BOARD OF STUDIES
NEW SOUTH WALES

General Mathematics

Stage 6

Support Document

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Contents

1 Introduction

This material is designed to assist teachers with the implementation and teaching of the *General Mathematics Stage 6 Syllabus*.

The support document does not attempt to cover all aspects of the syllabus. Its purpose is to raise awareness in particular areas of content and to provide advice about specific elements. This advice is neither prescriptive nor exhaustive, and it should be recognised that other approaches may be taken successfully.

Sections of this material may form the basis of professional development activities within and among schools and school systems.

2 Programming Overviews

This section demonstrates a range of ways in which both the Preliminary and HSC components of the course may be programmed. Each overview is based on 120 (indicative) hours for each of the Preliminary and HSC courses.

The overviews presented here have been prepared by teachers from a range of schools, and are examples only. They indicate some of the ways in which units of work may be grouped to form teaching sequences, and different ways of ordering them within the school program.

Each overview indicates periods for school assessment. These periods vary in both location and duration, and do not imply any specific number of tasks or style of assessment instrument. Decisions about number, timing and style of assessment tasks are made at the school level.

There is no predetermined order in which the course should be taught. This flexibility supports different programming arrangements to suit the requirements of different schools and classes of students. In schools where there are multiple classes in General Mathematics, flexible programming could reduce pressure on particular resources such as computing facilities, by allowing different classes access at different times.

Particular students may wish to transfer from the Mathematics Stage 6 course into General Mathematics in the early stages of the Preliminary course. This movement can be facilitated by maintaining commonality in the programming of the two courses for as long as possible, as illustrated in Overviews 2 and 3.

Overview 1

TERM 1

Weeks 1–5	**Weeks 6–10**
FM1 FM2 FM3	M1 M2 M3

TERM 2

Weeks 1–2	**Weeks 3–5**	**Weeks 6–8**	**Weeks 9–10**
M4	PB1 PB2	Review and school assessment period	AM1

TERM 3

Weeks 1–2	**Weeks 3–7**	**Weeks 8–10**
AM2	DA1 DA2 DA3 DA4	Review and school assessment period

TERM 4

Weeks 1–5	**Weeks 6–10**
AM3 AM4	FM4 FM5 FM6

TERM 1 (YEAR 12)

Weeks 1–7	**Weeks 8–10**
M5 M6 M7	Review and school assessment period

TERM 2

Weeks 1–7	**Weeks 8–10**
DA5 DA6 DA7	PB3 PB4

TERM 3

Weeks 1–2	**Weeks 3–10**
PB4 (cont.)	Trial HSC examination and preparation for final examination

Overview 2

TERM 1

Weeks 1–4	**Weeks 5–7**	**Weeks 8–9**	**Week 10**
AM1 AM2	M1 M2	DA1 DA2	Review and school assessment period

TERM 2

Weeks 1–5	**Weeks 6–10**
FM1 FM2 FM3	DA3 DA4

TERM 3

Weeks 1–5	**Weeks 6–9**	**Week 10**
M3 M4	PB1 PB2	Review and school assessment period

TERM 4

Weeks 1–4	**Weeks 5–8**	**Weeks 9–10**
M5 M6	FM4 FM5	Review and school assessment period

TERM 1 (YEAR 12)

Weeks 1–3	**Weeks 4–9**	**Week 10**
DA5	AM3 AM4	Review and school assessment period

TERM 2

Weeks 1–4	**Weeks 5–8**	**Weeks 9–10**
PB3 PB4	DA6 DA7	M7

TERM 3

Weeks 1–4	**Weeks 5–10**
FM6	Trial HSC examination and preparation for final examination

Overview 3

TERM 1

Weeks 1–3	**Weeks 4–5**	**Weeks 6–7**	**Weeks 8–9**	**Week 10**
M1 M2	AM1	PB1	M3	Review and school assessment period

TERM 2

Weeks 1–2	**Weeks 3–4**	**Weeks 5–6**	**Weeks 7–9**	**Week 10**
M4	AM2	PB2	DA1 DA2	FM1

TERM 3

Weeks 1–4	**Weeks 5–8**	**Weeks 9–10**
FM2 FM3	DA3 DA4	Review and school assessment period

TERM 4

Weeks 1–4	**Weeks 5–6**	**Weeks 7–9**	**Week 10**
M5 M6	AM3	DA5	Review and school assessment period

TERM 1 (YEAR 12)

Weeks 1–2	**Weeks 3–6**	**Weeks 7–9**	**Week 10**
DA6	PB3 PB4	FM4	Review and school assessment period

TERM 2

Weeks 1–3	**Weeks 4–5**	**Weeks 6–7**	**Week 8**	**Weeks 9–10**
FM5	DA7	M7	Review and school assessment period	FM6

TERM 3

Weeks 1–4	**Weeks 5–10**
AM4	Trial HSC examination and preparation for final examination

Overview 4

TERM 1

Weeks 1–6	Weeks 7–10
FM1 FM2 FM3	AM1 AM2

TERM 2

Week 1	Weeks 2–3	Weeks 4–9	Week10
DA1	Review and school assessment period	DA2 DA3 DA4	PB1

TERM 3

Weeks 1–3	Weeks 4–8	Weeks 9–10
PB1 (cont.) PB2	M1 M2 M3	Review and school assessment period

TERM 4

Weeks 1–2	Weeks 3–8	Weeks 9–10
M4	DA5 DA6 DA7	AM3

TERM 1 (YEAR 12)

Weeks 1–4	Weeks 5–8	Weeks 9–10
AM3 (cont.) AM4	FM4 FM5	Review and school assessment period

TERM 2

Weeks 1–3	Weeks 4–8	Weeks 9–10
FM6	M5 M6 M7	PB3

TERM 3

Weeks 1–2	Weeks 3–10
PB4	Trial HSC examination and preparation for final examination

3 Linking Outcomes to Units of Work

The following table indicates the individual units of work that will address particular outcomes.

Objectives Students will develop:	**Preliminary Outcomes** A student:	**HSC Outcomes** A student:
• appreciation of the relevance of mathematics	P1 develops a positive attitude to mathematics and appreciates its capacity to provide enjoyment and recreation **FM1, DA1, 2, 3, PB1**	H1 appreciates the importance of mathematics in her/his own life and its usefulness in contributing to society **FM4, 5, 6, DA5, 7, M5, 6, 7, AM4**
• ability to apply mathematical skills and techniques to interpret practical situations	P2 applies mathematical knowledge and skills to solving problems within familiar contexts **FM1, 2, 3, DA4, M1, 2, 3, 4, PB2, AM1**	H2 integrates mathematical knowledge and skills from different content areas in exploring new situations **FM4, 5, 6, DA5, 6, 7, M5, 6, 7, PB3, 4, AM3, 4**
• skills, knowledge and understanding in Algebraic Modelling	P3 develops rules to represent patterns arising from numerical and other sources **FM2, M4, PB1, AM1, 2**	H3 develops and tests a general mathematical relationship from observed patterns **M5, PB3, AM3, 4**
	P4 represents information in symbolic, graphical and tabular forms **DA3, 4, PB2, AM2**	H4 analyses representations of data in order to make inferences, predictions and conclusions **DA5, 6, 7, PB3, 4**
	P5 represents the relationships between changing quantities in algebraic and graphical form **FM3, M1, AM2**	H5 makes predictions about the behaviour of situations based on simple models **FM4, 5, 6, DA5, 6, 7, AM4**

Objectives Student will develop:	**Preliminary Outcomes** A student:	**HSC Outcomes** A student:
• skills, knowledge and understanding in Measurement	P6 performs calculations in relation to two-dimensional and three-dimensional figures **M2, 3, 4**	H6 analyses two-dimensional and three-dimensional models to solve practical and mathematical problems **M5, 6, 7**
	P7 determines the degree of accuracy of measurements and calculations **FM1, 2, 3, DA3, 4, M1, 2, 3, 4, AM1**	H7 interprets the results of measurements and calculations and makes judgements about reasonableness **M5, 6, 7, AM3**
• skills, knowledge and understanding in Financial Mathematics	P8 models financial situations using appropriate tools **FM1, 2, 3**	H8 makes informed decisions about financial situations **FM4, 5, 6**
• skills, knowledge and understanding in Data Analysis	P9 determines an appropriate form of organisation and representation of collected data **DA1, 2, 3**	H9 develops and carries out statistical processes to answer questions which she/he and others have posed **DA5, 6, 7**
• skills, knowledge and understanding in Probability	P10 performs simple calculations in relation to the likelihood of familiar events **PB1, 2**	H10 solves problems involving uncertainty using basic principles of probability **PB3, 4**
• ability to communicate mathematics in written and/or verbal form	P11 justifies his/her response to a given problem using appropriate mathematical terminology **FM1, 2, 3, DA1, 2, 3, 4, M4, PB1, 2**	H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others **FM4, 5, 6, DA5, 6, 7, M5, 6, 7, PB3, 4, AM3, 4**

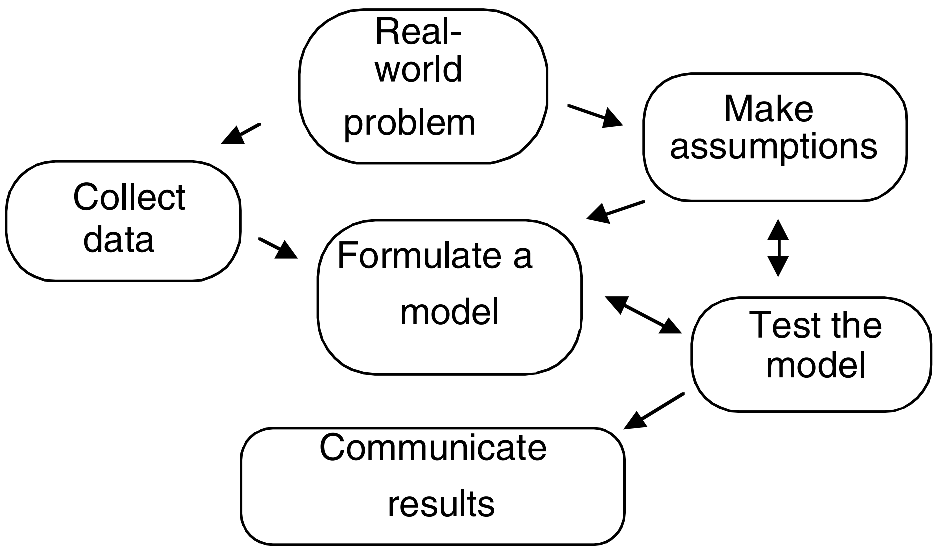
4 What is 'Modelling'?

Mathematical modelling

The *General Mathematics Stage 6 Syllabus* places emphasis on the process of using information to create, test and interpret simple mathematical models.

A mathematical model is a mathematical representation of a situation. Indeed, all applications of mathematics are based on mathematical models. People use simple mathematical models almost every day of their lives. A mathematical model does not need to be complex.

The following diagram illustrates the cyclic nature of the modelling process, while the associated text further explains each step and places it in the context of a specific example.



In the example, the modelling process is demonstrated in relation to a simple problem involving the relationship between speed, distance and time.

Real-world problem

Given that you are travelling to an important appointment and that you are only part-way there, will you be on time?

Collect the data

If the speed limit is 80 km/h and the sign indicates that it is 123 km to your destination, what additional information do you need?

Make and clarify assumptions

What assumptions do you need to make to solve the problem in real time[1]*?*
Is there sufficient petrol in the car, can you maintain a constant speed, will the speed limit change, will you round-off the values?

Formulate a simple mathematical model

Time = distance ÷ speed

Test the model

If you can maintain a speed of 80 km/h it will take about 1.5 hours to arrive.

Communicate results: explain, predict, recommend

Will you make the appointment? Can any elements of the situation be changed so that you will make it?

The processes involved in modelling are interrelated. Even with the very simple example described above, predicting from, and testing of, the model will generally go through several iterations.

In creating a mathematical model, assumptions and simplifications have to be made. Different models make different assumptions. Newton's laws of motion result in one mathematical model. Einstein's laws of motion involve different assumptions and result in a different mathematical model. Each model is in use and, for general applications, produces similar results. The choice of which model to use is then often governed by the simplicity of the calculations.

Mathematical modelling develops students' mathematical reasoning skills by requiring students to explore, conjecture, validate and convince others. Students use inductive reasoning when they make conjectures by generalising from data. They use deductive reasoning when they test those conjectures.

Learning activities should help students relate mathematical concepts to real-world situations. Students become aware of the usefulness of mathematics when mathematical ideas are connected to everyday experiences. This approach emphasises that mathematics helps solve problems, describes and models real-world phenomena, and communicates complex thoughts and information with conciseness and precision.

> Incorporating applications and modelling in the curriculum substantially assists the acquisition and understanding of mathematical ideas, concepts, methods and theories, and provides illustrations and interpretations of them. (Lee et al, 'Applications and Modelling', *Proceedings of the 5th International Congress on Mathematical Education (ICME)*, M Carss (ed), Boston Birkhauser Inc, Boston, 1986)

[1] If your method takes 2 hours, you cannot solve the problem in real time (ie before the answer is irrelevant).

Approaches to modelling

1. *Structured modelling*, in which the teacher directs the process and selects the model to be used.
2. *Open modelling*, in which students select the model to be used.

Depending upon the prior experiences of students, it may be appropriate to start with structured modelling and allow sufficient time to make the process of modelling explicit.

In teaching the modelling process, the following steps are recommended.

1 Understand the problem

What is the problem actually asking you to do? Encourage discussion, either on a whole class basis or in groups, and establish a shared view of the task at hand. Rephrase the problem in the students' own words.

2 Choose a model

Do you have enough information? Will you need to make any assumptions? What mathematical skills are needed? Is there more than one way to address the problem? What mathematical processes will be useful? Determine the 'best' model to use.

3 'Solve' the problem

Use the model chosen in Step 2 to produce a ***mathematical*** response to the problem. This could involve graphs, calculations, scale drawings or a range of other techniques and skills.

4 Interpret the solution

Explain the relationship between the 'answer' obtained in Step 3 and the solution to the problem as you described it in Step 1.

5 Evaluate the solution

Is your solution sensible? Does it address the issues and assumptions raised in Step 2? Were the assumptions you made valid? Did you miss something? At this point you may even decide to start all over again. An 'answer' is not the same thing as a solution to a real problem!

6 Report your results

Who needs to know the solution? How best can you communicate your findings to those who need to know? What style of reporting – verbal, written, audiovisual – will best suit your purpose? Prepare an appropriate report for the relevant audience.

5 Aspects of Financial Mathematics

The following advice is provided in relation to content presented in the Financial Mathematics area of study. It is not intended that it form part of classroom teaching.

5.1 Use of Formulae

The future value of an annuity

(Months will be used in this example, but other time periods may be used.)

Assume an investment of M dollars at the end of each month, for n months at a given percentage per month. Note that 'r' appears in the formula as the percentage interest rate per month, expressed as a decimal. For example, if the interest rate is 6% per annum, then the value of r in this formula is 0.06/12, that is, 0.005. Consider the balance of the account at the end of each month. After one month you have M dollars, and after two months you have $M + M(1+r)$ dollars since the original M dollars has grown by the product of M and r.

After three months you have $M + M(1+r) + M(1+r)^2$ dollars, and so on.

After n months, you have $M + M(1+r) + M(1+r)^2 + M(1+r)^3 + ... + M(1+r)^{n-1}$ dollars, which is the sum of n terms of a geometric series with first term M and common ratio $(1+r)$.

Use the formula for the sum of a geometric series. Perhaps first write the formula as follows:

$$M + MR + MR^2 + MR^3 + ... + MR^{n-1} = \frac{M\left(R^n - 1\right)}{R-1}$$

Then substitute $R = 1 + r$ and simplify to get the required result:

$$A = M\left(\frac{(1+r)^n - 1}{r}\right)$$

The present value of an annuity

To obtain N, the present value of an annuity, we use the idea of compound interest. The present value of anything is how much money you would have to invest *now* in order to have a certain future value, A, after a period of time denoted by n.

The relationship is
$future\ \ value\ = present\ \ value \times (1+r)^n$, or $A = N(1+r)^n$
Dividing through by $(1+r)^n$ gives $N = \dfrac{A}{(1+r)^n}$, or $N = A(1+r)^{-n}$.

Substituting the formula for A gives rise to the formula for present value (N) given on p 53 of the syllabus:

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

Contribution per period

Another formula needed is that for the monthly repayment on a home loan, or on any loan with 'reducible interest', where the principal, as well as the interest, is paid off during the course of the loan. This periodical repayment equates to M in the formula above when N is the principal borrowed.

Consider the situation from the point of view of the lender (eg bank, building society). The borrower is obliged to pay a monthly amount for a certain number of months. This accumulates in the same way as an annuity. So at the end of the loan, the lender has received an amount equivalent to an annuity over n months at M dollars per month. The *present value* of this 'annuity' corresponds to the amount borrowed. In the formula for present value above, making M the subject leads to the following formula for monthly repayments on a home loan. (N is replaced with P, the usual abbreviation for the principal or amount borrowed. Note that r in the formula has the same meaning as that described earlier, that is, it is the percentage interest rate per month, expressed as a decimal.)

$$M = \frac{P\,r}{1-(1+r)^{-n}}$$

This formula is not given in the syllabus. It is intended that students use the formula for the present value of an annuity (N), calculate the value in the braces, and then find M. For example, to find the monthly repayment on a home loan of \$100 000 at 12% pa, compounded monthly for 20 years, we use

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

Now r = 0.01, n = 240, so the value in the braces is 90.8194. Students will need practice in the calculator procedure to arrive at this result.
Hence $N = M \times 90.8194$
ie 100 000 = $M \times 90.8194$
so that M = 1101.09. The monthly repayment on this loan will be \$1101.09. This is checked in the table (representing a spreadsheet) overleaf.

Verifying a home loan calculation

Number of periods 240

Amount borrowed $100 000

Interest rate pa 12%

Amount of payment each period $1101.09

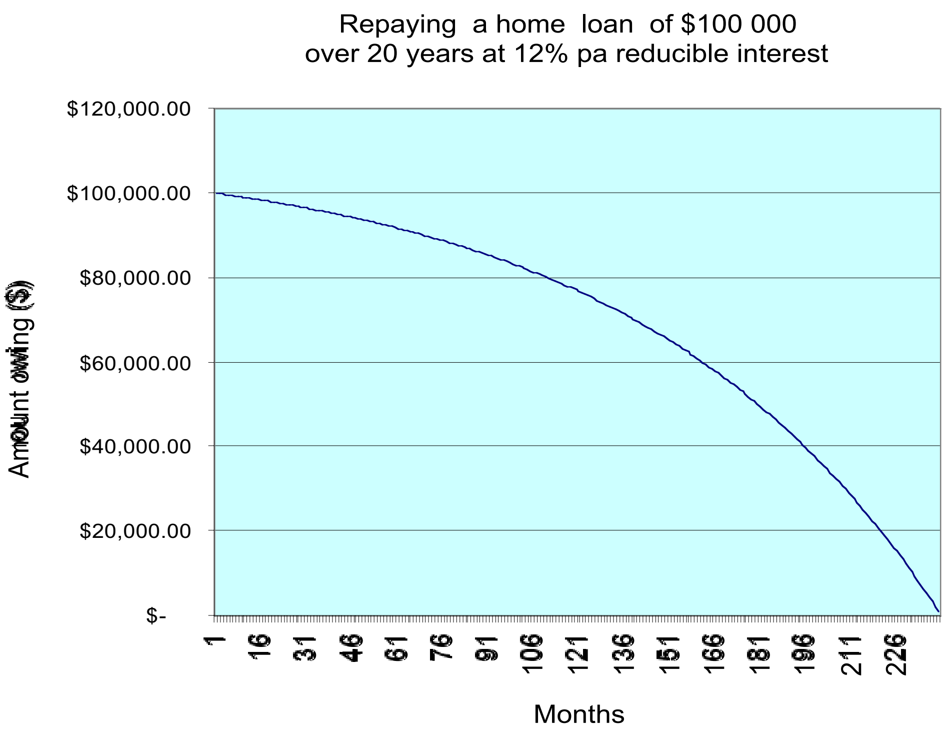
Period	Amount ($) owing at start of period	Interest ($) this period	Amount ($) paid at end of period	Amount ($) owing at end of period
1	100 000.00	1000.00	1101.09	99 898.91
2	99 898.91	998.99	1101.09	99 796.81
3	99 796.81	997.97	1101.09	99 693.69
4	99 693.69	996.94	1101.09	99 589.53
5	99 589.53	995.90	1101.09	99 484.34
6	99 484.34	994.84	1101.09	99 378.09
7	99 378.09	993.78	1101.09	99 270.78
8	99 270.78	992.71	1101.09	99 162.40
9	99 162.40	991.62	1101.09	99 052.94
10	99 052.94	990.53	1101.09	98 942.37

(Lines 11 to 234 omitted)

235	6377.74	63.78	1101.09	5340.43
236	5340.43	53.40	1101.09	4292.74
237	4292.74	42.93	1101.09	3234.58
238	3234.58	32.35	1101.09	2165.83
239	2165.83	21.66	1101.09	1086.40
240	1086.40	10.86	1101.09	**0.00**
TOTAL		164 257.78	264 261.60	

This table is available as a spreadsheet as part of the electronic support included with this publication.

Note the differences in presentation between the table and the spreadsheet — in particular, the continued use of commas in spreadsheets to separate thousands in large numbers.


Repaying a home loan of $100 000
over 20 years at 12% pa reducible interest
$120,000.00
$100,000.00
$80,000.00
$60,000.00
$40,000.00
$20,000.00
$-
Amount owing ($)
1
16
31
46
61
76
91
106
121
136
151
166
181
196
211
226
Months

5.2 Linking Financial Mathematics with Algebraic Modelling in the Syllabus

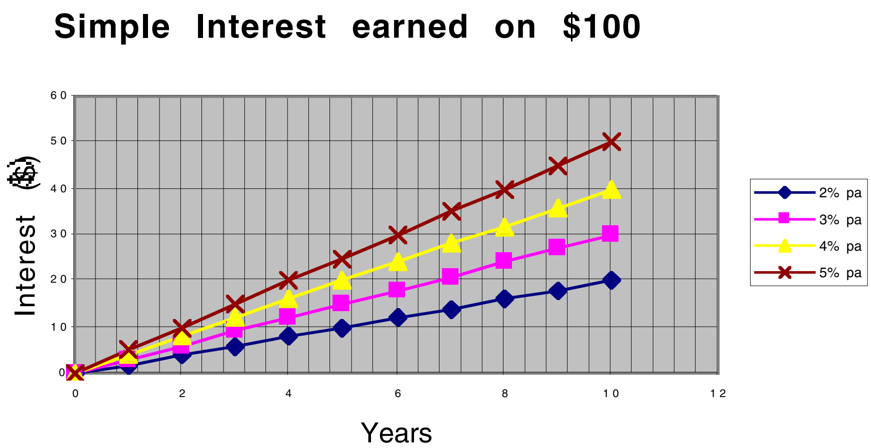
The General Mathematics Stage 6 Syllabus stresses the use of graphs to investigate various concepts in other parts of the course. Examples of how this may be achieved in the Financial Mathematics area of study are given below.

1 Investigating simple interest

Sam has earned $100 and wants to see how much interest it can earn over ten years at the following rates of interest pa: 2%, 3%, 4%, 5%. The following table is compiled (a spreadsheet could be used but is not essential).

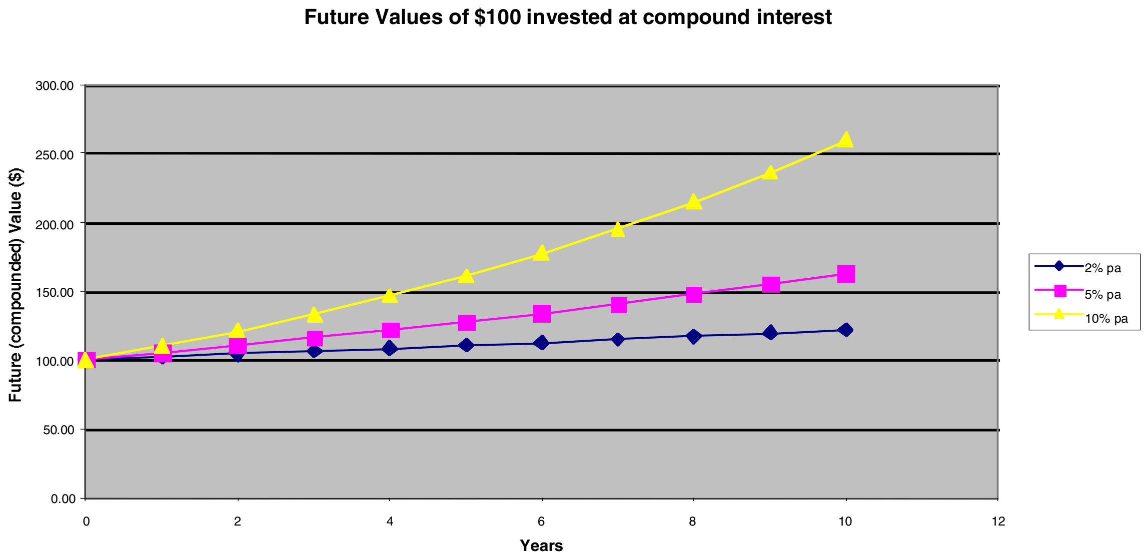
Years	1	2	3	4	5	6	7	8	9	10
Interest at 2%	$2	$4	$6	$8	$10	$12	$14	$16	$18	$20
Interest at 3%	$3	$6	$9	$12	$15	$18	$21	$24	$27	$30
Interest at 4%	$4	$8	$12	$16	$20	$24	$28	$32	$36	$40
Interest at 5%	$5	$10	$15	$20	$25	$30	$35	$40	$45	$50

From the table, graphs can be drawn showing the effect of the interest rate and the period on the amount of interest. Such graphs would clearly demonstrate the linear nature of the relationship. The graph below was produced using *Excel*, but graphs could also be drawn by hand. Note that the graphs are straight lines whose gradient is determined by the interest rate.



2 Investigating compound interest

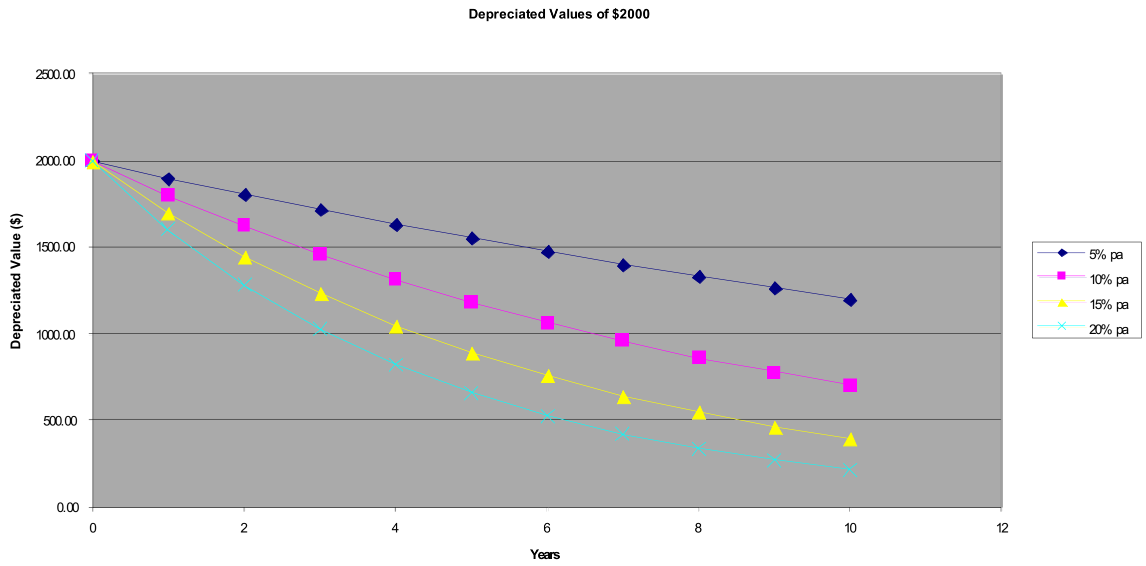
As for simple interest, graphs could be drawn showing the amount that $100 (say), becomes after a number of years invested at compound interest. It should be noted that, despite the appearance of the 2% pa curve below, these are examples of exponential graphs. The example below has been produced using spreadsheet software, but students could develop similar graphs by hand, especially in the initial stages.



3 Models of depreciation

The graph on the following page shows the way that $2000 depreciates at different rates, demonstrating depreciation as a case of exponential decay. The graphs could be drawn by hand or by using spreadsheet software. The table used to draw the graphs has been prepared on a spreadsheet.

Years Rate (pa)	0	1	2	3	4	5	6	7	8	9	10
5%	2000.00	1900.00	1805.00	1714.75	1629.01	1547.56	1470.18	1396.67	1326.84	1260.50	1197.47
10%	2000.00	1800.00	1620.00	1458.00	1312.20	1180.98	1062.88	956.59	860.93	774.84	697.36
15%	2000.00	1700.00	1445.00	1228.25	1044.01	887.41	754.30	641.15	544.98	463.23	393.75
20%	2000.00	1600.00	1280.00	1024.00	819.20	655.36	524.29	419.43	335.54	268.44	214.75


Depreciated Values of $2000
2500.00
2000.00
1500.00
1000.00
500.00
0.00
Depreciated Value ($)
0
2
4
6
8
10
12
Years
5% pa
10% pa
15% pa
20% pa

6 Aspects of Data Analysis

The following advice is provided in relation to content presented in the Data Analysis area of study. It is not intended that it form part of classroom teaching.

6.1 Stem-and-leaf Plots

A stem-and-leaf plot is a graph of a data set in which the graph is constructed out of the data points themselves. It is a simple representation of the data that provides a clear 'picture' of its distribution.

To make a stem-and-leaf plot, first separate each number in the data set into a stem (the first digit or digits) and a leaf (the final digit). The stems may consist of any number of digits, but the leaves can only be a single digit.

Example: The following data represents the heights (cm) of a group of Year 11 students.

171	185	167	174	150	173	192	170
173	169	162	162	182	186	164	178

First, arrange the stems in order, to the left of a vertical line.

15	
16	
17	
18	
19	

For each data point, add the leaf to the stem in the appropriate location, as follows:

15	0
16	94722
17	183304
18	526
19	2

Finally, arrange the leaves in ascending order, from the stem outwards. The completed stem-and-leaf plot appears overleaf.

15	0
16	22479
17	013348
18	256
19	2

In a stem-and-leaf plot the largest, smallest, and modal scores can clearly be seen. It is also relatively easy to find the median and the range.

It is important to look at the overall pattern, and then the particular characteristics of the distribution. Is the distribution a symmetrical shape? Where is the centre and how spread out are the scores? Is there one ‘peak’ or are there several?

Using back-to-back stem-and-leaf plots, it is possible to compare two data sets. Consider a second set of student heights from the same group of Year 11 students:

151	158	166	171	145	174	190	180
153	149	160	162	177	186	158	159

The following back-to-back plot may be used to investigate differences between the data sets, and to make conjectures about the source of the data.

95	14	
98831	15	0
620	16	22479
741	17	013348
60	18	256
0	19	2

What conclusions might be drawn about these two groups of students?

In some circumstances it may be necessary to round or truncate the numbers in the data set. This is useful when there are many stems with no leaves or just one leaf.

If the leaves are crowded onto too few stems then it might be useful to split each stem into two, one for leaves 0 to 4 and the other for leaves 5 to 9.

6.2 Five-number Summaries and Box-and-whisker Plots

The box-and-whisker plot is the graph of the five-number summary.
The five-number summary consists of:

- lower extreme
- first (lower) quartile
- median
- third (upper) quartile
- upper extreme.

These statistics for a data set provide information about the centre, spread and range.

To calculate the quartiles, first locate the median (Q_2) in the ordered data. The first quartile (Q_1) is the median of the data ***below*** Q_2, and the third quartile (Q_3) is the median of the data ***above***. Remember, if there is an even number of data points, the median is the average of the two middle data points.

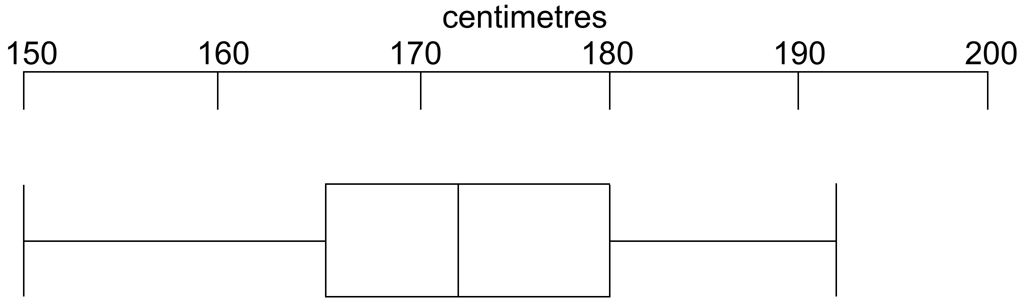
Example: Refer to the heights of the original group of Year 11 students.

171	185	167	174	150	173	192	170
173	169	162	162	182	186	164	178

In this data set Q_2, the median, is $\frac{171+173}{2}=172$ cm. Q_1 , the first quartile is $\frac{164+167}{2}=165.5$ cm and Q_3 , the third quartile is $\frac{178+182}{2}=180$ cm.

Hence, the five-number summary for this data set is (150, 165.5, 172, 180, 192).

The corresponding box-and-whisker plot is obtained by locating the ends of the 'box' at the quartiles Q_1 and Q_3. The median is marked by a vertical line within the box. The 'whiskers' are the two lines extending outside the box to the lower and upper extremes.



The range, median and spread of the distribution are immediately apparent from a box-and-whisker plot. The graph highlights the location of the central 50% of the data, and is useful in observing the symmetry of the distribution.

Box-and-whisker plots can be presented in either a vertical or horizontal orientation. They are most effective in comparing two or more distributions.

6.3 The Normal Distribution and z-scores

A large data set often assumes a distribution that is described as *normal*. Many sets of real data are approximately normally distributed. A frequency graph of data that is normally distributed is a symmetric, bell-shaped curve, which can be described exactly by the mean and standard deviation of the data set. The usual notation for the *mean* of a population that is normally distributed is μ, the Greek letter *mu*, and its *standard deviation* is σ, the Greek letter *sigma*. It is important to note that $\bar{x}$ and s are specifically used to represent the mean and standard deviation of a *sample* drawn from a population.

For a population that is normally distributed, the mean, μ, is the centre of symmetry. The standard deviation, σ, affects the shape of the curve of any distribution. Variation to μ simply moves the curve along the axis without changing its shape. Decreasing σ makes the curve 'taller' and less spread out, and increasing σ makes the curve 'flatter' and more spread out.

In any normal distribution:

- approximately 68% of the data points lie within σ of the mean
- approximately 95% of the data points lie within 2σ of the mean
- approximately 99.7% of the data points lie within 3σ of the mean.

Example: Scores on a particular personality inventory scale are assumed to be normally distributed with a mean score of 50 and a standard deviation of 10. Between what two scores would you expect approximately 95% of the scores to lie?

Solution:
Approximately 95% of scores lie in the interval $\mu \pm 2\sigma$, that is between $50 \pm (2 \times 10)$. Hence, approximately 95% of scores lie between 30 and 70.

It is best to observe their *standardised* values when comparing observations from different sets of normally distributed data. It is usual to use z for these standardised values where

$$z = \frac{x - \mu}{\sigma}$$

If x is an observation from a set of normally distributed data with mean μ and standard deviation σ, then the standardised value z is an observation from a derived normal distribution with mean 0 and standard deviation 1. This set of z values is called the *standard normal distribution*.

Example: On her school report Rebecca scores 75% in Mathematics, where the mean is 65% and the standard deviation is 5. On the same report she scores 80% in Music where the mean is 75% and the standard deviation is 5.
Which is the better result, given that the students are of similar ability and the scores are approximately normally distributed?

Solution:

z-score (Mathematics) = $\frac{75 - 65}{5} = 2.0$

z-score (Music) = $\frac{80 - 75}{5} = 1.0$

On comparing z-scores it is clear that the 75% in Mathematics is a much better result than the 80% in Music on this school report!

If Rebecca's Music result were to be comparable to her Mathematics result, what would it need to be?

Solution: If x represents Rebecca's Music result,

$$\frac{x-75}{5} = 2.0$$
$$x - 75 = 10$$
$$x = 85$$

Her result in Music would have to be 85% to be comparable to her Mathematics result.

6.4 Population Standard Deviation and Sample Standard Deviation

Summary Statistics (DA4) and *The Normal Distribution* (DA6) refer to population standard deviation (σ_n) and sample standard deviation (σ_{n-1}). (Students are

expected to know which concept applies in a given situation and to be able to use their calculators to obtain σ_n and σ_{n-1}.)

Population Standard Deviation (PSD)

$$\sigma_{\mathrm{n}} = \sqrt{\frac{\sum(\mathrm{x_i} - \mu)^2}{\mathrm{n}}} \qquad (1)$$

Sample Standard Deviation (SSD)

$$\sigma_{\mathrm{n}-1} = \sqrt{\frac{\sum(\mathrm{x_i} - \overline{\mathrm{x}})^2}{\mathrm{n}-1}} \qquad (2)$$

Equation (1) above is appropriate for describing the *actual* standard deviation for a given set of scores. However, if these scores constitute a sample drawn from a population of scores, then this sample information is typically used to estimate the corresponding value for the population – the sample scores are not considered for their own sake. This leads to the problem that the variability in the sample value tends to be less than the population variability, and hence the standard deviation for the sample underestimates the PSD, ie. the standard deviation for the sample is a biased estimate of the PSD.

This problem is solved by applying a correction factor to the *variance* (the square of the standard deviation*)* which inflates it slightly in order for it to become an unbiased estimate of the *population variance.* The correction factor is based on the sample size - the extent of the bias (underestimation) is worse in small samples and gradually improves as the sample size increases. The corrected variance is called the *sample variance* and is the square of the Sample Standard Deviation (2) above.

The appropriate correction factor is $\frac{n}{n-1}$. A larger sample (which requires less adjustment) results in a smaller correction factor.

The sample variance ($\sigma_{n-1}{}^2$) is obtained by multiplying the population variance ($\sigma_n{}^2$) by the correction factor.

That is,

$$\frac{\Sigma(\mathrm{x_i}-\mu)^2}{\mathrm{n}} \leftarrow \frac{n}{n-1} = \frac{\Sigma(\mathrm{x_i}-\bar{\mathrm{x}})^2}{\mathrm{n}-1}$$

Note also that the sample mean ($\bar{x}$) is an unbiased estimate of the population mean (μ).

In examples presented in the General Mathematics course:

- the sample standard deviation would be used where a sample of scores has been taken from a population of scores in order for inference to be made to that population;

- the population standard deviation would be used where

  – a set of scores is being considered with no inference to be made to a larger population
  – the entire population has been surveyed (census).

6.5 Correlation

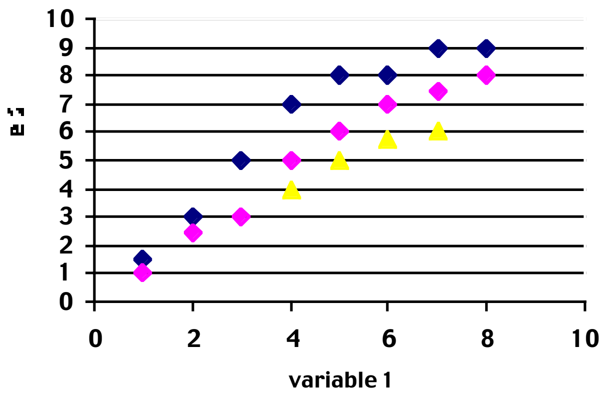
Correlation describes the association between the elements of the members of a set of ordered pairs. The correlation coefficient (r) is a measure of the strength of this association and the degree to which it varies from a linear relationship.

It is important to note that, for a perfect linear relationship, $r = 1 \ \ or \ \ -1$, and that the possible values for r lie in the range $-1 \le r \le 1$.

The calculation of correlation coefficients is not required in the General Mathematics course.

Representing ordered pairs as points in a scatterplot can illustrate the degree of correlation. The closer the 'cloud of points' is to a straight line, the closer the value of r is to ±1.

In the following scatterplot the correlation coefficient is ***positive*** (r >0).



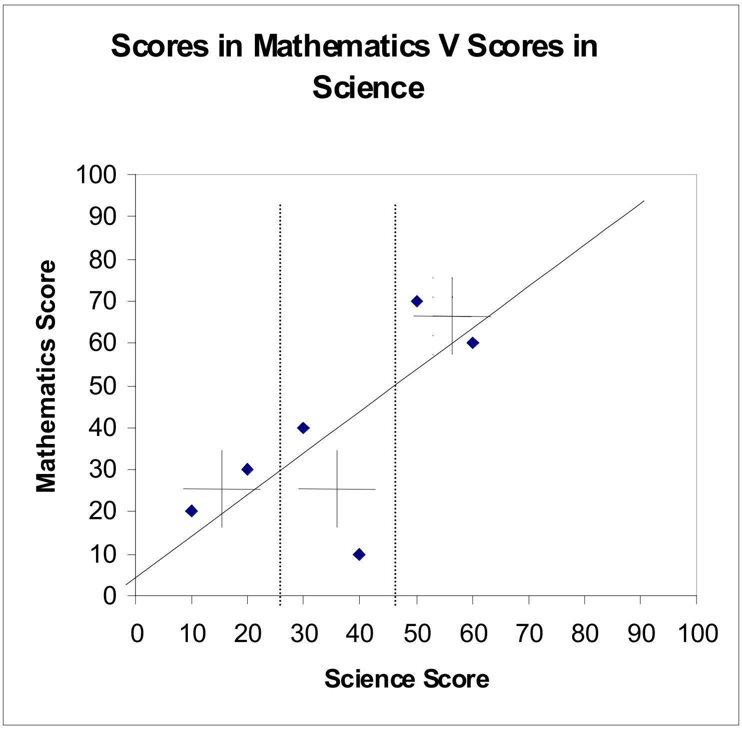
It is clear that as one variable increases, so does the other. In a similar scatterplot we could watch one variable decrease as the other increases. This is characteristic of a ***negative*** correlation.

Often we want to plot a 'line of fit' to a scatterplot that suggests a linear relationship, but whose points do not lie exactly on any particular straight line. The reason for finding a line of fit is to help us predict the value of one variable, given the value of the other.

To construct the *median regression line* of a scatterplot, first slice the plot into three vertical strips containing approximately equal numbers of points. If the number of data points is not exactly divisible by three, ensure that the extreme slices contain equal numbers of points. In each strip, determine and plot the point that represents the *medians* of both the *abscissae* and the *ordinates* (horizontal and vertical coordinates) of the points in that strip. Connect the median points in the extreme strips with a ruler. This determines the *gradient* (m) of the regression line. Slide the ruler one third of the way vertically towards the median point of the central strip, maintaining the existing gradient, and draw the line. Find the point where the line crosses the vertical axis (b). The equation for the line may then be written in the form $y = mx + b$. This process is illustrated on the following page, where a possible relationship between Mathematics (M) and Science (S) scores is examined. The equation of the resulting median regression line is $M = S + 5$.

In considering a line of fit, always construct a scatterplot first, to decide whether fitting a line makes sense. Be cautious of extrapolation outside the data set range. Lastly, it is important to note that a strong correlation does not always mean that there is a direct cause-and-effect link between the variables. A strong observed association can be due to direct cause-and-effect, but it can also be due to the effects of some other factor.

Example: Median regression line



7 Using Projects in Internal Assessment

This section provides some guidelines that may be helpful to teachers who are interested in using an investigative project as part of the school assessment program.

Syllabus outcomes specifically addressed by projects

A student

- P1 develops a positive attitude to mathematics and appreciates its capacity to provide enjoyment and recreation
- H1 appreciates the importance of mathematics in her/his own life and its usefulness in contributing to society
- P2 applies mathematical knowledge and skills to solving problems within familiar contexts
- H2 integrates mathematical knowledge and skills from different content areas in exploring new situations
- P11 justifies his/her response to a given problem using appropriate mathematical terminology
- H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Appropriate teaching and learning strategies

Teachers will need to consider Board of Studies requirements concerning submission of school assessment marks if planning the timing of a project in the HSC year. If a project is to be used as part of the assessment in the Preliminary year, it is suggested that it should be submitted towards the end of the course to enable students to choose from the full range of topics.

While students require clear exposition of standard problems and techniques, and straightforward directions for their investigations, these should not be the only teaching and learning methods employed. Teacher-centred lessons should be balanced by activities that encourage students to take more responsibility for their own learning.

These student-centred activities will be valuable to students in planning and carrying out independent investigative projects. However, students will require instruction in relation to the broad process to ensure that the specific outcomes are achieved.

Elements in developing a project or investigation

1 Choice of topic or area for investigation

Teacher support and advice at this stage are very important. Students lack experience in selecting topics that are appropriately challenging and that cover subject matter that is accessible to them.

2 Introduction

This is fundamentally a writing exercise, and should refer to the purpose of the project, the specific process(es) to be applied, and the anticipated outcome(s). Students generally have little experience in writing in mathematics, and will benefit from a gentle introduction to the idea through preparatory classwork.

3 Display and acknowledgement of sources of information and/or data

Investigative work is based on source material. Students may generate information through invention, through their own endeavours (such as a survey or questionnaire) or from existing sources such as Australian Bureau of Statistics (ABS) records or the Internet. Whatever the source, it must be clearly identified and the material displayed in a way that can be easily interpreted. This part of the process is a combination of writing and presenting mathematical information.

4 Mathematical calculations or other mathematical processes used in working with the material

The mathematics used in interpreting the source material and arriving at a conclusion must be shown clearly and logically. In the context of the original project topic, students should explain and justify their choice of mathematical processes at each stage and comment on their effectiveness. This is a skill that is often new to students, and preliminary work in class should highlight this aspect of working mathematically.

5 Conclusion

This is a significant element of the process, and will provide opportunities to assess the degree to which outcomes P11 and/or H11 have been achieved. Students may not be comfortable with the notion that their work does not appear to have ‘solved the problem’, but, if this is the case, then the conclusion should say so. Alternatively, if the work indicates that a different direction should have been taken, this should be stated in the conclusion. Independent project work often highlights the fact that a ‘problem’ may have **no** solution, or **one or more** solutions. The conclusion may simply be that more work needs to be done.

6 Critique

This is an opportunity for the student to reflect, in writing, on the work done, and to comment on any aspect of the process that was in any way noteworthy. Such comments may be in relation to the original choice of project topic, the way in which the information was generated, the mathematics or any other stage of the work. The process of reflection is an important part of independent work.

7 Bibliography and acknowledgements

Students are familiar with the need to give references and to acknowledge sources and support from their work in other subject areas. This is also essential in their project work in mathematics.

Each of these steps will benefit from specific instruction and the opportunity to practise any new skill prior to commencement of an actual project or investigation.

For the purposes of this course, a project or investigation should be:

- work that is original, from the student's perspective, in which a problem is posed, analysed mathematically and a solution or solutions presented; or

- a carefully compiled synthesis of available knowledge on a particular issue, preferably one which shows the key factors in a problem and which, from the student's point of view, shows the problem in a new light.

Possible project ideas

1. *Financial Mathematics*

   – To work or not to work: the competing issues of employment and government assistance
   – Easy start loans
   – Term and endowment insurance
   – Property purchases
   – Renting, buying or building a home: what are the options?
   – Costing and production in a particular business or rural industry
   – Consumer Price Index: calculation and movement over time
   – Canteen: profits versus labour
   – Stocks and shares: prices, trends, dividends, yields, profit/loss over time
   – General business applications: needs, fixed versus variable costs, profit margin, quotes, break-even analysis, depreciation (tables and graphs), straight line and reducing balance, effect on discount etc

2. *Data Analysis*

(a) Investigating distributions

   – City to Surf: are the times normally distributed? Are the times this year and last year the same?
   – Smoking habits
   – Eating habits
   – Student transport
   – Sport participation
   – TV viewing habits

(b) Investigating correlation

- Has unemployment reduced the road toll?
- Year 12 results compared with homework time, study time
- Correlation analysis arising out of sporting contests: average height versus scores (basketball), average weight versus winning times (rowing), shoe size versus tackle count (school rugby side) etc

3. *Measurement*

- Investigate the factors affecting the speed with which an individual is served in a supermarket
- Survey an area in two ways and compare the results
- Prepare and navigate an orienteering course
- Lay out a running track
- Design a watering/irrigation system for a home garden or property
- Global Positioning System (GPS) coordinates: use in satellites, airline navigation, taxi management

4. *Probability*

- Design and analyse a simple game of chance
- Are successful basketball shots random?
- Estimate and describe the size and nature of an unknown population
- Analysis of a gambling strategy
- What is the difference between selection with and without replacement?
- Comparison between 'systems' entries and individual entries in State-run lotteries

5. *Algebraic Modelling*

- Shortest path between two points on a cube, circle, cone, doughnut …
- A linear programming problem with constraints
- Networks: eg planning and carrying out pamphlet or mail deliveries
- Critical Path Analysis such as the optimal order for programming the sub-tasks of a complex operation, eg cooking a meal, organising a major public event

Management of Projects

1. Choice of topic

This may be managed in any of the following ways:

- The class chooses a theme, eg Olympics, the music industry, Hollywood, sport etc. Following a class brainstorming session, students choose a particular aspect of the theme for their projects
- The teacher prescribes a list of titles, from which students choose one
- Students nominate a project of their choice, which is relevant and of personal interest.

2. Teacher feedback

   Students should have a discussion with the teacher at each of the following stages:
   - Choice of topic and definition of scope
   - Outline of plan with major components identified and scheduled
   - Partial completion, at a pre-determined date
   - Pre-submission check.

   This dialogue provides support for the student, but also allows the teacher to be confident that the work is authentic.

   In providing feedback, the following issues may be discussed:
   - Has the student defined the problem/question?
   - Does the proposed project duplicate that of other students?
   - Is the scope of the project manageable?
   - Is the material gathered useful and relevant?
   - Is the product the student's own work?
   - Is the work presented clearly?
   - Does the work show what the student has learnt?
   - Is the mathematics clear in the work?
   - Is the mathematics used appropriate to the original question?
   - Is there any reference to the validity, reliability or accuracy of the findings?
   - Is there any discussion of the dynamics of the situation, eg what is the effect on the findings if one or more conditions change?

3. Student accountability

   It is essential that the teacher be confident that the submitted product is, in fact, the work of the student. There are a number of ways in which this issue may be dealt with. These include:

   - the processes of teacher/student dialogue and feedback referred to in point 2 above
   - student logbooks or diaries in which the progress of the work is documented, and which are checked by the teacher; these diaries may form part of the final work for assessment
   - the requirement that the final product for submission be prepared under teacher supervision
   - assessing some aspect(s) of the project through a sub-task administered under teacher supervision.

Length and duration of projects

In six to ten pages of work, students should be able to demonstrate their knowledge and understanding and provide adequate evidence upon which teachers can make judgements about achievement of outcomes. Those who choose to submit greater amounts of work should be neither advantaged nor disadvantaged in the assessment process.

If class time is to be allocated, a maximum of ten hours is suggested. Additional time out of class is at the discretion of the student, with advice from the teacher. However, it is suggested that the time period from choice of topic to submission of final product should not exceed three weeks.

Assessment

Since the inclusion of project work in the school assessment program is at the discretion of individual schools, the strategies used for assessment of projects will vary from school to school. Some suggestions for assessment of project work follow.

Marking models

Two possible marking models are given below. Neither is mandatory, nor is the list exclusive. However, the chosen model should be explained to students prior to their commencing work on the task, and a hard copy should be distributed to each student.

Model 1: A marking scheme based on the demonstration of outcomes through specific knowledge, skills and applications

Teachers could specify the number of marks for specific elements of the work, as measures of the extent to which targeted outcomes have been achieved. Some examples of possible aspects are given below, together with outcomes that they are most likely to inform:

- identification of important information (P2, H2, P4, H4)
- consideration of constraints and assumptions (P2–P5, H2–H5)
- clarity of content (P11, H11)
- accuracy of mathematics (P7, H7)
- knowledge of mathematics and mathematical understanding (P2, H2 and outcomes appropriate to content area)
- interpretation skills (P7, H7)
- appropriateness of mathematics used (P2, H2 and outcomes appropriate to content area)
- depth of investigation (P2–P10, H2–H10 as appropriate)
- analysis skills (P4, H4)
- organizational skills (P9, H9)
- skills of communication (P11, H11)
- appropriate use of technology (P2, H2, P4, H4, P5, H5)
- reflection (P11, H11)
- evaluation (P11, H11)
- conclusion (P11, H11)

Model 2: An extension of Model 1

This model involves assessing the work and allocating marks in relation to a series of established aspects of the project or investigation. Within each aspect, there is a range of possible levels of student achievement, each with an appropriate suggested score. In judging achievement within a particular aspect, the teacher should move from the *lowest* score upwards, until the description is *beyond* the student's achievement. The appropriate score is the one that precedes this higher description.

The following aspects and suggested mark allocations[2] exemplify the way in which this model could be used. The outcomes most likely to be informed by each aspect are listed alongside.

Aspect 1: Identification of important information, variables and constraints (P2–P5, H2–H5)

0: **inadequate information** in relation to the chosen starting point, or **excessive, indiscriminate information** presented

1: identifies **most** essential information, and makes **some reference** to limitations and/or constraints

2: makes **sensible judgements** about relevance of information, with **limited discussion** of assumptions and constraints

3: researches and uses appropriate information **for all stages of the activity**, identifies **most significant variables** and **discusses the effect of assumptions and constraints on the outcome of the activity**

4: identifies **all important variables**, provides **explicit discussion** of assumptions and constraints, and researches and uses **all essential information** to address all aspects of the activity **at a high level**.

[2] Adapted from *Major Project Guidelines – TPC Mathemathics*, Access Educational Services Division, TAFE NSW, Granville, 1996 and *Mathematical Methods, Standard Level*, International Baccalaureate, 1997.

Aspect 2: *Mathematical formulation of the problem, situation or issue (P2, H2 and outcomes appropriate to content)*

0: **no** relevant mathematical concepts recognised
1: **recognises or selects** a mathematical concept which is relevant to the activity
2: **recognises and attempts to use** a mathematical strategy which is relevant to the activity and **consistent with the standard of the course**
3: recognises and uses a mathematical strategy which is relevant and consistent with the standard of the course, **and makes few errors in applying mathematical techniques**
4: recognises and uses **successfully** a mathematical strategy which is relevant and consistent with the standard of the course, and applies mathematical techniques **correctly throughout the activity**
5: produces work distinguished by **precision, insight and a sophisticated level of mathematical understanding.**

Aspect 3: *Results and conclusions (P7, H7, P11, H11)*

0: **no conclusion** given, or **unreasonable** or **irrelevant** results presented
1: draws **partial conclusions**, or indicates **some consideration of reasonableness of results**
2: draws **adequate** conclusions or shows **some understanding** of **significance** and reasonableness of results
3: draws **full and relevant** conclusions, demonstrating **complete understanding** of significance, reasonableness or **possible limitations** of results.

Aspect 4: *Communication and use of mathematical notation and terminology (P4, H4, P11, H11)*

0: provides **neither diagrams nor explanations, does not use** appropriate notation and/or terminology
1: **uses some** appropriate notation and terminology in giving **minimal explanations**, uses **basic forms of representation**
2: uses appropriate notation and terminology **in a consistent manner throughout the activity.**

Aspect 5: Interpretation and evaluation of solution(s) (P11, H7, H11)

0: **minimal attempt** made to explain results, **little connection made** between results and aim of activity

1: **obvious trends** identified, **some comment** made on significance of results

2: **most significant observations** are made, **most aberrations** accounted for with **generally correct explanations,** general results **justified**, significance of results evaluated **with occasional errors**

3: **all significant observations made, all aberrations accounted for** with generally correct explanations, **general results justified** with reference to specific cases, **reasonableness of results** checked and **relevant limitations** identified and discussed

4: **clear explanation in both qualitative and quantitative terms** of solution and results obtained, **explicit discussion** of **relevance of data**, results **interpreted** with reference to carefully selected cases, **general results justified** in relation to specific cases, results **tested against real data** where possible.

8 Sample HSC Assessment Program

The following table represents a sample HSC school assessment program. The example uses four tasks only, a suggestion that reflects the advice provided in the Board of Studies' publication *The New Higher School Certificate: Assessment Support Document.* This publication indicates that '3–5 tasks are probably sufficient' (p 12), and that a range of tasks should be used.

Up to 30% of the HSC assessment may be based on material from the Preliminary course. The sample overleaf has a 25% Preliminary component, and shows how these 25 marks can be distributed across all tasks. It also demonstrates how the 40/60 weightings for knowledge and skills versus applications can be maintained through the total assessment program.

Task	Outcomes assessed	Type of task/task description	Total marks		Knowledge and skills weighting	Applications weighting
			Preliminary component	HSC component		
Task 1	P2, P6, P7 H6, H7	Practical task – outdoor surveying/measurement activity. Students survey part of the school grounds and complete a scale diagram and specify lengths, areas and angles. They analyse possible sources of error and make judgements about accuracy and reasonableness.	15		5	10
			5	10		
Task 2	H2, H4, H5, H8, H11	Project (extended task) – Financial Mathematics modelling and research task. Students research home loans and enter variables into a home loan spreadsheet or calculator to answer suitable 'what if' questions and make informed decisions about financial situations.	20		10	10
			—	20		
Task 3	H2, H4, H9, H11	Data Analysis investigation (extended task) – Students plan an investigation, collect, organise, display, analyse and interpret sets of data and draw conclusions. They use a variety of data displays and measures of location, spread and association as appropriate to their investigation.	25		5	20
			10	15		
Task 4	P2–P11 H2–H11	Trial HSC examination.	40		20	20
			10	30		
		TOTAL	**100**		40	60
			25	75		

9 Sample Teaching Unit

The following schematic may be useful in considering the elements involved in preparing a teaching and learning sequence based on syllabus units.

Unit: Introduction to Data Analysis Time: 10 hours

Incorporates syllabus units DA1, DA2

Outcomes:

P1 A student develops a positive attitude to mathematics and appreciates its capacity to provide enjoyment and recreation
P9 A student determines an appropriate form of organisation and representation of collected data
P11 A student justifies his/her response to a given problem using appropriate mathematical terminology

Resources	**Language**	
	bias	quality control
Syllabus pp 24–27	categorical	questionnaire
ABS website	census	random
Media (print, television)	continuous	sample
Stage 5 texts	data	sample size
	database	statistical inquiry
	discrete	statistics
	information	strata
	poll	stratified
	population	systematic

Suggested activities

1 Focus questions (class or group)
- Who collects data (information) in our society?
- What kinds of information are collected?
- Why is it collected?
- What is done with it?
- Why sample?
- How is sampling carried out?
- What are the steps in a statistical investigation?

2 Identify an issue that is of current interest to the class/group. Determine the role(s) that statistics can play in finding out more about it.

3 Arrange a visit from someone who uses statistics in her/his work, eg quality control engineer, public servant, business consultant.

4 Students locate newspaper articles that rely on statistics to make a point. They report to the class on the steps that must have been taken in order to produce the particular figures used.

5 Pairs or small groups choose a topic of interest and determine questions to investigate as part of a mini-project.

Content	**Linked units**
Syllabus units DA1, DA2 See syllabus pp 24–27	Measurement M1 Probability PB1
Assessment ideas	**Evaluation**
• Mini-project presentations. • Students make improvements to a poorly designed questionnaire, and write a report on their changes.	• Student views on unit. • Did assessment process reflect teaching and learning activities? • Was information gained about achievement of outcomes? • Did students receive feedback?

10 Notes to Accompany Spreadsheet Applications

The electronic spreadsheets that accompany this document have been developed to support teachers in the initial use of spreadsheets in teaching elements of General Mathematics. They are examples only, and their use is not mandatory. Teachers are encouraged to experiment with them in order to determine how they may be best used. They are designed for use with Microsoft *Excel 97* and should be compatible with later versions of Excel (eg Excel 98). They are *not* compatible with earlier versions of *Excel* such as *Excel 5.0* or with other spreadsheets such as *Lotus 123*.

Note: It is recommended that you do not change the filenames, as some of the macros within them rely on writing to the existing filename.

The five spreadsheets are:

1. BOS_Investing_Money.xls – this spreadsheet models the value of two ordinary annuities both as a table and graphically. In addition, simple and compound interest investments are compared both as a table and graphically.
   Syllabus link: FM2, FM6.

2. BOS_Mortgage_Analysis.xls – this spreadsheet allows the user to model home loan repayments. Principal, repayments, interest rate and period can be varied. Changes can be viewed graphically as well as in table form.
   Syllabus link: FM5 and FM6.

3. BOS_Probability_Sim.xls – this spreadsheet simulates the rolling of 36 dice. The concept of the long-run proportion can be demonstrated.
   Syllabus link: PB4.

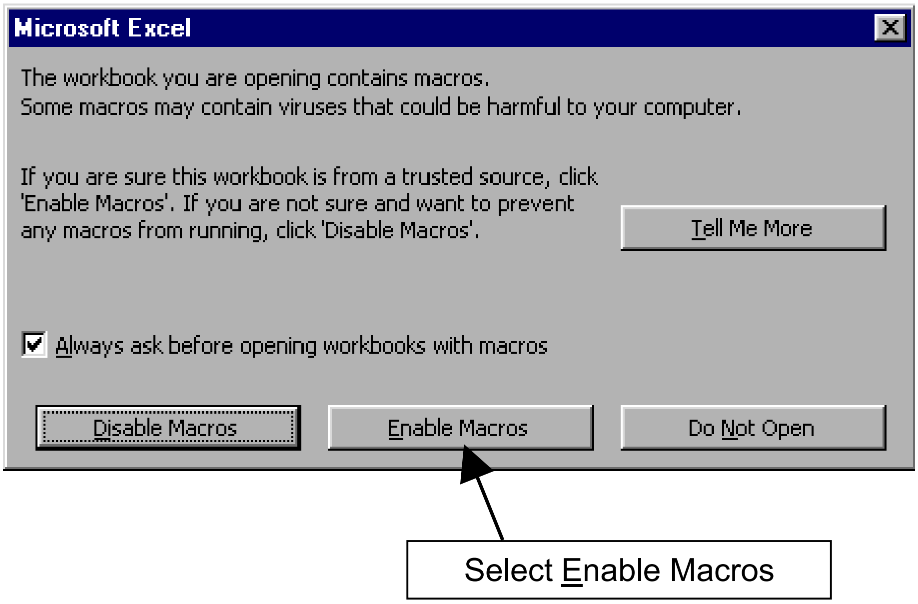
4. BOS_Income_Tax_Calc.xls – this model can be used to calculate income tax and the basic Medicare levy for the 2000 financial year and for the tax scales to be introduced for the 2001 financial year.
   Syllabus link: FM3.

5. BOS_Correlation_Model.xls – this spreadsheet allows the user to vary the y-coordinate(s) of some data points displayed on a scatter plot and to observe how the changes affect the line of best fit and the value of the correlation coefficient.
   Syllabus link: DA7.

   You can download the package of all five as a ZIP archive (300K).

Using the spreadsheets

When the spreadsheets are first loaded the 'Disable/Enable Macros' window appears:



Screenshot reprinted by permission from Microsoft Corporation.

Make sure that the Enable Macros option is selected. These spreadsheets function using macro programs.

The spreadsheets will work best if only one is open at a time, ie Select File | Close before opening a new spreadsheet. In particular, the spreadsheet 'BOS_Probability_Sim.xls' employs manual calculation whereas the others employ automatic calculation. The user does not need to select these options as the spreadsheets will select the appropriate settings when loaded.

Suggested teaching/learning ideas

1. BOS_Investing_Money.xls

Sheet1	Compare the difference in growth over ten periods for a principal earning simple interest and compound interest. View the growth on the chart. Simple interest is linear whereas compound interest is exponential. Also create a table showing the value for each period.
Sheet2	Compare two ordinary annuities. For example, 'Does a 30-year annuity of \$500 per annum at 15 % per annum catch up in value to a 30-year annuity of \$1000 per annum at 10 % per annum and what is the final difference in value? If yes, when does it catch up?'. A table and graph is shown.

2. BOS_Mortgage_Analysis.xls

Sheet1	Vary the amount borrowed, the interest rate, term of loan and monthly repayment. Create a table showing the principal.
Sheet2	Compare two levels of accelerated repayment with the minimum repayment.
Sheet3	Graphically compare two levels of accelerated repayment with the minimum repayment. For example, 'What difference does it make to the term of a 30-year loan of \$80 000 at a rate of 9% pa if the minimum repayment is increased by \$50 and \$100 per month?'.

3. BOS_Probability_Sim.xls

Sheet1	Simulate the rolling of 36 dice. Model the long-run proportion. Compare theoretical results with experimental results for n trials.

4. BOS_Income_Tax_Calc.xls

Sheet1	Calculate income tax and basic Medicare levy for the 2000 financial year.
Sheet2	Calculate income tax and basic Medicare levy for the 2001 financial year.
Sheet3	Compare the income tax payable for the 2000 financial year tax scales with the tax payable for the 2001 financial year tax scales. Create and compare your own tax scales.

5. BOS_Correlation_Model.xls

Sheet1	Observe changes to the correlation co-efficient and the gradient/*y*-intercept of the line of best fit as the *y*-coordinate of a single data point is varied.
Sheet2	Observe changes to the correlation co-efficient and the gradient/*y*-intercept of

	the line of best fit as the *y*-coordinate of a single data point is varied. Demonstrate the effect of an outlier on the line of best fit and the removal of outliers from the data (given the cause of the outlying value has been determined).
Sheet3	Observe changes to the correlation co-efficient and the gradient/*y*-intercept of the line of best fit as the *y*-coordinates of several data points are varied. Examine the pattern of the scatter plot when the line of best fit has a negative gradient, zero gradient or positive gradient. Graphically demonstrate the line of best fit when $r = 1$.