

# REFERENCE SHEET

– Mathematics –

– Mathematics Extension 1 –

– Mathematics Extension 2 –

# Mathematics

## Factorisation

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

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## Angle sum of a polygon

$$S = (n - 2) \times 180^\circ$$

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## Equation of a circle

$$(x - h)^2 + (y - k)^2 = r^2$$

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## Trigonometric ratios and identities

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

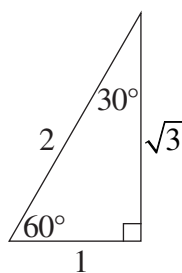
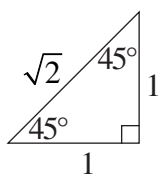
$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

## Exact ratios



## Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

## Cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

## Area of a triangle

$$\text{Area} = \frac{1}{2} ab \sin C$$

## Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

## Perpendicular distance of a point from a line

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

## Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

## Point-gradient form of the equation of a line

$$y - y_1 = m(x - x_1)$$

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## $n$ th term of an arithmetic series

$$T_n = a + (n - 1)d$$

## Sum to $n$ terms of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n - 1)d] \quad \text{or} \quad S_n = \frac{n}{2}(a + l)$$

## $n$ th term of a geometric series

$$T_n = ar^{n-1}$$

## Sum to $n$ terms of a geometric series

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

## Limiting sum of a geometric series

$$S = \frac{a}{1 - r}$$

## Compound interest

$$A_n = P \left( 1 + \frac{r}{100} \right)^n$$

# Mathematics (continued)

## Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

## Derivatives

$$\text{If } y = x^n, \text{ then } \frac{dy}{dx} = nx^{n-1}$$

$$\text{If } y = uv, \text{ then } \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\text{If } y = \frac{u}{v}, \text{ then } \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\text{If } y = F(u), \text{ then } \frac{dy}{dx} = F'(u) \frac{du}{dx}$$

$$\text{If } y = e^{f(x)}, \text{ then } \frac{dy}{dx} = f'(x)e^{f(x)}$$

$$\text{If } y = \log_e f(x) = \ln f(x), \text{ then } \frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$\text{If } y = \sin f(x), \text{ then } \frac{dy}{dx} = f'(x) \cos f(x)$$

$$\text{If } y = \cos f(x), \text{ then } \frac{dy}{dx} = -f'(x) \sin f(x)$$

$$\text{If } y = \tan f(x), \text{ then } \frac{dy}{dx} = f'(x) \sec^2 f(x)$$

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## Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## Sum and product of roots of a quadratic equation

$$\alpha + \beta = -\frac{b}{a} \quad \alpha\beta = \frac{c}{a}$$

## Equation of a parabola

$$(x-h)^2 = \pm 4a(y-k)$$

## Integrals

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$$

$$\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + C$$

$$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + C$$

$$\int \sec^2(ax+b) dx = \frac{1}{a} \tan(ax+b) + C$$

## Trapezoidal rule (one application)

$$\int_a^b f(x) dx \approx \frac{b-a}{2} [f(a) + f(b)]$$

## Simpson's rule (one application)

$$\int_a^b f(x) dx \approx \frac{b-a}{6} \left[ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

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## Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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## Angle measure

$$180^\circ = \pi \text{ radians}$$

## Length of an arc

$$l = r\theta$$

## Area of a sector

$$\text{Area} = \frac{1}{2} r^2 \theta$$

# Mathematics Extension 1

## Angle sum identities

$$\sin(\theta + \phi) = \sin\theta \cos\phi + \cos\theta \sin\phi$$

$$\cos(\theta + \phi) = \cos\theta \cos\phi - \sin\theta \sin\phi$$

$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta \tan\phi}$$

## t formulae

If  $t = \tan \frac{\theta}{2}$ , then

$$\sin\theta = \frac{2t}{1+t^2}$$

$$\cos\theta = \frac{1-t^2}{1+t^2}$$

$$\tan\theta = \frac{2t}{1-t^2}$$

## General solution of trigonometric equations

$$\sin\theta = a, \quad \theta = n\pi + (-1)^n \sin^{-1} a$$

$$\cos\theta = a, \quad \theta = 2n\pi \pm \cos^{-1} a$$

$$\tan\theta = a, \quad \theta = n\pi + \tan^{-1} a$$

## Division of an interval in a given ratio

$$\left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

## Parametric representation of a parabola

For  $x^2 = 4ay$ ,

$$x = 2at, \quad y = at^2$$

At  $(2at, at^2)$ ,

$$\text{tangent: } y = tx - at^2$$

$$\text{normal: } x + ty = at^3 + 2at$$

At  $(x_1, y_1)$ ,

$$\text{tangent: } xx_1 = 2a(y + y_1)$$

$$\text{normal: } y - y_1 = -\frac{2a}{x_1}(x - x_1)$$

Chord of contact from  $(x_0, y_0)$ :  $xx_0 = 2a(y + y_0)$

## Acceleration

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left( \frac{1}{2}v^2 \right)$$

## Simple harmonic motion

$$x = b + a \cos(nt + \alpha)$$

$$\ddot{x} = -n^2(x - b)$$

## Further integrals

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

## Sum and product of roots of a cubic equation

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

## Estimation of roots of a polynomial equation

Newton's method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

## Binomial theorem

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k} = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$