



# **Mathematics**

## **Stage 6**

### **Writing Brief**

**Courses: Mathematics Advanced  
Mathematics Extension 1  
Mathematics Extension 2**

**2007**

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Published by  
Board of Studies NSW  
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Sydney NSW 2001  
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ISBN 978 174147 6941

September 2007

2007542

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# **1 Writing Brief**

## 1 Writing Brief

Preparation of a writing brief takes place in Phase 2 of the Board of Studies' syllabus development process (see page 7).

### 1.1 Purpose

This writing brief will be used to guide the development of the new Mathematics Stage 6 courses: Mathematics Advanced, Mathematics Extension 1, and Mathematics Extension 2. (A further writing brief has been developed for the new Mathematics courses: Mathematics General 1 and Mathematics General 2.) The writing brief will provide the detailed 'blueprint' for the development of the courses.

The final syllabus documents will be developed during the next phase (*Syllabus Development Phase*) of the syllabus development process.

### 1.2 Structure

The writing brief is structured as follows:

Sections 1 to 3 of the document provide background reading on the development of the writing brief. Sections 4 and 5 provide details of the proposed structure of the new Mathematics Stage 6 courses, the place of the courses in the K–12 curriculum, and proposed elements of the Mathematics Advanced, Mathematics Extension 1, and Mathematics Extension 2 courses. Section 6 is a glossary of syllabus terms for the assistance of teachers. Section 7 provides information in relation to proposed syllabus support materials, while Section 8 (Appendix) contains the Broad Directions for the Mathematics Stage 6 syllabus writing briefs.

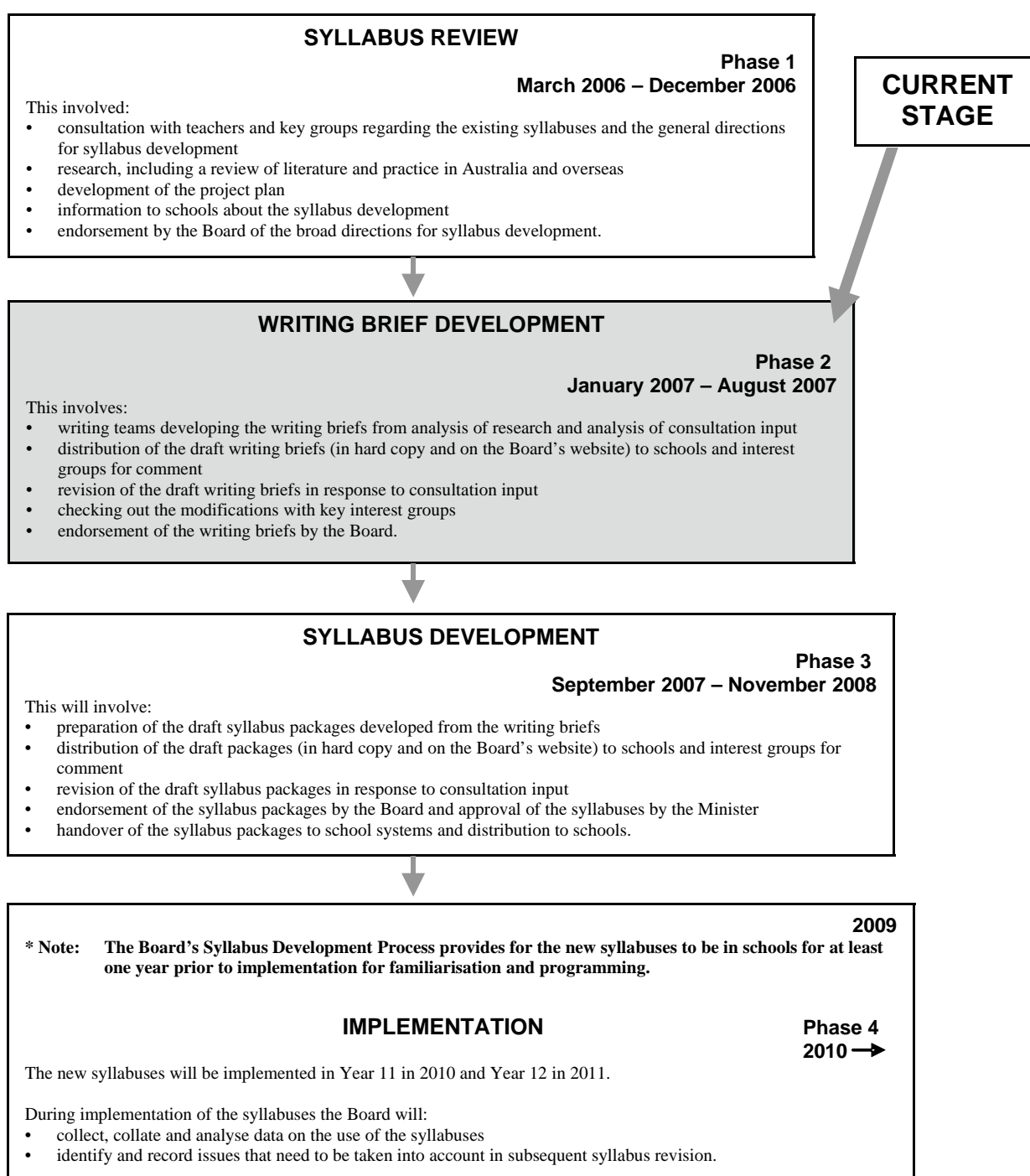
## **2 Syllabus Development**

## 2 Syllabus Development

### 2.1 Mathematics Stage 6 syllabus development process

Syllabuses in New South Wales are developed in accordance with the Board of Studies' syllabus development process. This process is detailed in the Board's *Syllabus Development Handbook* (July 2006), which is available on the Board's website ([www.boardofstudies.nsw.edu.au/manuals/#syl\\_develop\\_process](http://www.boardofstudies.nsw.edu.au/manuals/#syl_develop_process)).

The first phase, *Syllabus Review*, in the development of new Mathematics Stage 6 syllabuses, is now complete. This phase involved several key data-collection strategies, including oral and written submissions, a symposium, a survey of a sample of schools, and a literature and curriculum review.



## 2.2 Timeline for the development of the syllabus packages for Mathematics Stage 6 courses

The syllabus packages for the Mathematics Stage 6 courses will be developed in accordance with:

- the Board's *Syllabus Development Handbook*, a copy of which is available on the Board's website ([www.boardofstudies.nsw.edu.au/manuals/#syl\\_develop\\_process](http://www.boardofstudies.nsw.edu.au/manuals/#syl_develop_process))
- the syllabus development plan for the Mathematics Stage 6 syllabuses set out below.

| Steps in the syllabus development process   | Date   |
|---|--|
| <b>Syllabus Review</b> <ul style="list-style-type: none"> <li>• Oral submissions</li> <li>• Written submissions</li> <li>• Symposium</li> <li>• Survey of a sample of schools</li> <li>• Literature and curriculum review</li> </ul>  | 24–25 May 2006<br>closed 30 June 2006<br>19 August 2006<br>closed 18 September 2006<br>August–September 2006                         |
| <b>Writing Brief Development</b> <ul style="list-style-type: none"> <li>• Preparation of draft writing briefs and surveys</li> <li>• Consultation (five weeks)</li> <li>• Development of consultation reports and revision of draft writing briefs</li> </ul>   | January 2007 – April 2007<br>1 May 2007 – 1 June 2007<br>June 2007 – August 2007   |
| <b>Syllabus Development</b> <ul style="list-style-type: none"> <li>• Development of draft syllabuses and surveys</li> <li>• Consultation (eight weeks)</li> <li>• Development of consultation reports and revision of draft syllabuses</li> <li>• Distribution of syllabuses</li> <li>• Development and distribution of specimen examination papers and marking guidelines</li> </ul> | September 2007 – April 2008<br>5 May 2008 – 27 June 2008<br><br>July 2008 – October 2008<br>November 2008<br>(Date to be determined) |
| <b>Note: Support Materials</b><br>The nature of proposed support materials is presented in section 7.   |  |

## **3 Background Information**

### 3. Background Information

#### 3.1 Evaluation and review of the current Stage 6 Mathematics syllabuses

The current *Mathematics 2/3 Unit Syllabus – Years 11–12* was introduced in 1983, while the current *4 Unit Mathematics Syllabus* was introduced in 1980, with some amendment in 1989. The current General Mathematics course was introduced in 2000 as part of the New Higher School Certificate.

The Board of Studies initiated the revision of the suite of Stage 6 Mathematics courses, following the completion of new K–6 and Years 7–10 Mathematics syllabuses in 2002, to ensure an up-to-date and cohesive K–12 continuum of mathematics learning and teaching in New South Wales that meets the needs of the full range of students.

In 1998, the Board undertook the evaluation of all existing Higher School Certificate (HSC) courses against the specific criteria of the Government’s White Paper *Securing Their Future* (August 1997). For Stage 6 Mathematics, two evaluation reports were developed: *Mathematics 2/3/4 Unit* and *Non-Calculus-Based Mathematics (incorporates Mathematics in Society and Mathematics in Practice)*.

Following the release of these reports, the Board endorsed the development of a single, non-calculus-based course (which became General Mathematics) to replace Mathematics in Society and Mathematics in Practice and the maintenance of the 2, 3 and 4 Unit courses for the immediate future, with ‘minimal change to existing content’. The Board also agreed to ‘a longer term, comprehensive review of the present 2/3/4 Unit courses by an appropriately constituted expert committee’.

The *General Mathematics Syllabus* was released to schools in June 1999, with the study of the Preliminary course undertaken for the first time in 2000, and the first HSC examination undertaken in 2001. As an interim arrangement for the introductory years of the new HSC, the Board of Studies maintained the course content, internal assessment arrangements and examination specifications of the 2, 3 and 4 Unit Mathematics courses from the previous HSC. The new HSC standards-referencing procedures that were introduced for all new HSC Board-developed courses were also put in place for these Mathematics courses, which were renamed Mathematics, Mathematics Extension 1 and Mathematics Extension 2 respectively.

In July 2000, the NSW Government announced the review of Years 7–10 Mathematics following its 1999 commitment in *Literacy and Numeracy Plan: Focusing on the basics* that: ‘A new Years 7–10 Mathematics syllabus will be developed to ensure it provides students with the skills, knowledge and understanding required and appropriately prepares students for the New School Certificate and the New Higher School Certificate’. At about this time, the Board of Studies announced the review and development of the K–6 Mathematics syllabus.

The review and development of K–10 Mathematics consequently became the focus of Mathematics curriculum development in 2000–2002, with its completion a necessary prerequisite to the ‘longer term, comprehensive review of the present 2/3/4 Unit courses’ agreed to by the Board in 1998.

With the completion of the new K–6 and Years 7–10 syllabuses at the end of 2002, a plan was established for the review and development of the Stage 6 calculus-based Mathematics courses. In considering the plan, the Board felt that it would be necessary to review the full Stage 6 Mathematics provision, taking account of the needs of less able students as well as those who would undertake the higher-level calculus-based courses. The Board endorsed the plan for the review and development of the full Stage 6 Mathematics course provision in December 2004.

### 3.2 Scope of Mathematics Stage 6 review and development

The implementation of the plan in 2006, with the commencement of the *Mathematics Stage 6 Review and Development Project*, recognised the importance of teachers having appropriate time to focus on the implementation of the new *Mathematics Years 7–10 Syllabus*, and the value of obtaining feedback following 2004 and 2005, the initial years of implementation.

The first phase of the project, *Syllabus Review*, commenced in March 2006. The main purpose in undertaking the *Syllabus Review* phase was to review the existing Mathematics course provision and to establish Broad Directions for revision and development.

A range of strategies was used to gather data in the *Syllabus Review* phase. The most significant of these were:

- oral submissions
- written submissions
- a survey of a sample of schools
- a symposium
- a literature and curriculum review.

The information obtained through the data-gathering strategies used in the *Syllabus Review* phase was analysed to identify issues that need to be considered in the revision and development of Stage 6 Mathematics courses. Key findings were synthesised from the data and a set of draft Broad Directions for the revision and development compiled.

The Board's endorsement of the Broad Directions at its December 2006 meeting represented the conclusion of the *Syllabus Review* phase of the project. The Broad Directions have guided the development of the draft writing briefs for the calculus-based, and non-calculus-based, Stage 6 Mathematics courses.

(See Appendix for the Broad Directions.)

## **4 Structure of the Proposed Mathematics Stage 6 Courses**

## **4 Structure of the Proposed Mathematics Stage 6 Courses**

In the review and development of the Mathematics Stage 6 courses, it is proposed that there be five Board-developed Mathematics courses of study for the Higher School Certificate (in increasing order of difficulty): Mathematics General 1, Mathematics General 2, Mathematics Advanced, Mathematics Extension 1, and Mathematics Extension 2.

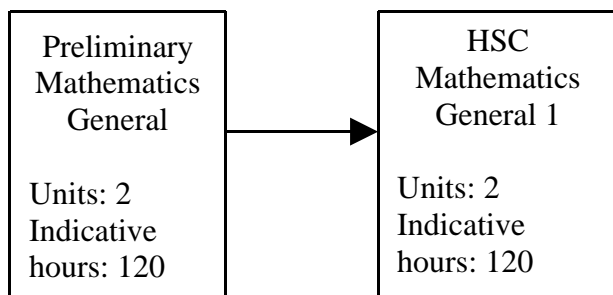
Mathematics General 1 represents an additional course of study in the suite of Mathematics Stage 6 courses, while Mathematics General 2 will replace the current General Mathematics course (on which it is largely modelled). The Mathematics General 1 course has been included in accordance with the Broad Direction (see section 8): ‘That the set of Stage 6 Mathematics courses include an additional offering to accommodate the purposes of students who wish to study a Board-developed Mathematics course in Stage 6 but who are currently choosing not to, as well as those whose purposes are not accommodated through the study of General Mathematics.’

It is proposed that students of the Mathematics General 1 and Mathematics General 2 courses study common Preliminary material within the course Preliminary Mathematics General, leading to the HSC Mathematics General 1 and HSC Mathematics General 2 courses.

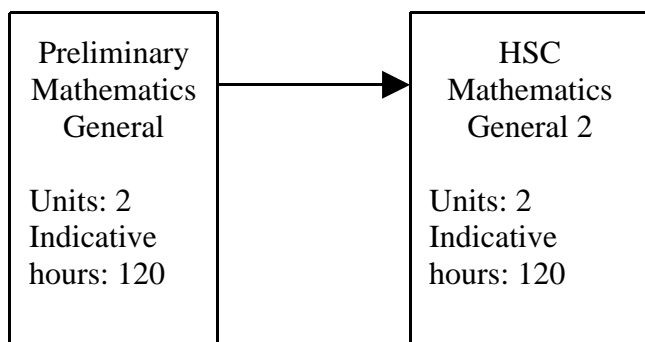
Mathematics Advanced replaces the current Mathematics (‘2 Unit’) course. It is proposed that it consist of the courses Preliminary Mathematics Advanced and HSC Mathematics Advanced. Mathematics Extension 1 and Mathematics Extension 2 replace the current Mathematics Extension 1 and Mathematics Extension 2 courses. It is proposed that students of these courses study Preliminary Extension course material prior to undertaking the study of HSC Mathematics Extension 1 and HSC Mathematics Extension 2.

The proposed Mathematics Stage 6 courses are illustrated below, in terms of their Preliminary and HSC course components.

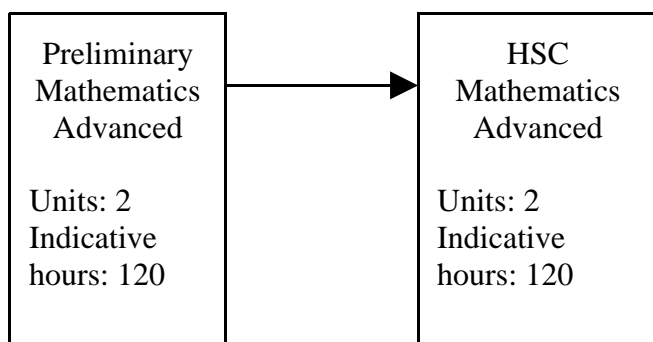
### Mathematics General 1



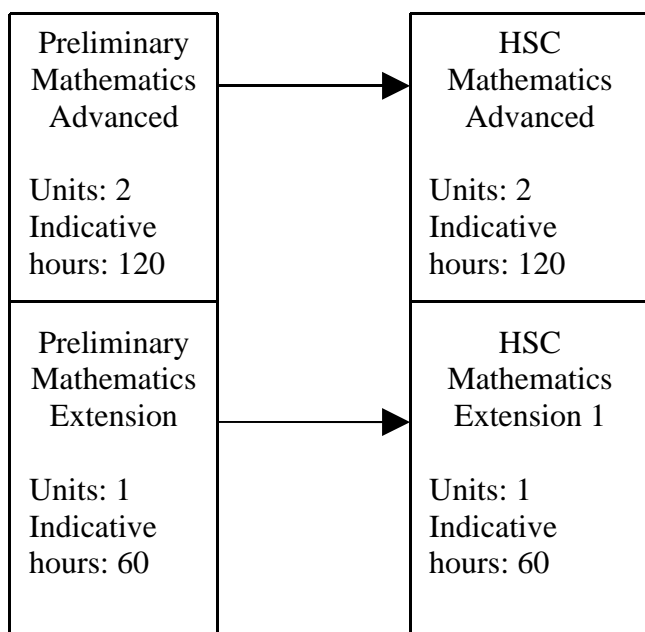
### Mathematics General 2



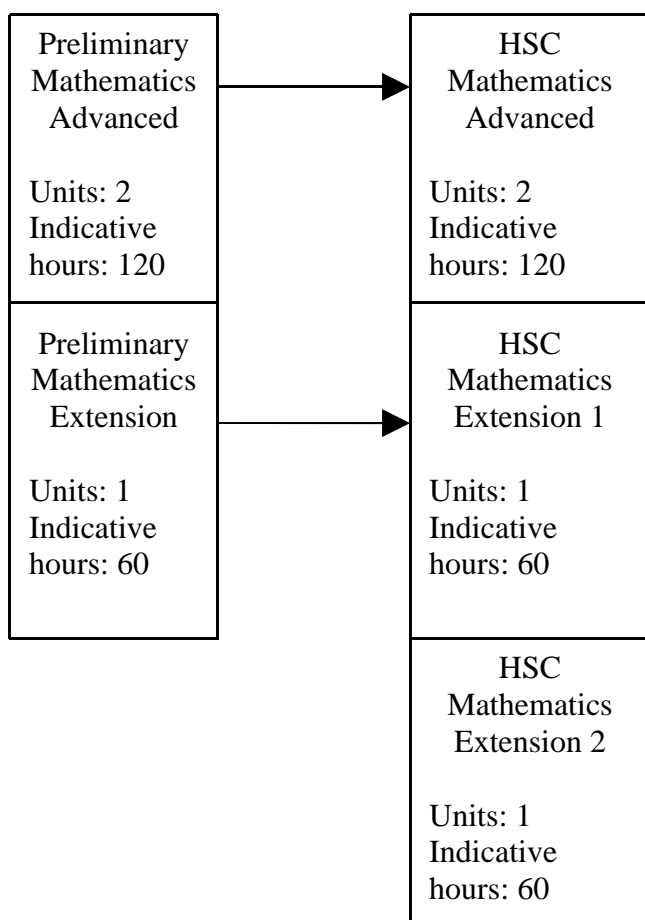
### Mathematics Advanced



### Mathematics Extension 1



### Mathematics Extension 2



**5 Mathematics Advanced,  
Mathematics Extension 1,  
Mathematics Extension 2 Courses**  
**– course elements**

## 5 Mathematics Advanced, Mathematics Extension 1, Mathematics Extension 2 Courses

### – course elements

This section of the writing brief is structured to address the course elements for the *Mathematics Advanced*, *Mathematics Extension 1* and *Mathematics Extension 2* courses:

- Rationale
- Aim
- Objectives
- Content organisation
- Outcomes
- Content
- Use of technology
- Assessment and HSC examination.

The material presented here in relation to the course elements has been amended in accordance with feedback received during consultation on the draft writing brief for these courses. Further revision of the material will occur as the full syllabuses are developed in the next phase (*Syllabus Development Phase*) of the syllabus development process.

### 5.1 Rationale

*The **rationale** describes the nature of the subject and relevant courses in broad terms. It explains the place and purpose of the subject and relevant courses in the curriculum.*

Mathematics is deeply embedded in modern society. From the numeracy skills required to manage personal finances, to making sense of data in various forms, to leading-edge technologies in the Sciences and Engineering, Mathematics provides the framework for interpreting, analysing and predicting, and the tools for effective participation in an increasingly complex society.

The need to interpret the large volumes of data made available through technology draws on skills in logical thought and the ability to check claims and assumptions in a systematic way. Mathematics is the appropriate training ground for the development of these skills and abilities. The thinking required to enhance further the power and usefulness of technology in real-world applications requires advanced mathematical training. The rapid advances in technology experienced in recent years have driven, and been driven by, advances in the discipline of Mathematics.

The development of Mathematics throughout history has been catalysed by its utility in explaining real-world phenomena and its inherent beauty. In this way, the discipline has continued to evolve through a process of observation, conjecture, proof and application.

The Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 courses provide the opportunity for students to acquire knowledge, skills and understanding in relation to important mathematical concepts, and applications in a range of contexts, that are appropriate to their continued experience of Mathematics as a coherent, interrelated, interesting and intrinsically valuable study that forms a basis for future learning. The introductory concepts and techniques of differential and integral calculus form a strong basis of the courses, and are developed and utilised across the courses, through a range of applications.

Students develop an appreciation of Mathematics as a study with high levels of internal structure that provide opportunities for the development of logical and disciplined thought. Through the learning experiences within the courses, students are able to progress from a knowledge and understanding of facts, procedures and applications in idealised contexts to facility in the use of mathematical models that situate the Mathematics in context and provide information on the behaviour of real-world systems, and to more advanced generalisations based on deductive and inductive reasoning processes. This involves the development and use of an increasingly sophisticated level of communication and literacy.

The courses provide students with the opportunity to study applications of Mathematics in a range of contexts relevant to contemporary professional practice, including examples from the Mathematics, Science, Engineering, Technology, Education, Business and Finance areas.

## 5.2 Aim

*The **aim** states the overall purpose of the courses. It indicates the educational benefits that are intended to accrue for students who satisfactorily complete programs of study based on the courses.*

The calculus-based Mathematics Advanced, Mathematics Extension 1, and Mathematics Extension 2 courses are designed to promote the development of knowledge, skills and understanding in relation to important concepts within areas of Mathematics that have applications in an increasing number of contexts. This includes the development of deductive and inductive reasoning and the ability to construct, solve and interpret mathematical models.

Students will learn to use a range of techniques and tools, including relevant technologies, in order to develop solutions to a wide variety of problems relating to their present and future needs and aspirations.

## 5.3 Objectives

***Objectives** provide more specific statements of the intent of the courses. They amplify the aim and provide direction to teachers on the learning and teaching process emerging from the course(s). They define in broad terms the knowledge, skills, understanding and values and attitudes fundamental to the key learning area/subject. They act as organisers of the intended outcomes. The same objectives may apply across Stages in a key learning area/subject.*

## **Knowledge, skills and understanding**

Students will develop:

- the ability to apply deductive and inductive reasoning, and the use of appropriate language, in the construction of proofs and mathematical arguments
- the ability to use concepts and techniques, including technological applications, in the solution of problems
- the ability to construct, interpret, and use mathematical models in a range of contemporary contexts
- the ability to interpret and communicate Mathematics in a variety of forms.

## **Values and attitudes**

Students will develop:

- appreciation of the scope, usefulness, power and elegance of Mathematics.

## **5.4 Content organisation**

*The **Content organisation** section describes how the course content is to be organised.*

It is proposed that the course content for the Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 courses be organised into topics, and be presented in the following format:

### **Name of topic**

A brief summary of the content/purpose of the topic.

**Assumed Stage 4 and Stage 5 outcomes** [This section only appears for topics in the Preliminary course].

Outcomes from Stage 4/Stage 5 that students should have achieved to engage successfully with the topic.

### **Outcomes addressed**

A list of the course outcomes that will be addressed in the study of the topic.

### **Students learn and acquire the following knowledge, skills and understanding**

The mathematical content to be addressed in the topic.

**Terminology introduced in this topic**

A list of words and/or phrases that may be new to students, and which may be used in relevant assessment tasks.

**Technology that may be used in support of this topic**

Advice about the nature and use of technology that is appropriate to the topic.

**Applications, considerations, examples**

The provision of examples defining the range and style of applications used to introduce and illustrate the mathematical content of the topic, as well as important considerations for the learning and teaching of the topic.

## 5.5 Outcomes

*Syllabus outcomes express the specific intended student learning that results from the teaching of the course(s). They are derived from the objectives and content of the syllabus. Outcomes provide clear statements of the knowledge, skills and understanding expected to be gained by most students as a result of effective learning and teaching by the end of a Stage.*

A proposed arrangement and set of outcomes for the Mathematics Stage 6 calculus-based courses are presented in the table below. The outcomes, together with the content, determine the breadth and depth of study to be undertaken by students in a course.

| Objectives  | Mathematics Advanced  |   | Mathematics Extension 1  |   | Mathematics Extension 2  |
|---|---|---|--|---|--|
|   | Preliminary Outcomes  | HSC Outcomes  | Preliminary Outcomes   | HSC Outcomes  | HSC Outcomes   |
| Students will develop:<br>the ability to apply deductive and inductive reasoning, and the use of appropriate language, in the construction of proofs and mathematical arguments | A student:<br>PA1<br>provides reasoning to support conclusions which are appropriate to the context | A student:<br>HA1<br>constructs arguments to prove and justify results in a variety of contexts | A student:<br>PX1<br>uses multi-step deductive reasoning to solve problems/<br>prove results in circle geometry<br>PX2<br>uses algebraic techniques to solve inequalities and prove results and identities | A student:<br>HX1.1<br>uses the binomial theorem and techniques from algebra and calculus to prove identities | A student:<br>HX2.1<br>chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings<br>HX2.2<br>applies algebraic, graphical and calculus techniques in the construction of proofs involving inequalities |

| Objectives  | Mathematics Advanced   |   | Mathematics Extension 1   |   | Mathematics Extension 2  |
|---|--|---|---|---|--|
|   | Preliminary Outcomes   | HSC Outcomes  | Preliminary Outcomes  | HSC Outcomes  | HSC Outcomes   |
| Students will develop:  | A student:<br><br>PA2  | A student:<br><br>HA2   | A student:<br><br>PX3   | A student:<br><br>HX1.2<br>HX1.3  | A student:<br><br>HX2.3<br>HX2.4   |
| the ability to use concepts and techniques, including technological applications, in the solution of problems | applies Stage 5 concepts in algebra, functions, graphs and coordinate geometry                             | manipulates algebraic expressions involving logarithmic and exponential functions | applies the relationship between the algebraic and geometric forms of a function to problems, including those involving transformations | uses inductive reasoning in the construction of proofs<br>applies the concept of an inverse function to problems involving tests for existence of an inverse, and algebraic and graphical representations | uses inductive reasoning in the construction of proofs involving inequalities<br>combines the ideas of algebra and calculus to determine the important features of the graphs of a wide variety of functions |
|   | PA3<br>applies a range of counting strategies to solve problems involving ordered and unordered selections |   |   | HX1.4<br>demonstrates understanding of the significance of the binomial coefficients in counting, expansion of algebraic expressions, and probability calculations  |  |

| Objectives             | Mathematics Advanced  |                            | Mathematics Extension 1            |   | Mathematics Extension 2  |
|------------------------|---|----------------------------|------------------------------------|---|--|
|                        | Preliminary Outcomes  | HSC Outcomes               | Preliminary Outcomes               | HSC Outcomes  | HSC Outcomes   |
| Students will develop: | A student:<br>PA4<br>applies the concepts of complementary, mutually exclusive and independent<br>PA5<br>applies basic concepts in trigonometry to the solution of theoretical and practical problems<br>PA6<br>uses the concept of radian measure to solve problems involving area and arc length in circles | A student:<br>HSC Outcomes | A student:<br>Preliminary Outcomes | A student:<br>HSC Outcomes  | A student:<br>HSC Outcomes   |
|                        |   |                            |                                    | HX1.5<br>uses the properties of trigonometric functions to derive formulae, and construct proofs, and solve theoretical and practical problems, including problems involving trigonometry in three dimensions | HX2.5<br>performs arithmetic operations on complex numbers and applies De Moivre's theorem to problems involving powers and roots<br><br>HX2.6<br>uses the relationship between algebraic and geometric representations of complex numbers |

| Objectives             | Mathematics Advanced   |   | Mathematics Extension 1   |   | Mathematics Extension 2  |
|------------------------|--|---|---|---|--|
|                        | Preliminary Outcomes   | HSC Outcomes  | Preliminary Outcomes  | HSC Outcomes  | HSC Outcomes   |
| Students will develop: | <p>A student:</p> <p>PA7 solves trigonometry problems involving circular angle measure using algebraic and graphical techniques</p> <p>PA8 determines the derivative of a function through the routine application of the rules of differentiation</p> | <p>A student:</p> <p>HA3 applies the graphical relationship between a function and its derivative to curve sketching and practical problems</p> | <p>A student:</p> <p>PX4 uses the language, notation and theory of polynomial functions</p> | <p>A student:</p> <p>HX1.6 investigates the properties of polynomial functions using a variety of algebraic and calculus techniques</p> | <p>A student:</p> <p>HX2.7 applies concepts from the theory of polynomials and complex numbers to investigate the factors, roots and coefficients of polynomial functions</p> <p>HX2.8 uses standard techniques to solve first-order and second-order ordinary linear differential equations</p> |

| Objectives             | Mathematics Advanced   |   | Mathematics Extension 1 |              | Mathematics Extension 2 |
|------------------------|--|---|-------------------------|--------------|-------------------------|
|                        | Preliminary Outcomes   | HSC Outcomes  | Preliminary Outcomes    | HSC Outcomes | HSC Outcomes            |
| Students will develop: | A student:<br><br>PA9<br>uses different variable types and uses appropriate techniques from descriptive statistics to present and interpret data | A student:<br><br>HA4<br>applies techniques of differentiation and integration to logarithmic, exponential and trigonometric functions<br><br>HA5<br>applies the concept of a z-score, standardises normal random variables and solves related probability problems | A student:              | A student:   | A student:              |

| Objectives             | Mathematics Advanced   |  | Mathematics Extension 1 |   | Mathematics Extension 2  |
|------------------------|--|--|-------------------------|---|--|
|                        | Preliminary Outcomes   | HSC Outcomes   | Preliminary Outcomes    | HSC Outcomes  | HSC Outcomes   |
| Students will develop: | A student:<br>PA10<br>derives general results for arithmetic and geometric series, and applies these results to the solution of theoretical and practical problems<br>PA11<br>uses the relationship between the primitive and derivative of a function and determines primitives for functions involving powers of $x$ | A student:<br>HA6<br>uses techniques of integration to calculate definite integrals, areas and volumes | A student:              | A student:<br>HX1.7<br>evaluates integrals using given substitutions and trigonometric identities | A student:<br>HX2.9<br>uses integral calculus to solve problems requiring the use of integration tables, identification and use of appropriate substitutions, partial fractions, integration by parts, and recurrence formulae |

| Objectives  | Mathematics Advanced   |  | Mathematics Extension 1  |   | Mathematics Extension 2   |
|---|--|--|--|---|---|
|   | Preliminary Outcomes   | HSC Outcomes   | Preliminary Outcomes   | HSC Outcomes  | HSC Outcomes  |
| Students will develop:  | A student:<br><br>PA12<br>uses algebraic, numerical and graphical representations of linear and quadratic functions in mathematical modelling situations and interprets results in context | A student:<br><br>HA7<br>uses differential and integral calculus to interpret mathematical models of linear motion, exponential growth and decay, and financial situations | A student:<br><br>PX5<br>uses the language and theory of difference equations in the solution of problems in a variety of practical contexts | A student:<br><br>HX1.8<br>applies differential and integral calculus to mathematical modelling situations involving related rates, linear and projectile motion, and modified exponential growth and decay | A student:<br><br>HX2.10<br>uses the techniques of slicing, cylindrical shells and similar cross-sections to calculate volumes by integration<br><br>HX2.11<br>formulates and solves ordinary differential equations arising in mathematical modelling situations |
| the ability to construct, solve and interpret mathematical models in a range of contemporary contexts |  |  |  |   |   |

| Objectives   | Mathematics Advanced  |   | Mathematics Extension 1   |   | Mathematics Extension 2  |
|--|---|---|---|---|--|
|  | Preliminary Outcomes  | HSC Outcomes  | Preliminary Outcomes  | HSC Outcomes  | HSC Outcomes   |
| Students will develop:   | A student:<br><br>PA13 interprets and uses the language of Mathematics in a variety of contexts | A student:<br><br>HA8 applies series techniques to the solution of financial problems and interprets results<br><br>HA9 Interprets mathematical models in context and communicates using mathematical language, notation, diagrams and graphs | A student:<br><br>PX6 makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations | A student:<br><br>HX1.9 evaluates and interprets mathematical solutions to a wide range of practical problems and communicates the solutions in an appropriate form | A student:<br><br>HX2.12 communicates abstract ideas and relationships using appropriate notation and logical argument |
| (Values and attitudes)<br>appreciation of the scope, usefulness, power and elegance of Mathematics | PA/VA demonstrates confidence in using Mathematics to obtain realistic solutions to problems    | HA/VA seeks to apply mathematical techniques to problems in a wide range of practical contexts  | PX/VA demonstrates confidence in extending known concepts to derive results with broader applicability  | HX1/VA appreciates interrelationships between ideas drawn from different areas of Mathematics   | HX2/VA appreciates the creativity, power and usefulness of Mathematics to solve a broad range of problems              |

## 5.6 Content

**Content** describes the knowledge, skills, understanding and values and attitudes to be studied and developed by students over a Stage or Stages in a course and the development of processes of learning so that students are encouraged to be effective learners.

**Note:** In the information presented below, proposed course topics shown in *italics* are new topics in the calculus-based courses or are new in the particular course described.

Appropriate arrangement of the proposed course topics into Preliminary/HSC courses will be undertaken in the next phase (*Syllabus Development Phase*) of the syllabus development process.

### Mathematics Advanced

(For each topic, the necessary assumed knowledge, skills and understanding are to be identified for review.)

Proposed course topics:

- *Counting techniques*
- *Probability (simple and counting methods)*
- Real functions and their graphs
- Trigonometry
  - Right-angle triangle trigonometry
  - Introduction to trigonometric functions (using radians)
  - Calculus of trigonometric functions (including applications)
- Differential calculus
  - Introduction
  - Geometrical applications of differentiation
- *Data analysis*
  - *Types of variables, measures of centre and variability, graphical representations of data*
  - *Simple discrete and continuous probability models, expected value and the Normal distribution*
- Sequences and series
- Integral calculus
  - Introduction
  - Areas and volumes
- Logarithmic and exponential functions (including applications of calculus)
- *Mathematical modelling*
  - Applications of calculus

## Mathematics Extension 1 (includes Preliminary Mathematics Extension)

Proposed course topics:

- Circle geometry
- Further algebra (including sum and product of roots of quadratic equations, quadratic identities)
- *Transformations of graphs*
- Other inequalities
- Polynomials
  - Polynomial equations, graphs
  - *Multiple roots of polynomials*
- *Elementary difference equations and the discrete logistic growth model*
- Mathematical induction (series and divisibility only)
- The binomial theorem, binomial identities and the binomial probability distribution (*including expected value*)
- Further trigonometry (sums and differences, general solutions, auxiliary angle, and angle between two lines)
- Methods of integration (including substitution, the primitive of  $\sin^2 x$  and  $\cos^2 x$ )
- Inverse functions (including inverse trigonometric functions)
- Further applications of calculus involving *mathematical modelling* (including motion, modified growth and decay, and Newton's method)

## Mathematics Extension 2

Proposed course topics:

- Further inequalities
  - induction with inequalities
  - proof using graphs and calculus
- Complex numbers and polynomials over the complex field
  - geometric representation
  - vectors
  - powers and roots
  - curves and regions
- Graphs
  - sketching basic curves
  - addition, subtraction, multiplication, division and reflection
  - general approach to curve sketching
- Integration techniques
  - *t*-formulae
  - partial fractions
- Volumes
  - slicing
  - cylindrical shells
- *First and second order ordinary differential equations and modelling*
  - including aspects of mechanics
  - simple harmonic motion

## 5.7 Use of technology

### (a) in learning and teaching, and school-based assessment

The appropriateness, viability and level of use of different types of technology in the learning and teaching of courses within the Mathematics Key Learning Area are decisions for students, teachers and schools. However, the Broad Directions developed during the first phase of the *Mathematics Stage 6 Review and Development Project* included: ‘That the use of technology with capabilities beyond the level of scientific calculators be encouraged in the learning and teaching, and school-based assessment, of all Stage 6 Board-developed Mathematics courses.’

The final syllabuses will provide a range of opportunities for the use of calculators and computer software packages in learning and teaching. This will include opportunities to utilise the graphing functions and financial and statistical capabilities of calculators, and dynamic geometry and statistics software packages.

### (b) in the HSC examinations

The Broad Directions include: ‘That the use of technology in HSC examinations for the calculus-based courses be further investigated and clarified in the Writing Brief phase.’ In accordance with this Broad Direction, it is proposed that in HSC examinations for these courses, candidates be permitted to use only calculators manufactured to meet a clear set of Board-prescribed calculator functions and capabilities. These functions and capabilities, which will be beyond the level of scientific calculators, will be consistent with and support the knowledge and skills that students should be able to demonstrate after completing the Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 courses. For this reason, the functions and capabilities will be determined in parallel with the development of the content for the courses in the next phase (*Syllabus Development Phase*) of the syllabus development process.

## **5.8 Assessment and HSC examination**

The general assessment and reporting advice in the Mathematics Stage 6 calculus-based syllabuses will focus on the role of assessment in improving learning and teaching.

The following requirements and advice in relation to internal assessment and external examinations will be contained in the syllabus:

- Preliminary course components and weightings (advisory) and HSC course components and weightings (mandatory). The components will be based on related outcome groupings (eg knowledge and skills, interpretation and communication), and the associated weightings will reflect the relative importance of the outcomes within the courses.
- HSC examination specifications.

The following materials will be developed for each course:

- performance band descriptions
- a specimen HSC examination, including a mapping grid linking specimen examination questions to syllabus outcomes, content and targeted performance bands, and sample marking guidelines.

## **6 Glossary**

## 6 Glossary

The **Glossary** explains terms that will assist teachers in the interpretation of the Mathematics Stage 6 calculus-based syllabuses.

### Syllabus terminology

#### Content

Content describes the knowledge, skills, understanding and values and attitudes to be studied and developed by students over a Stage or Stages in a syllabus and the development of processes of learning so that students are encouraged to be effective learners.

#### Outcomes

Syllabus outcomes express the specific intended student learning that results from the teaching of the course(s). They are derived from the objectives and content of the syllabus. Outcomes provide clear statements of the knowledge, skills and understanding expected to be gained by most students as a result of effective learning and teaching by the end of a Stage.

#### Standards

The term *standards* refers to the knowledge, skills and understanding expected to be learned by:

- students as a result of studying a subject – the *content standards*
- the levels of achievement of the knowledge, skills and understanding – the *performance standards*.

Both *content standards* and *performance standards* are based on the aims, objectives, outcomes and content of a course. Together they specify what is to be learned and how well it is to be achieved.

*Content standards* specify what students are expected to know, understand and be able to do as a result of studying a course. Teacher understanding of *content standards* comes from their consideration of the aims, objectives, outcomes and content of the syllabus.

*Performance standards* are the different levels of achievement demonstrated by students.

## **Subject**

A subject is a name given to a defined area of knowledge. There may be several courses offered in a subject.

## **Syllabus**

A document that describes for a key learning area or a course of study what students are expected to learn in terms of aims, objectives, outcomes, content and assessment requirements.

A **syllabus package** includes a syllabus document with additional information on assessment and examination, and support materials.

## **7 Support Materials**

## **7 Support Materials**

It is proposed that a range of support materials be produced to assist teachers with the implementation of the new Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 courses.

In accordance with the Broad Direction: ‘That the syllabus documents within the Stage 6 Mathematics syllabus package incorporate applications, implications and considerations for the teaching of the syllabus content, including in relation to depth of coverage’, it is proposed that each of the courses include an ‘applications, considerations and examples’ section within the syllabus document for each of the topic areas.

The support materials could include:

- sample learning and teaching units
- advice on programming
- program overviews
- advice in relation to teaching ‘new’ areas of course content
- teaching suggestions to assist the development of meaningful and engaging units of work
- sample HSC assessment programs
- advice on the utilisation of different types of assessment tasks
- sample assessment tasks
- suggestions for the utilisation of technology in learning and teaching
- suggested applications that relate to real-world problems.

## **8 Appendix**

## 8 Appendix

### **Broad Directions for the Mathematics Stage 6 Writing Briefs – endorsed by the Board of Studies on 12 December 2006.**

**Note:** *The Board of Studies endorsed these Broad Directions on 12 December 2006. They reflect the consultation undertaken in Phase 1 of the syllabus development process and inform the development of Section 4 of the Draft Writing Brief, which is subject to consultation. The Broad Directions are not subject to consultation.*

Broad Directions for the Mathematics Stage 6 Writing Briefs:

- That the set of Stage 6 Mathematics courses include an additional offering to accommodate the purposes of students who wish to study a Board-developed Mathematics course in Stage 6 but who are currently choosing not to, as well as those whose purposes are not accommodated through the study of General Mathematics.
- That in the revision, due attention be given to clarifying the purpose of each course and identifying future learning or vocational pathways of the intended candidatures.
- That the nested structure of the current Mathematics ('2 Unit'), Mathematics Extension 1 and Mathematics Extension 2 courses, and the calculus-basis to these courses, be retained.
- That any revision or development of the calculus-based courses maintain the current rigour and level of challenge of the courses.
- That the amount of content prescribed for Stage 6 Mathematics courses reflect the amount that can be taught, and learnt by the typical student, in the indicative time.
- That in reviewing the content of Stage 6 Mathematics courses, particular attention be given to the purpose of the Mathematics ('2 Unit') course for Mathematics ('2 Unit')-only students, and the appropriateness and relevance of the course content for those students.
- That the inclusion of additional study of statistics be considered, while addressing implications in relation to the extent of relevant teacher expertise, professional development, future pathways of students, school Mathematics staffing, and school timetabling.
- That the current General Mathematics course material be largely maintained within the structure of Stage 6 non-calculus-based Mathematics courses.
- That, in reviewing the content of the calculus-based courses, the appropriateness and relevance of the applications within the courses be explored, with a view to ensuring that they are contemporary and that they meet the needs of students.
- That, in the consideration of the use of technology in Stage 6 Mathematics courses, due regard must be given to the related access and equity issues.

- That the use of technology with capabilities beyond the level of scientific calculators be encouraged in the learning and teaching, and school-based assessment, of all Stage 6 Board-developed Mathematics courses.
  - That the non-calculus-based Stage 6 Mathematics courses be developed with the view that technology with capabilities beyond the level of scientific calculators will need to be utilised for aspects of the associated HSC examinations.
  - That the use of technology in HSC examinations for the calculus-based courses be further investigated and clarified in the Writing Brief phase.
  - That the appropriateness of the current processes for the examination of Stage 6 Mathematics courses be reviewed, with particular emphasis on the examination of Mathematics ('2 Unit')-only candidates.
  - That the syllabus documents within the Stage 6 Mathematics syllabus package incorporate applications, implications and considerations for the teaching of the syllabus content, including in relation to depth of coverage.
  - That each Stage 6 Mathematics course be named so as to avoid confusion with the discipline itself.
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