaterals should be given, including quadrilaterals containing a reflex angle and figures presented in different orientations.

Dynamic geometry software or prepared applets are useful tools for investigating properties of geometrical figures. Computer drawing programs enable students to prepare tessellation designs and to compare these with other designs such as those of M.C. Escher.

Language:

Scalene means ‘uneven’ (Greek word ‘skalenos’: uneven): our English word ‘scale’ comes from the same word. Isosceles comes from the two Greek words ‘isos’: equals and ‘skelos’: leg; equilateral comes from the two Latin words ‘aequus’: equal and ‘latus’: side; equiangular comes from ‘aequus’ and another Latin word ‘angulus’: corner.

Mathematics • Stage 4

Measurement and Geometry
Properties of Geometrical Figures 2

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.3 recognises and explains mathematical relationships using reasoning
4.8 identifies and uses angle relationships and properties of plane shapes, including transformations and congruent figures

Students:

Define congruence of plane shapes using transformations

- identify congruent figures by superimposing them through a combination of rotations, reflections and translations [L] [CCT]
  - recognise congruent figures in tessellations, art and design work (Understanding) [N] [CCT]
  - recognise that area, length of matching sides and angle sizes are preserved in congruent figures (Understanding, Reasoning) [N] [CCT]
- match sides and angles of two congruent polygons [N] [CCT]
- name the vertices in matching order when using the symbol $\equiv$ in a congruence statement [L]
- determine the condition for two circles to be congruent (equal radii) [CCT]

Develop the conditions for congruence of triangles

- develop the four tests describing the minimum conditions needed for two triangles to be congruent [L] [CCT]
  - If three sides of one triangle are respectively equal to three sides of another triangle, then the two triangles are congruent (SSS rule)
  - If two sides and the included angle of one triangle are respectively equal to two sides and the included angle of another triangle, then the two triangles are congruent (SAS rule)
  - If two angles and one side of one triangle are respectively equal to two angles and the matching side of another triangle, then the two triangles are congruent (AAS rule)
  - If the hypotenuse and a second side of one right-angled triangle are respectively equal to the hypotenuse and a second side of another right-angled triangle, then the two triangles are congruent (RHS rule)
  - use dynamic geometry software and/or geometrical instruments to investigate what information is needed to show that two triangles are congruent (Fluency, Problem Solving) [ICT] [CCT]
  - explain why the angle in the SAS test must be the included angle (Reasoning) [CCT]
  - determine why AAA is not a sufficient condition for congruence of triangles (Understanding, Reasoning) [CCT]
- use the congruency tests to identify a pair of congruent triangles from a selection of triangles or from triangles embedded in a diagram [CCT]

Mathematics • Stage 4

Measurement and Geometry

Properties of Geometrical Figures 2

Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning

- apply the properties of congruent triangles to find an unknown side and/or angle in a diagram, giving a reason (Problem Solving) [N] [CCT]
- use transformations of congruent triangles to verify some of the properties of special quadrilaterals, including properties of the diagonals,
  eg the diagonals of a parallelogram bisect each other [CCT]

Background Information:

For some students formal setting out of proofs of congruent triangles could be introduced.

Congruent figures are embedded in a variety of designs (eg tapa cloth, Aboriginal designs, Indonesian ikat designs, Islamic designs, designs used in ancient Egypt and Persia, window lattice, woven mats and baskets).

Dynamic geometry software or prepared applets are useful tools for investigating properties of congruent figures through transformations.

Language:

The term ‘corresponding’ is often used in relation to congruent and similar figures to refer to angles or sides in the same position, but it also has a specific meaning when used to describe a pair of angles in relation to lines cut by a transversal. This syllabus has used ‘matching’ to describe angles and sides in the same position; however, the use of the word ‘corresponding’ is not incorrect.

The term ‘superimpose’ is used to describe the placement of one figure upon another in such a way that the parts of one coincide with the parts of the other.

Mathematics • Stage 4

Statistics and Probability
Single Variable Data Analysis 1

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.3 recognises and explains mathematical relationships using reasoning
4.9 collects, represents, analyses and interprets single data sets using appropriate statistical displays and measures of location

Students:

Identify and investigate issues involving continuous or large count data collected from primary and secondary sources

- collect, read and interpret information on an issue of interest from secondary sources presented as tables, charts and/or graphs,
  eg the relationship between wealth or education and the health of populations from different countries [L] [N]
  - analyse a variety of data displays used in the media and in other school subject areas (Understanding) [N] [CCT]
- formulate key questions to generate data for a problem of interest [L] [CCT]
  - work in a group to design and conduct an investigation: decide on an issue, decide whether to use a census or sample, choose appropriate methods of presenting questions (yes/no, tick a box, a scale of 1 to 5, open-ended, etc), analyse and present the data, draw conclusions (Understanding, Fluency, Reasoning, Problem Solving) [L] [N] [CCT] [PSC]
- refine key questions after a trial [L] [CCT]
- use spreadsheets or statistical packages to tabulate and graph data [ICT]
  - discuss ethical issues that may arise from collecting and representing data (Reasoning) [CCT] [EU]

Construct and compare a range of data displays including stem-and-leaf plots and dot plots

- choose appropriate scales on the horizontal and vertical axes when drawing graphs [N] [CCT]
- recognise data as quantitative (either discrete or continuous) or categorical [L]
- use a tally to organise data into a frequency distribution table [N]
- draw and use frequency histograms and polygons, dot plots and stem-and-leaf plots [N]
  - choose appropriate forms to display data (Fluency) [N] [CCT]
  - compare the strengths and weaknesses of different forms of data display (Reasoning, Understanding) [N] [CCT]

Explore the practicalities and implications of obtaining representative data using a variety of investigative processes

- identify issues for which it may be difficult to obtain representative data either from primary or secondary sources [CCT]

Mathematics • Stage 4

Statistics and Probability
Single Variable Data Analysis 1

  - discuss constraints that may limit the collection of data or result in unreliable data, eg proximity to the location where data could be collected, access to ICT, cultural sensitivities that may influence the results (Understanding, Reasoning) [CCT] [EU] [IU]

- investigate and question the selection of data used to support a particular viewpoint, eg the selective use of data in product advertising [CCT] [EU]

Background Information:

It is important that students have the opportunity to gain experience with a wide range of tabulated and graphical data.

Advantages and disadvantages of different representations of the same data should be explicitly taught.

Data may be quantitative (discrete or continuous) or categorical, eg gender (male, female) is categorical; height (measured in cm) is quantitative, continuous; quality (poor, average, good, excellent) is categorical; school population (measured in individuals) is quantitative, discrete.

Mathematics • Stage 4

Statistics and Probability
Single Variable Data Analysis 2

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.3 recognises and explains mathematical relationships using reasoning
4.9 collects, represents, analyses and interprets single data sets using appropriate statistical displays and measures of location

Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data

- collect data using a random process, eg numbers from a page in a phone book, or from a random number generator [N]
- find measures of location (mean, mode, median) and the range for small sets of data [N]
  - calculate summary statistics using a spreadsheet and use these to interpret data displayed in a spreadsheet [N] [ICT]
- use a calculator with statistics functions to determine the mean of a set of scores [ICT]

Describe and interpret data displays and the relationship between the median and mean

- interpret the findings displayed in a graph, eg the graph shows that the heights of all children in the class are between 140 cm and 175 cm and that most are in the group 151–155 cm [N] [CCT]
- use measures of location (mean, mode, median) and the range to analyse data that is displayed in a frequency distribution table, stem-and-leaf plot, or dot plot [N] [CCT]
  - draw conclusions based on the analysis of data using the mean, mode and/or median, and range, eg a survey of the school canteen food (Fluency, Reasoning) [N] [CCT]
- explain the difference between the mean and median in the context of the data being examined [CCT]

Explore the variation of means and proportions in representative data

- investigate differences which may occur in means and frequencies of scores for different samples of a population [N] [CCT]
  - discuss reasons for differences in measures of location where the data was obtained from different groups (Understanding, Reasoning) [N] [CCT]
  - recognise that summary statistics may vary from sample to sample (Reasoning) [CCT]

Investigate the effect of individual data values, including outliers, on the mean and median

- use the terms ‘cluster’ and ‘outlier’ when describing data [L] [N]
- investigate the effect of outliers on the mean, median, mode and range by considering small data sets and calculating each measure with and without the inclusion of the outlier [N] [CCT]
  - determine situations when it is more appropriate to use the mode or median, rather than the mean, when analysing data, eg median for property prices (Understanding) [N] [CCT]

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Statistics and Probability
Single Variable Data Analysis 2

- analyse collected data to identify any obvious errors and justify the inclusion of any scores that differ remarkably from the rest of the data collected [N] [CCT]

Background Information:

Many opportunities occur in this topic for students to strengthen their skills in:

- collecting, analysing and organising information
- communicating ideas and information
- planning and organising activities
- working with others and in teams
- using mathematical ideas and techniques
- using technology, including spreadsheets

Language:

Students need to be provided with opportunities to discuss what information can be drawn from the data presented. Students need to think about the meaning of the information and to put it into their own words.

Language to be developed would include superlatives, comparatives and other language such as ‘prefer ….over’ etc.

Mathematics • Stage 4

Statistics and Probability
Probability 1

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.10 represents and calculates probabilities of simple and compound events

Students:

Construct sample spaces for single-step experiments with equally likely outcomes

- list all possible outcomes included in a single-step experiment [N]
- use the term 'sample space' to denote all possible outcomes,
eg for rolling a fair six-sided die, the sample space is 1, 2, 3, 4, 5, 6 [L] [N]

Assign probabilities to the outcomes of events and determine probabilities for events

- assign a probability of zero to events that are impossible and a probability of one to events that are certain [L] [N]
  - explain the meaning of a probability of 0, $\frac{1}{2}$ and 1 in a given situation (Understanding) [N] [CCT]
- assign probabilities to simple events by reasoning about equally likely outcomes,
eg the probability of a 5 resulting from the throw of a fair six-sided die is $\frac{1}{6}$ [N] [CCT]
- express the probability of an event A given a finite number of equally likely outcomes as
$P(A)=\frac{\text{number of favourable outcomes}}{n}$
where n is the total number of outcomes in the sample space [L] [N]
  - interpret and use probabilities expressed as percentages or decimals (Understanding, Fluency) [N] [CCT]
- solve probability problems involving single-step experiments such as card, dice and other games (Problem Solving) [N]
- recognise that the sum of the probabilities of all possible outcomes of a single-step experiment is 1 [N]

Identify complementary events and use the sum of probabilities to solve problems

- identify the complement of an event,
eg 'The complement of drawing a red card from a deck of cards is drawing a black card.' [L]
  - compare the use of 'complement' and 'complementary' to describe events with their use in geometry to describe angle pairs (Understanding) [L] [N] [CCT]
- establish that the sum of the probability of an event and its complement is 1
ie $P(\text{event A})+P(\text{complement of A})=1$ [N]
- calculate the probability of a complementary event using the fact that the sum of the probabilities of complementary events is 1 [N]

Mathematics • Stage 4

Statistics and Probability
Probability 1

Background Information:

In a probability experiment, such as rolling a fair six-sided die, an event is a collection of outcomes. For instance an event might be that we roll an odd number, which would include the outcomes 1, 3 and 5. A simple event has outcomes that are equally likely.

Mathematics • Stage 4

Statistics and Probability
Probability 2

Outcomes
A student:
4.1 represents mathematical ideas using appropriate terminology, diagrams and symbolism
4.2 applies appropriate mathematical techniques to solve problems
4.3 recognises and explains mathematical relationships using reasoning
4.10 represents and calculates probabilities of simple and compound events

Students:

Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and'

- recognise the difference between mutually exclusive and non-mutually exclusive events [L]
- describe compound events using the following terms: [L] [N] [CCT]
  - ‘at least’, eg rolling a 4, 5 or 6 on a single six-sided die is described as rolling at least 4
  - ‘at most’, eg rolling a 1, 2, 3 or 4 on a six-sided die is described as rolling at most 4
  - ‘not’, eg choosing a black card from a pack is described as choosing a card that is not red
  - ‘and’, eg choosing a card that is black and a king means the card must have both attributes

  – explain why using ‘not’ is another way of describing complementary events (Reasoning) [CCT] [L]

  – pose problems that involve the use of these terms, and solve problems posed by others (Understanding, Problem Solving) [N] [CCT] [L]

  – describe the effect of the use of AND and OR when using internet search engines (Understanding) [ICT] [L]

- classify compound events using inclusive and exclusive ‘or’,
  eg ‘choosing a male or a female’ is exclusive as one cannot be both, whereas ‘choosing a male or someone left-handed’ could imply exclusivity or inclusivity [L] [CCT]

  – recognise that the word ‘or’ on its own often needs a qualifier, such as ‘both’ or ‘not both’, to determine inclusivity or exclusivity (Understanding) [L] [CCT]

Represent such events in two-way tables and Venn diagrams and solve related problems

- interpret Venn diagrams involving two variables [L] [N]

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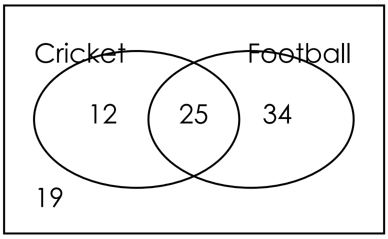
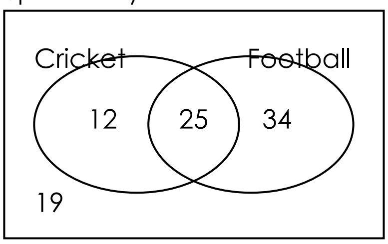
Statistics and Probability
Probability 2

- describe individual or combinations of areas in a Venn diagram using the language of 'and', exclusive 'or', inclusive 'or', 'neither' and 'not',
  eg the Venn diagram below represents the sports played by 90 people; there are 25 people who play Cricket and Football; there are $12+34=46$ people who play Cricket or Football but not both; there are 19 people who play neither sport. (Understanding, Fluency) [N] [CCT]
  Sports Played



- construct a Venn diagram to represent mutually exclusive events using an appropriate example [N] [CCT]
- construct Venn diagrams to represent non-mutually exclusive events from given or collected data [N]
- solve simple probability problems involving Venn diagrams (Problem Solving) [L] [N] [CCT]
- calculate probabilities of non-mutually exclusive events from Venn diagrams,
  eg the Venn diagram below compares the sports played by students in a particular year group − what is the probability that a randomly chosen student plays Football but not Cricket? [N] [CCT]
  Sports Played



- interpret given two-way tables representing non-mutually exclusive events [L] [N] [CCT]
  - use the language 'and', exclusive 'or', inclusive 'or', 'neither' and 'not' to describe relationships displayed in two-way tables,
    eg the table below represents data collected for 300 athletes and compares height with weight − there are 150 people who are short and light, ie 150 people are neither tall nor heavy; there are $180+50=230$ who are short or light or both. (Understanding, Fluency) [L] [N] [CCT]

	Heavy	Light	
Tall	70	50	120
Short	30	150	180
	100	200	300

- construct two-way tables to represent non-mutually exclusive events involving two variables [N]

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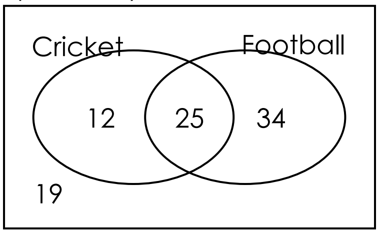
Statistics and Probability

Probability 2

– use given data to determine missing values in a two-way table (Fluency) [N]

– recognise that data represented in a Venn diagram can also be represented in a two-way table (Understanding) [N] [CCT],
eg

Sports Played



	Cricket	Not Cricket	
Football	25	34	59
Not Football	12	19	31
	37	53	90

- calculate probabilities of non-mutually exclusive events represented in two-way tables, eg the table below represents data collected on 300 athletes and compares height with weight – what is the probability that a randomly chosen athlete is light AND short? [N] [CCT]

	Heavy	Light	
Tall	70	50	120
Short	30	150	180
	100	200	300

Background Information:

John Venn (1834−1923) was a British mathematician best known for his diagrammatic way of representing sets, their unions and intersections.

It is not expected that students should study Venn diagrams with three variables; however, Venn diagrams with three variables could be included for more able students.

Language:

A compound event is an event which can be expressed as a combination of simple events, eg drawing a card that is black or a King, throwing at least 5 on a fair six-sided die.