7.7 Content for Stage 5

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Mathematics • Stage 5 (5.1 pathway)

Number and Algebra
Financial Mathematics

Outcomes
A student:
5.1.1 communicates mathematical ideas using appropriate terminology, diagrams and symbolism
5.1.2 selects and uses appropriate strategies to solve problems
5.1.3 provides reasoning to support conclusions which are appropriate to the context
5.1.4 operates with numbers of any magnitude and performs calculations regarding earning, spending and investing money

Students:

Solve problems involving earning money

- calculate wages given an hourly rate of pay, including overtime and special rates for Sundays and public holidays [N] [WE] [PSC]
  - use classifieds and online advertisements to compare pay rates and conditions for different positions (Understanding) [N] [WE] [PSC]
  - read and interpret examples of pay slips (Understanding) [N] [WE] [PSC]
- calculate earnings for various time periods from non-wage sources including salary, commission and piecework [N] [WE] [PSC]
- calculate leave loading as 17.5% of normal pay for up to four weeks [N] [WE] [PSC]
  - research the reasons for inclusion of leave loading provisions in many awards (Understanding, Reasoning) [N] [WE]
- determine the weekly, fortnightly or monthly tax to be deducted from a worker's pay [N] [WE] [PSC]
- determine taxable income by subtracting allowable deductions and use current rates to calculate the amount of tax payable for the financial year [N] [WE] [PSC]
  - determine a worker's tax refund or liability by comparing the tax payable for a financial year with the tax already paid under the Australian PAYG system (Fluency, Problem Solving) [N] [WE] [PSC]
  - investigate how rebates and levies, including the Medicare levy and family tax rebate, affect different workers' taxable incomes (Problem Solving) [N] [WE]

Solve problems involving simple interest

- calculate simple interest using the formula $I = PRN$
  where I is the interest, P is the principal, R is the interest rate per time period (expressed as a fraction or decimal) and N is the number of time periods

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Number and Algebra
Financial Mathematics

- apply the simple interest formula to problems related to investing money at simple interest rates (Problem Solving) [N]
  - find the total value of a simple interest investment after a given time period (Fluency) [N]
  - calculate the principal/time needed to earn a particular amount of interest given the simple interest rate (Fluency) [N]
- calculate the cost of buying expensive items by paying an initial deposit and making regular repayments that include simple interest [N] [PSC]
  - investigate other fees and charges related to 'buy today, no more to pay until…' promotions (Problem Solving, Understanding) [N] [CCT] [PSC]
  - compare total cost of buying on terms to paying cash (Understanding, Fluency) [N] [PSC]
  - recognise that repossession does not remove financial debt (Understanding) [N] [PSC]

Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies

- calculate compound interest for two or three years using repetition of the formula for simple interest
  - connect the calculation of the total value of a compound interest investment to repeated multiplication using a calculator, eg a rate of 5% per annum leads to repeated multiplication by 1.05 (Understanding, Fluency) [N] [ICT]
  - compare simple interest with compound interest in practical situations, eg to determine the most beneficial investment or loan (Understanding, Fluency, Reasoning) [N] [PSC]
  - compare simple interest with compound interest on an investment over various time periods using tables, graphs or spreadsheets (Fluency, Understanding) [L] [N] [ICT] [PSC]

Background Information:

Pay-as-you-go (PAYG) is the Australian taxation system for withholding taxation from employees in their regular payments from employers. The appropriate level of taxes are withheld from an employee's payment and passed on to the Australian Taxation Office. Deduction amounts will reduce the taxation debt that may be payable following submission of a tax return, or alternatively will be part of the refund given for overpayment.

Simple interest is commonly used for short-term investments or loans. As such, time periods may be in months or even days. Students should be encouraged to convert the interest rate per period to a decimal, eg 6% per month $= 0.005$, which is then used as R in the formula.

It is not intended at this Stage for students to use the formula for compound interest.

Internet sites may be used to find commercial rates for home loans and 'home loan calculators'.

Mathematics • Stage 5 (5.1 pathway)

Number and Algebra
Indices

Outcomes
A student:
5.1.1 communicates mathematical ideas using appropriate terminology, diagrams and symbolism
5.1.3 provides reasoning to support conclusions which are appropriate to the context
5.1.5 generalises index laws to operate with algebraic expressions

Students:

Extend and apply the index laws to variables, using positive integral indices and the zero index

- use the index laws previously established for numbers to develop the index laws in algebraic form
  eg $2^2 \times 2^3 = 2^{2+3} = 2^5$ $\quad a^m \times a^n = a^{m+n}$
  $2^5 \div 2^2 = 2^{5-2} = 2^3$ $\quad a^m \div a^n = a^{m-n}$
  $\left(2^2\right)^3 = 2^6$ $\quad \left(a^m\right)^n = a^{mn}$
  - explain why a particular algebraic sentence is incorrect
    eg explain why $a^3 \times a^2 = a^6$ is incorrect (Reasoning) [CCT]
- establish that $a^0 = 1$ using the index laws [N]
  eg $a^3 \div a^3 = a^{3-3} = a^0$
  and $a^3 \div a^3 = 1$
  $\therefore$ $a^0 = 1$
  - explain why $x^0 = 1$ (Reasoning) [CCT]
- simplify expressions that involve the zero index, eg $5x^0 + 3 = 8$

Simplify algebraic products and quotients using index laws

- simplify expressions that involve the product and quotient of simple algebraic terms containing indices, eg $2x^2 \times 3x^3 = 6x$, $15a^6 \div 3a^2 = 5a^4$, $\dfrac{3a^2}{15a^6} = \dfrac{1}{5a^4}$
  - compare expressions such as $3a^2 \times 5a$ and $3a^2 + 5a$ by substituting values for a (Understanding, Fluency) [CCT]

Apply index laws to numerical expressions with integer indices

- establish the meaning of negative indices, eg by patterns [N]

3^3	3^2	3^1	3^0	3^{-1}	3^{-2}
27	9	3	1	$\frac{1}{3}$	$\frac{1}{3^2} = \frac{1}{9}$

- evaluate numerical expressions with a negative index by first rewriting with a positive index
  eg $3^{-4} = \dfrac{1}{3^4} = \dfrac{1}{81}$
- translate numbers to index form (integer indices only) and vice versa

Mathematics • Stage 5 (5.1 pathway)

Number and Algebra
Linear Relationships

Outcomes
A student:
5.1.1 communicates mathematical ideas using appropriate terminology, diagrams and symbolism
5.1.3 provides reasoning to support conclusions which are appropriate to the context
5.1.6 calculates distance, midpoint and gradient on the number plane and graphs linear and simple non-linear relationships

Students:

Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software

- determine the midpoint of an interval from a diagram [N]
- use the concept of 'average' to find the midpoint, M, of the interval joining two points on the number plane [N]
  - explain how the concept of average is used to calculate the midpoint of an interval (Understanding) [L] [CCT]
- graph two points to form an interval on the number plane and form a right-angled triangle by drawing a vertical side from the higher point and a horizontal side from the lower point
- use a right-angled triangle drawn between two points on the number plane and the relationship $\text{gradient} = \dfrac{\text{rise}}{\text{run}}$ to find the gradient of the interval joining two points [N]
  - describe the meaning of the gradient of a line joining two points and explain how it can be found (Understanding, Fluency) [L] [CCT]
  - distinguish between positive and negative gradients from a diagram (Understanding) [CCT]
- use graphing software to find the midpoint and gradient of an interval [ICT]

Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software

- use a right-angled triangle drawn between two points on the number plane and Pythagoras' theorem to determine the length of the interval joining the two points [N]
  - describe how the length of an interval joining two points can be calculated using Pythagoras' theorem (Understanding, Fluency) [L] [N]
- use graphing software to find the distance between two points on the number plane [ICT]

Sketch linear graphs using the coordinates of two points

- construct tables of values and use coordinates to graph vertical and horizontal lines such as $x=3$, $x=-1$, $y=2$, $y=-3$ [N] [CCT]
- identify the x- and y- intercepts of graphs [N]
- identify the x- axis as the line $y=0$ and the y- axis as the line $x=0$ [L] [N]
  - explain why the x- and y -axes have these equations (Reasoning) [L]

Mathematics • Stage 5 (5.1 pathway)

Number and Algebra
Linear Relationships

- graph a variety of linear relationships on the number plane with and without ICT
  eg $y=3-x$, $y=\frac{x+1}{2}$, $x+y=5$, $x-y=2$, $y=\frac{2}{3}x$ [N] [ICT] [CCT]
  - compare and contrast equations of lines that have a negative gradient and equations of lines that have a positive gradient (Reasoning) [CCT]
- determine whether a point lies on a line by substituting

Solve problems involving parallel and perpendicular lines

- determine that parallel lines have equal gradients
  - graph a set of lines using ICT, including parallel lines, and identify similarities and differences (Fluency, Understanding) [N] [ICT] [CCT]
  - determine how to tell whether two equations will produce parallel lines by comparing graphs of known parallel lines with their respective equations (Understanding, Reasoning) [N] [CCT]

Background Information:

The Cartesian plane is named after Descartes who was one of the first to develop analytical geometry on the number plane. He shared this honour with Fermat. Descartes and Fermat are recognised as the first modern mathematicians.

The process of forming right-angled triangles to find gradients is applied in a variety of other situations, eg finding angles of elevation or depression, gradients in calculus in Stage 6, and plotting courses using compass bearings.

Mathematics • Stage 5 (5.1 pathway)

Number and Algebra
Non-linear Relationships

Outcomes
A student:
5.1.1 communicates mathematical ideas using appropriate terminology, diagrams and symbolism
5.1.3 provides reasoning to support conclusions which are appropriate to the context
5.1.6 calculates distance, midpoint and gradient on the number plane and graphs linear and simple non-linear relationships

Students:

Sketch simple non-linear relations with and without the use of digital technologies

- complete tables of values to graph simple non-linear relationships and compare with graphs drawn using ICT, eg $y = x^2$, $y = x^2 + 2$, $y = 2^x$ [N] [ICT]

Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate

- use ICT to graph simple quadratics, circles and exponentials,
  eg $y = x^2$, $y = -x^2$, $y = x^2 + 1$, $y = x^2 - 1$ [N] [ICT]
  $y = 2^x$, $y = 3^x$, $y = 4^x$
  $x^2 + y^2 = 1$, $x^2 + y^2 = 4$
  - describe and compare a range of simple non-linear relationships (Understanding, Reasoning) [N] [CCT]
  - connect the shape of a non-linear graph with the distinguishing features of its equation (Understanding, Reasoning) [N] [CCT]

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra

Financial Mathematics ◊

Outcomes

A student:

5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions

5.2.2 analyses mathematical or real-life situations, systematically applying appropriate strategies to solve problems

5.2.4 constructs arguments to prove and justify results

Students:

Connect the compound interest formula to repeated applications of simple interest use appropriate digital technologies

- determine and use the formula for compound interest,

  $A = P(1+R)^n$

  where A is the total amount, P is the principal, R is the interest rate per compounding period as a decimal and n is the number of compounding periods

  - calculate and compare investments for different compounding periods, eg compare the interest earned on a $\$5000$ investment at 6% pa with the same amount invested at 5.5% pa with interest compounded monthly (Fluency, Understanding) [N] [CCT] [PSC]

- solve problems involving compound interest (Problem Solving) [N]

  - calculate the principal needed to obtain a particular total amount (Fluency) [N]
  - use a 'guess and refine' strategy to determine the number of time periods required to obtain a particular total amount (Fluency, Problem Solving) [N] [CCT]
  - compare the cost of loans using flat and reducible interest for a small number of repayment periods (Understanding, Fluency) [N] [CCT] [PSC]

- use the compound interest formula to calculate depreciation [N] [PSC]

Background Information:

Internet sites may be used to find commercial rates for home loans and 'home loan calculators'.

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Proportion

Outcomes
A student:
5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions
5.2.2 analyses mathematical or real-life situations, systematically applying appropriate strategies to solve problems
5.2.7 graphs and interprets a range of linear and non-linear relationships

Students:

Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems

- convert rates from one set of units to another, eg kilometres per hour to metres per second [N]
- identify and describe everyday examples of direct proportion [N],
  eg as the number of hours worked increases, earnings also increase
- identify and describe everyday examples of inverse (indirect) proportion [N],
  eg as speed increases, the time taken to travel a particular distance decreases
- recognise direct and inverse proportion from graphs [N]
  - distinguish between positive and negative gradients from a graph (Understanding) [N] [CCT]
- interpret and use conversion graphs to convert from one unit to another [N],
  eg conversions between different currencies or metric and imperial measures
- use the equation $y = kx$ to model direct linear proportion where k is the constant of proportionality [N]
  - given the constant of proportionality, establish an equation and use it to find an unknown quantity (Problem Solving) [N]
  - calculate the constant of proportionality, given appropriate information, and use this to find unknown quantities (Problem Solving, Fluency) [N]
- use graphing software or a table of values to graph equations of linear direct proportion [ICT]

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Algebraic Techniques

Outcomes
A student:
5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions
5.2.3 constructs arguments to prove and justify results
5.2.5 selects and applies appropriate algebraic techniques to simplify and operate with quadratic expressions and algebraic fractions

Students:

Apply the four operations to simple algebraic fractions with numerical denominators

- simplify expressions that involve algebraic fractions with numerical denominators,
  eg $\frac{a}{2}+\frac{a}{3}$, $\frac{2x}{5}-\frac{x}{3}$, $\frac{3x}{4}\times\frac{2x}{9}$, $\frac{3x}{4}\div\frac{9x}{2}$
  - connect the processes for simplifying algebraic fractions with the processes for numerical fractions (Understanding, Fluency) [CCT]

Apply the four operations to algebraic fractions

- simplify algebraic fractions, including those with indices,
  eg $\frac{10a^4}{5a^2}$, $\frac{9a^2b}{3ab}$, $\frac{3ab}{9ab^2}$
  - explain the difference between expressions such as $\frac{3a}{9}$ and $\frac{9}{3a}$ (Understanding) [L] [CCT]
- simplify expressions that involve algebraic fractions, including those with algebraic denominators and/or indices,
  eg $\frac{2ab}{3}\times\frac{6}{2b}$, $\frac{3x^2}{8y^5}\div\frac{15x^3}{4y}$, $\frac{a^2b^4}{6}\times\frac{9}{a^2b^2}$, $\frac{3}{x}-\frac{1}{2x}$

Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate

- expand algebraic expressions by removing grouping symbols and collecting like terms where applicable,
  eg $2y(y-5)+4(y-5)$, $4x(3x+2)-(x-1)$, $-3x^2(5x^2+2xy)$

Factorise algebraic expressions by taking out a common algebraic factor

- factorise algebraic expressions by determining common factors,
  eg $3x^2-6x$, $14ab+12a^2$, $21xy-3x+9x^2$, $15p^2q^3-12pq^4$
  - recognise that expressions such as $24x^2y+16xy^2=4xy(6x+4y)$ are partially factorised and that further factorisation is necessary (Understanding, Fluency) [CCT]

Expand binomial products and factorise monic quadratic expressions using a variety of strategies

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Algebraic Techniques

- expand binomial products, including those involving negatives and indices, eg $-3x^2\left(5x^2+2xy\right)$
- expand binomial products by finding the area of rectangles [N]

  eg

	x	8
x	x^2	$8x$
3	$3x$	24

  hence, $\left(x+8\right)\left(x+3\right)=x^2+8x+3x+24$
  $=x^2+11x+24$
- use algebraic methods to expand binomial products,
  eg $\left(x+2\right)\left(x-3\right)$, $\left(4a-1\right)\left(3a+2\right)$
- factorise monic quadratic expressions, eg x^2+5x+6, x^2+2x-8
  - connect algebra with the commutative property of arithmetic
    ie $(a+b)(c+d)=(c+d)(a+b)$ (Understanding) [CCT]
  - explain why a particular algebraic expansion or factorisation is incorrect [CCT],
    eg Why is the factorisation $x^2-6x-8=\left(x-4\right)\left(x-2\right)$ incorrect? (Fluency, Understanding, Reasoning) [L]

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Indices

Outcomes
A student:
5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions
5.2.3 constructs arguments to prove and justify results
5.2.5 selects and applies appropriate algebraic techniques to simplify and operate with quadratic expressions and algebraic fractions

Students:

Simplify algebraic products and quotients using index laws

- use index notation and the index laws to establish that $a^{-1}=\frac{1}{a},\ a^{-2}=\frac{1}{a^2},\ a^{-3}=\frac{1}{a^3},\ldots$ [CCT]
  - explain the difference between similar pairs of algebraic expressions, eg What is the difference between x^{-2} and $-2x$? (Understanding) [CCT]
- apply the index laws to simplify algebraic expressions involving indices, eg $\left(3y^2\right)^3$, $4b^{-5}\times 8b^{-3}$, $9x^{-4}\div 3x^3$, $3x^{\frac{1}{2}}\times 5x^{\frac{1}{2}}$, $6y^{\frac{1}{3}}\div 4y^{\frac{1}{3}}$
  - state whether particular equivalences are true or false and give reasons, eg Are the following true or false? Why?
    $5x^0=1$, $9x^5\div 3x^5=3x$, $a^5\div a^7=a^2$, $2c^{-4}=\frac{1}{2c^4}$ (Understanding, Reasoning) [CCT]
  - determine and justify whether a simplified expression is correct by substituting numbers for variables (Reasoning) [CCT]
  - evaluate a fraction raised to the power of -1, leading to $\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$ (Fluency, Understanding) [CCT]

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Equations

Outcomes
A student:
5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions
5.2.2 analyses mathematical or real-life situations, systematically applying appropriate strategies to solve problems
5.2.3 constructs arguments to prove and justify results
5.2.6 applies appropriate techniques to solve linear and simple quadratic equations, inequalities and simultaneous equations

Students:

Solve linear equations involving grouping symbols

- solve linear equations, including equations that involve grouping symbols, eg $3(a+2)+2(a-5)=10$, $3(2m-5)=2m+5$

Solve linear equations involving simple algebraic fractions

- solve linear equations involving one or more fractions, eg $\frac{x-2}{3}+5=10$, $\frac{2x+5}{3}=10$, $\frac{2x}{3}+5=10$, $\frac{x}{3}+\frac{x}{2}=5$, $\frac{2x+5}{3}=\frac{x-1}{4}$
  - compare and contrast different methods of solving linear equations and justify a choice for a particular case (Fluency, Reasoning) [N] [CCT]

Solve simple quadratic equations using a range of strategies

- solve simple quadratic equations of the form $ax^2=c$, leaving answers in exact form or as decimal approximations
  - explain why quadratic equations could be expected to have two solutions (Understanding, Reasoning) [CCT]
  - recognise and explain that $x^2=c$ does not a have solution if c is a negative number (Understanding) [CCT]

Substitute values into formulas to determine an unknown

- solve equations arising from substitution into formulae, eg given $P=2l+2b$ and $P=20, l=6,$ solve for b
  - substitute into formulae from other strands of the syllabus or in other subjects to solve problems and interpret solutions, eg $A=\frac{1}{2}xy$, $v=u+at$, $C=\frac{5}{9}(F-32)$, $V=\pi r^2h$ (Fluency, Understanding) [N]

Solve problems involving linear equations, including those derived from formulas

- translate word problems into equations, solve the equations and interpret the solutions (Problem Solving) [N]
  - state clearly the meaning of introduced unknowns as ‘the number of …’ (Understanding)

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Equations

- solve word problems involving familiar formulas, eg 'The area of a triangle is 30 square centimetres and base length 12 centimetres, find the perpendicular height of the triangle (Problem Solving) [L] [N]
- explain why the solution to a linear equation generated from a word problem may not be a solution to that problem (Reasoning) [CCT]

Solve linear inequalities and graph their solutions on a number line

- represent simple inequalities on the number line, eg $x \le 4$, $m > -3$ [N]
- recognise that an inequality has an infinite number of solutions [N]
- solve linear inequalities, including reversing the direction of the inequality when multiplying or dividing by a negative number, and graph the solutions [N],
  eg $3x - 1 < 9$, $2(a+4) \ge 24$, $\dfrac{t+4}{5} > -3$, $1 - 4y \le 6$
  - use a numerical example to justify the need to reverse the direction of the inequality when multiplying or dividing by a negative number (Reasoning) [CCT]
  - verify the direction of the inequality sign by substituting a value within the solution range (Fluency) [CCT]

Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology

- solve linear simultaneous equations by finding the point of intersection of their graphs with and without ICT [ICT] [N]
- solve linear simultaneous equations using an appropriate algebraic method,
  eg solve $\begin{cases} 3a + b = 17 \\ 2a - b = 8 \end{cases}$

- generate and solve linear simultaneous equations from word problems and interpret the results (Problem Solving) [N] [CCT]

Background Information:

Graphics calculators and graphing software enable students to graph two linear equations and display the coordinates of the point of intersection.

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra
Linear Relationships

Outcomes
A student:
5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions
5.2.3 constructs arguments to prove and justify results
5.2.7 graphs and interprets a range of linear and non-linear relationships

Students:

Apply the gradient/intercept form to interpret and graph straight lines

- graph straight lines of the form $y = mx + b$ (gradient/intercept form) [N]
- recognise equations of the form $y = mx + b$ as representing straight lines and interpret the x- coefficient (m) as the gradient and the constant (b) as the y- intercept [L] [N]
- rearrange $ax + by + c = 0$ (general form) to gradient/intercept form
- graph equations of the form $y = mx + b$ using the y- intercept and gradient [N]
  - use graphing software to graph a variety of equations of straight lines, and describe the similarities and differences between them,
    eg $y = -3x$, $y = -3x + 2$, $y = -3x - 2$
    $y = \frac{1}{2}x$, $y = -2x$, $y = 3x$
    $x = 2$, $y = 2$
    (Fluency, Understanding) [N] [ICT] [CCT]
  - predict whether a particular equation will have a similar graph to another equation and graph both lines to test the conjecture (Understanding, Reasoning, Fluency) [CCT]
  - explain the effect on the graph of a line of changing the gradient or y- intercept and demonstrate using ICT (Understanding, Fluency) [CCT]
- find the gradient and the y- intercept of a straight line from the graph and use them to determine the equation of the line [N]
  - match equations of straight lines to graphs of straight lines and justify choices (Understanding, Reasoning) [N] [CCT]

Solve problems involving parallel and perpendicular lines

- determine that lines are perpendicular if the product of their gradients is -1
  - graph a set of lines using ICT, including perpendicular lines, and identify similarities and differences (Fluency, Understanding) [N] [ICT] [CCT]
  - recognise that when lines are perpendicular, the gradient of one line is the negative reciprocal of the other (Understanding) [CCT]
- find the equation of a straight line parallel or perpendicular to another given line using $y = mx + b$ [N]

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra

Non-linear Relationships ◊

Outcomes

A student:

5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions

5.2.3 constructs arguments to prove and justify results

5.2.7 graphs and interprets a range of linear and non-linear relationships

Students:

Sketch simple non-linear relations with and without the use of digital technologies

- graph parabolic relationships of the form $y = ax^2$, $y = ax^2 + c$ [N]
  - identify parabolic shapes in the environment (Understanding) [N]
  - describe the effect on the graph of $y = x^2$ of multiplying by or adding different constants, including negatives (Fluency, Understanding) [N] [CCT]
  - determine the equation of a parabola given a graph with the main features clearly marked (Understanding, Reasoning) [L] [N] [CCT]
- sketch, compare and describe the key features of simple exponential curves [L] [N] [CCT], eg $y = 2^x$, $y = -2^x$, $y = 2^{-x}$, $y = -2^{-x}$, $y = 2^x + 1$, $y = 2^x - 1$
  - describe exponentials in terms of what happens to the y- values as x becomes very large or very small and what occurs at $x = 0$ (Understanding, Reasoning) [N] [CCT]
- recognise and describe the algebraic equations that represent circles with centre the origin and radius r [L] [N]
  - use Pythagoras' theorem to establish the equation of a circle, centre the origin, radius r and graph equations of the form $x^2 + y^2 = r^2$ (Reasoning, Fluency) [N]
- sketch circles of the form $x^2 + y^2 = r^2$ where r is the radius [L] [N]

Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate

- identify graphs of straight lines, parabolas, circles and exponentials [L] [N]
- match graphs of straight lines, parabolas, circles and exponentials to the appropriate equations [N] [CCT]
  - sort and classify a set of graphs, match each graph to an equation, and justify each choice (Reasoning, Understanding) [N] [CCT]

Mathematics • Stage 5 (5.2 pathway)

Number and Algebra

Non-linear Relationships ◊

Background Information:

Graphics calculators and graphing software facilitate the investigation of the shapes of curves and the effect on the equation of multiplying by, or adding, a constant.

This substrand could provide opportunities for modelling. For example, $y = 1.2^x$ for $x \geq 0$, models the growth of a quantity beginning at 1 and increasing 20% for each unit increase in x.

Mathematics • Stage 5 (5.3 pathway)

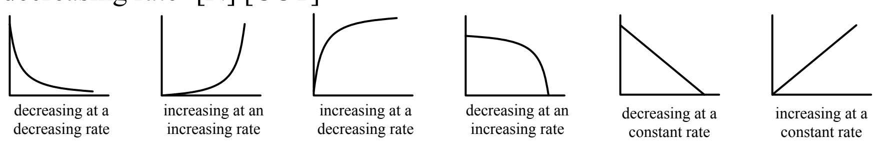
Number and Algebra
Proportion

Outcomes
A student:
5.3.1 uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
5.3.2 connects and generalises mathematical ideas and techniques to analyse and solve problems efficiently
5.3.3 uses deductive reasoning in presenting arguments and formal proofs
5.3.7 uses and interprets appropriate formulae to graph and analyse linear and non-linear relationships

Students:

Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems

- interpret distance/time graphs when the speed is variable [N]
  - match a set of distance/time graphs to situations, and explore the likelihood that they are accurate, appropriate, and whether they are possible (Understanding, Fluency) [N]
  - match a set of distance/time graphs to a set of descriptions and give reasons for choices (Understanding, Reasoning) [L] [N] [CCT]
  - record the distance of a moving object from a fixed point at equal time intervals and draw a graph to represent the situation, eg move along a measuring tape for 30 seconds using a variety of activities that include variable speeds such as running fast, walking slowly, and walking slowly then speeding up (Understanding, Problem Solving) [N]
- analyse the relationship between variables as they change over time [N] [CCT], eg draw graphs to represent the relationship between the depth of water in containers of different shapes when they are filled at a constant rate
- interpret graphs, making sensible statements about rate of increase or decrease, the initial and final points, constant relationships as denoted by straight lines, variable relationships as denoted by curved lines, etc [N] [CCT]
  - decide whether a particular graph is a suitable representation of a given physical phenomenon (Understanding, Fluency) [N] [CCT]
- describe qualitatively the rate of change of a graph using terms such as 'increasing at a decreasing rate' [N] [CCT]



- sketch a graph from a simple description given a variable rate of change [N]

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Proportion

Background Information:

Rate of change is considered as it occurs in practical situations, including population growth and travel. Simple linear models have a constant rate of change. In other situations, the rate of change is variable.

This work is intended to provide experiences for students that will give them an intuitive understanding of rates of change and will assist the development of appropriate vocabulary. No quantitative analysis is needed at this Stage.

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Algebraic Techniques §

Outcome
A student:
5.3.1 uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
5.3.5 systematically selects and applies appropriate algebraic techniques to operate fluently with algebraic expressions

Students:

Add and subtract algebraic fractions with binomial numerators and numerical denominators

- add and subtract algebraic fractions, including those with binomial numerators, eg $\dfrac{2x+5}{6}+\dfrac{x-4}{3}$, $\dfrac{x}{3}-\dfrac{x+1}{5}$

Expand binomial products and factorise monic quadratic expressions using a variety of strategies

- recognise and apply the special product, $(a-b)(a+b)=a^2-b^2$
  - recognise and name appropriate expressions as the 'difference of two squares' (Understanding) [L] [CCT]
- recognise and apply the special products, $\left\{\begin{matrix}(a+b)^2=a^2+2ab+b^2\\(a-b)^2=a^2-2ab+b^2\end{matrix}\right\}$
  - recognise and name appropriate expressions as 'perfect squares' (Understanding) [L] [CCT]
- use algebraic methods to expand a variety of binomial products, eg $(2y+1)^2$, $(3a-1)(3a+1)$, $(3x+1)(2-x)+2x+4$, $(x-y)^2-(x+y)^2$

Factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts

- factorise algebraic expressions by choosing an appropriate strategy [CCT]
  - common factors
  - difference of two squares
  - grouping in pairs for four-term expressions
  - trinomials
- use a variety of strategies to factorise expressions, eg $3d^3-3d$, $2a^2+12a+18$, $4x^2-20x+25$, $t^2-3t+st-3s$, $2a^2b-6ab-3a+9$ [CCT]
- factorise and simplify complex algebraic expressions involving algebraic fractions, eg $\dfrac{x^2+3x+2}{x+2}$, $\dfrac{4}{x^2+x}-\dfrac{3}{x^2-1}$, $\dfrac{3m-6}{4}\times\dfrac{8m}{m^2-2m}$, $\dfrac{4}{x^2-9}+\dfrac{2}{3x+9}$

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Surds and Indices §

Outcome
A student:
5.3.3 uses deductive reasoning in presenting arguments and formal proofs
5.3.4 operates with fractional indices, surds and logarithms

Students:

Define rational and irrational numbers and perform operations with surds and fractional indices

- define real numbers: *a real number is any number that can be represented by a point on the number line*. [L]
- define rational and irrational numbers: *a rational number is the ratio* $\frac{a}{b}$ *of two integers* a *and* b *where* $b \neq 0$. *An irrational number is a real number that is not rational.* [L]
  - recognise that all rational and irrational numbers are real (Understanding) [CCT]
  - explain why all integers and recurring decimals are rational numbers (Reasoning) [L]
  - explain why rational numbers can be expressed in decimal form (Reasoning) [L]
- distinguish between rational and irrational numbers [CCT]
  - demonstrate that not all real numbers are rational (Understanding, Problem Solving) [CCT]
- write recurring decimals in fraction form using calculator and non-calculator methods, eg $0.\dot{2}, 0.\dot{2}\dot{3}, 0.2\dot{3}$
  - justify why $0.\dot{9} = 1$ (Reasoning) [CCT]
- define surds: *a surd is a numerical expression involving one or more irrational roots of numbers.* . [L]
  - use a pair of compasses and a straight edge to construct simple rationals and surds on the number line [N]
- establish that $\sqrt{x}$ is undefined for $x < 0$, $\sqrt{x} = 0$ for $x = 0$, and $\sqrt{x}$ is the positive square root of x when $x > 0$ [L]
- use the following results for $x, y > 0$: $\left(\sqrt{x}\right)^2 = x = \sqrt{x^2}$

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$
$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

- apply the four operations of addition, subtraction, multiplication and division to simplify expressions involving surds
  - explain why a particular sentence is incorrect, eg $\sqrt{3} + \sqrt{5} = \sqrt{8}$ (Reasoning) [CCT]
- expand expressions involving surds, eg $\left(\sqrt{3} + \sqrt{5}\right)^2$, $\left(2 - \sqrt{3}\right)\left(2 + \sqrt{3}\right)$

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Surds and Indices §

- connect operations with surds to algebraic techniques (Understanding, Fluency) [CCT]

- rationalise the denominators of surds of the form $\dfrac{a\sqrt{b}}{c\sqrt{d}}$
  - investigate methods of rationalising surdic expressions with binomial denominators, making appropriate connections to algebraic techniques (Problem Solving)
- recognise that a surd is an exact value that can be approximated by a rounded decimal
  - use surds to solve problems where a decimal answer is insufficient [N], eg find the exact perpendicular height of an equilateral triangle (Fluency, Problem Solving)
- establish that $\left(\sqrt{a}\right)^2 = \sqrt{a} \times \sqrt{a} = \sqrt{a \times a} = \sqrt{a^2} = a$ [CCT]
- apply index laws to assist with the definition of the fractional index for the square root,

  eg $\left(\sqrt{a}\right)^2 = a$ [L]

  and $\left(a^{\frac{1}{2}}\right)^2 = a$

  $\therefore \quad \sqrt{a} = a^{\frac{1}{2}}$
  - explain why finding the square root of an expression is the same as raising the expression to the power of a half (Understanding, Reasoning) [CCT]
- use the index laws to demonstrate the reasonableness of the following definitions for fractional indices: $x^{\frac{1}{n}} = \sqrt[n]{x}$, $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ [L]
- translate expressions in surd form to expressions in index form and vice versa
- use the $\boxed{x^{\frac{1}{y}}}$ key on a scientific calculator
- evaluate numerical expressions involving fractional indices, eg $27^{\frac{2}{3}}$

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Surds and Indices §

Background Information:

Operations with surds are applied when simplifying algebraic expressions.

This substrand can be linked with graphing $y = \sqrt{x}$.

Having expanded binomial products and rationalised denominators of surds of the form $\dfrac{a\sqrt{b}}{c\sqrt{d}}$, students could rationalise denominators of surds with binomial denominators.

Early Greek mathematicians believed that the length of any line would always be given by a rational number. This was proved to be false when Pythagoras and his followers found the length of the hypotenuse of an isosceles right-angled triangle with side length one unit.

Some students may enjoy a demonstration of the proof, by contradiction, that $\sqrt{2}$ is irrational.

Language:

There is a need to emphasise how to read and articulate surds and fractional indices, eg $\sqrt{x}$ is 'the square root of *x*' or 'root *x*'.

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Equations

Outcome
A student:
5.3.2 connects and generalises mathematical ideas and techniques to analyse and solve problems efficiently
5.3.3 uses deductive reasoning in presenting arguments and formal proofs
5.3.6 applies appropriate techniques to solve linear and non-linear equations

Students:

Solve complex linear equations involving algebraic fractions

- solve a range of linear equations, including equations that involve two or more fractions, eg $\frac{2x-5}{3}-\frac{x+7}{5}=0$, $\frac{y-1}{4}-\frac{2y+3}{3}=\frac{1}{2}$, $3(2a-6)=5-(a+2)$

Factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts

- solve equations of the form $ax^2+bx+c=0$ using factors, by completing the square and using the quadratic formula
- use the quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
- solve a variety of quadratic equations, eg $3x^2=4$, $x^2-8x-4=0$, $x(x-4)=4$, $(y-2)^2=9$
  - choose the most appropriate method to solve a particular quadratic equation (Fluency) [CCT]
- check the solutions of quadratic equations by substituting [CCT]
- identify whether a given quadratic equation has no solution, one solution or two solutions [N]
  - predict the possible number of solutions for any quadratic equation (Understanding) [CCT]
  - connect the value of b^2-4ac to the number of possible solutions of $ax^2+bx+c=0$ and explain the significance of this connection (Understanding) [L]
- solve quadratic equations resulting from substitution into formulae
- create quadratic equations to solve a variety of problems and check solutions (Problem Solving) [N]
  - explain why the solution to a quadratic equation generated from a word problem may not be a solution to that problem (Reasoning) [CCT]
- use variable substitution to simplify higher order equations into quadratic equations, eg substitute u for x^2 to solve $x^4-13x^2+36=0$

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Equations

Solve literal equations

- change the subject of formulae, including examples from other strands and other subjects, eg make a the subject of $v = u + at$; make r the subject of $\frac{1}{x} = \frac{1}{r} + \frac{1}{s}$; make b the subject of $x = \sqrt{b^2 - 4ac}$ [N]
  - determine restrictions on the values of variables implicit in the original formula and after rearrangement of the formula, eg 'consider what restrictions there would be on the variables in the equation $Z = ax^2$ and what additional restrictions are assumed if the equation is rearranged to $x = \sqrt{\frac{Z}{a}}$', (Understanding, Reasoning) [N] [CCT]

Solve simultaneous equations, using algebraic and graphical techniques

- use analytical methods to solve a variety of simultaneous equations, including where one equation is quadratic, eg $\begin{cases} 3x - 4y = 2 \\ 2x + y = 3 \end{cases}$, $\begin{cases} y = x^2 \\ y = x \end{cases}$, $\begin{cases} y = x^2 - x - 2 \\ y = x + 6 \end{cases}$
  - choose an appropriate method to solve a pair of simultaneous equations (Fluency) [N]
  - compare the use of graphing software with algebraic methods in solving simultaneous equations (Understanding, Fluency) [N] [ICT] [CCT]

Background Information:

Derivation of the quadratic formula can be demonstrated for more capable students.

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Linear Relationships §

Outcome
A student:
5.3.1 uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
5.3.2 connects and generalises mathematical ideas and techniques to analyse and solve problems efficiently
5.3.3 uses deductive reasoning in presenting arguments and formal proofs
5.3.7 uses and interprets appropriate formulae to graph and analyse linear and non-linear relationships

Students:

Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software

- use the concept of 'average' to establish the formula for the midpoint, M, of the interval joining two points (x_1, y_1) and (x_2, y_2) on the number plane: $M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ [N]
  - explain the meaning of each of the variables in the formula for midpoint (Understanding) [CCT]
- use the formula to find the midpoint of the interval joining two points on the number plane
- use the relationship $\text{gradient} = \frac{\text{rise}}{\text{run}}$ to establish the formula for the gradient, m, of an interval joining two points (x_1, y_1) and (x_2, y_2) on the number plane: $m = \frac{y_2 - y_1}{x_2 - x_1}$ [N]
- use the formula to find the gradient of an interval joining two points on the number plane
  - explain why the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$ gives the same solution as $m = \frac{y_2 - y_1}{x_2 - x_1}$ (Reasoning, Understanding) [CCT]

Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software

- use Pythagoras' theorem to establish the formula for the distance, d, between two points (x_1, y_1) and (x_2, y_2) on the number plane: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ [N]
  - explain the meaning of each of the variables in the formula for distance (Understanding) [CCT]
- use the formula to find the distance between two points on the number plane
  - explain why the formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ gives the same solution as $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (Reasoning, Understanding) [CCT]

Sketch linear graphs using the coordinates of two points

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Linear Relationships §

- sketch the graph of a line by using its equation to find the x- and y- intercepts [N]

Solve problems using various standard forms of the equation of a straight line

- describe the equation of a line as the relationship between the x- and y- coordinates of any point on the line [N]
  - recognise from a list of equations those that result in straight line graphs (Understanding) [CCT]
- find the equation of a line passing through a point (x_1, y_1), with a given gradient m, using:
  $y - y_1 = m(x - x_1)$
  $y = mx + b$
- find the equation of a line passing through two points
- recognise and find the equation of a line in the general form: $ax + by + c = 0$
- rearrange linear equations into the general form

Solve problems involving parallel and perpendicular lines

- find the equation of a line that is parallel or perpendicular to a given line
- determine whether two given lines are perpendicular
  - use gradients to show that two given lines are perpendicular (Understanding, Problem Solving) [CCT]
- solve a variety of problems by applying coordinate geometry formulae and reasoning (Problem Solving) [N]
  - derive the formula for the distance between two points (Fluency, Reasoning) [CCT]
  - show that two intervals with equal gradients and a common point form a straight line and use this to show that three points are collinear (Fluency, Reasoning) [N] [CCT]
  - use coordinate geometry to investigate and describe the properties of triangles and quadrilaterals (Problem Solving, Understanding) [N] [CCT]
  - use coordinate geometry to investigate the intersection of the perpendicular bisectors of the sides of acute-angled triangles (Problem Solving, Understanding) [N] [CCT]
  - show that four specified points form the vertices of particular quadrilaterals (Fluency, Understanding) [N] [CCT]
  - prove that a particular triangle drawn on the number plane is right-angled (Fluency, Reasoning) [N] [CCT]

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Non-linear Relationships

Outcome
A student:
5.3.1 uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
5.3.3 uses deductive reasoning in presenting arguments and formal proofs
5.3.7 uses and interprets appropriate formulae to graph and analyse linear and non-linear relationships

Students:

Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations

- find x- and y- intercepts for the graph of $y = ax^2 + bx + c$ given a, b and c [N]
- graph a range of parabolas, including where the equation is given in the form $y = ax^2 + bx + c$ for various values of a, b and c [N]
  - use ICT to investigate and describe features of the graphs of parabolas of the following forms for both positive and negative values of a and k
    eg $y = ax^2$
    $y = ax^2 + k$
    $y = (x + a)^2$
    $y = (x + a)^2 + k$
    (Fluency, Understanding) [N] [ICT] [CCT]
  - describe features of a parabola by examining its equation (Understanding) [CCT]
- determine the equation of the axis of symmetry of a parabola [L]
  - find the midpoint of the interval joining the points at which the parabola cuts the x- axis to determine the equation of the axis of symmetry for a parabola [N] [CCT]
  - use the formula $x = -\frac{b}{2a}$ to determine the equation of the axis of symmetry of a parabola [N]
- find the coordinates of the vertex of a parabola [L]
  - find the midpoint of the interval joining the points at which the parabola cuts the x- axis and substitute to find the coordinates of its vertex [N]
  - use the formula for the axis of symmetry to obtain the x- coordinate and substitute to obtain the y- coordinate of the vertex of a parabola [N]
  - find the coordinates of the vertex of a parabola by completing the square [N] [CCT]
- identify and use features of parabolas and their equations to assist in sketching quadratic relationships, eg x- and y- intercepts, vertex, axis of symmetry and concavity [N]
- determine quadratic expressions to describe particular number patterns, eg generate the equation $y = x^2 + 1$ for the table [N]

x	0	1	2	3	4	5
y	1	2	5	10	17	26

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Non-linear Relationships

- graph hyperbolic relationships of the form $y = \frac{k}{x}$ for integer values of k [N]
  - describe the effect on the graph of $y = \frac{1}{x}$ of multiplying by different constants (Fluency, Understanding) [N] [CCT]
  - explain what happens to the y- values of the points on the hyperbola $y = \frac{k}{x}$ as the x- values get very large (Understanding) [L] [N] [CCT]
  - explain what happens to the y- values of the points on the hyperbola $y = \frac{k}{x}$ as the x- values get closer to zero (Understanding) [CCT]
  - explain why it may be useful to choose small and large numbers when constructing a table of values for a hyperbola (Reasoning, Understanding) [CCT]
- graph a range of hyperbolic curves, including where the equation is given in the form $y = \frac{k}{x} + c$ or $y = \frac{k}{x-b}$ for integer values of k, b, and c [N]
  - determine the equations of the asymptotes of a hyperbola in the form $y = \frac{k}{x} + c$ or $y = \frac{k}{x-b}$ (Understanding, Fluency) [N] [CCT]
  - identify features of hyperbolas from their equations to assist in sketching their graphs, eg asymptotes, orientation, x- and/or y- intercepts where they exist (Understanding, Fluency) [N] [CCT]
  - describe hyperbolas in terms of what happens to the y- values as x becomes very large or very small, whether there is a y- value for every x- value, and what occurs near or at $x = 0$ (Understanding, Reasoning) [N] [CCT]
- recognise and describe the algebraic equations that represent circles with centre (h, k) and radius r [N] [CCT]
  - establish the equation of the circle centre (h, k), radius r, and graph equations of the form $(x-h)^2 + (y-k)^2 = r^2$ (Understanding, Reasoning) [N] [CCT]
  - determine whether a particular point is inside, on, or outside a circle (Fluency, Reasoning) [N] [CCT]
  - find the centre and radius of a circle whose equation is in the form $x^2 + fx + y^2 + gy = c$ by completing the square (Fluency)

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Non-linear Relationships

- identify a variety of graphs from their equations, eg $(x-2)^2+y^2=4$, $y=(x-2)^2-4$, $y=4^x+2$, $y=x^2+2x-4$, $y=\frac{2}{x-4}$ [N] [CCT]
  - determine how to sketch a particular curve by determining its features from the equation (Fluency, Understanding) [N] [CCT]
  - identify equations that have a graph that is symmetrical about the y- axis (Fluency) [N] [CCT]
- determine a possible equation from a given graph and check using ICT [N] [ICT]
  - compare and contrast a mixed set of graphs and determine possible equations from key features, eg $y=2$, $y=2-x$, $y=(x-2)^2$, $y=2^x$, $(x-2)^2+(y-2)^2=4$, $y=\frac{1}{x-2}$, $y=2x^2$ [N] [CCT] [ICT]
- determine the points of intersection of a line with a parabola, circle or hyperbola, graphically and algebraically [N] [ICT]
  - compare methods of finding points of intersection of curves and justify choice of method for a particular example (Reasoning, Understanding) [CCT]

Describe, interpret and sketch cubic functions, other curves and their transformations

- graph and compare features of the graphs of cubic equations of the form
  $y=ax^3$
  $y=ax^3+d$
  $y=a(x-r)(x-s)(x-t)$
  describing the effect on the graph of different values of a and d [L] [N] [CCT]
- graph a variety of equations of the form $y=ax^n$ for $n>0$, describing the effect of n being odd or even on the shape of the curve [N] [CCT]
- graph curves of the form $y=ax^n+k$ from curves of the form $y=ax^n$ by vertical transformations [N] [CCT]
- graph curves of the form $y=a(x-r)^n$ after first graphing curves of the form $y=ax^n$ and then using horizontal transformations [N] [CCT]

Background Information:

Links to other subjects and real life examples of graphs, eg exponential graphs used for population growth in demographics, radioactive decay, town planning, etc.

This topic could provide opportunities for modelling. For example, the hyperbola $y=\frac{1}{x}$ for $x>0$, models sharing a prize between x people, or length of a rectangle given area k and breadth x.

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Polynomials #

Outcome
A student:
5.3.1 uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
5.3.2 connects and generalises mathematical ideas and techniques to analyse and solve problems efficiently
5.3.3 uses deductive reasoning in presenting arguments and formal proofs
5.3.7 uses and interprets appropriate formulae to graph and analyse linear and non-linear relationships

Students:

Investigate the concept of a polynomial and apply the factor and remainder theorems to solve problems

- recognise a polynomial expression $a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$ and use the terms 'degree', 'leading term', 'leading coefficient', 'constant term' and 'monic polynomial' appropriately [L]
- use the notation $P(x)$ for polynomials and $P(c)$ to indicate the value of $P(x)$ for $x = c$ [L]
- add and subtract polynomials and multiply polynomials by linear expressions
- divide polynomials by linear expressions to find the quotient and remainder, expressing the polynomial as the product of the linear expression and another polynomial plus a remainder, eg $p(x) = (x - a).Q(x) + c$ [N]
- verify the remainder theorem and use it to find factors of polynomials [N]
- use the factor theorem to factorise certain polynomials completely [N]
  ie if $(x - a)$ is a factor of $P(x)$, then $P(a) = 0$
- use the factor theorem and long division to find all zeros of a simple polynomial and hence solve $P(x) = 0$ (degree ≤ 4) [N]
- state the number of zeros that a polynomial of degree n can have [L] [N]

Apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation

- sketch the graphs of quadratic, cubic and quartic polynomials by factorising and finding the zeros [N]
  - recognise linear, quadratic and cubic expressions as examples of polynomials and relate sketching of these curves to factorising polynomials and finding the zeros (Reasoning, Understanding) [CCT]
  - use ICT to graph polynomials of odd and even degree and investigate the relationship between the number of zeros and the degree of the polynomial (Fluency, Understanding) [N] [ICT] [CCT]

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Polynomials #

  - connect the roots of the equation to the x- intercepts and connect the constant term to the y- intercept (Understanding) [N] [CCT]
  - determine the importance of the sign of the leading term of the polynomial on the behaviour of the curve as $x \to \pm\infty$ (Understanding) [CCT]
- determine the effect of single, double and triple roots of a polynomial equation on the shape of the curve [CCT]
- use the leading term, the roots of the equation and the x- and y- intercepts to sketch polynomials [L] [N]
  - describe the key features of a polynomial and draw its graph from the description (Understanding, Fluency) [N] [CCT]
- use the sketch of $y = P(x)$ to sketch $y = -P(x)$, $y = P(-x)$, $y = P(x) + c$, $y = aP(x)$ [N] [CCT]
  - explain the similarities and differences between the graphs of two polynomials, such as $y = x^3 + x^2 + x$, $y = x^3 + x^2 + x + 1$ (Understanding) [N] [CCT]

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Logarithms #

Outcome
A student:
5.3.1 uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
5.3.3 uses deductive reasoning in presenting arguments and formal proofs
5.3.4 operates with fractional indices, surds and logarithms

Students:

Use the definition of a logarithm to establish and apply the laws of logarithms

- define logarithms as indices and translate index statements into equivalent statements using logarithms,
  eg $9 = 3^2, \ \therefore \log_3 9 = 2$ [L] [N]

  $\frac{1}{2} = 2^{-1}, \ \therefore \log_2 \frac{1}{2} = -1$

  $4^{\frac{3}{2}} = 8, \ \therefore \log_4 8 = \frac{3}{2}$

- deduce the following laws of logarithms from the laws of indices:
  $\log_a x + \log_a y = \log_a (xy)$ [N]

  $\log_a x - \log_a y = \log_a (\frac{x}{y})$

  $\log_a x^n = n \log_a x$

- establish and use the following results:
  $\log_a a^x = x$ [N] [CCT]

  $\log_a a = 1$

  $\log_a 1 = 0$

  $\log_a \left(\frac{1}{x}\right) = -\log_a x$

- apply the laws of logarithms to evaluate simple expressions,
  eg $\log_2 8$, $\log_{81} 3$, $\log_{10} 25 + \log_{10} 4$, $3\log_{10} 2 + \log_{10}(12.5)$, $\log_2 18 - 2\log_2 3$, [N]

- simplify expressions using the laws of logarithms, eg simplify $5\log_a a - \log_a a^4$ [N]

- draw the graphs of the inverse functions $y = a^x$ and $y = \log_a x$ [N]

  – relate logarithms to practical scales that use indices, eg Richter, decibel and pH (Understanding) [N]

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Logarithms #

– compare and contrast a set of exponential and logarithmic graphs drawn on the same axes, eg $y = 2^x$, $y = \log_2 x$, $y = 3^x$, $y = \log_3 x$ [N] [CCT] (Reasoning)

Solve simple exponential equations

- Solve simple equations that involve exponents or logarithms, eg $2^t = 8$, $4^{t+1} = \dfrac{1}{8\sqrt{2}}$, $\log_{27} 3 = x$, $\log_4 x = -2$ [N]

Background Information:

Logarithm tables were used to assist with calculations before the use of hand-held calculators. They converted multiplication and division to addition and subtraction thus simplifying the calculations.

Language:

Teachers need to emphasise the correct language used in connection with logarithms,

eg $\log_a a^x = x$ is 'log to the base *a* of *a* to the power of *x* equals *x*'.

Mathematics • Stage 5 (5.3 pathway)

Number and Algebra
Functions and Other Graphs #

Outcome
A student:
5.3.1 uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
5.3.3 uses deductive reasoning in presenting arguments and formal proofs
5.3.7 uses and interprets appropriate formulae to graph and analyse linear and non-linear relationships

Students:

Describe, interpret and sketch functions

- Define a function as a rule or relationship where for each input there is only one output, or that associates every member of one set with exactly one member of the second set [L] [N]
- use the vertical line test on a graph to decide whether it represents a function [N]
  - explain whether a given graph represents a function (Understanding, Reasoning) [CCT]
  - decide whether a straight line graph always, sometimes or never represents a function (Understanding, Reasoning) [CCT]
- use the notation $f(x)$ [L]
- use $f(c)$ notation to determine that value of $f(x)$ when $x = c$ [L]
- find the permissible x- and y- values for a variety of functions (including functions represented by straight lines, parabolas, exponentials and hyperbolas) [N]
- determine the inverse functions for a variety of functions and recognising their graphs as reflections in the line $y = x$ [L] [N]
  - describe the conditions for a function to have an inverse function (Understanding, Reasoning) [CCT]
  - recognise and describe the restrictions that need to be placed on quadratic functions so that they have an inverse function (Understanding, Reasoning) [CCT]
- sketch the graph of $y = f(x) + k$ and $y = f(x - a)$ given the graph of $y = f(x)$ [N] [CCT]
  - sketch graphs to model relationships that occur in practical situations and explain the relationship between the variables represented in the graph (Fluency, Understanding) [N]
  - consider a graph that represents a practical situation and explain the relationship between the two variables (Understanding) [N] [CCT]

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Area & Surface Area

Outcome
A student:
5.1.1 communicates mathematical ideas using appropriate terminology, diagrams and symbolism
5.1.2 selects and uses appropriate strategies to solve problems
5.1.7 selects and applies appropriate formulae to calculate areas and surface areas of prisms, and uses trigonometry

Students:

Calculate the areas of composite shapes

- develop and use the formula to find the area of a trapezium
  Area of trapezium $= \frac{1}{2}h(a+b)$
  where h is the perpendicular height and a and b are the lengths of the parallel sides [N] [CCT]
  - identify the perpendicular height of various trapeziums in different orientations (Understanding) [CCT]
- calculate the area of composite figures by dissection into triangles, special quadrilaterals, quadrants, semicircles and sectors [N]
  - identify different possible dissections for a given composite figure and select an appropriate dissection to facilitate calculation of the area (Fluency) [CCT]
- solve practical problems involving area of quadrilaterals and composite shapes (Problem Solving) [N]
  - apply properties of geometrical shapes to assist in finding areas, eg symmetry (Fluency) [N] [CCT]
  - calculate the area of an annulus (Fluency) [N]

Solve problems involving the surface area and volume of right prisms

- identify the surface area and edge lengths of rectangular and triangular prisms [L] [CCT]
- visualise and name a common solid, given its net [L] [CCT]
  - recognise whether a diagram is a net of a right prism (Understanding) [CCT]
- visualise and sketch the nets of right prisms [CCT]
- find the surface area of rectangular and triangular prisms, given its net [N]
- calculate the surface area of rectangular and triangular prisms [N]
  - apply Pythagoras' theorem to assist with finding the surface area of triangular prisms (Fluency) [N]
- solve problems involving the surface area of rectangular and triangular prisms (Problem Solving) [N]

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Area & Surface Area

Background Information:

The dissection of composite figures into special quadrilaterals and triangles links with work on the properties of special shapes in the Space and Geometry strand.

It is important that students can visualise rectangular and triangular prisms in different orientations before they find the surface area. Properties of solids are treated in Stage 3. They should be able to sketch other views of the object.

Language:

The abbreviation m^2 is read ‘square metre(s)’ and not ‘metre squared’ or ‘metre square’.

The abbreviation cm^2 is read ‘square centimetre(s)’ and not ‘centimetre squared’ or ‘centimetre square’.

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Numbers of Any Magnitude

Outcome
A student:
5.1.1 communicates mathematical ideas using appropriate terminology, diagrams and symbolism
5.1.2 selects and uses appropriate strategies to solve problems
5.1.3 provides reasoning to support conclusions which are appropriate to the context
5.1.4 operates with numbers of any magnitude and performs calculations regarding earning, spending and investing money

Students:

Investigate very small and very large time scales and intervals

- use the language of estimation appropriately, including:
  - rounding
  - approximate
  - level of accuracy [L] [N]
- identify significant figures [L] [N]
- round numbers to a specified number of significant figures
- determine the effect of truncating or rounding during calculations on the accuracy of the results [N] [CCT]
- interpret the meaning of common prefixes such as ‘milli’, ‘centi’, ‘kilo’ [L]
- interpret the meaning of prefixes for very large and very small units of measurement such as ‘nano’, ‘micro’, ‘mega’, ‘giga’, ‘tera’ [L]
  - investigate the use of metric prefixes in the measurement of digital information (Understanding, Fluency) [N] [ICT]
  - investigate appropriate units of time for very small or very large time intervals (Understanding) [L] [N] [CCT]
- describe the limits of accuracy of measuring instruments (± 0.5 unit of measurement) [L] [N]
  - explain why measurements are never exact (Reasoning) [N] [CCT]
  - appreciate the importance of the number of significant figures in a given measurement (Understanding, Reasoning) [N] [CCT]
  - choose appropriate units of measurement based on the required degree of accuracy (Fluency) [N] [CCT]
  - consider the degree of accuracy needed when making measurements in practical situations or as a result of calculations (Fluency) [N] [CCT]

Express numbers in scientific notation

- recognise the need for a notation to express very large or very small numbers [L] [CCT]
- express numbers in scientific notation
- enter and read scientific notation on a calculator

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Numbers of Any Magnitude

- explain the difference between numerical expressions such as 2×10^4 and 2^4, particularly with reference to calculator displays (Understanding, Reasoning) [N] [CCT]

- use index laws to make order of magnitude checks for numbers in scientific notation, eg $\left(3.12\times10^4\right)\times\left(4.2\times10^6\right)\approx12\times10^{10}\approx1.2\times10^{11}$ [N]
- convert numbers expressed in scientific notation to decimal form
- order numbers expressed in scientific notation [N]
- solve problems involving scientific notation (Problem Solving) [N]
  - communicate and interpret technical information using scientific notation (Understanding) [N]

Language:

The metric prefixes milli-, centi-, deci- for units smaller than the base SI unit derive from the Latin words: 'mille' meaning thousand, 'centum' meaning hundred and 'decimus' meaning tenth. The metric prefixes kilo-, hecto-, deca- for units larger than the base SI unit derive from the Greek words: 'khilioi' meaning thousand, 'hekaton' meaning hundred, and 'deka' meaning ten.

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Right-Angled Triangles (Trigonometry)

Outcome
A student:
5.1.1 communicates mathematical ideas using appropriate terminology, diagrams and symbolism
5.1.2 selects and uses appropriate strategies to solve problems
5.1.3 provides reasoning to support conclusions which are appropriate to the context
5.1.7 selects and applies appropriate formulae to calculate areas and surface areas of prisms, and uses trigonometry

Students:

Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles

- identify the hypotenuse, adjacent and opposite sides with respect to a given angle in a right-angled triangle in any orientation [L]
  - label sides of right-angled triangles in different orientations in relation to a given angle (Fluency, Understanding) [L] [CCT]
- label the side lengths of a right-angled triangle in relation to a given angle, eg the side c is opposite angle C [L] [CCT]
- recognise that the ratio of matching sides in similar right-angled triangles is constant for equal angles [N] [CCT]
  - explain why the ratio of matching sides in similar right-angle triangles is constant for equal angles (Reasoning) [N] [CCT]
- define the sine, cosine and tangent ratios for angles in right-angled triangles [L] [N]
- use trigonometric notation, eg $\sin C$ [L]
- use a calculator to find approximations of the trigonometric ratios of a given angle measured in degrees [N]
- use a calculator to find an angle correct to the nearest degree, given one of the trigonometric ratios of the angle [N]

Apply trigonometry to solve right-angled triangle problems

- select and use appropriate trigonometric ratios in right-angled triangles to find unknown sides, including the hypotenuse [N]
- select and use appropriate trigonometric ratios in right-angled triangles to find unknown angles correct to the nearest degree [N]

Solve right-angled triangle problems including those involving direction and angles of elevation and depression

- identify angles of elevation and depression [L]
  - interpret diagrams in questions involving angles of elevation and depression (Understanding) [CCT]
- solve problems involving angles of elevation and depression when given a diagram (Problem Solving) [N]

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Right-Angled Triangles (Trigonometry)

- solve problems in practical situations involving right-angled triangles, eg finding the pitch of a roof (Problem Solving) [N]

Background Information:

The definitions of the trigonometric ratios rely on the angle test for similarity, and trigonometry is, in effect, automated calculations with similarity ratios. The content is thus strongly linked with ratio and with scale drawing.

The fact that the other angles and sides of a right-angled triangle are completely determined by giving two other measurements is justified by the four standard congruence tests.

Trigonometry is introduced through similar triangles with students calculating the ratio of two sides and realising that this remains constant for a given angle.

Trigonometry has practical and analytical applications in surveying, navigation, meteorology, architecture, engineering and electronics. It is important to emphasise real-life applications of trigonometry, eg building construction and surveying.

Language:

The word trigonometry is derived from two Greek words meaning 'triangle' and 'measurement'.

Emphasis should be placed on correct pronunciation of sin as 'sine'.

The origin of the word 'cosine' is from the Latin 'complementi sinus', meaning 'complement of sine', so that $\cos 40° = \sin 50°$.

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Properties of Geometrical Figures

Outcome
A student:
5.1.1 communicates mathematical ideas using appropriate terminology, diagrams and symbolism
5.1.2 selects and uses appropriate strategies to solve problems
5.1.3 provides reasoning to support conclusions which are appropriate to the context
5.1.8 recognises and applies the properties of similar figures and scale drawings

Students:

Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar

- use the term 'similar' for any two figures that have the same shape but not necessarily the same size [L]
  - find examples of similar figures embedded in designs from many cultures and historical periods (Understanding) [N] [IU] [DD]
  - explain why any two equilateral triangles, or any two squares, are similar, and explain when they are congruent (Reasoning, Understanding) [CCT]
  - investigate whether any two rectangles, or any two isosceles triangles, are similar (Problem Solving) [CCT]
- match the sides and angles of similar figures [N] [CCT]
- name the vertices in matching order when using the symbol $\cong$ in a similarity statement [L]
- use the enlargement transformation and measurement to determine that shape, angle size and the ratio of matching sides are preserved in similar figures [N] [CCT]
  - use dynamic geometry software to investigate the properties of similar figures (Fluency, Problem Solving) [ICT] [CCT]
- develop the four tests describing the minimum conditions needed for two triangles to be similar [L] [CCT]
  - if the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are similar
  - if two sides of one triangle are proportional to two sides of another triangle, and the included angles are equal, then the two triangles are similar
  - if two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar
  - if the hypotenuse and a second side of a right-angled triangle are proportional to the hypotenuse and a second side of another right-angled triangle, then the two triangles are similar.
- determine whether two triangles are similar using an appropriate test [CCT]

Solve problems using ratio and scale factors in similar figures

- choose an appropriate scale in order to enlarge or reduce a diagram [N]
  - enlarge diagrams such as cartoons and pictures (Fluency, Understanding) [N]
- construct scale drawings [N]

Mathematics • Stage 5 (5.1 pathway)

Measurement and Geometry
Properties of Geometrical Figures

  - investigate different methods of producing scale drawings, including ICT (Fluency, Understanding) [ICT] [CCT]
  - interpret and use scales in photographs, plans and drawings found in the media and/or other subjects (Understanding, Fluency) [N] [CCT]
- determine the scale factor for pairs of similar polygons and circles [N]
- calculate dimensions of similar figures using the scale factor [N]
  - apply similarity to finding lengths in the environment where it is impractical to measure directly, eg heights of trees, buildings (Fluency, Understanding) [N] [CCT]
- apply the scale factor to find unknown sides in similar triangles [N]
- calculate unknown sides in a pair of similar triangles using a proportion statement [N]

Background Information:

This syllabus uses the symbol $\cong$ in similarity statements; however use of the symbol $|||$ is not incorrect.

The definitions of the trigonometric ratios depend upon the similarity of triangles, eg any two right-angled triangles in which another angle is 30º must be similar.

Mathematics • Stage 5 (5.2 pathway)

Measurement and Geometry
Area & Surface Area

Outcome
A student:
5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions
5.2.2 analyses mathematical or real-life situations, systematically applying appropriate strategies to solve problems
5.2.8 selects and applies appropriate formulae to calculate surface areas and volumes of cylinders and composite solids, and applies bearings to right-angled triangles

Students:

Calculate the surface area and volume of cylinders and solve related problems

- develop and use the formula to find the surface area of closed right cylinders
  Surface area of cylinder $= 2\pi r^2 + 2\pi rh$
  where r is the length of the radius and h is the perpendicular height [N] [CCT]
- solve practical problems involving surface area of cylinders,
  eg find the area of the label of a cylindrical can (Problem Solving) [N]

Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids

- calculate the surface area of composite solids involving right cylinders and prisms [N]
- solve practical problems related to surface area,
  eg compare the amount of packaging material needed for different objects (Problem Solving) [N]
  - interpret the given conditions of a problem to determine whether a prism or cylinder is open or closed (Understanding, Problem Solving) [CCT]

Mathematics • Stage 5 (5.2 pathway)

Measurement and Geometry
Volume

Outcome
A student:
5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions
5.2.2 analyses mathematical or real-life situations, systematically applying appropriate strategies to solve problems
5.2.8 selects and applies appropriate formulae to calculate surface areas and volumes of cylinders and composite solids, and applies bearings to right-angled triangles

Students:

Solve problems involving the surface area and volume of right prisms

- calculate the volume of prisms with cross-sections that are composite figures that may be dissected into triangles and special quadrilaterals [N]
  - solve practical problems related to volume and/or capacity of composite prisms (Problem Solving) [N]
  - compare the surface areas of prisms with the same volume (Understanding, Fluency) [N] [CCT]
  - find the volume and/or capacity of various everyday containers such as water tanks or moving cartons (Problem Solving) [N]

Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids

- find the volume of solids that have sectors as the uniform cross-section [N]
- find the volume of simple composite solids such as a cylinder on top of a rectangular prism [N]
  - dissect composite solids into several simpler solids to find volume (Fluency, Understanding) [N]
- solve practical problems related to volume and/or capacity of prisms, cylinders and composite solids (Problem Solving) [N]

Mathematics • Stage 5 (5.2 pathway)

Measurement and Geometry

Right-Angled Triangles (Trigonometry) ◊

Outcome

A student:

5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions

5.2.2 analyses mathematical or real-life situations, systematically applying appropriate strategies to solve problems

5.2.8 selects and applies appropriate formulae to calculate surface areas and volumes of cylinders and composite solids, and applies bearings to right-angled triangles

Students:

Apply trigonometry to solve right-angled triangle problems

- use a calculator to find trigonometric ratios of a given approximation for angles measured in degrees and minutes [N]
- use a calculator to find an approximation for an angle in degrees and minutes, given the trigonometric ratio of the angle [N]
- find unknown sides in right-angled triangles where the given angle is measured in degrees and minutes [N]
- use trigonometric ratios to find unknown angles in degrees and minutes in right-angled triangles [N]

Solve right-angled triangle problems including those involving direction and angles of elevation and depression

- use three-figure bearings (eg $035°$, $225°$) and compass bearings (eg SSW) [L] [N]
  - interpret directions given as bearings and represent them in diagrammatic form (Understanding) [N] [CCT]
- solve word problems involving bearings or angles of elevation and/or depression (Problem Solving) [N]
  - draw diagrams to assist in solving practical problems involving bearings, angles of elevation and depression (Understanding) [CCT]
  - check the reasonableness of answers to trigonometry problems (Problem Solving) [N] [CCT]

Background Information:

Students may need encouragement to set out their solutions carefully and to use the correct mathematical language and suitable diagrams.

When setting out their solutions related to finding unknown lengths and angles, students should be advised to give a simplified exact answer, eg $25\sin 42°$ metres or $\sin A = \frac{4}{7}$, then give an approximation correct to a specified or sensible level of accuracy.

Students could have practical experience in using clinometers for finding angles of elevation and depression and in using magnetic compasses for bearings. Students need to recognise the 16 points of a mariner's compass (eg SSW) for comprehension of compass bearings in everyday life, eg weather reports.

Students studying circle geometry will be able to apply their trigonometry to many problems, making use of the right-angles between a chord and a radius bisecting it, between a tangent and a radius at the point of contact, and in a semicircle.

Language:

The language of bearings needs to be taught explicitly, eg the meaning of the word 'of' can be different depending on the question.

Mathematics • Stage 5 (5.2 pathway)

Measurement and Geometry
Properties of Geometrical Figures

Outcome
A student:
5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions
5.2.2 analyses mathematical or real-life situations, systematically applying appropriate strategies to solve problems
5.2.3 constructs arguments to prove and justify results
5.2.9 calculates the angle sum of any polygon and applies results for proving triangles are congruent or similar

Students:

Formulate proofs involving congruent triangles and angle properties

- write formal proofs of congruence of triangles preserving matching order of vertices [L] [CCT]
- apply congruent triangle results to prove properties of isosceles and equilateral triangles (Problem Solving) [CCT]
  Consider properties such as:
  - *if two sides of a triangle are equal, then the angles opposite the equal sides are equal*
  - *conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal*
  - *if three sides of a triangle are equal then each interior angle is* $60°$
- use congruent triangles to prove properties of the special quadrilaterals (Problem Solving) [CCT]
  Consider properties such as:
  - *opposite angles of a parallelogram are equal*
  - *diagonals of a parallelogram bisect each other*
  - *diagonals of a rectangle are equal in length*

Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes

- apply geometrical facts, properties and relationships to find unknown sides and angles in diagrams, providing appropriate reasons (Problem Solving) [N] [CCT]
  - compare different solutions for the same problem to determine the most efficient method (Reasoning, Fluency) [CCT]
  - analyse a diagram to determine the most efficient method of solution (Understanding, Fluency) [CCT]
  - apply the properties of congruent and similar triangles, justifying the results (Fluency, Reasoning) [CCT]
- define the exterior angle of a convex polygon [L]
- establish that the sum of the exterior angles of any convex polygon is 360° [CCT]
  - use dynamic geometry software to investigate the constancy of the exterior angle sum of polygons for different polygons (Understanding) [ICT] [CCT]
- apply the result for the interior angle sum of a triangle to find, by dissection, the interior angle sum of polygons with more than three sides [N]

Mathematics • Stage 5 (5.2 pathway)

Measurement and Geometry
Properties of Geometrical Figures

- use dynamic geometry software to investigate the angle sum of different polygons (Fluency, Understanding) [ICT] [CCT]
- express in algebraic terms the interior angle sum of a polygon with n sides, eg Interior Angle Sum $= (n-2) \times 180°$ (Fluency) [N] [CCT]

- apply angle sum results to find unknown angles in polygons (Problem Solving) [N] [CCT]

Background Information:

Students are expected to give reasons when proving properties of plane shapes using congruence.

Dynamic geometry software or prepared applets are useful tools for investigating the interior and exterior angle sums of polygons.

This work may be extended to interpreting the sum of the exterior angles of a convex polygon as the amount of turning during a circuit of the boundary, and generalising to circles and any closed curve.

Comparing the perimeters of inscribed and circumscribed polygons leads to an approximation for the circumference of a circle. This is the method Archimedes used to develop an approximation for the ratio of the circumference to the diameter, that is, π .

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Area & Surface Area

Outcome
A student:
5.3.1 uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
5.3.2 connects and generalises mathematical ideas and techniques to analyse and solve problems efficiently
5.3.8 calculates surface areas and volumes of pyramids, cones, spheres and their composites

Students:

Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids

- identify the perpendicular and slant height of pyramids and right cones [L]
- apply Pythagoras' theorem to find slant height, base length or perpendicular height of pyramids and right cones [N]
- devise and use methods to calculate the surface area of pyramids [N]
- develop and use the formula to calculate the surface area of cones
  Curved surface area of a cone $= \pi rl$
  where r is the length of the radius and l is the slant height [N] [CCT]
- use the formula to calculate the surface area of spheres
  Surface area of a sphere $= 4\pi r^2$
  where r is the length of the radius [N]
- solve problems involving the surface area of solids (Problem Solving) [N]
  - find surface area of composite solids, eg a cone with a hemisphere on top (Problem Solving) [N]
  - find the dimensions of solids given their surface area by substitution into a formula to generate an equation (Problem Solving, Fluency) [N]

Background Information:

Pythagoras' theorem is applied here to right-angled triangles in three-dimensional space.

The focus in this section is on right cones and right pyramids. Dealing with the oblique version of these objects is difficult and is mentioned only as a possible extension.

The area of the curved surface of a hemisphere is $2\pi r^2$ which is twice the area of its base. This may be a way of making the formula for the surface area of a sphere look reasonable to students. Deriving the relationship between the surface area and the volume of a sphere by dissection into infinitesimal pyramids may be an extension activity for some students. Similarly, some students may investigate as an extension, the surface area of a sphere by projection of infinitesimal squares onto a circumscribed cylinder.

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Volume

Outcome
A student:
5.3.1 uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
5.3.2 connects and generalises mathematical ideas and techniques to analyse and solve problems efficiently
5.3.3 uses deductive reasoning in presenting arguments and formal proofs
5.3.8 calculates surface areas and volumes of pyramids, cones, spheres and their composites

Students:

Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids

- develop and use the formula for the volume of pyramids and cones

  $$\text{Volume of pyramid/cone} = \frac{1}{3}Ah$$

  where A is the base area and h is the perpendicular height [N] [CCT]

  – recognise that a pyramid/cone has one-third the volume of a prism/cylinder with the same base and the same perpendicular height (Understanding) [N] [CCT]

  – deduce that the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$ (Reasoning) [N] [CCT]

- use the formula to find the volume of spheres

  $$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

  where r is the length of the radius [N]

- find the volume of composite solids that include right pyramids, right cones and hemispheres, eg find the volume of a cylinder with cone on top [N]

- solve problems relating to volume and/or capacity of right pyramids, cones and spheres [N]

  – apply Pythagoras' theorem as needed to calculate volumes of pyramids and cones (Fluency) [N]

  – find the dimensions of solids given their volume by substitution into a formula to generate an equation,
  eg find the length of the radius of a sphere given the volume (Fluency) [N]

Background Information:

The formulae for the volume of solids mentioned here depend only on the perpendicular height and apply equally well to the oblique case. The volume of oblique solids may be included as an extension for some students.

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Volume

A more systematic development of the volume formulae for spheres, cones and pyramids can be given after integration is developed in Stage 6 (where the factor $\frac{1}{3}$ emerges essentially because the primitive of x^2 is $\frac{1}{3}x^3$).

At this Stage, the relationship could be demonstrated by practical means, eg filling a pyramid with sand and pouring into a prism with the same base and perpendicular height and repeating until the prism is filled.

Some students may undertake the following exercise: visualise a cube of side length $2a$ dissected into six congruent pyramids with a common vertex at the centre of the cube, and hence prove that each of these pyramids has volume $\frac{4}{3}a^3$, which is $\frac{1}{3}$ of the enclosing rectangular prism.

The problem of finding the edge length of a cube that has twice the volume of another cube is called ‘the duplication of the cube’, and is one of three famous problems left unsolved by the ancient Greeks. It was proved in the 19th century that this cannot be done with straight edge and compasses, essentially because the cube root of 2 cannot be constructed on the number line.

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Trigonometry & Pythagoras §

Outcome
A student:
5.3.1 uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
5.3.2 connects and generalises mathematical ideas and techniques to analyse and solve problems efficiently
5.3.3 uses deductive reasoning in presenting arguments and formal proofs
5.3.9 uses and graphs trigonometric relationships and calculates attributes of non-right-angled triangles

Students:

Apply Pythagoras' theorem and trigonometry to solving three-dimensional problems in right-angled triangles

- calculate side lengths and/or the length of the diagonal of a rectangular prism [N]
- use a given diagram to solve problems involving right-angled triangles in three dimensions [N]
- draw diagrams and use them to solve word problems involving right-angled triangles in three dimensions, including bearings and angles of elevation or depression (Problem Solving) [N] [CCT]
  - check the reasonableness of answers to trigonometry problems in three dimensions (Problem Solving) [N] [CCT]

Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies

- prove that the tangent ratio can be expressed as a ratio of the sine and cosine ratios $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ [N] [CCT]
- use the unit circle and ICT to investigate the sine, cosine and tangent curves for at least $0° \le A \le 360°$ and sketch the results [L] [N] [CCT]
  - compare features of these trigonometric curves including periodicity and symmetry (Understanding) [N] [CCT]
  - describe how trigonometric ratios change as the angle increases from 0° to 360° (Understanding) [N] [CCT]
- use the unit circle or graphs of trigonometric functions to establish and use the following relationships for obtuse angles, where $0° \le A \le 90°$:
  $\sin A = \sin(180 - A)$
  $\cos A = -\cos(180 - A)$ [CCT]
  $\tan A = -\tan(180 - A)$
  - recognise that if $\sin A \ge 0$ then there are two possible values for A, given $0° \le A \le 180°$ (Understanding) [N] [CCT]
- find the angle of inclination, θ, of a line in the coordinate plane by establishing and using the relationship $\text{gradient} = \tan\theta$ [N] [CCT]

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Trigonometry & Pythagoras §

Solve simple trigonometric equations

- determine and use exact sine, cosine and tangent ratios for angles of $30°$, $45°$ and $60°$ [L] [N]
  - solve problems in right-angled triangles using exact trigonometric ratios for $30°$, $45°$ and $60°$ (Fluency, Problem Solving) [N]
- prove and use the relationship between the sine and cosine ratios of complementary angles in right-angled triangles
  $\sin A = \cos(90° - A)$ [CCT]
  $\cos A = \sin(90° - A)$
- find the possible acute and/or obtuse angles, given a trigonometric ratio [N]

Establish the sine, cosine and area rules for any triangle and solve related problems

- prove the sine rule: *In a given triangle ABC, the ratio of a side to the sine of the opposite angle is a constant.*
  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (Problem Solving) [L] [CCT]
- use the sine rule to find unknown sides and angles of a triangle, including in problems in which there are two possible solutions for an angle [N]
  - recognise that if given two sides and a non-included angle then two triangles may result, leading to two solutions when the sine rule is applied (Understanding, Reasoning) [N] [CCT]
- prove the cosine rule: *In a given triangle ABC*
  $a^2 = b^2 + c^2 - 2bc\cos A$
  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ [L] [CCT]
- use the cosine rule to find unknown sides and angles of a triangle [N]
- prove and use the area rule to find the area of a triangle: In a given triangle ABC
  Area of triangle $= \frac{1}{2}ab\sin C$ [L] [CCT]
- select and apply the appropriate rule to find unknowns in non-right-angled triangles [N] [CCT]
  - explain what happens if the sine, cosine and area rules are applied in right-angled triangles (Understanding) [N] [CCT]
- draw diagrams and use them to solve word problems that involve non-right-angled triangles (Problem Solving) [N] [CCT]
  - use appropriate trigonometric ratios and formulae to solve two-dimensional trigonometric problems that require the use of more than one triangle, where the diagram is provided, and where a verbal description is given (Problem Solving) [N] [CCT]
  - solve problems, including practical problems, involving the sine and cosine rules and the area rule, eg problems related to surveying or orienteering (Problem Solving) [N]

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Trigonometry & Pythagoras §

Background Information:

Pythagoras' theorem is applied here to right-angled triangles in three-dimensional space.

The sine and cosine rules and the area rule are closely linked with the standard congruence tests for triangles. These are the most straightforward ways to proceed:

- Given an SAS situation, use the cosine rule to find the third side.
- Given an SSS situation, use the alternative form of the cosine rule to find an angle.
- Given an AAS situation, use the sine rule to find each unknown side.

Given an ambiguous ASS situation (the angle non-included), use the sine rule to find the sine of the unknown angle opposite the known side - there may then be two solutions for this angle. Alternatively, use the cosine rule to form a quadratic equation for the unknown side.

The cosine rule is a generalisation of Pythagoras' theorem. The sine rule is linked to the circumcircle and to circle geometry.

The definitions of the trigonometric functions in terms of a circle provide the link between Cartesian and polar coordinates. Note that the angle concerned is turned anti-clockwise from the positive x-axis (East). This is not the same as the angle used in navigation (clockwise from North).

The formula $\text{gradient} = \tan\theta$ is a formula for gradient in the coordinate plane.

Circle geometry and the trigonometric functions are closely linked. First, Pythagoras' theorem becomes the equation of a circle in the coordinate plane, and such a circle is used to define the trigonometric functions for general angles. Secondly, the sine and cosine rules are closely linked with the circle geometry theorems concerning angles at the centre and circumference and cyclic quadrilaterals. Many formulae relating the sides, diagonals, angles and area of cyclic quadrilaterals are now accessible.

The trigonometric functions here could be redefined for the general angle using a circle in the coordinate plane. This allows the sine and cosine functions to be plotted for a full revolution and beyond so that their wave nature becomes clear. The intention, however, of this section is for students to become confident using the sine and cosine rules and area rule in practical situations. For many students it is therefore more appropriate to justify the extension of the trigonometric functions to obtuse angles only, either by plotting the graphs and continuing them in the obvious way, or by taking the identities for $180° - \theta$ as definitions. Whatever is done, experimentation with the calculator should be used to confirm this extension.

Students are not expected to reproduce proofs of the sine, cosine and area rules.

Students at this level should realise that when the triangle is right-angled, the cosine rule becomes Pythagoras' theorem, the area formula becomes the simple 'half base times perpendicular height' formula, and the sine rule becomes a simple application of the sine function in a right-angled triangle.

Students studying circle geometry will be able to apply their trigonometry to many problems in circles involving non-right-angled triangles, making use of the supplementary angles at opposite vertices of a cyclic quadrilateral, the equal angles in the same segment, and the alternate segment theorem.

The graphs of the trigonometric functions mark the transition from understanding trigonometry as the study of lengths and angles in triangles (as the word trigonometry implies) to the study of waves, as will be developed in the Stage 6 calculus courses. Waves are fundamental to a vast range of physical

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Trigonometry & Pythagoras §

and practical phenomena, like light waves and all other electromagnetic waves, and to periodic phenomena like daily temperatures and fluctuating sales over the year, and the major importance of trigonometry lies in the study of these waves.

Brahmagupta, an Indian mathematician and astronomer (c. 598–665 CE), showed that the area of a cyclic quadrilateral is $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where a, b, c and d are the lengths of the sides of the cyclic quadrilateral and s is the semiperimeter $s = \frac{a+b+c+d}{2}$.

This is a generalisation of Heron's formula for the area of a triangle mentioned in Stage 4 Area, as can be seen by putting $d = 0$ so that the quadrilateral becomes a triangle.

The unit circle is part of the history of trigonometry and explains the derivations of the terms sine, cosine and tangent. A semi-chord in a unit circle subtending at the centre an angle of θ, has length $\sin\theta$ and is distant $\cos\theta$ from the centre. The table of values of the sine function was originally called a table of semi-chords. A tangent subtending an angle θ at the centre of a unit circle has length $\tan\theta$ and hence the name tan for this function.

Language:

The origin of the word 'cosine' is from the Latin 'complementi sinus', meaning 'complement of sine', so that $\cos 40° = \sin 50°$.

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Properties of Geometrical Figures §

Outcome
A student:
5.3.1 uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
5.3.2 connects and generalises mathematical ideas and techniques to analyse and solve problems efficiently
5.3.3 uses deductive reasoning in presenting arguments and formal proofs
5.3.10 determines properties of plane shapes using deductive reasoning and formulates proofs using formal geometric arguments

Students:

Formulate proofs involving congruent triangles and angle properties

- construct and write geometrical arguments to prove a general geometrical result, giving reasons at each step of the argument,
  eg prove that the angle in a semicircle is a right angle (Problem Solving) [CCT]

Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes

- write formal proofs of similarity of triangles in the standard four- or five-line format, preserving the matching order of vertices, identifying the similarity factor when appropriate, and drawing relevant conclusions from this similarity [L] [CCT]
  – prove that the interval joining the midpoints of two sides of a triangle is parallel to the third side and half its length and its converse (Problem Solving) [CCT]
- establish and apply the fact that in two similar figures with similarity ratio $1:k$ [N] [CCT]
  – matching angles have the same size
  – matching intervals are in the ratio $1:k$
  – matching areas are in the ratio $1:k^2$
  – matching volumes are in the ratio $1:k^3$
  – solve problems involving the similarity ratio and areas and volumes (Problem Solving) [N]
- state a definition as the minimum amount of information needed to identify a particular figure [L] [CCT]
- prove properties of isosceles and equilateral triangles and special quadrilaterals from the formal definitions of shapes (Problem Solving) [L] [CCT]
  Definitions:
  – a scalene triangle is a triangle with no two sides equal in length
  – an isosceles triangle is a triangle with two sides equal in length
  – an equilateral triangle is a triangle with all sides equal in length
  – a trapezium is a quadrilateral with at least one pair of opposite sides parallel
  – a parallelogram is a quadrilateral with both pairs of opposite sides parallel
  – a rhombus is a parallelogram with two adjacent sides equal in length
  – a rectangle is a parallelogram with one angle a right angle
  – a square is a rectangle with two adjacent sides equal.
  – use dynamic geometry software to investigate and test conjectures about geometrical figures (Problem Solving, Reasoning) [ICT] [CCT]

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Properties of Geometrical Figures §

- prove and apply theorems and properties related to triangles and quadrilaterals such as:
    - the sum of the interior angles of a triangle is 180°
    - the exterior angle of a triangle is equal to the sum of the two interior opposite angles
    - if two sides of a triangle are equal, then the angles opposite those sides are equal
    - conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal
    - each angle of an equilateral triangle is equal to 60°
    - the sum of the interior angles of a quadrilateral is 360°
    - the opposite angles of a parallelogram are equal
    - the opposite sides of a parallelogram are equal
    - the diagonals of a parallelogram bisect each other
    - the diagonals of a rhombus bisect each other at right angles
    - the diagonals of a rhombus bisect the vertex angles through which they pass
    - the diagonals of a rectangle are equal. (Problem Solving) [N] [CCT]

  - reason that any result proven for a parallelogram would also hold for a rectangle (Reasoning) [CCT]

  - give reasons why a square is a rhombus, but a rhombus is not necessarily a square (Reasoning) [CCT]

  - use a diagram or flow chart to show the relationships between different quadrilaterals (Understanding) [CCT]

- prove and apply tests for quadrilaterals:
    - if both pairs of opposite angles of a quadrilateral are equal, then it is a parallelogram
    - if both pairs of opposite sides of a quadrilateral are equal, then it is a parallelogram
    - if all sides of a quadrilateral are equal, then it is a rhombus.
      (Problem Solving) [CCT]

- solve numerical and non-numerical problems in Euclidean geometry based on known assumptions and proven theorems (Problem Solving) [N] [CCT]

  - state possible converses of known results, and examine whether or not they are also true (Understanding, Reasoning) [CCT]

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Properties of Geometrical Figures §

Background Information:

In Circle Geometry, similarity of triangles is used to prove further theorems on intersecting chords, secants and tangents.

Attention should be given to the logical sequence of theorems and to the types of arguments used. Memorisation of proofs is not intended. Every theorem presented could be preceded by a straight-edge-and-compasses construction to confirm it, and then proven, in a manner appropriate to the student's work level, by way of an exercise or an investigation.

In Euclidean geometry, congruence is the method by which symmetry arguments are constructed. It is often helpful intuitively to see exactly what transformation, or sequence of transformations, will map one triangle into a congruent triangle. For example, the proof that the opposite sides of a parallelogram are equal involves constructing a diagonal and proving that the resulting triangles are congruent - these two triangles can be transformed into each other by a rotation of 180º about the midpoint of the diagonal.

In the universe of Einstein's general theory of relativity, three-dimensional space is curved, and as a result, the sum of the angles of a physical triangle of cosmological proportions is not 180º. Abstract geometries of this nature were developed by Gauss, Bolyai, Lobachevsky and Riemann and others in the early 19^{th} century, amid suspicions that Euclidean geometry may not be the correct description of physical space.

The Elements of Euclid (c 325-265 BCE) gives an account of geometry written almost entirely as a sequence of axioms, definitions, theorems and proofs. Its methods have had an enormous influence on mathematics. Students could read some of Book 1 for a far more systematic account of the geometry of triangles and quadrilaterals.

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Circle Geometry #

Outcome
A student:
5.3.1 uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
5.3.2 connects and generalises mathematical ideas and techniques to analyse and solve problems efficiently
5.3.3 uses deductive reasoning in presenting arguments and formal proofs
5.3.10 determines properties of plane shapes using deductive reasoning and formulates proofs using formal geometric arguments

Students:

Prove and apply angle and chord properties of circles

- identify and name parts of a circle (centre, radius, diameter, circumference, sector, arc, chord, secant, tangent, segment, semicircle) [L]
- use terminology associated with angles in circles such as subtend, standing on the same arc, angle at the centre, angle at the circumference, angle in a segment [L]
- identify the arc on which an angle at the centre or circumference stands [CCT]
- demonstrate that at any point on a circle there is a unique tangent to the circle, and that this tangent is perpendicular to the radius at the point of contact [CCT]
- prove the following chord properties of circles:
  - chords of equal length in a circle subtend equal angles at the centre and are equidistant from the centre
  - the perpendicular from the centre of a circle to a chord bisects the chord
  - conversely, the line from the centre of a circle to the midpoint of a chord is perpendicular to the chord
  - the perpendicular bisector of a chord of a circle passes through the centre
  - given any three non-collinear points, the point of intersection of the perpendicular bisectors of any two sides of the triangle formed by the three points is the centre of the circle through all three points
  - when two circles intersect, the line joining their centres bisects their common chord at right angles. (Problem Solving) [CCT]

  - use dynamic geometry software to investigate chord properties of circles (Fluency, Understanding) [ICT] [CCT]

- prove the following angle properties involving circles:
  - the angle at the centre of a circle is twice the angle at the circumference standing on the same arc
  - tthe angle in a semicircle is a right angle
  - angles at the circumference, standing on the same arc, are equal
  - the opposite angles of cyclic quadrilaterals are supplementary
  - an exterior angle at a vertex of a cyclic quadrilateral is equal to the interior opposite angle. (Problem Solving, Reasoning) [CCT]

  - use dynamic geometry software to investigate angle properties of circles (Fluency, Understanding) [ICT] [CCT]

Mathematics • Stage 5 (5.3 pathway)

Measurement and Geometry
Circle Geometry #

- apply circle theorems to prove that the angle in a semicircle is a right angle (Problem Solving, Reasoning) [CCT]

- prove the following tangent and secant properties involving circles:
  - the two tangents drawn to a circle from an external point are equal in length
  - the angle between a tangent and a chord drawn to the point of contact is equal to the angle in the alternate segment
  - when two circles touch, their centres and the point of contact are collinear
  - the products of the intercepts of two intersecting chords of a circle are equal
  - the products of the intercepts of two intersecting secants to a circle from an external point are equal
  - the square of a tangent to a circle from an external point equals the product of the intercepts of any secants from the point. (Problem Solving, Reasoning) [CCT]

  - use dynamic geometry software to investigate tangent properties of circles (Fluency, Understanding) [ICT] [CCT]

- apply circle theorems to find unknown angles and sides in diagrams (Problem Solving, Reasoning) [N] [CCT]

Background Information:

As well as solving arithmetic and algebraic problems in circle geometry, students should be able to reason deductively within more theoretical arguments. Diagrams would normally be given to students, with the important information labelled on the diagram to aid reasoning. Students would sometimes need to produce a clear diagram from a set of instructions.

Attention should be given to the logical sequence of theorems and to the types of arguments used. Memorisation of proofs is not intended. Ideally, every theorem presented should be preceded by a straight-edge-and-compasses construction to confirm it, and then proven, in a manner appropriate to the student's work level, by way of an exercise or an investigation.

The tangent-and-radius-theorem is difficult to justify at this Stage, and is probably better taken as an assumption as indicated above.

Circle Geometry may be extended to examining the converse of some of the theorems related to cyclic quadrilaterals, leading to a series of conditions for points to be concyclic. However, students may find these results difficult to prove and apply.

The angle in a semicircle theorem is also called Thales' theorem because it was traditionally ascribed to Thales (c 624–548 BCE) by the ancient Greeks, who reported that it was the first theorem ever proven in mathematics.

Mathematics • Stage 5 (5.1 pathway)

Statistics and Probability
Single Variable Data Analysis

Outcome
A student:
5.1.1 communicates mathematical ideas using appropriate terminology, diagrams and symbolism
5.1.2 selects and uses appropriate strategies to solve problems
5.1.3 provides reasoning to support conclusions which are appropriate to the context
5.1.9 investigates and evaluates techniques of large-data collection, and compares data sets using statistical displays and measures

Students:

Investigate techniques for collecting data, including census, sampling and observation

- recognise the differences between collecting data by observation, a census and a sample [L] [N] [CCT]
  - recognise examples of variables for which data could be collected by observation, a census and a sample,
    eg data about daily temperature can be collected by observation; data about the income of Australians can be collected by census or sample (Understanding) [N] [CCT]
  - discuss the practicalities of collecting data through a census compared to a sample (Problem Solving, Fluency) [CCT]
- explore issues involved in constructing and conducting surveys such as sample size, bias and ethics [L] [N] [EU]
  - detect bias in the selection of a sample (Understanding) [N] [CCT]

Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources

- identify and investigate relevant issues involving at least one numerical and at least one categorical variable using information gained from secondary sources,
  eg the number of hours in a working week for different professions in Australia; the annual rainfall in various parts Australia compared with other countries in the Asia-Pacific region [N] [CCT]

Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bi modal’

- compare data sets using back-to-back stem-and-leaf plots [N] [CCT]
- use the terms ‘skewed’, ‘symmetric’ or ‘bi-modal’ when describing the shape of distributions [L]
  - use histograms and stem-and-leaf plots to describe the shape of a distribution (Understanding) [N] [CCT]
  - recognise when a distribution is symmetrical or skewed, and discuss possible reasons for its shape (Understanding, Reasoning) [CCT]

Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread

Mathematics • Stage 5 (5.1 pathway)

Statistics and Probability
Single Variable Data Analysis

- compare two sets of data displayed in back-to-back stem-and-leaf plots, histograms, or double column graphs, using mean, median and range [N] [CCT]
- compare two sets of numerical data by finding the mean, mode and/or median, and range of both sets [N] [CCT]

Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data

- interpret media reports and advertising that quote various statistics, eg media ratings, house prices [L] [N]
- analyse graphical displays to recognise features that may cause a misleading interpretation, eg displaced zero, irregular scales [N] [CCT]
  - explain and evaluate the effect of misleading features on graphical displays (Understanding, Reasoning) [N] [CCT]
- critically evaluate statements on chance and probability appearing in the media and/or in other subjects [N] [CCT]
- investigate the use of probability by governments and companies, eg in demography, insurance, planning for roads (Problem Solving) [N]
- consider informally the reliability of conclusions from statistical investigations, such as factors which may have masked the results, the accuracy of measurements taken, and whether the results can be generalised to other situations [N] [CCT]

Background Information:

No specific analysis of the relative positions of mean, mode and median in skewed distributions is required. Recognition of the general shape and lack of symmetry (only) needs to be considered.

Meteorologists use probability to predict the weather (chance of rain). Insurance companies use probability to determine premiums (chance of particular age groups having accidents).

Mathematics • Stage 5 (5.1 pathway)

Statistics and Probability
Probability

Outcome
A student:
5.1.1 communicates mathematical ideas using appropriate terminology, diagrams and symbolism
5.1.2 selects and uses appropriate strategies to solve problems
5.1.3 provides reasoning to support conclusions which are appropriate to the context
5.1.10 calculates relative frequencies to estimate probabilities of simple and compound events

Students:

Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'

- repeat an experiment a number of times to determine the relative frequency of an event [L]
  - model probability experiments using random number generators or other digital simulators (Understanding, Fluency) [N] [ICT]
  - recognise randomness in chance situations (Understanding) [N] [CCT]
  - recognise that probability estimates become more stable as the number of trials increases (Reasoning) [N] [CCT]
  - explain theoretical probability as being the likelihood of occurrence under ideal circumstances (Understanding) [N] [CCT]
  - explain the relationship between the relative frequency of an event and its theoretical probability (Understanding) [N] [CCT]
- predict the probability of a compound event from experimental data using relative frequencies [N]
  - apply relative frequency to predict future experimental outcomes (Reasoning, Problem Solving) [N] [CCT]
  - design a device to produce a specified relative frequency, eg a four-coloured circular spinner (Problem Solving) [N] [CCT]

Background Information:

ICT could be used for simulation experiments to demonstrate that the relative frequency gets closer and closer to the theoretical probability as the number of trials increases.

Students may not appreciate the significance of a simulation experiment, eg they may not transfer random number generator results for tossing a die to the situation of actually tossing a die a number of times.

Mathematics • Stage 5 (5.2 pathway)

Statistics and Probability

Single Variable Data Analysis ◊

Outcome

A student:

5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions

5.2.3 constructs arguments to prove and justify results

5.2.10 represents, describes and compares single variable and bivariate data sets using statistical displays and measures

Determine quartiles and interquartile range

- determine the upper and lower quartiles for a set of scores [L]
- determine the interquartile range for a set of scores [L]
  - recognise that the interquartile range is a measure of spread of the middle 50% of data (Understanding) [N] [CCT]
- compare the relative merits of the range and interquartile range as measures of spread [N] [CCT]

Construct and interpret box plots and use them to compare data sets

- construct a box plot using the median, the upper and lower quartiles and the extreme values (the 'five-point summary') [N]
- compare two or more sets of data using parallel box plots drawn on the same scale [N] [CCT]
  - describe similarities and differences between two data sets displayed in parallel box plots (Understanding) [N] [CCT]

Compare shapes of box plots to corresponding histograms and dot plots

- represent the same data set using a box plot and either a histogram or dot plot [CCT]
  - compare the relative merits of a box plot with its corresponding histogram or dot plot (Understanding, Reasoning) [CCT]

Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians

- investigate survey data reported in the digital media and elsewhere to critically evaluate the reliability/validity of the source of the data and its usefulness [ICT] [CCT] [EU]
- make predictions from a sample that may apply to the whole population [N]
  - consider the size of the sample when making predictions about the population (Reasoning) [N] [CCT]

Background Information:

Graphics calculators and other statistical software will display box-and-whisker plots for entered data, but students should be aware that results will not always be the same since the technologies use varying methods of creating the plots.

Mathematics • Stage 5 (5.2 pathway)

Statistics and Probability
Bivariate Data Analysis

Outcome
A student:
5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions
5.2.3 constructs arguments to prove and justify results
5.2.10 represents, describes and compares single variable and bivariate data sets using statistical displays and measures

Investigate and describe bivariate numerical data where the independent variable is time

- recognise the difference between an independent variable and its dependent variable [L]
- distinguish bivariate data from single variable data [L]
- investigate an issue of interest, representing the dependent numerical variable against the independent variable, time, in an appropriate graphical form [N]
  - describe changes in the dependent variable over time (Understanding) [N] [CCT]
  - suggest reasons for changes in the dependent variable over time with reference to relevant world or national events (Reasoning) [N] [CCT]
- interpret data displays representing two or more dependent numerical variables against time, eg compare the population of Australia with oil production over time; compare daily food intake of different countries over time [N] [CCT]

Use scatter plots to investigate and comment on relationships between two continuous variables

- make predictions from a given scatter diagram or graph [N] [CCT]
- describe informally the strength and direction of the relationship between two variables in a scatter plot,
  eg strong positive relationship, weak negative relationship, no association [N] [CCT]
- investigate an issue of interest involving two continuous variables and construct a scatter plot, with or without the use of ICT, to determine and comment on the relationship between them, eg height versus handspan; reaction time versus hours of sleep [N] [ICT] [CCT]

Mathematics • Stage 5 (5.2 pathway)

Statistics and Probability
Probability

Outcome
A student:
5.2.1 selects appropriate notations and conventions to communicate mathematical ideas and solutions
5.2.2 analyses mathematical or real-life situations, systematically applying appropriate strategies to solve problems
5.2.3 constructs arguments to prove and justify results
5.2.11 describes and determines probabilities for multi-step events

Students:

List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events

- sample with and without replacement in two-step experiments,
  eg drawing two counters from a bag containing 3 blue, 4 red and 1 white counter [L] [N]
  - compare results between an experiment undertaken firstly with replacement and then without (Reasoning) [N] [CCT]
- analyse two-step events through constructing organised lists, tables and tree diagrams [N] [CCT]
- solve two-step probability problems including instances of sampling with and without replacement [N]
  - explain the effect of knowing the result of the first step on the probability of two-step events with or without replacement (Reasoning, Understanding) [N] [CCT]

Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence

- distinguish informally between dependent and independent events [L]
  - evaluate the likelihood of winning a prize in lotteries and other competitions (Problem Solving, Reasoning) [N] [CCT]
- sample with and without replacement in three-step experiments,
  eg tossing three coins or drawing three counters from a bag containing 3 blue, 4 red and 1 white counter [N]
- analyse three-step events through constructing lists, tables or tree diagrams [N] [CCT]
- solve three-step probability problems including sampling with and without replacement [N]

Use the language of ‘if ....then’, ‘given’, ‘of’, ‘knowing that’ to investigate conditional statements and identify common mistakes in interpreting such language

- critically evaluate conditional statements used in descriptions of chance situations,
  eg explain why if you toss a coin and obtain a head, then the chance of obtaining a head on the next toss remains the same [N] [CCT]
  - describe the validity of conditional statements used in descriptions of chance situations with reference to dependent and independent events (Understanding) [N] [CCT]

Mathematics • Stage 5 (5.2 pathway)

Statistics and Probability
Probability

- identify common misconceptions related to chance events,
  eg if you obtain a tail in each of four consecutive tosses of a coin, then there is a greater chance of obtaining a head with the next cross (Reasoning) [N] [CCT]

- determine whether the converses of particular mathematical statements are true,
  eg 'if two angles are vertically opposite then they are equal' and 'if two angles are equal then they are vertically opposite' [N] [CCT]

Background Information:

Meteorologists use probability to predict the weather (chance of rain). Insurance companies use probability to determine premiums (chance of particular age groups having accidents).

The mathematical analysis of probability was prompted by the French gentleman gambler, the Chevalier de Méré. Over the years, the Chevalier had consistently won money betting on at least one six in four rolls of a die. He felt that he should also win betting on at least one double six in 24 rolls of two dice, but in fact regularly lost.

In 1654 he asked his mathematician friend Pascal to explain why. This question led to a famous correspondence between Pascal and the renowned mathematician Fermat. The Chevalier's change of fortune is explained by the fact that the chance of at least one six in four rolls of a die is 51.8%, while the chance of at least one double six in 24 rolls of two dice is 49.1%.

Mathematics • Stage 5 (5.3 pathway)

Statistics and Probability
Single Variable Data Analysis

Outcome
A student:
5.3.1 uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
5.3.11 compares and quantifies statistical relationships for single variable and bivariate data sets, evaluating the usefulness of statistics in prediction and planning

Students:

Calculate and interpret the mean and standard deviation of data and use these to compare data sets

- find the standard deviation of a set of scores [L] [ICT]
- use the mean and standard deviation to compare two sets of data [N] [CCT]
  - compare data sets with the same mean and different standard deviations (Understanding) [N] [CCT]
- compare the relative merits of the range, interquartile range and standard deviation as measures of spread [N] [CCT]

Background Information:

It is intended that students develop a feeling for the concept of standard deviation being a measure of spread of a symmetrical distribution without going into detailed analysis. When using a calculator the $\boxed{\sigma_n}$ button for standard deviation of a population will suffice.

Use of ICT enables 'what if' questions to be asked and explored, eg what happens to the standard deviation if a score of zero is added, or if three is added to each score, or if each score is doubled?

Mathematics • Stage 5 (5.3 pathway)

Statistics and Probability
Bivariate Data Analysis

Outcome
A student:
5.3.2 connects and generalises mathematical ideas and techniques to analyse and solve problems efficiently
5.3.3 uses deductive reasoning in presenting arguments and formal proofs
5.3.11 compares and quantifies statistical relationships for single variable and bivariate data sets, evaluating the usefulness of statistics in prediction and planning

Students:

Use information technologies to investigate bivariate numerical data sets. Where appropriate use a straight line to describe the relationship allowing for variation

- use ICT to display bivariate numerical data and draw the line of best fit by eye [N] [ICT]
- use ICT to construct a line of best fit for bivariate numerical data [N] [ICT]
  - investigate different methods of constructing the line of best fit using ICT (Problem Solving) [ICT] [CCT]
- use lines of best fit to estimate what might happen between known data values (interpolation) and predict what might happen beyond known data values (extrapolation) [N] [ICT] [CCT]
  - compare predictions obtained from different lines of best fit (Problem Solving, Understanding) [N] [ICT] [CCT]

Investigate reports of studies in digital media and elsewhere for information on the planning and implementation of such studies, and the reporting of variability

- investigate and evaluate the appropriateness of sampling methods and sample size in reports where statements about population are based on a sample [N] [CCT]
- critically review surveys, polls and media reports [N] [CCT]
  - identify, describe and evaluate issues such as the misrepresentation of data, apparent bias in reporting or sampling techniques, or issues with the questions posed to collect the data (Reasoning, Problem Solving) [N] [CCT] [EU]
  - discuss issues to be considered in the implementation of policies or procedures that result from data reported in the media or elsewhere (Problem Solving) [N] [CCT] [EU]
- investigate the use of statistics and associated probabilities in shaping decisions made by governments and companies,
  eg setting of insurance premiums or the use of demographic data to determine where and when hospitals and schools may be built [N] [CCT]