

BOARD OF STUDIES  
NEW SOUTH WALES

# 2001 HSC Specimen Paper

**Mathematics  
Extension 1**

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# Mathematics Extension 1

## **Introduction**

This booklet contains the specimen examination paper for the 2001 Higher School Certificate examination in Mathematics Extension 1. A mapping grid is also included, showing how each question in the examination relates to the syllabus outcomes and content, and to the performance bands.

The specimen paper shows the format of the New HSC examination. It has been printed on A4 paper and side-stapled to make it convenient for use in schools. Actual examination papers will be produced as A4 booklets. All New HSC papers will be printed on white paper.

The 2001 HSC specimen papers have been produced in accordance with the Board's *Principles for Setting HSC Examinations in a Standards-Referenced Framework*, published in Board Bulletin Volume 8 Number 9 (Nov/Dec 99). Questions are closely related to the outcomes of the course, and the paper as a whole is structured to allow for appropriate differentiation of student performance at all levels on the performance scale.

The papers have been designed so that students have a clear understanding of what they are required to do in each question and in working through the paper. Instructions have been standardised, and the demands of the questions have been made explicit. Key words in questions, such as 'discuss', 'analyse', and 'explain', have been used consistently in accordance with the glossary published in the Board's *Assessment Support Document*.

This specimen paper is an example of the type of examination that could be prepared within the current examination specifications. Examinations will be based on the syllabus, and will test a representative sample of syllabus outcomes. Therefore, the range and balance of outcomes tested in HSC examinations in 2001 and subsequent years may differ from those addressed in the specimen paper.

The mapping grid is an important feature of the development of the examination. It aids in ensuring that the examination as a whole samples a range of content and outcomes, and allows all students the opportunity to demonstrate their level of achievement. Where courses have components in the examination other than written papers, the grid indicates the wider range of outcomes that are assessed by including these other components.

In considering the Mathematics Extension 1 specimen paper, it should be noted that the paper is based on the 1999 Mathematics 3 Unit (Additional) and 3/4 Unit (Common) HSC Examination, with minor changes to ensure consistency with the 2001 HSC Examination format and the Board's *Principles for Setting HSC Examinations in a Standards-Referenced Framework*.

# Mathematics Extension 1

## HSC Specimen Examination Mapping Grid

For each item in the examination, the grid shows the marks allocated, the syllabus content and syllabus outcomes it relates to, and the bands it is targeting on the Draft Mathematics Extension 1 performance scale. The range of bands shown indicates the performance candidates may be able to demonstrate in their responses. That is, if an item is shown as targeting Bands E2 – E3, it indicates that candidates who demonstrate performance equivalent to the Band E2 descriptions should be able to score some marks on the item, while those who perform at Band E3 or above could reasonably be expected to gain high marks. In the case of one-mark items, candidates who demonstrate performance at or above the bands shown generally could be expected to answer the item correctly.

Question	Marks	Content	Syllabus outcomes	Targeted performance bands
1(a)	2	Inverse Functions and the Inverse Trigonometric Functions	HE4	E2 – E3
1(b)	2	The Tangent to a Curve and the Derivative of a Function; The Trigonometric Functions	P7, H5	E2 – E3
1(c)	2	Linear Functions and Lines	P4	E2 – E3
1(d)	1	Real Functions of a Real Variable and Their Geometrical Representation	P5	E2 – E3
1(e)	2	Polynomials	PE3	E2 – E3
1(f)	3	Integration	HE6	E2 – E3
2(a)	2	Permutations, Combinations and Further Probability	PE3	E2 – E3
2(b)	4	Trigonometric Ratios; The Trigonometric Functions; Real Functions of a Real Variable and Their Geometrical Representation	H5, PE2	E2 – E4
2(c)(i)	1	Logarithmic and Exponential Functions	P5, H9	E2 – E3
2(c)(ii)	2	Geometrical Applications of Differentiation; Logarithmic and Exponential Functions	P6, H2, H6	E2 – E3
2(c)(iii)	1	Real Functions of a Real Variable and Their Geometrical Representation; Logarithmic and Exponential Functions	P5, H2	E2 – E3
2(c)(iv)	2	Polynomials; Logarithmic and Exponential Functions	PE3	E2 – E3
3(a)	4	Integration; The Trigonometric Functions	H8	E2 – E3
3(b)	2	Permutations, Combinations and Further Probability	HE3	E3 – E4
3(c)	2	Plane Geometry	PE2, PE3	E3 – E4
3(d)(i)	2	Trigonometric Ratios; Basic Arithmetic and Algebra	P4	E2 – E3
3(d)(ii)	2	Integration; The Trigonometric Functions; Logarithmic and Exponential Functions	HE6	E3 – E4
4(a)	1	Series and Applications	H5, H9	E2 – E3
4(b)(i)	2	The Quadratic Polynomial and the Parabola	PE3, PE4	E2 – E3
4(b)(ii)	2	The Quadratic Polynomial and the Parabola	PE3	E2 – E3
4(b)(iii)	2	The Quadratic Polynomial and the Parabola	PE3	E3 – E4
4(c)(i)	1	Logarithmic and Exponential Functions	H8	E2 – E3
4(c)(ii)	2	Integration	H8	E2 – E3
4(c)(iii)	2	Logarithmic and Exponential Functions	H3	E3 – E4

5(a)	3	Series and Applications	HE2	E3 – E4
5(b)(i)	3	Logarithmic and Exponential Functions; Geometrical Applications of Differentiation	P7, H6	E2 – E4
5(b)(ii)	1	Logarithmic and Exponential Functions; Geometrical Applications of Differentiation	H2	E3 – E4
5(b)(iii)	2	Logarithmic and Exponential Functions; Geometrical Applications of Differentiation	P5, H9	E2 – E3
5(b)(iv)	1	Logarithmic and Exponential Functions; Inverse Functions and the Inverse Trigonometric Functions	HE4	E2 – E3
5(b)(v)	1	Logarithmic and Exponential Functions; Inverse Functions and the Inverse Trigonometric Functions	P5, HE4	E2 – E3
5(b)(vi)	1	Inverse Functions and the Inverse Trigonometric Functions; Real Functions of a Real Variable and Their Geometrical Representation	P5, H2, HE4	E3 – E4
6(a)(i)	1	Applications of Calculus to the Physical World	H5, HE3	E3 – E4
6(a)(ii)	1	Applications of Calculus to the Physical World	HE3	E3 – E4
6(a)(iii)	2	Applications of Calculus to the Physical World	H5, HE3	E3 – E4
6(a)(iv)	1	Applications of Calculus to the Physical World	HE3	E3 – E4
6(a)(v)	1	Applications of Calculus to the Physical World	HE3	E3 – E4
6(b)(i)	2	Plane Geometry	P4, PE3	E2 – E3
6(b)(ii)	1	The Trigonometric Functions	H5	E3 – E4
6(b)(iii)	3	Trigonometric Ratios; The Trigonometric Functions	P4, H5	E3 – E4
7(a)(i)	4	Applications of Calculus to the Physical World	HE3, HE7	E2 – E3
7(a)(ii)	2	Applications of Calculus to the Physical World	HE3	E3 – E4
7(a)(iii)	2	Applications of Calculus to the Physical World	HE3	E3 – E4
7(b)	4	Binomial Theorem	PE3, PE6, HE7	E3 – E4



## Sample marking guidelines for Mathematics Extension 1

The following marking guidelines have been developed for selected questions from the 2001 HSC Specimen Examination in Mathematics Extension 1. These guidelines indicate the approach that would be taken to marking questions.

For each question, the following are typically included:

1. The syllabus outcomes that are targeted by the question.
2. The assessment rubric from the specimen paper, where there is one, listing the set of general criteria that are used to assess responses.
3. The marking guidelines, which show the criteria to be applied to responses along with the marks to be awarded in line with the quality of the responses. For extended-response questions, performance is described at a number of levels of performance, each covering a range of marks.
4. A sample answer or some points that answers might include. Sample answers indicate the scope and depth of treatment expected, and are not intended to be prescriptive. Similarly, the points that could be included in answers are not intended to be an exhaustive list, but rather an indication of the considerations that students could include in their responses.

Marking guidelines will generally require some refinement at the Marking Centre to take account of unanticipated responses that students present. For essay-type questions, the standard described at each mark range will be made clear during pilot-marking by the selection of sample scripts.

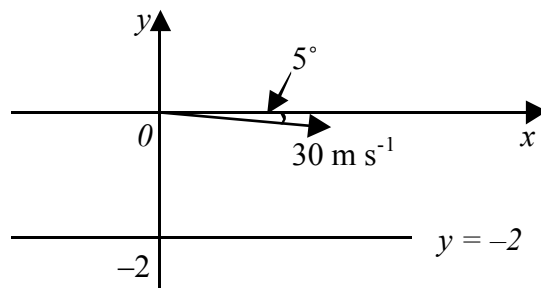
In a standards-referenced framework, examination questions are closely linked to syllabus content and outcomes. Expectations of the question are to be clear in the wording of the question. Marking guidelines will be developed at the same time as the examination questions, by examination committees. The development of marking guidelines will be guided by the Board's *Principles for Developing Marking Guidelines in a Standards-Referenced Framework*, published in Board Bulletin Volume 9 Number 3 (May 2000).

## Sample Marking Guidelines – Mathematics Extension 1

Question 7 (12 marks)

Marks

(a)



A cricket ball leaves the bowler's hand 2 metres above the ground with a velocity of  $30 \text{ m s}^{-1}$  at an angle of  $5^\circ$  below the horizontal. The equations of motion for the ball are

$$\ddot{x} = 0 \quad \text{and} \quad \ddot{y} = -10$$

Take the origin to be the point where the ball leaves the bowler's hand.

- (i) Using calculus, prove that the coordinates of the ball at time  $t$  are given by 4

$$x = 30t \cos(5^\circ), \text{ and}$$
$$y = -30t \sin(5^\circ) - 5t^2.$$

**Outcomes assessed: HE3, HE7**

### MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"><li>• One mark for integration of <math>\ddot{x} = 0</math> and substitution of initial conditions to obtain <math>\dot{x} = 30 \cos 5^\circ</math></li><li>• One mark for correct integration of their <math>\dot{x}</math>, including the initial conditions</li><li>• One mark for integration of <math>\ddot{y} = -10</math> and substitution of initial conditions to obtain <math>\dot{y} = -10t - 30 \sin 5^\circ</math></li><li>• One mark for correct integration of their <math>\dot{y}</math>, including the initial conditions</li></ul>	4

Sample answer:

When  $t = 0$ ,  $x = 0$ ,  $y = 0$ ,  $\dot{x} = 30 \cos 5^\circ$ ,  $\dot{y} = -30 \sin 5^\circ$

$$\ddot{x} = 0$$

$$\dot{x} = C \text{ (constant)}$$

$$\dot{x} = 30 \cos 5^\circ$$

$$x = \int 30 \cos 5^\circ dt$$

$$x = 30t \cos 5^\circ + C' \text{ (constant)}$$

When  $t = 0$ ,  $x = 0$ ,  $\therefore C' = 0$

$$\therefore x = 30t \cos (5^\circ)$$

$$\ddot{y} = -10$$

$$\dot{y} = -10t + K \text{ (constant)}$$

When  $t = 0$ ,  $\dot{y} = -30 \sin 5^\circ$

$$\therefore \dot{y} = -10t - 30 \sin 5^\circ$$

$$y = \int (-10t - 30 \sin 5^\circ) dt$$

$$= -5t^2 - 30t \sin 5^\circ + K' \text{ (constant)}$$

When  $t = 0$ ,  $y = 0$   $\therefore K' = 0$

$$\therefore y = -30t \sin (5^\circ) - 5t^2$$

**Marks**

(ii) Find the time at which the ball strikes the ground.

**2****Outcome assessed: HE3****MARKING GUIDELINES**

Criteria	Marks
<ul style="list-style-type: none"> <li>Substitutes <math>y = -2</math> into their equation for <math>y</math> from part (i)</li> <li>Correctly calculates <math>t</math> from their equation</li> </ul>	2
<ul style="list-style-type: none"> <li>Substitutes <math>y = -2</math> into their equation for <math>y</math> from part (i)</li> </ul>	1

Sample answer:

$$-2 = -5t^2 - 30t \sin 5^\circ$$

$$5t^2 + 30t \sin 5^\circ - 2 = 0$$

$$t = \frac{-30 \sin 5^\circ \pm \sqrt{(30 \sin 5^\circ)^2 - 4(5)(-2)}}{10}$$

$$= 0.4229... \text{ seconds (negative answer discarded)}$$

$\therefore$  Ball strikes ground after 0.42 seconds (correct to 2 decimal places).

(iii) Calculate the angle at which the ball strikes the ground.

**2****Outcome assessed: HE3****MARKING GUIDELINES**

Criteria	Marks
<ul style="list-style-type: none"> <li>Calculates the correct angle at which the ball strikes the ground from their answers to parts (i) and (ii)</li> </ul>	2
<ul style="list-style-type: none"> <li>Writes an expression for <math>\tan \theta</math> or <math>\frac{dy}{dx}</math> in terms of <math>\dot{y}</math> and <math>\dot{x}</math>, such as <math>\tan \theta = \frac{\dot{y}}{\dot{x}}</math></li> </ul>	1

Sample answer:

$$\dot{x} = 30 \cos 5^\circ$$

$$\dot{y} = -30 \sin 5^\circ - 10t$$

$$\tan \theta = \frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{-30 \sin 5^\circ - 10t}{30 \cos 5^\circ}$$

Substitute  $t = 0.4229\dots$  from part (ii)

$$\tan \theta = -0.228\ 99\dots$$

$$\theta \approx 167^\circ 6' \text{ (with positive direction of } x \text{ axis)}$$

(Or  $\theta \approx 180^\circ - 167^\circ 6' = 12^\circ 54'$  if considering acute angle.)

(b) By considering  $(1-x)^n \left(1 + \frac{1}{x}\right)^n$ , or otherwise, express

4

$$\binom{n}{2}\binom{n}{0} - \binom{n}{3}\binom{n}{1} + \dots + (-1)^n \binom{n}{n}\binom{n}{n-2}$$

in simplest form.

**Outcomes assessed: PE3, PE6, HE7**

**MARKING GUIDELINES**

Criteria	Marks
<ul style="list-style-type: none"> <li>One mark for the correct binomial expansion of <math>(1-x)^n</math> or <math>\left(1 + \frac{1}{x}\right)^n</math> or <math>\left(\frac{1}{x} - x\right)^n</math> or <math>(1-x^2)^n</math></li> <li>One mark for recognising that the coefficient of <math>x^2</math> is given by <math>\binom{n}{2}\binom{n}{0} - \binom{n}{3}\binom{n}{1} + \dots + (-1)^n \binom{n}{n}\binom{n}{n-2}</math></li> <li>One mark for <math>2r - n = 2</math> or equivalent (see Sample answer)</li> <li>One mark for the correct coefficient expression</li> </ul>	4

Sample answer:

$$\begin{aligned}(1-x)^n &= \binom{n}{0}1^n(-x)^0 + \binom{n}{1}1^{n-1}(-x)^1 + \binom{n}{2}1^{n-2}(-x)^2 + \dots + \binom{n}{n}1^0(-x)^n \\ &= \binom{n}{0} - \binom{n}{1}x + \binom{n}{2}x^2 - \dots + (-1)^n \binom{n}{n}x^n\end{aligned}$$

$$\text{and } \left(1 + \frac{1}{x}\right)^n = \binom{n}{0}\left(\frac{1}{x}\right)^0 + \binom{n}{1}\left(\frac{1}{x}\right)^1 + \dots + \binom{n}{n}\left(\frac{1}{x}\right)^n$$

So the term in  $x^2$  in  $(1-x)^n\left(1 + \frac{1}{x}\right)^n$  is given by

$$\binom{n}{2}\binom{n}{0}x^2\left(\frac{1}{x}\right)^0 - \binom{n}{3}\binom{n}{1}x^3\left(\frac{1}{x}\right)^1 + \binom{n}{4}\binom{n}{2}x^4\left(\frac{1}{x}\right)^2 - \dots + (-1)^n \binom{n}{n}\binom{n}{n-2}x^n\left(\frac{1}{x}\right)^{n-2}$$

ie the coefficient of  $x^2$  in  $(1-x)^n\left(1 + \frac{1}{x}\right)^n$  is

$$\binom{n}{2}\binom{n}{0} - \binom{n}{3}\binom{n}{1} + \dots + (-1)^n \binom{n}{n}\binom{n}{n-2}$$

$$\begin{aligned}\text{Now } (1-x)^n\left(1 + \frac{1}{x}\right)^n &= \left[(1-x)\left(1 + \frac{1}{x}\right)\right]^n \\ &= \left(\frac{1}{x} - x\right)^n\end{aligned}$$

$$\begin{aligned}\text{The general term in } \left(\frac{1}{x} - x\right)^n &\text{ is } \binom{n}{r}\left(\frac{1}{x}\right)^{n-r}(-x)^r = \binom{n}{r}(-1)^r x^{r-n}x^r \\ &= \binom{n}{r}(-1)^r x^{2r-n}\end{aligned}$$

The term in  $x^2$  occurs when  $2r - n = 2$

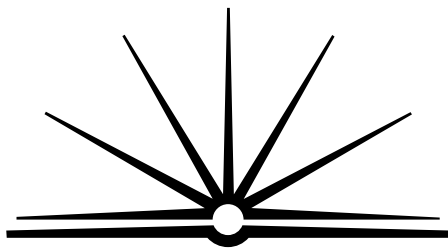
$$\text{ie } r = \frac{n+2}{2}$$

So if  $n$  is odd there is no term involving  $x^2$  (ie the coefficient of  $x^2$  is zero). If  $n$  is even, the

$$\text{coefficient of } x^2 \text{ is } \binom{n}{\frac{n+2}{2}}(-1)^{\frac{n+2}{2}}.$$

$$\therefore \binom{n}{2} \binom{n}{0} - \binom{n}{3} \binom{n}{1} + \dots + (-1)^n \binom{n}{n} \binom{n}{n-2} = \binom{n}{\frac{n+2}{2}} (-1)^{\frac{n+2}{2}} \text{ when } n \text{ is even}$$

= 0 when  $n$  is odd.



**B O A R D O F S T U D I E S**  
N E W S O U T H W A L E S

**2001**  
**HIGHER SCHOOL CERTIFICATE**  
**SPECIMEN EXAMINATION**

# Mathematics

## Extension 1

### **General Instructions**

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on page 10
- All necessary working should be shown in every question

Total marks **(84)**

- Attempt Questions 1 – 7
- All questions are of equal value

**Total marks (84)**

**Attempt Questions 1 – 7**

**All questions are of equal value**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

---

	<b>Marks</b>
<b>Question 1</b> (12 marks) Use a SEPARATE writing booklet.	
(a) Evaluate $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}$ .	<b>2</b>
(b) Differentiate $\sin^3 x$ .	<b>2</b>
(c) The interval $AB$ has end points $A(-2, 7)$ and $B(8, -8)$ .  Find the coordinates of the point $P$ which divides the interval $AB$ internally in the ratio $2 : 3$ .	<b>2</b>
(d) Write down the equation of the vertical asymptote of $y = \frac{4x}{(x-3)}$ .	<b>1</b>
(e) Find the remainder when the polynomial $P(x) = x^3 - 4x$ is divided by $x + 3$ .	<b>2</b>
(f) Use the substitution $u = \tan x$ to evaluate $\int_0^{\frac{\pi}{3}} \tan^2 x \sec^2 x \, dx$ .	<b>3</b>

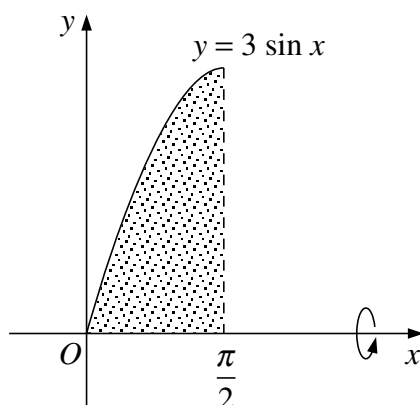
**Question 2** (12 marks) Use a SEPARATE writing booklet.

- (a) The staff in an office consists of 4 males and 7 females. **2**  
How many committees of 5 staff can be chosen which contain exactly 3 females?
- (b) Find all values of  $\theta$  in the range  $0 \leq \theta \leq 2\pi$  for which  $\cos \theta + \sqrt{3} \sin \theta = 1$ . **4**
- (c) Let  $f(x) = x + \log_e x$ .
- (i) Write down the natural domain for  $f(x)$ . **1**
- (ii) Show that, for all values of  $x$  in the natural domain,  $y = f(x)$  is increasing. **2**
- (iii) Show that the curve  $y = f(x)$  cuts the  $x$  axis between  $x = 0.5$  and  $x = 1$ . **1**
- (iv) Use Newton's method with a first approximation of  $x = 0.5$  to find a second approximation to the root of  $x + \log_e x = 0$ . **2**

**Please turn over**

**Question 3** (12 marks) Use a SEPARATE writing booklet.

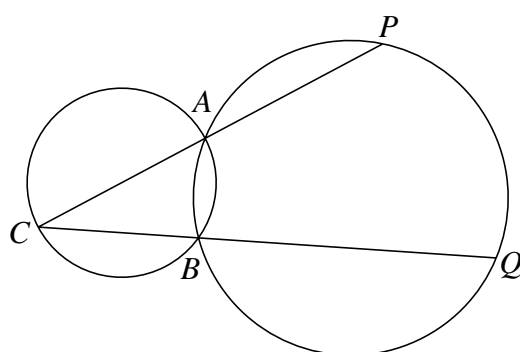
(a) 4



The shaded region bounded by  $y = 3 \sin x$ , the  $x$  axis and the line  $x = \frac{\pi}{2}$  is rotated about the  $x$  axis to form a solid. Calculate the volume of the solid.

(b) A fair, six-sided die is thrown seven times. What is the probability that a '6' occurs on exactly 2 of the 7 throws? 2

(c) 2



Two circles intersect at two points  $A$  and  $B$  as shown in the diagram. The diameter of one circle is  $CA$  and this line intersects the other circle at  $A$  and  $P$ . The line  $CB$  intersects the second circle at  $B$  and  $Q$ .

Copy or trace the diagram into your writing booklet.

Prove that  $\angle CPQ$  is a right angle.

**Question 3 continues on page 5**

## Question 3 (continued)

- (d) (i) By equating the coefficients of  $\sin x$  and  $\cos x$ , or otherwise, find constants  $A$  and  $B$  satisfying the identity 2

$$A(2\sin x + \cos x) + B(2\cos x - \sin x) \equiv \sin x + 8\cos x.$$

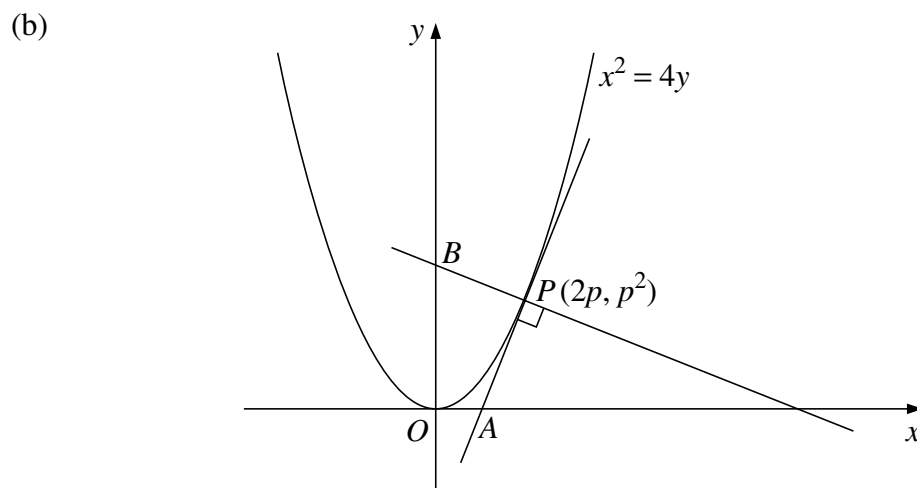
- (ii) Hence evaluate  $\int \frac{\sin x + 8\cos x}{2\sin x + \cos x} dx$  . 2

**End of Question 3**

**Please turn over**

**Question 4** (12 marks) Use a SEPARATE writing booklet.

- (a) Evaluate  $\sum_{k=2}^5 (-1)^k k$ . 1



The diagram shows the graph of the parabola  $x^2 = 4y$ . The tangent to the parabola at  $P(2p, p^2)$ ,  $p > 0$ , cuts the  $x$  axis at  $A$ . The normal to the parabola at  $P$  cuts the  $y$  axis at  $B$ .

- (i) Derive the equation of the tangent  $AP$ . 2
- (ii) Show that  $B$  has coordinates  $(0, p^2 + 2)$ . 2
- (iii) Let  $C$  be the midpoint of  $AB$ . Find the cartesian equation of the locus of  $C$ . 2
- (c) (i) Evaluate  $\int_1^2 \frac{dx}{x}$ . 1
- (ii) Use Simpson's rule with 3 function values to approximate  $\int_1^2 \frac{dx}{x}$ . 2
- (iii) Use your results to parts (i) and (ii) to obtain an approximation for  $e$ . 2  
Give your answer correct to 3 decimal places.

**Question 5** (12 marks) Use a SEPARATE writing booklet.

- (a) Prove by induction that, for all integers  $n \geq 1$ , **3**

$$(n+1)(n+2) \cdots (2n-1)2n = 2^n [1 \times 3 \times \cdots \times (2n-1)].$$

- (b) Consider the function  $f(x) = e^x - 1 - x$ .

- (i) Show that the minimum of  $f(x)$  occurs at  $x = 0$ . **3**
- (ii) Deduce that  $f(x) \geq 0$  for all  $x$ . **1**
- (iii) On the same set of axes, sketch  $y = e^x - 1$  and  $y = x$ . **2**
- (iv) Find the inverse function of  $g(x) = e^x - 1$ . **1**
- (v) State the domain of  $g^{-1}(x)$ . **1**
- (vi) For what values of  $x$  is  $\log_e(1+x) \leq x$ ? Justify your answer. **1**

**Please turn over**

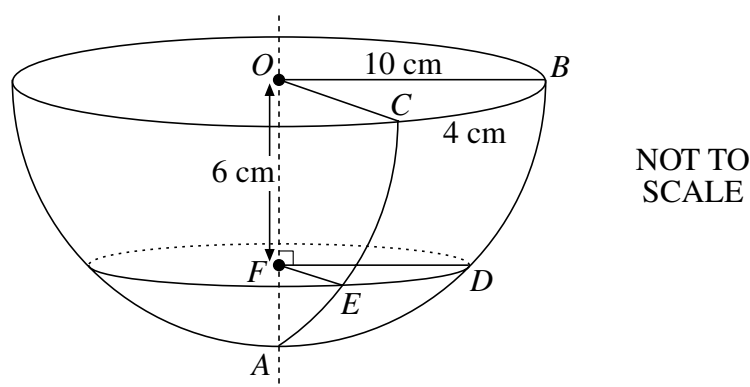
**Question 6** (12 marks) Use a SEPARATE writing booklet.

- (a) A particle moves in a straight line and its displacement  $x$  metres from the origin after  $t$  seconds is given by

$$x = \cos^2 3t, \quad t > 0.$$

- (i) When is the particle first at  $x = \frac{3}{4}$ ? 1
- (ii) In what direction is the particle travelling when it is first at  $x = \frac{3}{4}$ ? 1
- (iii) Express the acceleration of the particle in terms of  $x$ . 2
- (iv) Hence, or otherwise, show that the particle is undergoing simple harmonic motion. 1
- (v) State the period of the motion. 1

- (b)



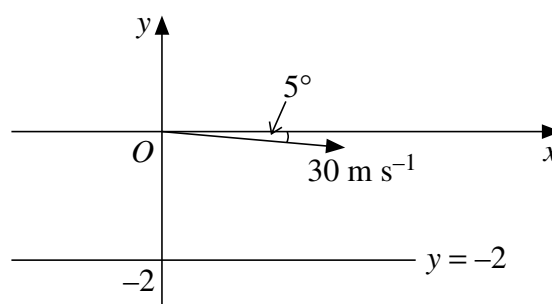
The diagram shows a hanging basket in the shape of a hemisphere with radius 10 cm. Let  $O$  be the centre of the sphere and let  $OA$  be the central axis. Two vertical wire supports,  $AB$  and  $AC$ , are shown on the diagram. The length of the arc  $BC$  is 4 cm.

A horizontal wire support is placed around the surface of the basket. This wire meets  $AB$  at  $D$  and  $AC$  at  $E$ . The plane through  $DE$  parallel to the plane  $OBC$  cuts  $OA$  at  $F$ . The length  $OF$  is 6 cm. Note that  $\angle BOC = \angle DFE$ .

- (i) Show that the length of  $FD$  is 8 cm. 2
- (ii) Find  $\angle DFE$  in radians. 1
- (iii) Find the size of  $\angle DOE$  in radians, correct to 3 decimal places. 3

**Question 7** (12 marks) Use a SEPARATE writing booklet.

(a)



A cricket ball leaves the bowler's hand 2 metres above the ground with a velocity of  $30 \text{ m s}^{-1}$  at an angle of  $5^\circ$  below the horizontal. The equations of motion for the ball are

$$\ddot{x} = 0 \quad \text{and} \quad \ddot{y} = -10.$$

Take the origin to be the point where the ball leaves the bowler's hand.

- (i) Using calculus, prove that the coordinates of the ball at time  $t$  are given by 4

$$x = 30t \cos(5^\circ), \text{ and}$$

$$y = -30t \sin(5^\circ) - 5t^2.$$

- (ii) Find the time at which the ball strikes the ground. 2
- (iii) Calculate the angle at which the ball strikes the ground. 2

- (b) By considering  $(1-x)^n \left(1 + \frac{1}{x}\right)^n$ , or otherwise, express 4

$$\binom{n}{2} \binom{n}{0} - \binom{n}{3} \binom{n}{1} + \dots + (-1)^n \binom{n}{n} \binom{n}{n-2}$$

in simplest form.

**End of paper**

## STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left( x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left( x + \sqrt{x^2 + a^2} \right)$$

NOTE :  $\ln x = \log_e x, \quad x > 0$