

Sample HSC Examination Questions for the Draft Stage 6 Mathematics Courses

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Introduction

This publication contains sample HSC examination questions for the draft Stage 6 Mathematics courses. A mapping grid is also included, showing how each question relates to the syllabus outcomes and content. The purpose of these sample questions is to show the type of questions that could be asked in HSC examinations, based on the examination specifications in the draft syllabuses.

The examination specifications state that course objectives and HSC course outcomes will be the focus of HSC examination questions. The sample questions, accompanied by their mapping to outcomes and content, illustrate the statement in the examination specifications that ‘questions focusing on HSC course outcomes may relate to knowledge, skills and understanding from the Preliminary course’. Questions targeting HSC course outcomes need not be based only on content from the HSC course. That is, questions could be based on content from both the HSC and Preliminary courses and, where applicable, on content from any other Mathematics course that was assumed knowledge for the course.

Good assessment practice recommends a variety of item types in any examination. For this reason, the examination specifications in all the Mathematics courses include objective-response items. The objective-response formats currently used in the Board’s Higher School Certificate examinations and School Certificate tests include multiple-choice, multiple correct/incorrect and constrained response questions. The sample questions provide examples of items in all of these formats for each course, to illustrate how such items could appear in the HSC examinations.

Multiple correct/incorrect questions can be used in Mathematics in situations where there may be more than one approach and/or one correct answer to a mathematical problem or situation. In these items, it is possible to award part marks. The multiple correct/incorrect question format used directs students to indicate whether each alternative is correct or incorrect. Note that, in the HSC examinations, multiple correct/incorrect questions would appear in the question booklet and students would write their answers on the objective-response answer sheet, as is currently the case for School Certificate Mathematics tests.

Constrained response questions are similar in appearance to short free-response questions. Students write their answers to constrained-response questions in particular answer spaces on an objective-response answer sheet. Typically these answer spaces are a series of boxes for each letter, numeral or symbol. Note that while these answer spaces have been placed in this document to illustrate the item type and answer spaces, in the HSC examinations, constrained response questions would appear in the question booklet and students would write their answers in the answer spaces provided on the objective-response answer sheet, as is currently the case for School Certificate Science tests.

Specimen HSC examinations for each of the Mathematics General 1, Mathematics General 2, Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 courses will be published after the final Stage 6 Mathematics syllabuses are produced.

Mathematics General 1

Objective-response questions

Marks

Multiple choice

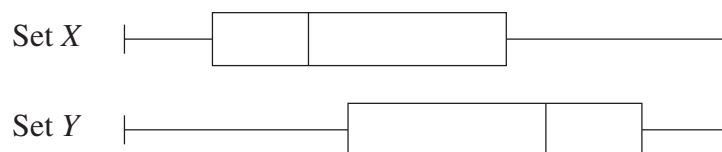
- 1** Paul plays a game in which he tosses two coins. He gains \$5 if two heads appear and \$1 if a head and a tail appear, but loses \$6 if two tails appear. **1**

What is his financial expectation?

- (A) -\$1.00
- (B) \$0.00
- (C) \$0.25
- (D) \$1.00

Multiple correct/incorrect

- 2** Consider the box-and-whisker plots for data sets X and Y . **1**

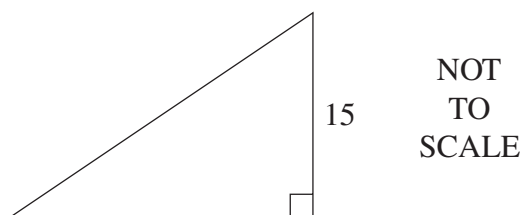


Indicate whether each of the following is Correct or Incorrect.

- a. Each set has the same range.
- b. Each set has the same median.
- c. Each set has the same interquartile range.
- d. Each set has a skewed distribution.

Marks
1

- 3 Consider the right-angled triangle.



Indicate whether each of the following is Correct or Incorrect.

A Pythagorean triad which could represent the side lengths of this triangle is

- a. 9 12 15
 - b. 10 15 20
 - c. 15 20 25
 - d. 15 36 39
- 4 A survey of 2925 people was conducted. Some of the results are displayed in the two-way table. **2**

	Men	Women	Totals
Smokers	1053	702	1755
Non-smokers	936	234	1170
Totals	1989	936	2925

Indicate whether each of the following is Correct or Incorrect.

- a. 25% of the women surveyed were smokers.
- b. 32% of the people surveyed were women.
- c. 40% of the smokers surveyed were women.
- d. 60% of the people surveyed were smokers.

Marks

Constrained response

- 5** It is estimated that, after age 30, the height of a person decreases at the rate of 0.06 cm per year. **1**

My 85-year-old grandmother is 154.2 cm tall.

Use the information given to determine an estimate of her height at age 30. Give your answer in centimetres to one decimal place.

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- 6** The annual interest rate on a credit card is 20.74%. Express the equivalent daily interest rate as a percentage, correct to four decimal places. **1**

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- 7** The formula for the size of the interior angle (θ) of a regular polygon with n sides is **1**

$$\theta = 180\left(1 - \frac{2}{n}\right).$$

Calculate the size of the interior angle of a regular polygon with 20 sides.

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Free-response questions

- 8 Three athletes are competing for Olympic selection in the long jump. The lengths (in metres) of their six trial jumps are shown in the table.

	Fabrice	John	Timothy
	7.25	7.80	7.67
	7.99	7.88	7.78
	8.28	7.61	7.92
	6.95	7.97	7.62
	7.66	7.75	7.85
	7.81	7.59	7.73
<i>Mean</i>	7.66	7.77	7.76
<i>Standard deviation</i>	0.49	0.15	0.11
<i>Longest jump</i>	8.28	7.97	7.92

- (a) Which athlete has the greatest mean length over the six trial jumps? **1**
- (b) Which athlete was the most consistent over the six trial jumps? Give reasons for your answer. **1**
- (c) Olympic long jump competitions are decided by the longest jump recorded, with each competitor having three jumps. **1**

Which one of the three athletes should be selected? Give reasons for your answer.

- 9 A media article reported that a new fuel efficiency measure would cost \$1500 per vehicle and would improve fuel efficiency for a typical family car by 1 litre per 100 kilometres (eg from 8 litres per 100 kilometres to 7 litres per 100 kilometres). **3**

The article claimed: “Do the mathematics: It would take a typical driver 10 years before saving enough in petrol costs to pay for the initial fuel conversion measure!”

Evaluate this claim. Assume that a typical family car is driven for 20 000 kilometres per year, and fuel costs \$1.50 per litre. Justify your answer using appropriate mathematical calculations.

Mathematics General 2

Objective-response questions

Marks

Multiple choice

- 1** Paul plays a game in which he tosses two coins. He gains \$5 if two heads appear and \$1 if a head and a tail appear, but loses \$6 if two tails appear. **1**

What is his financial expectation?

- (A) -\$1.00
- (B) \$0.00
- (C) \$0.25
- (D) \$1.00

- 2** Washing removes 20% of a deep stain at each wash. How many washes are needed to reduce a stain to approximately 40% of its original amount? **1**

- (A) 2
- (B) 3
- (C) 4
- (D) 5

Multiple correct/incorrect

- 3** Sydney has approximate location (34° S, 151° E) and Perth (32° S, 116° E). **2**

Indicate whether each of the following is Correct or Incorrect. Ignore time zones and daylight saving.

- a. Perth is further south than Sydney.
- b. The sun rises in Sydney before it rises in Perth.
- c. The approximate local time difference between Sydney and Perth is $2\frac{1}{3}$ hours.
- d. The distance from Sydney to Perth is given by $d = \frac{(151 - 116)}{360} \times 2\pi(6400)$.

Marks

Constrained response

4 Use the table of x and y values to answer this question.

1

x	0	1	2	3
y	5	8	11	14

Write down an equation involving x and y that is satisfied by the values shown.

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Free-response questions

- 5 A student creates a spreadsheet comparing the rainfall of Sydney and Melbourne. (Figures are in millimetres.)

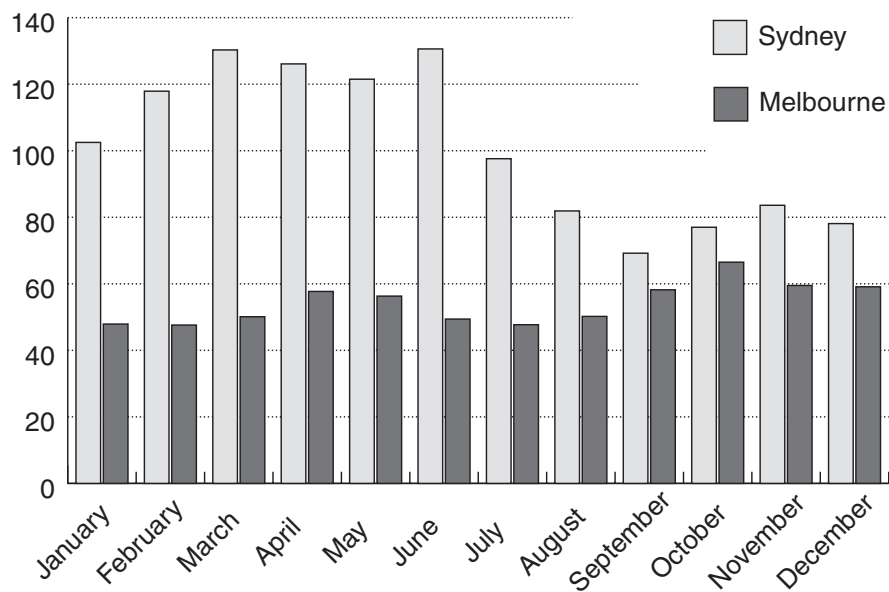
	A	B	C
1		Sydney Rainfall	Melbourne Rainfall
2	January	102.5	47.9
3	February	117.9	47.6
4	March	130.3	50.1
5	April	126.1	57.7
6	May	121.5	56.3
7	June	130.6	49.4
8	July	97.6	47.7
9	August	81.9	50.2
10	September	69.2	58.2
11	October	77.0	66.5
12	November	83.6	59.5
13	December	78.1	59.1
14	Total	1216.3	650.2
15	Mean (Jan-Dec)	101.4	54.2
16	Standard deviation	23.1	6.2

- (a) What does the value in cell B14 indicate? 1
- (b) What calculation is needed to obtain the value in cell B15?
(You may answer in sentence form, or by using a mathematical equation or a spreadsheet formula.) 1

Question 5 continues on page 8

Question 5 (continued)

- (c) The information in the spreadsheet was displayed in a column graph, as shown. 2



Using the information given in the spreadsheet and column graph, describe the main differences between the rainfall patterns of Sydney and Melbourne.

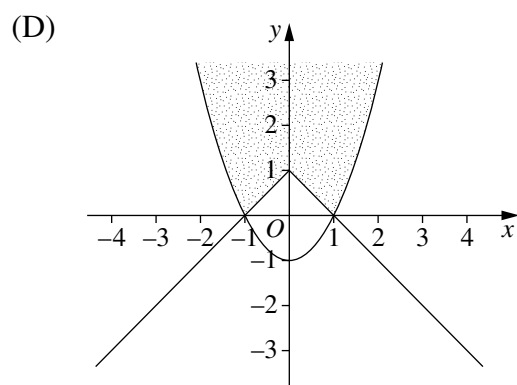
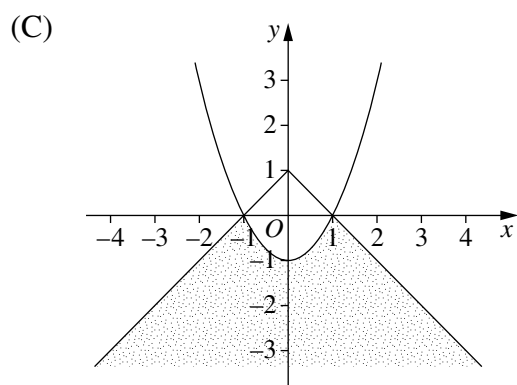
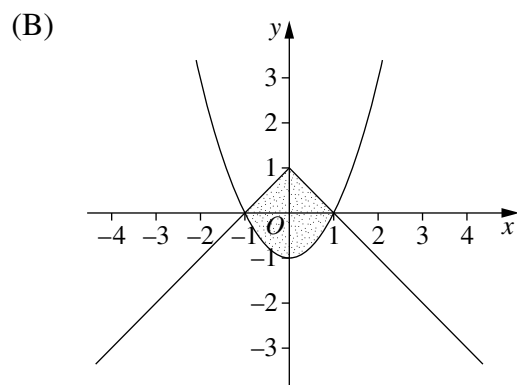
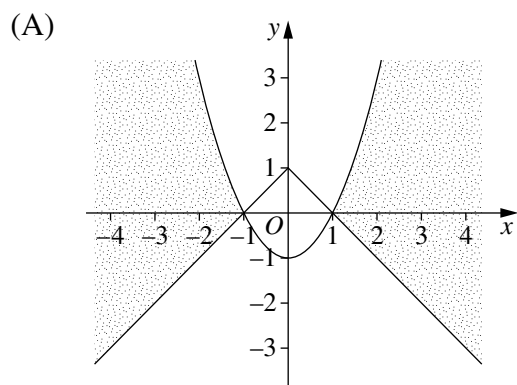
Mathematics Advanced

Objective-response questions

Marks

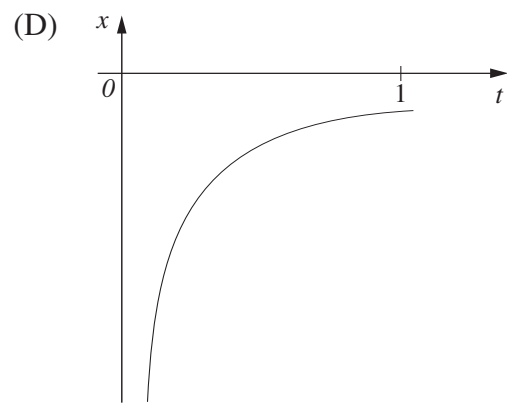
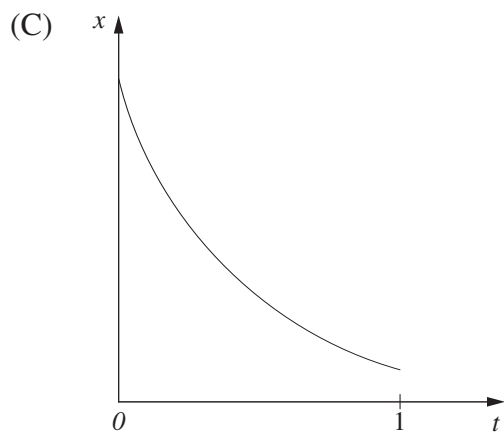
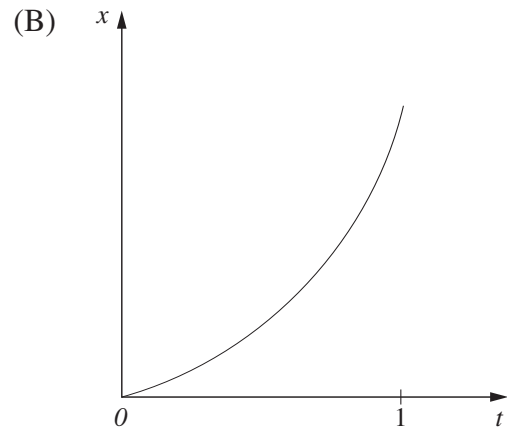
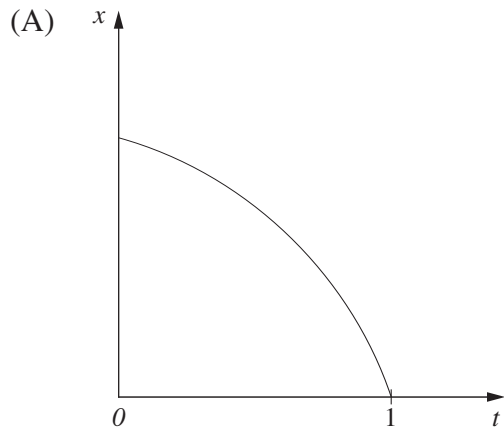
Multiple choice

- 1 Choose the diagram which shows the region satisfying $y \leq x^2 - 1$ and $y \geq 1 - |x|$. 1



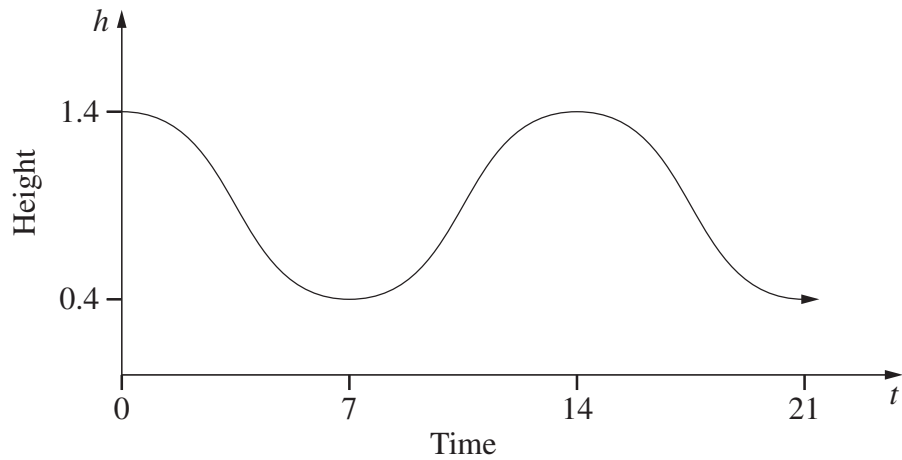
- 2 A particle is moving such that, for $0 < t < 1$, its velocity is negative and its acceleration is positive. **Marks**
1

Which graph could represent the displacement function of this particle?



Marks
1

3 Consider the graph below.

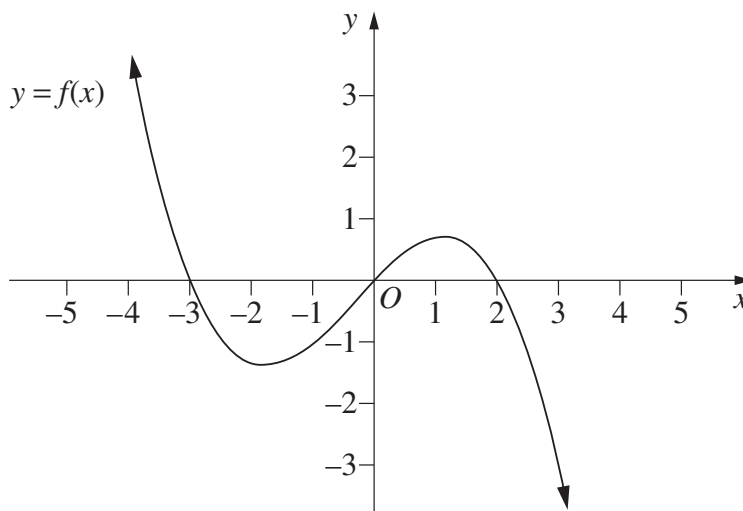


Which of the following functions does this graph represent?

- (A) $h = 0.9 + \cos \frac{\pi t}{14}$
- (B) $h = 0.9 + \cos \frac{\pi t}{7}$
- (C) $h = 0.9 + 0.5 \cos \frac{\pi t}{7}$
- (D) $h = 0.9 + 0.5 \cos \frac{\pi t}{14}$

Multiple correct/incorrect

- 4 Consider the area enclosed by the curve and the x -axis in the diagram below. 2



Indicate whether each of the following is Correct or Incorrect.

The area can be found by evaluating

- | | |
|---|--|
| a. $\left \int_{-3}^2 f(x) dx \right $. | b. $\int_{-3}^0 f(x) dx + \int_0^2 f(x) dx$. |
| c. $\int_{-3}^2 f(x) dx$. | d. $\left \int_{-3}^0 f(x) dx \right + \int_0^2 f(x) dx$. |

- 5 Indicate whether each of the following is Correct or Incorrect. 2

$\sin \theta$ always has the same value as

- | | |
|---------------------------|--|
| a. $\sin(\pi - \theta)$. | b. $\cos\left(\frac{\pi}{2} - \theta\right)$. |
| c. $\sin(\pi + \theta)$. | d. $\cos\left(\theta - \frac{\pi}{2}\right)$. |

Marks

Constrained response

- 6** In a Mathematics assessment task, the mean mark was 52 and the standard deviation was 8. Mimi scored 54. **1**

Write Mimi's z -score as a decimal.

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Free-response questions

- 7 The relationship between a person’s recommended maximum heart rate (in beats/minute) and the person’s age (in years) was expressed by the following formula: 4

$$R_1 : \text{Recommended maximum heart rate} = 220 - \text{age}$$

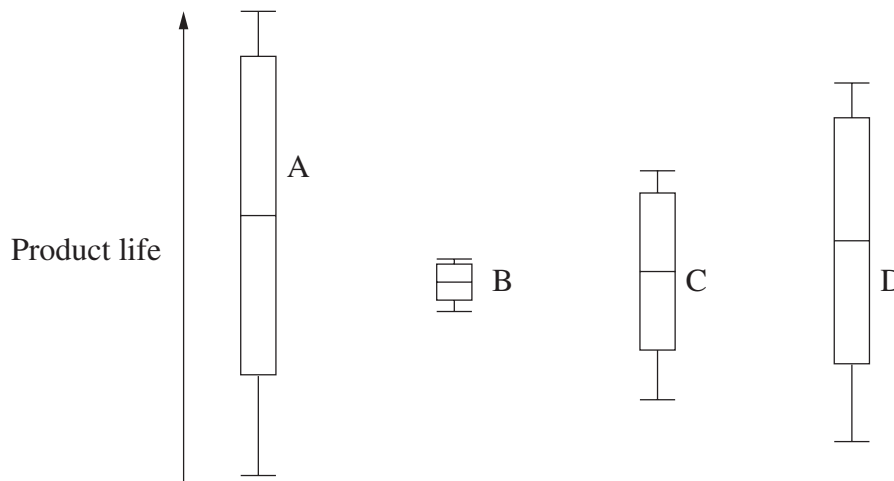
Recent research has shown that this formula should be modified to:

$$R_2 : \text{Recommended maximum heart rate} = 208 - (0.7 \times \text{age})$$

Does the new formula have the effect of increasing or decreasing the recommended maximum heart rate for each person’s age, or does it increase for some ages and decrease for others?

Support your answer with relevant calculations and/or graphs.

- 8 A company is considering four replacement models for a critical component in its assembly line. Each of the suppliers A, B, C and D has provided product life expectancy data in the form of a box-and-whisker plot. All models have approximately the same price. 2



With reference only to the life expectancy data provided, which replacement model would be the best for the company to choose? Justify your recommendation.

Mathematics Extension 1

Objective-response questions

Marks

Multiple choice

- 1** A multiple-choice test consists of ten questions. Each question has four choices. **1**

Peta knows the correct answers to six of the questions, but does not know the answers to the other four. She randomly guesses the answers to these four questions.

What is the probability that Peta gets exactly nine of the questions correct?

(A) ${}^{10}C_9 \left(\frac{1}{4}\right)^9 \left(\frac{3}{4}\right)$

(B) ${}^{10}C_9 \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^9$

(C) ${}^4C_3 \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^3$

(D) ${}^4C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)$

Marks

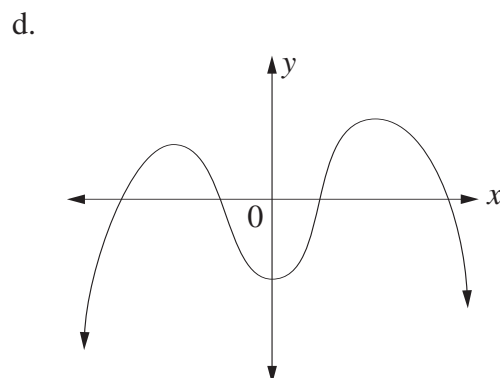
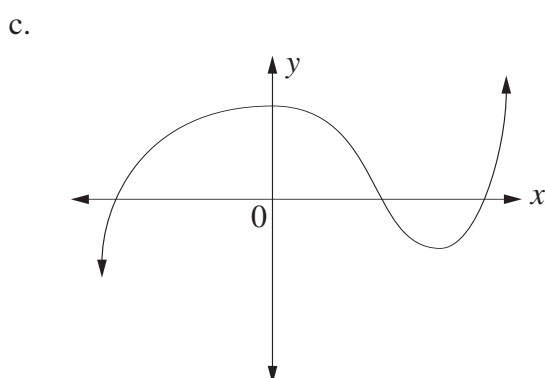
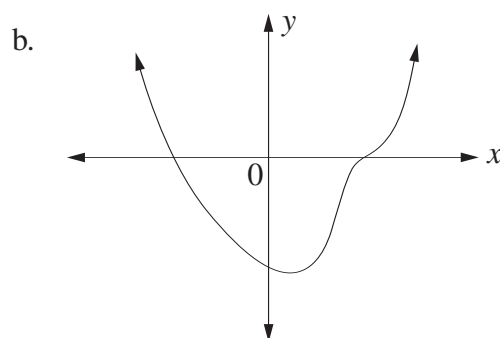
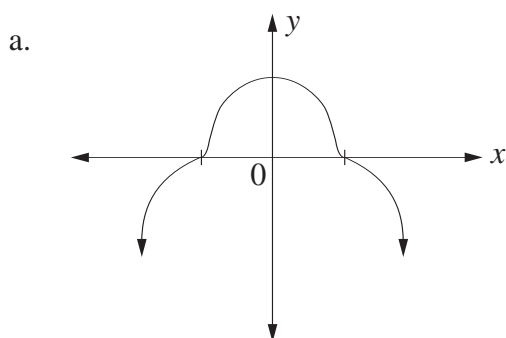
Multiple correct/incorrect

- 2** For the polynomial $P(x)$, $P(2) = 0$, and when $P(x)$ is divided by $(x - 1)$ the remainder is 5. **2**

Indicate whether each of the following is Correct or Incorrect.

$P(x)$ could be

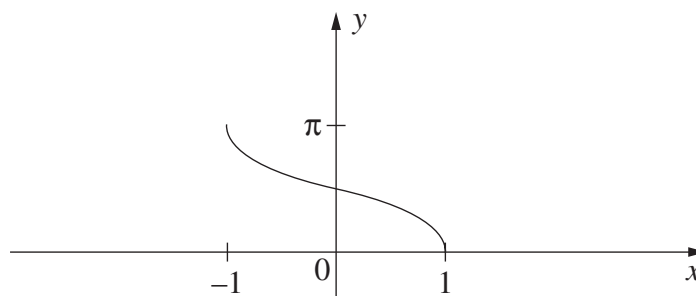
- a. $(x - 2)^2(x + 4)$.
 - b. $(x - 2)(x + 1) + 5$.
 - c. $(x - 2)(x - 6)$.
 - d. $(x - 2)(2x - 3)(3x + 2)$.
- 3** Which of the following graphs has exactly two points of inflexion? **1**



Marks

Constrained response

Use the graph of $y = \cos^{-1}x$ below to answer Questions 4 and 5.



- 4** Write down the domain of $y = 2 \cos^{-1} \frac{x}{2}$. **1**

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- 5** Write down the range of $y = 2 \cos^{-1} \frac{x}{2}$. **1**

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Mathematics Extension 2

Objective–response questions

Marks

Multiple choice

- 1** An object of mass m is projected downwards from point P with an initial velocity of 1 ms^{-1} . The object then falls under the influence of gravity in a medium which provides resistance proportional to velocity. Take the initial position as $y = 0$ and downwards as the positive direction. **1**

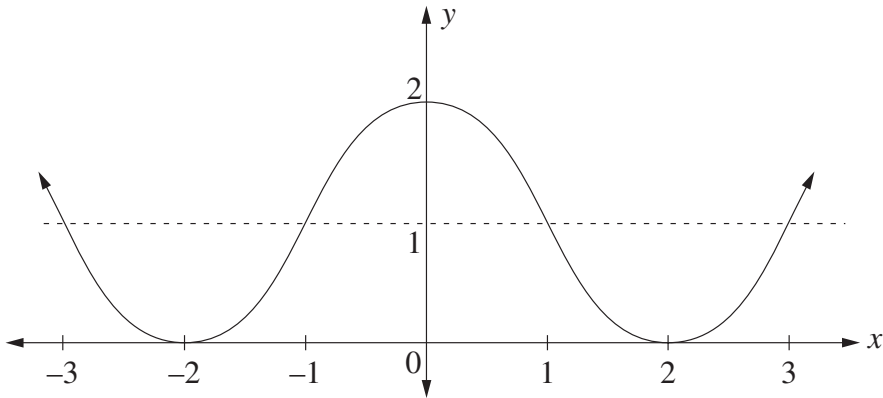
Which of the following initial value problems most accurately reflects this situation? (The acceleration due to gravity is g , and k is a positive constant).

- (A) $\ddot{y} + k\dot{y} = mg; \quad y(0) = 0, \quad \dot{y}(0) = 1$
- (B) $\ddot{y} + k\dot{y} = g; \quad y(0) = 0, \quad \dot{y}(0) = 1$
- (C) $\ddot{y} - k\dot{y} = mg; \quad y(0) = 0, \quad \dot{y}(0) = 1$
- (D) $\ddot{y} + k\dot{y} = g; \quad y(0) = 0, \quad \dot{y}(0) = -1$

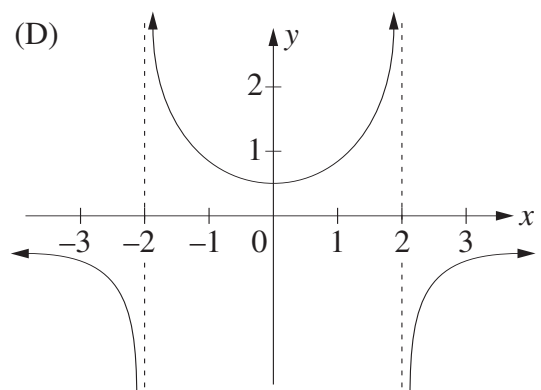
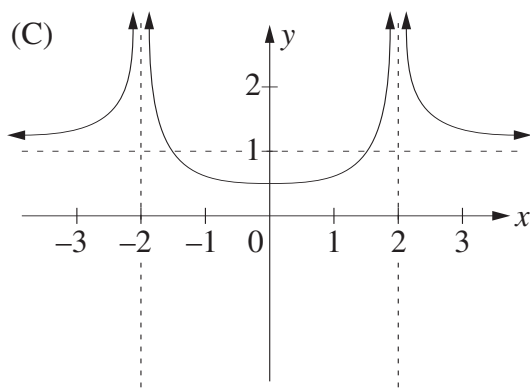
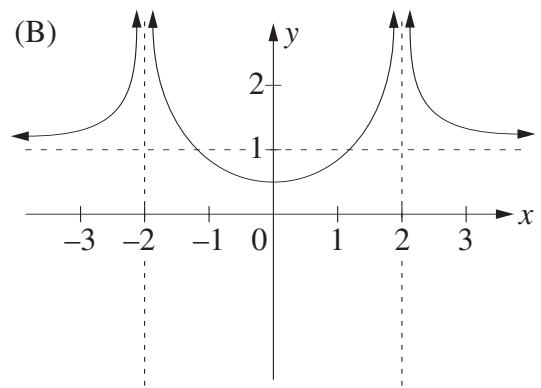
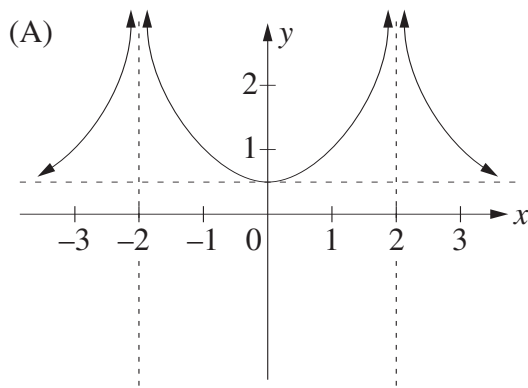
Marks

2 The graph of the function $y = f(x)$ is shown below.

1

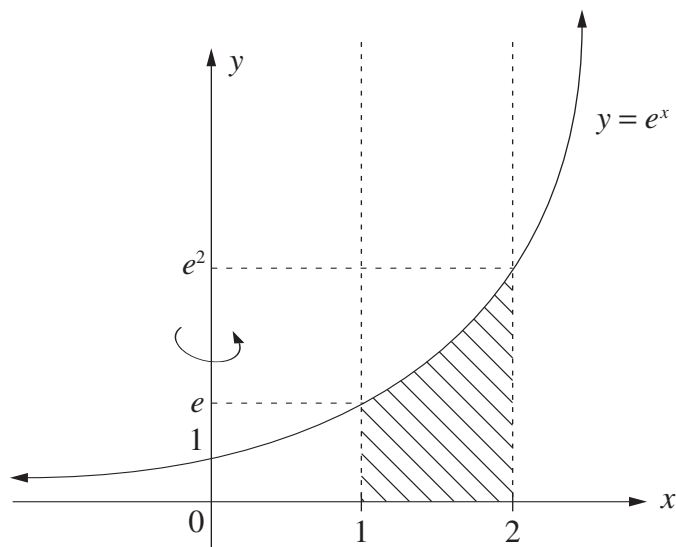


Which graph shows the most accurate representation of the graph of $y = \frac{1}{f(x)}$?



Multiple correct/incorrect

- 4 The region bounded by the curve $y = e^x$, the x -axis, and the lines $x = 1$ and $x = 2$ is rotated around the y -axis to form a solid with volume V . 2

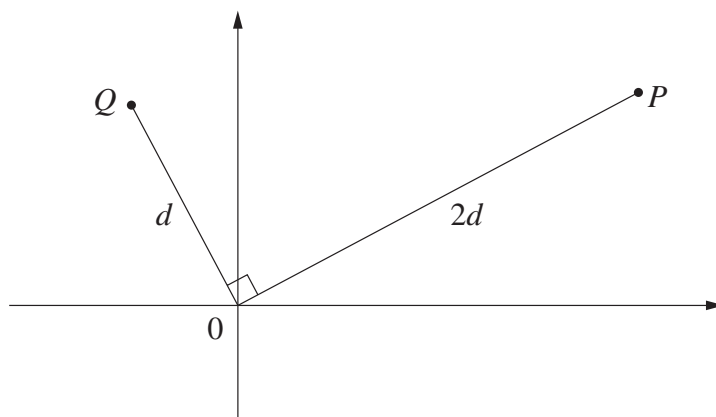


Indicate whether each of the following is Correct or Incorrect.

- a. $V = \pi \int_e^{e^2} [4 - (\ln y)^2] dy$
- b. $V = 3\pi e + \pi \int_e^{e^2} [4 - (\ln y)^2] dy$
- c. $V = 3\pi e + \pi \int_e^{e^2} (4 - 2 \ln y) dy$
- d. $V = 2\pi \int_1^2 x e^x dx$

Marks

Use the information below to answer Questions 5 and 6.



NOT
TO
SCALE

In the Argand diagram, the points P and Q represent the complex numbers z and w , respectively. Distance OP is $2d$ units, and distance OQ is d units. Angle QOP is a right angle.

5 Indicate whether each of the following is Correct or Incorrect. 2

a. $w = \frac{iz}{2}$

b. $w = -\frac{iz}{2}$

c. $z = i\bar{w}$

d. $w = -\frac{z}{2i}$

6 Indicate whether each of the following is Correct or Incorrect. Assume that all arguments are given as principal values. 2

a. $\arg(z) + \arg(\bar{z}) = 0$

b. $\arg(w) - \arg(z) = \frac{\pi}{2}$

c. $\arg(z) - \arg(w) = \frac{\pi}{2}$

d. $\arg(z) + \arg(\bar{w}) = -\frac{\pi}{2}$

Marks

Constrained response

7 Let $z = 4 + i$ and $w = \bar{z}$. Find each of the following in the form $x + iy$.

(a) w

1

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(b) z^2

1

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Marks**Free-response questions**

- 8** A mass-spring system is modelled by the second order differential equation **4**

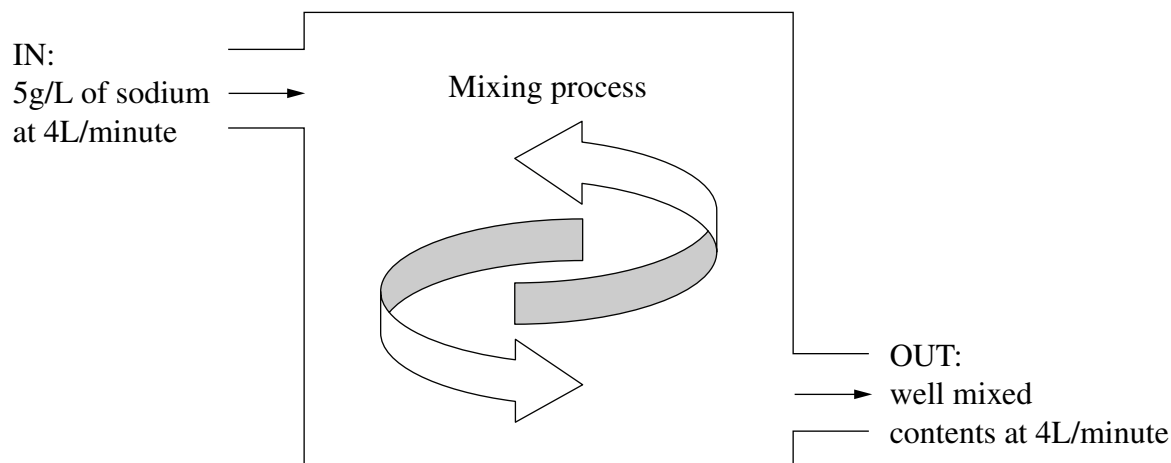
$$2\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 5x = 0$$

where $x = x(t)$ gives the displacement of the mass at time t seconds.

Solve this differential equation, given the initial conditions are given by $x(0) = -1$ and $\dot{x}(0) = 0$.

Marks

- 9 In a mixing process, a solution containing 5 grams of sodium per litre is pumped into a mixing tank at a rate of 4 litres per minute. The fluid in the tank is mixed before being pumped out at a rate of 4 litres per minute. The tank initially contains 240 litres of fresh water.



Let the amount of sodium in the tank at time t minutes be A grams.

- (a) By considering the rate of change of sodium in the tank as the difference between the rates of sodium input and output, show that the differential equation for the amount of sodium in the tank at time t minutes is **2**

$$\frac{dA}{dt} = \frac{1200 - A}{60}.$$

- (b) Solve the differential equation given in part (a), given that there is initially no sodium in the tank. **3**
- (c) Find the concentration of sodium in the tank after 3 hours. Give your answer to 2 decimal places. **1**
- (d) Find $\lim_{t \rightarrow \infty} A(t)$ and explain the meaning of your answer in the context of the problem. **2**
- (e) Make a neat sketch of $A(t)$ versus t . **2**
- (f) Find the maximum value of $\frac{dA}{dt}$ and explain the meaning of your answer in the context of the problem. **2**

Question Mapping

Question	Marks	Outcomes	Content
Mathematics General 1			
1	1	HG1.3	MG1.7: Multi-stage events and applications of probability
2	1	PG2 HG1.2	PMG5: Displaying single data sets MG1.2: Interpreting sets of data
3	1	HG1.4	MG1.5: Right-angled triangles (Review)
4	2	HG1.2	MG1.2: Interpreting sets of data
5	1	HG1.3	FS5.2: Body measurements
6	1	HG1.6	MG1.1: Credit cards
7	1	HG1.3	MG1.8: Further algebraic skills
8(a)	1	HG1.2	MG1.2: Interpreting sets of data
8(b)	1	HG1.10	PMG6: Summary statistics
8(c)	1	HG1.3, HG1.10	MG1.2: Interpreting sets of data
9	3	PG6, HG1.3, HG1.6 HG1.10	FS2.2: Running costs
Mathematics General 2			
1	1	HG2.3	MG2.10: Applications of probability
2	1	HG2.3	MG2.13: Modelling linear and non-linear relationships
3	2	HG2.4	MG2.8: Spherical geometry
4	1	PG3, HG2.3	PMG13: Modelling linear relationships
5(a)	1	HG2.2	MG2.3: Interpreting sets of data
5(b)	1	HG2.9	MG2.3: Interpreting sets of data
5(c)	2	HG2.2	MG2.3: Interpreting sets of data
Mathematics Advanced			
1	1	PA2, PA12, HA9	PMA3.4: Region and inequality
2	1	HA3	MA2.2: The second derivative and its geometrical significance, curve sketching
3	1	PA5, HA9	PMA4.2: The trigonometric functions and their graphs
4	2	HA7	MA1.4: Applications of integration: areas and volumes of solids of revolution
5	2	PA5, PA6, HA9	PMA4.1: Introduction to circular measure, exact ratios, angles of any magnitude
6	1	HA5	PMA6.3: The normal distribution
7	4	PA1, PA2, PA11, HA9	PMA3.5: Modelling with real functions
8	2	PA1, PA8, HA9	PMA6.1: Types of variables, measures of location and spread (variability), graphical and tabular representations of data

Mathematics Extension 1			
1	1	HX4	MX1.2: Binomial probabilities and the binomial distribution
2	2	PX4, HX6	PMX4.2 The remainder and factor theorems
3	1	PX4, HX9	PMX4.1: Definitions of polynomial, degree, polynomial equation; graphs of simple polynomials
4	1	HX3	MX5.2: The inverse trigonometric functions and their graphs
5	1	HX3	MX5.2: The inverse trigonometric functions and their graphs
Mathematics Extension 2			
1	1	HXX11	MXX6.2: Resisted motion
2	1	HXX4	MXX3.2: Sketching functions by multiplication and division of ordinates
3	1	HXX4	MXX3.2: Sketching functions by multiplication and division of ordinates
4	2	HXX10	MXX5.1: Volumes
5	2	HXX6	MXX2.2: Geometric representation of a complex number as a point and as a vector
6	2	HXX6	MXX2.2: Geometric representation of a complex number as a point and as a vector
7(a)	1	HXX5	MXX2.1: Arithmetic of complex numbers and solving quadratic equations
7(b)	1	HXX5	MXX2.1: Arithmetic of complex numbers and solving quadratic equations
8	4	HXX8, HXX12	MXX7.3: Second-order linear differential equations
9(a)	2	HXX11, HXX12	MXX7.2: First-order linear differential equations
9(b)	3	HXX8, HXX11, HXX12	MXX7.2: First-order linear differential equations
9(c)	1	HXX8	MXX7.2: First-order linear differential equations
9(d)	2	HXX11, HXX12	MXX7.2: First-order linear differential equations
9(e)	2	HXX11, HXX12	MXX7.2: First-order linear differential equations
9(f)	2	HXX11, HXX12	MXX7.2: First-order linear differential equations