



7 Content

7.1 Organisation of content

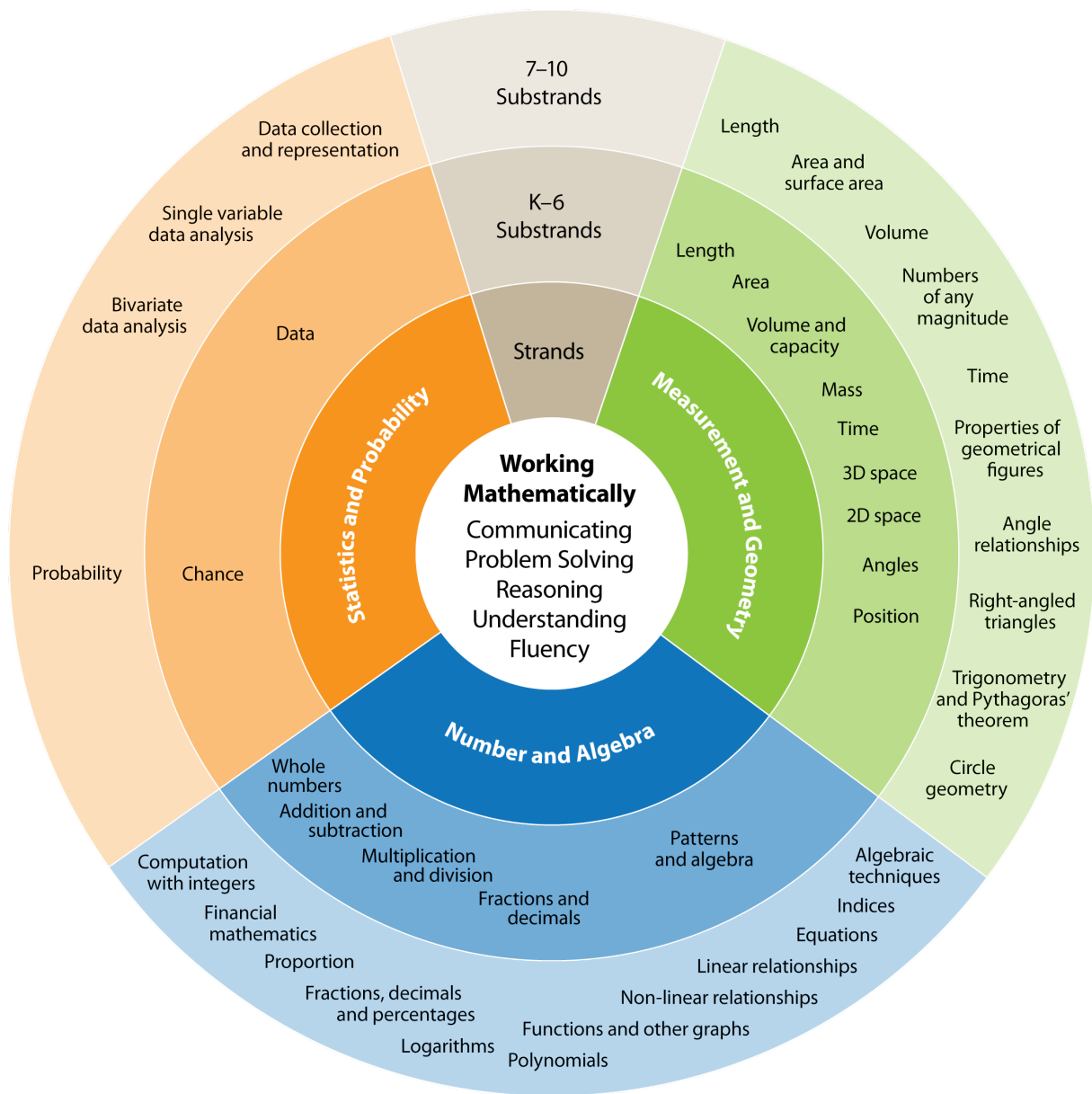
 for your information

Content specifies the expected learning for students as they work to achieve the outcomes and describes the subject matter that is to be studied. Syllabus content reflects a balance between the acquisition of knowledge and the specific processes of learning in Mathematics so that students are encouraged to engage in, take responsibility for, and continue their own learning.

The knowledge, understanding and skills described provide a sound basis for students to successfully move to the next stage of learning. Teachers will make decisions about the sequence of learning and the emphasis to be given to particular content, based on the needs of their students.

 consult

Essential content



The graph represents the relationships between the strands and substrands only. It is not intended to indicate the amount of time spent studying each strand or substrand.

The content presented in a stage represents the knowledge, understanding and skills that are to be achieved by a typical student by the end of that stage. It is acknowledged that students learn at different rates and in different ways, so that there will be students who have not achieved the outcomes for the stage/s prior to that identified with their stage of schooling. For example, some students will achieve Stage 3 outcomes during Year 5, while the majority will achieve them by the end of Year 6. Other students might not develop the same knowledge, understanding and skills until Year 7 or later.

The *Mathematics K–10 Syllabus* is organised into a Working Mathematically strand and three content strands: Number and Algebra; Measurement and Geometry; and Statistics and Probability. The syllabus is written with the flexibility to enable students to work at different

stages in different strands. For example, students could be working on Stage 4 content in one strand and Stage 3 content in another.

Working Mathematically

Working Mathematically relates to the syllabus objective:

Students understand and connect related mathematical concepts, choosing, applying and communicating approaches in order to investigate and solve problems.

As an essential part of the learning process, the Working Mathematically strand provides students with the opportunity to engage in genuine mathematical activity and develop the skills to become flexible and creative users of mathematics.

In this syllabus, Working Mathematically encompasses five interrelated components:

Communicating

Students develop the ability to use a variety of representations, in written, oral or graphical form, to formulate and express mathematical ideas. Students are communicating mathematically when they describe, represent and explain mathematical situations, concepts, methods and solutions to problems, using appropriate language, terminology, tables, diagrams, graphs, symbols, notation and conventions.

Problem Solving

Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, design investigations and plan their approaches, apply their existing strategies to seek solutions, and verify that their answers are reasonable.

Reasoning

Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, deduce and justify strategies used and conclusions reached, adapt the known to the unknown, transfer learning from one context to another, prove that something is true or false, and compare and contrast related ideas and explain their choices.

Understanding

Students build a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the ‘why’ and the ‘how’ of mathematics. Students build understanding when they connect related ideas, represent concepts in different ways, identify commonalities and differences between aspects of content, describe their thinking mathematically, and interpret mathematical information.

Fluency

Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, recognise robust ways of answering questions, choose appropriate methods and approximations, recall definitions and regularly use facts, and manipulate expressions and equations to find solutions.

The five components of Working Mathematically describe how content is explored or developed – that is, the thinking and doing of mathematics. They provide the language to build in the developmental aspects of the learning of mathematics. The components come into play when developing new skills and concepts, and also when applying existing knowledge to solve routine and non-routine problems both within and beyond mathematics.

In addition to its explicit link to one syllabus objective, the Working Mathematically strand has a separate set of outcomes for the components Communicating, Problem Solving and Reasoning. This approach has been adopted to ensure students' mastery of these components becomes increasingly sophisticated over the years of schooling.

Separate syllabus outcomes have not been developed for Understanding and Fluency. Understanding and Fluency are encompassed in the development of knowledge, understanding and skills across the range of syllabus strands, objectives and outcomes.

The five components emphasise the active engagement that students are to undertake with the content. At times the focus may be on a particular component of the Working Mathematically strand or a group of the components, but often the components overlap. While not all of the Working Mathematically components apply to all content, they indicate the breadth of mathematical actions that teachers need to emphasise.

Teachers are encouraged to extend students' mastery of the components by creating opportunities for their development through the learning experiences that they design.

Number and Algebra

The knowledge, understanding and skills developed in the Number and Algebra strand are fundamental to the other strands of this syllabus and are developed across the stages from Early Stage 1 to Stage 5.3.

Numbers, in their various forms, are used to quantify and describe the world. From Early Stage 1 there is an emphasis on the development of number sense, and confidence and competence in using mental, written and calculator techniques for solving appropriate problems. Algorithms are introduced after students have gained a firm understanding of basic concepts, including place value, and have developed mental strategies for computing with two-digit and three-digit numbers. Approximation is important and the systematic use of estimation is to be encouraged at all times. Students should always check that their answers ‘make sense’ in the contexts of the problems they are solving.

In the early stages, students explore number and pre-algebra concepts by pattern-making, and discussing, generalising and recording their observations. This demonstrates the importance of early number learning in the development of algebraic thinking and the algebra concepts that follow.

The use of mental-computation strategies should be developed in all stages. Information and communication technology (ICT) can be used to investigate number patterns and relationships, and facilitate the solution of real problems with measurements and quantities not easy to handle with mental or written techniques.

In Stages 2 to 5, students apply their number skills to a variety of financial and other situations and practical problems, developing a range of life skills important for numeracy. Ratio and rates underpin proportional reasoning needed for problem solving and the development of concepts and skills in other areas of mathematics, such as trigonometry, similarity and gradient.

Following the development of foundational number skills and pre-algebra concepts through patterning, a concrete approach to algebra is continued when students generalise their understanding of the number system to the algebraic symbol system. Students develop an understanding of the notion of a variable, establish the equivalence of expressions, apply algebraic conventions, and graph relationships on the number plane.

Students recognise that graphing is a powerful tool that enables algebraic relationships to be visualised. The use of ICT for graphing provides an opportunity for students to compare and investigate these relationships dynamically. By the end of Stage 5.3, students have the opportunity to develop knowledge and understanding of the shapes of graphs of different relationships and the effects of performing transformations on these graphs.

Algebra has strong links with the other strands in the syllabus, particularly when situations are to be generalised symbolically.

Measurement and Geometry

Measurement enables the identification and quantification of attributes of objects so that they can be compared and ordered, while Geometry is the study of spatial forms and involves representation of shape, size, pattern, position and movement of objects in the three-dimensional world or mind of the learner. The study of geometry enables the investigation of three-dimensional objects and two-dimensional shapes as well as the concepts of position, location and movement. The presentation of Measurement and Geometry as a single strand recognises and emphasises their interrelationship.

Important and critical skills for students to acquire are those of recognising, visualising and drawing shapes and describing the features and properties of three-dimensional objects and two-dimensional shapes in static and dynamic situations. Manipulation of a variety of real objects and shapes is crucial to the development of appropriate levels of imagery, language and representation. ICT, and dynamic geometry software in particular, can be used to facilitate the exploration and manipulation of shapes, geometric relationships and two-dimensional representations of three-dimensional objects.

Geometry uses systematic classification of angles, triangles, regular polygons and polyhedra. The ability to classify is a trait of human cultural development and an important aspect of education. Class inclusivity is a powerful tool in reasoning and determining properties. Justification and reasoning in both an informal and, later, in a formal way are fundamental to Geometry in Stages 4 and 5.

When classifying quadrilaterals for example, students need to begin to develop an understanding of inclusivity within the classification system. Quadrilaterals are inclusive of the parallelograms, trapeziums and kites, while parallelograms are inclusive of the rectangles and rhombuses, which are inclusive of the squares.

Measurement involves the application of number and geometry understandings and skills when quantifying and solving problems in practical situations. Students need to make reasonable estimates for quantities, be familiar with commonly used units for length, area, volume and capacity, and be able to convert between these units. They should develop an idea of the levels of accuracy that are appropriate to particular situations. Competence in applying Pythagoras' theorem to solve practical problems is developed in Stage 4 and applied throughout the topics involving measurement.

Statistics and Probability

In the Statistics and Probability strand, Statistics and Probability are developed initially in parallel, with the links between them then built progressively across the stages.

The study of Statistics within the strand includes the collection, organisation, display and analysis of data. Early experiences are based on real-life contexts using concrete materials. This leads to data collection methods and the display of data in a variety of ways. Students are encouraged to ask questions relevant to their experiences and interests, and to design ways of investigating their questions. They should be aware of the extensive use of statistics in society and be encouraged to critically evaluate claims based on statistical evidence. Data from a variety of sources, including print-based materials and the Internet, can be analysed and evaluated. Electronic tools, such as spreadsheets and other software packages, may be used where appropriate to organise, display and analyse data.

The study of chance is introduced from Stage 1 to enable the development of understanding of chance concepts from an early age. Early emphasis is on understanding the notion of chance and the use of informal language associated with chance. The understanding of chance situations is further developed through the use of simple experiments which produce data, so that students can make comparisons of the likelihood of events occurring, and begin to order chance expressions on a scale from zero to one. In later stages, students link chance concepts to numerical probabilities, and explore increasingly sophisticated methods of determining the likelihoods of events, using experimental and theoretical approaches. Emphasis should be placed on developing skills in representing outcomes of events in ways that facilitate the calculation of probabilities.

Students need to develop the ability to use the language of statistics and probability, distinguishing between such concepts as simple and compound events, mutually exclusive and non-mutually-exclusive events, discrete and categorical data, and independent and dependent variables. Appropriate analysis of data and the solution of associated problems depend on sound knowledge and understanding of such terms.

Skills in evaluation, and the ability to produce reasoned judgements, lead to students building further skills in critical evaluation of statistical information. In our contemporary society, there is a constant need to understand, interpret and analyse information displayed in tabular or graphical forms. Students need to recognise ways in which information can be displayed in a misleading manner, resulting in false conclusions.



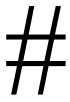
Pathways of learning in Stage 5

The arrangement of content in Stage 5 acknowledges the wide range of achievement of students in Mathematics by the time they reach the end of Year 8. Three Stage 5 pathways (Stages 5.1, 5.2 and 5.3) have been identified and made explicit in the syllabus:

- the Stage 5.1 pathway is designed to assist in meeting the needs of students who are continuing with Stage 4 outcomes when they enter Year 9
- the Stage 5.2 pathway builds on and includes the content of Stage 5.1 and is designed to assist in meeting the needs of students who have achieved Stage 4 outcomes, generally by the end of Year 8
- the Stage 5.3 pathway builds on and includes the content of Stage 5.2 and is designed to assist in meeting the needs of students who have achieved Stage 4 outcomes before the end of Year 8.

A multitude of other pathways, and therefore ‘endpoints’, are also possible; for example, some students may achieve all of the Stage 5.2 outcomes and a selection of the Stage 5.3 outcomes before the end of Year 10.

When planning learning experiences for students in Years 9 and 10, teachers need to consider courses of study that the students plan to follow beyond Stage 5. The table below outlines these considerations for current Stage 6 Mathematics Board Developed Courses. Other students may access Stage 6 Mathematics through Board Endorsed Courses or Life Skills.

Stage 5 pathway	Intended Stage 6 Board Developed Course	Recommended content from pathway above (if not all of the content)
Stage 5.1	General Mathematics  (substrands identified by ◇)	Stage 5.2 substrands: <i>Financial Mathematics</i> <i>Non-linear Relationships</i> <i>Right-Angled Triangles (Trigonometry)</i> <i>Single Variable Data Analysis</i>
Stage 5.2	Mathematics (‘2 Unit’)  (substrands identified by §)	Stage 5.3 substrands: <i>Surds and Indices</i> <i>Algebraic Techniques</i> <i>Equations</i> <i>Linear Relationships</i> At least some of the Stage 5.3 substrands: <i>Trigonometry and Pythagoras’ Theorem</i> <i>Properties of Geometrical Figures</i>
Stage 5.3	Mathematics Extension 1  (substrands identified by #)	Stage 5.3 substrands: <i>Polynomials</i> <i>Logarithms</i> <i>Circle Geometry</i> <i>Functions and Other Graphs</i>

Additional content

In addition to the essential content that relates to the outcomes listed in each of the strands, teachers may wish to include in their teaching and learning programs other material in order to broaden and deepen students' knowledge, understanding and skills to meet students' interests, or to stimulate student interest in other areas of mathematics.

The following list contains possible topics for inclusion as additional content in teaching and learning programs. This additional content is not essential, nor is it required as a prerequisite for other content in the K–12 Mathematics curriculum. The list of additional content provided below is not exhaustive; teachers are encouraged to investigate other appropriate content to meet the capabilities and interests of their students.

Additional Content: Number and Algebra	
Early Stage 1 – Stage 3	Stage 4 – Stage 5
Other monetary systems	Cube root formula Algorithm for finding square roots Matrices and vectors Linear programming Finite differences Three-dimensional coordinate geometry Polar coordinates
Number bases other than 10 Other number systems such as Aboriginal, Babylonian, Chinese (rod numerals), Mayan Calculating methods and devices, eg abacus, Napier's Bones, Russian Peasant Method, Egyptian Method Exploration of different types of numbers, such as perfect numbers and amicable numbers Construction of magic squares Logic puzzles Number theory Codes	

Additional Content: Measurement and Geometry	
Early Stage 1 – Stage 3	Stage 4 – Stage 5
The history of the calendar Other measuring devices such as sundials Alternate measurement systems Unusual units of measurement Perspective drawing Pentominoes	Heron’s formula for the area of a triangle Non-metric units of measurement Surveying Knots Networks Topology Planes of symmetry of solids Construction of inscribed, circumscribed and described circles for a triangle Constructions using ruler and compasses Golden section Golden mean construction Non-Euclidean geometry
Fractals Navigation – latitude and longitude Further tessellations (including semi-regular tessellations) Semi-regular polyhedrons; truncated, snub-nosed and stellated solids	

Additional Content: Statistics and Probability	
Early Stage 1 – Stage 3	Stage 4 – Stage 5
Further exploration of random number generators	The normal distribution
	Set theory

Content presentation


Sections 7.2 – 7.7 contain the essential content for Early Stage 1 to Stage 5. Within each Stage, the outcomes, content, background information and advice about language, are organised into substrands within the three content strands. The content is comprised of the statements of knowledge, understanding and skills and the associated components of Working Mathematically.

There are some substrands that contain the development of several concepts within a Stage. To assist programming, the content has been separated into two parts. The first part typically contains early concept development, while the second part represents further development of these concepts.

Life Skills


For some students with special education needs, particularly those students with an intellectual disability, it may be determined that the Stage 4 and 5 outcomes and content are not appropriate. For these students, Life Skills outcomes and content can provide a relevant and meaningful program. Refer to section 1 for further information regarding curriculum options for students with special education needs. Years 7–10 Life Skills outcomes and content are in section 8.

Learning across the curriculum

 for your information

The Board of Studies has identified important learning for all students that can be developed across the syllabuses. In K–10 syllabuses, the identified areas will be embedded in the descriptions of content. Content relation the learning across the curriculum addresses issues, perspectives and policies that will assist students to achieve the broad learning outcomes defined in the Board of Studies *K–10 Curriculum Framework*. These areas take account of the general capabilities and cross-curriculum priorities in the Australian curriculum.

This content will be included, where appropriate, while ensuring that subject integrity is maintained.

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Aboriginal and Torres Strait Islander histories and cultures [AHC]

Aboriginal and Torres Strait Islander Peoples have a unique sense of Identity which can be demonstrated through the interconnected aspects of Country and Place, People, and Culture.

Mathematics is a representation of the world that has developed over thousands of years through many diverse cultural contexts. Aboriginal and Torres Strait Islander cultures have a rich understanding of mathematics that includes a broad range of applications of mathematical concepts.

The NSW K–10 Mathematics curriculum values Aboriginal and Torres Strait Islander perspectives of mathematics and provides opportunities for students to investigate various aspects of number, measurement and geometry, including time, location and relationships, in Aboriginal and Torres Strait Islander contexts. Students can deepen and extend their

understanding of the lives of Aboriginal and Torres Strait Islander Peoples through the application and evaluation of statistical data.

Asia and Australia's engagement with Asia [A]

The Asia and Australia's engagement with Asia priority provides a regional context for learning in all areas of the curriculum. An understanding of Asia underpins the capacity of Australian students to be active and informed citizens working together to build harmonious local, regional and global communities, and build Australia's social, intellectual and creative capital. This priority is concerned with Asia literacy for all Australian students. Asia literacy develops knowledge, skills and understanding about the histories, geographies, cultures, arts, literatures and languages of the diverse countries of our region. It fosters social inclusion in the Australian community. It enables students to communicate and engage with the peoples of Asia so they can effectively live, work and learn in the region.

In their study of the NSW K–10 Mathematics curriculum, students investigate the concept of chance using Asian games and can explore the way Asian societies apply other mathematical concepts such as patterns and symmetry in art and architecture. Investigations involving data collection and representation can be used to examine issues pertinent to the Asia region.

Critical and creative thinking [CCT]

Critical and creative thinking are key to the development of mathematical understanding. Students use critical and creative thinking as they learn to generate and evaluate knowledge, ideas and possibilities, and when seeking new pathways or solutions. Mathematical reasoning and logical thought are fundamental elements of critical and creative thinking. They are integral to mathematical problem solving as students identify similarities and differences in mathematical situations and engage in reasoning and thinking about solutions to problems, and the strategies needed to find these solutions.

In their study of Mathematics in K–10, students use critical and creative thinking in such activities as exploring properties of shapes, setting up statistical investigations, comparing actual to expected results, approximating and estimating, interpreting data displays, examining misleading data, and interpolating and extrapolating. Critical and creative thinking are also of fundamental importance in such aspects of the Mathematics curriculum as posing problems, modelling situations, justifying choices and strategies used, and giving reasons to explain mathematical ideas.

Ethical understanding [EU]

Students develop ethical understanding as they learn about, and learn to act in accordance with, ethical principles, values, integrity and regard for others. There are various opportunities in the K–10 Mathematics curriculum for students to develop and apply ethical understanding when, for example, collecting and displaying data, interpreting misleading graphs and displays, examining selective use of data by individuals and organisations, and detecting and eliminating bias in the reporting of information.

Information and communication technology [ICT]

Information and communication technology [ICT] includes digital technologies such as calculators, spreadsheets, dynamic geometry software, and computer algebra and graphing software. Students use ICT effectively and appropriately when investigating, creating and communicating ideas and information, including in representing mathematics in a variety of

ways to aid understanding. ICT can be utilised by students to solve problems and to perform previously onerous tasks more readily.

In the Number and Algebra strand in the NSW K–10 Mathematics curriculum, students can utilise ICT in such topic areas as creating patterns, creating and interpreting graphs, investigating compound interest, and solving equations graphically. In the Measurement and Geometry strand of the curriculum, students can utilise ICT in such areas as exploring properties of angles and shapes including symmetry; creating designs that involve shapes and transformations; representing, visualising and manipulating three-dimensional objects; investigating congruency and similarity; representing position and paths; making informal measures of length and area; and developing formulas for perimeter and area; while in Statistics and Probability students can utilise ICT in such areas as recording and displaying data in various forms, comparing data sets, calculating measures of location and spread, modelling probability experiments, and using the internet to gather and analyse data presented by the media.

Intercultural understanding [IU]

Students develop intercultural understanding as they learn to understand themselves in relation to others. This involves students valuing their own cultures and beliefs and those of others, and engaging with people of diverse cultures in ways that recognise commonalities and differences, create connections, and cultivate respect.

Intercultural understanding can be enhanced if students are exposed to a range of cultural traditions in mathematics. It may be demonstrated, for example, through examining Aboriginal and Torres Strait Islander People's perceptions of time and weather patterns, and the networks embedded in family relationships, as well as in such activities as examining patterns in art and design, learning about culturally-specific calendar days, comparing currencies, examining current and historical number systems, and showing awareness of cultural sensitivities when collecting data.

Literacy [L]

Students become literate as they develop the skills to learn and communicate confidently. These skills include listening, reading and viewing, writing, speaking and creating print, visual and digital materials accurately and purposefully within and across all learning areas.

Literacy is an important aspect of mathematics. Students need to understand written problems and instructions; including the use of common words with a specific meaning in a mathematical context and metaphorical language used to express mathematics concepts and processes.

In their K–10 mathematics learning, students are provided with opportunities to learn mathematical vocabulary and the conventions for communicating mathematics in written form, including through its symbols and structures, as well as verbally through description and explanation. Mathematical literacy also extends to interpreting information from mathematical texts such as tables, graphs and other representations.

Numeracy [N]

Numeracy involves drawing on knowledge of particular contexts and circumstances in deciding when to use mathematics, choosing the mathematics to use and critically evaluating its use. To be numerate is to use mathematics effectively to meet the general demands of life at home, in work and for participation in community and civic life. Students become numerate as they develop the capacity to recognise and understand the role of mathematics in the world around them and the confidence, willingness and ability to apply mathematics to their lives in constructive and meaningful ways. Highly numerate students interpret, apply and critically evaluate mathematical strategies, and communicate mathematical reasoning in a range of practical situations.

Mathematics makes a special contribution to the development of numeracy in a manner that is more explicit and foregrounded than is the case in other learning areas. It is important that the Mathematics curriculum provides the opportunity to apply mathematical understanding and skills in context, both in other learning areas and in real world contexts. The NSW K–10 Mathematics curriculum provides students with opportunities to use numerical, spatial, graphical, statistical and algebraic concepts and skills in a variety of contexts and involves the critical evaluation, interpretation, application and communication of mathematical information in a range of practical situations.

The key role that teachers of mathematics play in the development of numeracy includes teaching students specific skills and providing them with opportunities to select, use, evaluate and communicate mathematical ideas in a range of situations. Students' numeracy and underlying mathematical understanding will be enhanced through engagement with a variety of applications of mathematics to real-world situations and problems in other key learning areas.

Personal and social competence [PSC]

Students develop personal and social competence as they learn to understand and manage themselves, their relationships and their lives, more effectively. This includes establishing positive relationships, making responsible decisions, working effectively in teams and handling challenging situations constructively.

The elements of personal and social competence relevant to mathematics include the application of mathematical skills for personal purposes, such as giving and following directions, visualisation and mapping skills, interpreting timetables and calendars, calculating with money and the Goods and Services Tax (GST), budgeting, using price comparison websites, evaluating discount offers, investigating payment on terms and conducting statistical investigations in teams.

Sustainability and environment [SE]

Sustainability is concerned with the ongoing capacity of Earth to maintain all life. Sustainable patterns of living meet the needs of the present without compromising the ability of future generations to meet their needs. Education for sustainability develops the knowledge, skills and values necessary for people to act in ways that contribute to more sustainable patterns of living.

The Australian curriculum: Mathematics provides the foundation for the exploration of issues of sustainability. It equips students with the skills of measurement, mathematical modelling, and data collection, representation and analysis. These skills are needed to investigate data, evaluate and communicate findings and to make predictions based on those findings.

Mathematical understandings and skills are necessary to monitor and quantify both the impact of human activity on ecosystems and changes to conditions in the biosphere.

The NSW K–10 Mathematics curriculum provides students with knowledge, understanding and skills to observe, record, organise and analyse data, and to engage in investigations regarding sustainability. The curriculum supports students in early stages to build connections with the natural world and their local community. In later stages, students can use probability concepts, mathematical and computer modelling, chance and probability, multiple data sets and statistical analysis to understand more complex concepts relevant to sustainability.

Students can measure and evaluate sustainability changes over time and develop a deeper appreciation of the world around them through such aspects of mathematics as patterning, three-dimensional space, symmetry and tessellations.

Work and enterprise [WE]

Students develop work-related knowledge, understanding and skills through a variety of experiences. These experiences may include constructing budgets, calculating wage and salary earnings, investigating and determining leave loadings, using deductions and ‘pay as you go’ (PAYG) instalments to calculate a tax liability or refund, and investigating tax rebates and levies. Students perform calculations involving discounts, profit and loss; use statistics to predict future earnings; monitor inventories; and use information gained from surveys.