

B O A R D O F S T U D I E S
NEW SOUTH WALES

Preliminary Mathematics General

Stage 6

**Draft
Course Content**

2008

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Mathematics General 2 Stage 6 Draft Syllabus 2008 for HSC
Mathematics General Course Content.

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9 Preliminary Mathematics General Course Content

Hours shown are indicative only.

Areas of Study **80 hours**

Financial Mathematics

- PMG1 Earning money
- PMG2 Investing money
- PMG3 Taxation

Data Analysis

- PMG4 Statistics and society, data collection and sampling
- PMG5 Displaying single data sets
- PMG6 Summary statistics

Measurement

- PMG7 Units of measurement and applications
- PMG8 Applications of area and volume
- PMG9 Similarity of two-dimensional figures, right-angled triangles

Probability

- PMG10 The language of chance
- PMG11 Relative frequency and probability

Algebra and Modelling

- PMG12 Basic algebraic skills
- PMG13 Modelling linear relationships

Focus Studies **40 hours**

- 1. **Mathematics and Communication** **(20 hours)**
- 2. **Mathematics and Driving** **(20 hours)**

Total indicative hours **120 hours**

Area of Study 1: Financial Mathematics

In this Area of Study, students investigate ways in which individuals earn, manage and invest money, and perform a range of calculations, including calculations in relation to taxation and the value of investments.

Assumed Stage 5.1 outcomes

NS 5.1.2

Outcomes addressed

A student:

- PG1 uses Mathematics and statistics to compare alternative solutions to contextual problems
- PG2 represents information in symbolic, graphical and tabular forms
- PG3 represents the relationships between changing quantities in algebraic and graphical form
- PG5 demonstrates awareness of issues in practical measurement, including accuracy, and the choice of relevant units
- PG6 models financial situations relevant to the student's current life using appropriate tools
- PG9 uses appropriate technology to organise information from a limited range of practical and everyday contexts
- PG10 justifies his/her response to a given problem using appropriate mathematical terminology.

Content summary

- PMG1 Earning money
- PMG2 Investing money
- PMG3 Taxation

Terminology

| | |
|------------------------------|-----------------------|
| annual leave | group certificate |
| annual leave loading | income tax |
| compounding period | Medicare levy |
| deductions | PAYG |
| dividend | royalty |
| dividend yield | shares |
| double time | tax deduction |
| present value | taxable income |
| future value | time and a half |
| goods and services tax (GST) | value added tax (VAT) |
| gross pay/gross wage | |

Use of technology

Spreadsheets are widely used in the workplace, especially in the business and finance sectors. Students should construct and use spreadsheets to perform tasks, including calculation of wages and overtime, preparation of personal and family budgets, and tracking of a share portfolio.

Spreadsheets allow the user to perform 'what if' analysis, eg to compare the interest earned on fixed-term deposits for various periods and interest rates.

Students can use prepared spreadsheets to determine PAYG income tax and the Medicare levy for different taxable incomes.

The internet should also be used as a source of up-to-date information, eg current income tax scales and interest rates.

Area of Study notes

Teaching and learning in this Area of Study should be supported by the use of real and up-to-date data, eg calculations of wages and salaries should include calculations in relation to current awards and work contracts.

Calculations involving Youth Allowance, or its equivalent, should be based on the current published rates and conditions. Similarly, the current advertised rates should be used in interest-rate calculations.

Tax and levy rates, as published on the Australian Tax Office (ATO) website, should be used in the calculation of income tax and the Medicare levy.

PMG1: Earning money

The principal focus of this topic is ways in which individuals earn and manage money.

Outcomes addressed

PG1, PG5, PG6, PG10

Students develop the following knowledge, skills and understanding

- calculation of monthly, fortnightly, weekly, daily and hourly payments from salary
- calculation of wages incorporating hourly rate and penalty rates such as overtime
- calculation of annual leave loading
- calculation of earnings based on commission, piecework, royalties
- calculation of payments based on government allowances and pensions, such as Youth Allowance, Austudy, ABSTUDY, Fares Allowance, and the Age Pension
- determination of deductions such as union fees, superannuation contributions, health fund instalments and tax instalments
- calculation and comparison of costs associated with maintaining accounts with financial institutions
- calculation of net pay following deductions.

Applications and considerations

- Prepare a household budget for given income and expenditure.
- Review a previously prepared budget to reallocate funds for a sudden contingency.
- Calculations of wages should include special allowances, eg allowances for wet work, confined spaces, toxic substances, heat, heights.
- Select a career and prepare a report that includes the educational requirements, job conditions and remuneration. Students can obtain information from sources including newspapers, online job advertisements, and recruitment agencies. The report might also include calculation of net pay after tax.
- Calculate earnings based on prepared timesheets.
- Make a comparison of wages in various countries or vocations.

PMG2: Investing money

The principal focus of this topic is the use of formulae and tables to perform calculations related to the value of investments over a period of time.

Outcomes addressed

PG1, PG2, PG3, PG5, PG6, PG10

Students develop the following knowledge, skills and understanding

- calculation of simple interest using $I = Prn$, where P = principal, r = percentage interest rate per period expressed as a decimal (eg if the rate is quoted as 8.2%, then $r = 0.082$, and n = number of periods)
- using tables of values for fixed values of P , and hence drawing and describing graphs of I against n for different values of r
- calculation of monthly, quarterly, six-monthly interest rates based on quoted rates per annum (pa)
- calculation of the final balance, interest and principal with pen and paper using the compound interest formula $A = P(1 + r)^n$
where A (amount) = final balance
 P (principal) = initial quantity
 n = number of compounding periods
 r = interest rate per compounding period
- calculation of the dividend paid on a shareholding and the dividend yield, excluding franked dividends
- recording and graphing the price of a share over time
- calculation of the future and present value of an investment from prepared tables
- calculation of the price of goods following inflation
- calculation of the appreciated value of items such as stamp collections and other memorabilia.

Applications and considerations

- Use a table to calculate the value of money invested in compound interest accounts.

| Compounded values of \$1 | | | | | |
|--------------------------|--------------------------|-------|-------|-------|-------|
| Periods | Interest rate per period | | | | |
| | 1% | 5% | 10% | 15% | 20% |
| 1 | 1.010 | 1.050 | 1.100 | 1.150 | 1.200 |
| 2 | 1.020 | 1.103 | 1.210 | 1.323 | 1.440 |
| 3 | 1.030 | 1.158 | 1.331 | 1.521 | 1.728 |
| 4 | 1.041 | 1.216 | 1.461 | 1.750 | 2.074 |
| 5 | 1.051 | 1.276 | 1.611 | 2.011 | 2.488 |
| 6 | 1.062 | 1.340 | 1.772 | 2.313 | 2.986 |

For example, to calculate the future value of \$3000 invested for 6 years compounded annually at 5%,

$$A = 3000 \times 1.340 = \$4020$$

- If the interest rate is quoted as 6% pa, what amount needs to be invested in order for the investment to be worth \$850 at year's end?
- Jan and Bob wish to save \$10 000 for their granddaughter's university expenses. They wish to have this amount available in 8 years' time. Calculate the single sum to be invested at 5% pa compounded annually.
- Determine the single sum to be deposited if \$10 000 is required in 5 years' time and a rate of 3% pa (compounded quarterly) is available.
- A principal of \$1000 is to be invested for 3 years. Determine which is the better investment option?
 - (i) 6% pa simple interest, (ii) 5.9% pa compounded annually or (iii) 5.85% pa compounded half-yearly.
- An investor purchased 1000 shares in a company at a price of \$3.98 per share, with a dividend yield of 5.5%. Brokerage costs were 1% of the purchase price. One year later, the shares were sold for \$4.80 per share. Calculate the total earnings over the year, after costs.
- For students intending to proceed to the Mathematics General 2 HSC course, teachers could introduce the following terms: *future value (FV)* and *present value (PV)* in the context of the compound interest formula. **Note:** In the financial world, the compound interest formula quoted above is generally presented as $FV = PV(1 + r)^n$.

PMG3: Taxation

The principal focus of this topic is the calculation of tax payable on income and on goods and services.

Outcomes addressed

PG1, PG2, PG3, PG5, PG6, PG10

Students develop the following knowledge, skills and understanding

- calculation of the amount of allowable deductions from gross income
- calculation of taxable income
- calculation of Medicare levy (basic levy only — see Tax Pack for details)
- calculation of PAYG (Pay As You Go) tax payable or refund owing, using current tax scales
- given rates of tax from a range of countries, calculation of the value added tax (VAT) payable on a range of goods and services
- calculation of the goods and services tax (GST) payable on a range of goods and services
- creating graphs to illustrate and describe different tax rates.

Applications and considerations

- Calculate tax refund (or amount payable) based on a sample group certificate, taking into account gross income, deductions, taxable income, tax payable on taxable income, Medicare levy and tax already paid as per the group certificate.
- Students complete a tax return form (as included in the Tax Pack) using a typical PAYG employee's earnings and deductions. The aim is to calculate the 'refund from' or 'amount owed to' the Australian Taxation Office (ATO).
- It should be noted that a graph of tax paid against taxable income is a piecewise linear function (see PMG13).
- GST calculations should include finding the original cost of goods before GST was added.
- Various online tax calculators can be accessed on the internet, eg from the ATO website.

(The output following the use of a particular online tax calculator is shown below.)

| | A | B | C | D | E | F |
|----|--|-----------|------------|-------------------------------|----|-----------|
| 1 | Tax Calculator Version 4.5 | | | Created by C. Books 22/2/2008 | | |
| 2 | | | | | | |
| 3 | This spreadsheet calculates the yearly, weekly and fortnightly tax based | | | | | |
| 4 | on yearly taxable income not including the Medicare levy | | | | | |
| 5 | | | | | | |
| 6 | Variables | | | Results | | |
| 7 | Taxable Income | \$89,988 | | Yearly Tax | \$ | 23,095.20 |
| 8 | | | | Weekly Tax | \$ | 444.14 |
| 9 | | | | Fortnightly Tax | \$ | 888.28 |
| 10 | | | | | | |
| 11 | Calculation Section | | | | | |
| 12 | Bracket | | | Tax Payable | | |
| 13 | 1 | \$ 1 | \$ 6,000 | | | |
| 14 | 2 | \$ 6,001 | \$ 30,000 | | | |
| 15 | 3 | \$ 30,001 | \$ 75,000 | | | |
| 16 | 4 | \$ 75,001 | \$ 150,000 | \$ 23,095.20 | | |
| 17 | 5 | \$150,001 | | | | |

Area of Study 2: Data Analysis

The topics within this Area of Study focus on the importance of statistical processes and inquiry in society, the planning and management of data collection, the preparation of a variety of data displays, and the calculation of summary statistics for single data sets and their use in interpretation.

Assumed Stage 5.1 outcomes

DS5.1.1

Outcomes addressed

A student:

- PG1 uses Mathematics and statistics to compare alternative solutions to contextual problems
- PG2 represents information in symbolic, graphical and tabular forms
- PG7 determines an appropriate form of organisation and representation of collected data
- PG9 uses appropriate technology to organise information from a limited range of practical and everyday contexts
- PG10 justifies his/her response to a given problem using appropriate mathematical terminology.

Content summary

- PMG4 Statistics and society, data collection and sampling
- PMG5 Displaying single data sets
- PMG6 Summary statistics

Terminology

| | |
|----------------------|----------------------|
| bias | questionnaire |
| box-and-whisker plot | radar chart |
| categorical data | random sample |
| census | relative frequency |
| continuous | sample size |
| decile | sector graph |
| discrete | standard deviation |
| dot plot | statistical inquiry |
| five-number summary | stem-and-leaf plot |
| grouped data | strata |
| interquartile range | stratified sample |
| ogive | summary statistics |
| poll | systematic sample |
| population | upper/lower extreme |
| quality control | upper/lower quartile |
| quartile | |

Use of technology

Technology such as statistical software and spreadsheets should be used by teachers and students. Spreadsheets are widely used in the workplace and are a suitable tool to tabulate and graph data and to calculate summary statistics.

Technology should be used to create frequency tables and statistical graphs, including box-and-whisker plots.

Teachers should demonstrate the effect of changing value(s) in a data set and the effect on the summary statistics. Outlying values could be examined in this way.

Large and small data sets from real-life situations should be accessed. These data sets may be obtained online.

Area of Study notes

Teaching, learning and assessment materials should use current information from a range of sources including, but not limited to, surveys, newspapers, journals, magazines, bills and receipts, and the internet.

Students may have greater interest in data that relates to their life experiences or to data that they have generated themselves. This data may be collected by survey, measurement or simple experiment. A student survey may represent the entire population of interest or may be a sample.

Suitable sets of data are available from various websites. Some data sets from the web could form the basis of a prepared spreadsheet that the teacher could use to illustrate a variety of statistical methods and concepts.

Students could collect, display and analyse data pertaining to another Key Learning Area, eg fitness data, or attitude data, collected in PDHPE, or results from a scientific experiment.

PMG4: Statistics and society, data collection and sampling

The principal focus of this topic is the planning and management of data collection. Data for this topic may be obtained from published sources or by conducting a survey. The topic includes the use of sampling techniques in order to make inferences about a population. Although the emphasis is on quantitative data, students should be aware of processes related to categorical data.

Outcomes addressed

PG2, PG7, PG10

Students develop the following knowledge, skills and understanding

- the importance of analysing data in planning and decision-making by governments and businesses
- the process of statistical inquiry, including the following steps: posing questions, collecting data, organising data, summarising and displaying data, analysing data and drawing conclusions, and writing a report
- identification of the target population to be investigated
- determining whether data for the whole population is available (eg the results of a round of a sporting competition), or if sampling is necessary
- recognising that the purpose of a sample is to provide an estimate for a particular population characteristic when the entire population cannot be accessed
- classification of data as: quantitative (either discrete or continuous) or categorical
- distinguishing between the following sample types: random, stratified, systematic
- determination which of the above sample types is appropriate for a given situation
- relating sample selection to population characteristics, eg if 20% of the Australian population is aged under 20, a selected sample should include 20% of under 20s.

Applications and considerations

- Relevant discussions include:
 - the role of statistical methods in quality control in manufacturing industries
 - issues of privacy and ethics in data collection and analysis
 - the role of organisations that collect and/or use statistics, including the Australian Bureau of Statistics (ABS), the United Nations (UN), and the World Health Organisation (WHO).
- Examples of classification of data could include: gender (male, female) *categorical*; height (measured in cm) *quantitative, continuous*; quality (poor, average, good, excellent) *categorical*; school population (measured in individuals) *quantitative, discrete*.
- Prepare questionnaires and discuss consistency of presentation and possible different interpretations of the questions.
- Discuss when it is appropriate to use a sample rather than a census.
- Determine the best method of sampling in a range of situations.

PMG5: Displaying single data sets

The principal focus of this topic is to have students gain experience in preparing a variety of data displays. The power of statistical displays both to inform and misinform should be emphasised.

Outcomes addressed

PG2, PG5, PG7, PG9, PG10

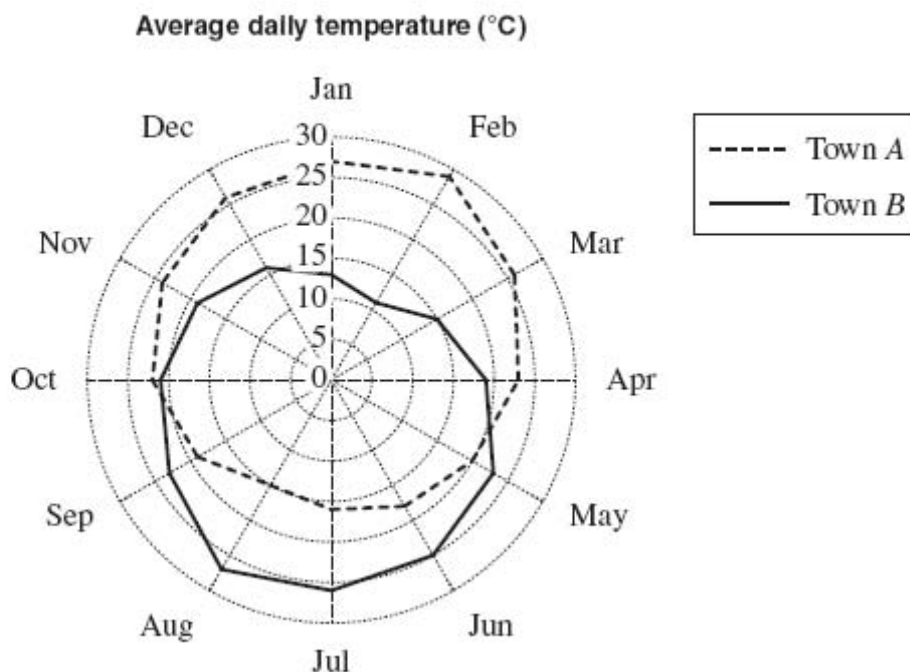
Students develop the following knowledge, skills and understanding

- drawing a radar chart to display data such as sales figures, temperature, or rainfall readings
- division of data into deciles, quartiles and percentiles
- review of calculation and interpretation of the range and interquartile range as measures of the spread of a data set
- construction of frequency tables from cumulative frequency graphs (ogives)
- determining the median and upper and lower quartiles of a data set from a cumulative frequency polygon
- establishment of a five-number summary for a data set (lower extreme, lower quartile, median, upper quartile, upper extreme)
- development of a box-and-whisker plot from a five-number summary
- linking type of data with appropriate display, eg continuous quantitative data is best represented using a histogram; categorical data is best represented using a bar graph or sector graph (pie chart).

Applications and considerations

- Teachers may find it necessary to revise the following:
 - construction of tally charts and frequency tables for ungrouped and grouped data
 - the appropriate selection of dot plots, sector graphs, bar graphs, stem-and-leaf plots, box-and-whisker plots, histograms, or line graphs, to display data sets
 - selection of suitable scales for the above displays.
- Suggest possible reasons for the occurrence of any unusual scores in a data set, particularly in relation to the students' collection of real data sets.
- Use technology to organise the data gathered, and to draw graphs.
- Perform experiments to record and analyse data to determine whether or not performance of a simple task can be improved with practice.
- Students could collect examples of misleading statistical displays and prepare accurate versions of each. They could describe the inaccuracies, and their corrected version, in appropriate mathematical language.
- Use published percentile charts (eg Infant Length for Age percentile charts) to assist in the development of mathematical literacy.
- Compare the suitability of different types of statistical displays, eg a radar chart can be used to illustrate change of a quantity over time, a line graph is useful to show trends in data over equal time intervals.

This radar chart was used to display the average daily temperatures each month for two different towns.



PMG6: Summary statistics

The principal focus of this topic is the calculation of summary statistics for single data sets and their use in interpretation.

Outcomes addressed

PG1, PG2, PG7, PG9, PG10

Students develop the following knowledge, skills and understanding

- calculation of the median, eg from stem-and-leaf plots and cumulative frequency polygons
- calculation of the measures of location mean, mode and median, for grouped data presented in table or graphical form
- determination of the mean for larger data sets of either grouped or ungrouped data using the statistical functions of a calculator
- informal description standard deviation as a measure of the spread of data in relation to the mean
- calculation of the population standard deviation using the σ_n button, or otherwise, on a calculator
- selection of and use of the appropriate statistic (mean, median or mode) to describe features of a data set, eg median house price, modal shirt size
- comparison of the summary statistics of various samples from the same population.

Applications and considerations

- Teachers may find it necessary to revise the following:
 - application of median and mode
 - calculation of the mean for *small* data sets, using the formulae

$$\bar{x} = \frac{\sum x}{n}, \bar{x} = \frac{\sum fx}{\sum f}$$
 where \bar{x} represents the mean of the sample
 - determination of the median and mode(s) of a data set, either from a list or from a frequency table.
- Interpret and evaluate data from students' own data sets and draw conclusions that can be justified.
- Use a spreadsheet to examine the effect on the calculated summary statistics of changing the value of a score. The spreadsheet below provides such an example. The two data sets are the same except that, for the 5th student in Set 2, the outlying value of 183 cm has the effect of increasing the mean and standard deviation, while leaving the median unchanged.

| | A | B | C | D | E | F |
|----|---------|---------|---|---------|---------|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | Set 1 | | | Set 2 | | |
| 4 | Student | Heights | | Student | Heights | |
| 5 | 1 | 134 | | 1 | 134 | |
| 6 | 2 | 145 | | 2 | 145 | |
| 7 | 3 | 150 | | 3 | 150 | |
| 8 | 4 | 151 | | 4 | 151 | |
| 9 | 5 | 153 | | 5 | 183 | |
| 10 | | | | | | |
| 11 | Mean: | 146.6 | | Mean: | 152.6 | |
| 12 | Median: | 150 | | Median: | 150 | |
| 13 | S.D.: | 7.64 | | S.D.: | 18.28 | |
| 14 | | | | | | |
| 15 | | | | | | |

- For students intending to proceed to the Mathematics General 2 HSC course, teachers could differentiate between population standard deviation σ_n and sample standard deviation σ_{n-1} , including using a calculator to determine both measures for small sets of data.

Area of Study 3: Measurement

In this Area of Study, students investigate metric units of measurement and their application, and solve a range of problems involving rates and ratios, area and volume, the application of similarity properties, and right-angled triangles.

Assumed Stage 5.1 outcomes

MS 5.1.1, MS 5.1.2, NS 5.1.1

Outcomes addressed

A student:

- PG2 represents information in symbolic, graphical and tabular forms
- PG3 represents the relationships between changing quantities in algebraic and graphical form
- PG4 performs calculations in relation to two-dimensional and three-dimensional figures
- PG5 demonstrates awareness of issues in practical measurement, including accuracy, and the choice of relevant units
- PG9 uses appropriate technology to organise information from a limited range of practical and everyday contexts
- PG10 justifies his/her response to a given problem using appropriate mathematical terminology.

Content summary

- PMG7 Units of measurement and applications
- PMG8 Applications of area and volume
- PMG9 Similarity of two-dimensional figures, right-angled triangles

Terminology

| | |
|---------------------|---------------------|
| adjacent | percentage error |
| angle of depression | prefix |
| angle of elevation | reduction |
| capacity | scale factor |
| concentration | significant figures |
| congruent | similar |
| cosine | sine |
| elevation | tangent |
| enlargement | trigonometry |
| hypotenuse | unitary method |
| opposite | |

Use of technology

Students should use suitable software applications for drawing accurate figures. They should investigate geometrical properties, as well as creating both two-dimensional (2D) and three-dimensional (3D) drawings.

Students should use a spreadsheet to investigate maximising perimeter and areas of shapes.

Online conversion calculators could be used to investigate conversion of units or change of units of volume to units of capacity.

Online maps (eg GoogleEarth) may be used to enhance visualisation of scale, to calculate distances, and to make estimations of travelling times. Such resources could also be used to make estimations and calculations of areas of land using field diagrams.

Area of Study notes

The Focus Study topics provide many opportunities for students to apply the skills developed in this Area of Study to context-based problems. Teachers can draw upon problems from a wide variety of sources to reinforce the skills developed.

The application of such skills is consolidated further in the HSC course content of both Mathematics General 1 and Mathematics General 2, providing a range of opportunities to revise the skills developed in the Preliminary Course. For example, work on units of measurement is revised and extended in the Preliminary Course Focus Studies, Mathematics of Communication and Mathematics of Driving. Rates and ratios are further addressed, in context, in Focus Studies 5, 6, 7 and 8.

Where possible, students should be given the opportunity to gain practical experiences in relation to the Mathematics addressed in this Area of Study. Where it is not possible to provide practical experiences, the problems posed should be relevant to the lives of students.

PMG7: Units of measurement and applications

The principal focus of this topic is on metric units of measurement, and rates and ratios. Students learn to make judgements about measurement errors.

Outcomes addressed

PG3, PG4, PG5, PG9, PG10

Students develop the following knowledge, skills and understanding

- repeating and averaging measurements to reduce likelihood of error
- determination of the number of significant figures to be used in recording measurements, in relation to the accuracy of the measuring instrument being used
- use of scientific notation and standard prefixes (milli, micro, etc) in the context of measurement
- using positive and negative powers of ten when expressing measurements in scientific notation
- calculation with ratios, including: finding the ratio of two quantities, dividing quantities in a given ratio, and using unitary method to solve problems
- conversion between units for rates, eg km/h to m/s
- conversion of units for area and volume, eg cm^2 to m^2 , $1\text{ha} = 10\,000\text{m}^2$
- determining the overall change in a quantity following repeated percentage changes, eg an increase of 20% followed by a decrease of 20%.

Applications and considerations

- Teachers may find it necessary to revise the following:
 - appropriate units of measure, conversion between commonly used units using standard prefixes
 - accuracy of physical measurement being limited to $\pm \frac{1}{2}$ of the smallest unit of which the measuring instrument is capable, including determination of possible sources of error in measuring.
- Modify given recipes by varying quantities to provide for various numbers of people.
- Calculate the quantity of each component needed for a fertilising operation, given the ratio of each component in the mixture.
- Calculate and compare freight costs for a variety of modes of transport.
- Calculate the rates of application of chemicals used in agriculture, such as those for pesticides and feed additives.
- Calculate distances and travelling time from maps.
- For students intending to proceed to the Mathematics General 2 HSC course, teachers could:
 - calculate the percentage error in a measurement
eg if the measured height was 155 cm \pm 0.5 cm (ie to the nearest cm), the percentage error for this measurement is $\pm \left(\frac{0.5}{155} \right) \times 100\%$

Students could measure their heights and calculate the percentage error in the measurement.
 - calculate concentrations expressed as weight/weight, weight/volume or volume/volume.
 - eg a patient needs 3 litres of fluid per day. One millilitre of fluid contains 15 drops. Find the rate at which the intravenous drip must run, expressing the answer in the form: number of drops fed to the patient per minute.

PMG8: Applications of area and volume

The principal focus of this topic is the calculation and application of area and volume in the solution of problems.

Outcomes addressed

PG3, PG4, PG5, PG9, PG10

Students develop the following knowledge, skills and understanding

- calculation of the area of triangles, quadrilaterals and circles (review only)
- calculation of the area of irregularly-shaped blocks of land using a field diagram
- calculation of the surface area of right prisms, square pyramids, and rectangular pyramids, using appropriate formulae
- calculation of the volume of right prisms, cylinders, pyramids, cones and spheres, using appropriate formulae
- converting between units of capacity and units of volume.

Applications and considerations

- Survey an irregularly shaped area using offsets and use this information to calculate its approximate area.
- Discuss and report on possible sources of error, eg experimental, instrumental and constant error in the above activity.
- Students apply their algebraic skills to find a missing dimension given the area or volume of a shape.
- Estimate the painted surface area of a classroom.
- Plan and prepare a budget for the redecoration of a favourite room at home.
- Investigate the dimensions that maximise the area for a given shape and perimeter, such as in the design of playpens and stock paddocks.
- Students make calculations of areas, including limiting the amount of wastage. For example, calculate the area of a backyard and decide upon the best way to lay turf in order to minimise the amount of wastage and number of cuts that need to be made. This example could be extended to include calculations of the amounts of preparation materials required and estimated labour costs.

PMG9: Similarity of two-dimensional figures, right-angled triangles

The principal focus of this topic is to solve a range of practical problems, involving the application of similarity properties and right-angled triangles.

Outcomes addressed

PG3, PG4, PG5, PG9, PG10

Students develop the following knowledge, skills and understanding

- calculation of sine, cosine and tangent ratios
- use of trigonometric ratios to find the length of an unknown side in a right-angled triangle, when the unknown side is in the numerator or denominator
- use of trigonometric ratios to find the size of an unknown angle in a right-angled triangle using a calculator to approximate the angle to the nearest degree
- obtaining measurements from plans of buildings and rooms
- interpretation of commonly used symbols on house plans
- transferring measurements between floor plans and elevations
- determining whether an answer seems reasonable by using a diagram drawn roughly in proportion.

Applications and considerations

- Teachers may find it necessary to revise the following:
 - calculating scale factors of objects and images
 - applying Pythagoras' theorem to practical applications
 - using compass bearings (eight points only) and true bearings (three-figure bearings) in problem solving related to maps and charts involving right-angled triangles
- Calculate the scale factor of enlargements obtained using an overhead projector.

Is there a relationship between the distance of the projector from the screen and the scale factor of the resulting projection?

- By measuring the shadow thrown by a metre-rule, students use similarity and shadow lengths to find the height of tall objects, eg tree, flag pole, etc.
- Students accurately construct a scaled floor plan of the classroom.
- Find ceiling heights from plans.
- Use house plans to cost carpeting, tiling, painting rooms, etc.
- Use a grid over a free-form diagram to draw an enlargement or reduction.
- Use the properties of its diagonals to determine if a room is rectangular.
- Find possible lengths of two sides of a right-angled triangle, given the length of the hypotenuse.
- Investigate the trigonometric ratios for angles of, say, 30° , 45° , 60° in a number of similar right-angled triangles.
- In groups, students use a clinometer to find the heights of school buildings etc. Different groups compare and discuss their answers.

Area of Study 4: Probability

In this Area of Study, students gain further experience in the use of the language of probability and compare relative frequency and calculated probability.

Assumed Stage 5.1 outcomes

NS 5.1.3

Outcomes addressed

A student:

- PG1 uses Mathematics and statistics to compare alternative solutions to contextual problems
- PG3 represents the relationships between changing quantities in algebraic and graphical form
- PG8 performs simple calculations in relation to the likelihood of familiar events
- PG9 uses appropriate technology to organise information from a limited range of practical and everyday contexts
- PG10 justifies his/her response to a given problem using appropriate mathematical terminology.

Content summary

- PMG10 The language of chance
- PMG11 Relative frequency and probability

Terminology

complementary events
equally likely
event
frequency
multi-stage event

outcomes
percentage chance
probability
relative frequency
sample space

Use of technology

Students should make use of contextual data, as available on the internet, eg the Australian Bureau of Statistics (ABS) Yearbooks and the Australian Bureau of Meteorology are useful sources of data.

Students should create a spreadsheet to record the results of an experiment and use it to display the results.

Area of Study notes

Students may have greater interest in probability contexts that relate to their life experiences, eg calculating the risk of injury as a probability for activities such as driving a car or playing contact sport.

Student-generated data, obtained from activities such as rolling dice and tossing coins, provides suitable data for analysis.

PMG10: The language of chance

The principal focus of this topic is the use of the language of probability, counting outcomes, and describing the sample space of an event.

Outcomes addressed

PG2, PG8, PG10

Students develop the following knowledge, skills and understanding

- ordering everyday events from the very unlikely to the almost certain
- using a list or table to identify the sample space (set of possible outcomes) of a simple experiment or game
- performing experiments and determining if the outcomes are equally likely
- determining the number of outcomes for a multi-stage event by multiplying the number of choices at each stage, eg the total number of ways of placing three different letters in three envelopes is $3 \times 2 \times 1$
- using systematic lists to verify the total number of outcomes for simple multi-stage events.

Note: factorial notation is not required.

Applications and considerations

- Practical experiments could involve coin tossing, dice rolling, selecting cards from a pack of cards.
- Comment critically on statements involving probability, such as: 'Since it either rains or is fine, the probability of a fine day is 50–50'.
- Investigate expressions used in other disciplines and in everyday life to describe likely or unlikely events, eg 'once in a blue moon', 'a one in 300-year flood', or 'a 75% chance of recovery following a medical operation'.
- Statements involving the language of probability could be collected from various media (newspapers, magazines, radio, television, internet, etc) and discussed.
- Investigate the number of different meals that can be chosen from a menu.
- Determine the number of combinations of raised dots that are possible in the Braille system for reading and writing. Investigate whether or not they are all used. Repeat for Morse Code.
- Investigate limitations on the number of postcodes, car registration plates or telephone numbers that can be used.

PMG11: Relative frequency and probability

The principal focus of this topic is to compare relative frequency and calculated probability.

Outcomes addressed

PG2, PG8, PG10

Students develop the following knowledge, skills and understanding

- estimating the relative frequency of events from recorded data
- performing simple experiments to obtain relative frequencies from recorded results
- using relative frequencies to obtain approximate probabilities
- using the following definition of the probability of an event *where outcomes are equally likely*:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- calculating probabilities using fractions, decimals and percentages
- demonstrating the range of possible probabilities, $0 \leq P(E) \leq 1$, by examination of a variety of results
- comparing calculated probabilities with experimental results
- illustrating the results of experiments using statistical graphs and displays (see PMG6)
- identifying the complement of an event and using the relationship

$$P(\text{an event does not occur}) = 1 - P(\text{the event does occur}).$$

Applications and considerations

- Teachers may find it necessary to revise the calculation of probabilities using fractions, decimals and percentages.
- Data could be generated from simple experiments, and also obtained from other sources, eg weather and sporting statistics from newspapers, the Australian Bureau of Statistics (ABS) Yearbooks, websites.
- Statements involving the language of probability could be collected from various media (newspapers, magazines, radio, television, internet, etc) and discussed.
- If it is appropriate for the school population, plan a school raffle for fundraising.
- Experiments could be carried out in which the probability is not intuitively obvious, eg probability of a drawing pin landing point up.
- Students could investigate the relative frequency of each different-coloured lolly when selecting a lolly from a packet of multi-coloured lollies.
- Examine the birth notices on a particular day in a daily newspaper. Record the number of boys and the number of girls. On this basis, estimate the probability that a child born is (a) male, (b) female. Compare these results with those published by the Australian Bureau of Statistics (ABS).

Area of Study 5: Algebra and Modelling

In this Area of Study, students apply algebraic skills and techniques to construct and evaluate simple mathematical models of real situations.

Assumed Stage 5.1 outcomes

PAS5.1.2

Outcomes addressed

A student:

- PG1 uses Mathematics and statistics to compare alternative solutions to contextual problems
- PG2 represents information in symbolic, graphical and tabular forms
- PG3 represents the relationships between changing quantities in algebraic and graphical form
- PG9 uses appropriate technology to organise information from a limited range of practical and everyday contexts
- PG10 justifies his/her response to a given problem using appropriate mathematical terminology

Content summary

- PMG12 Basic algebraic skills
- PMG13 Modelling linear relationships

Terminology

| | |
|-------------------------|---------------------------|
| cubic | linear |
| decreasing function | parameters |
| dependent variable | piecewise linear function |
| direct linear variation | power |
| exponential function | proportional to |
| extrapolate | quadratic |
| function | rate of change |
| gradient | relation |
| increasing function | step function |
| independent variable | y -intercept |
| interpolate | |

Use of technology

A student will require access to appropriate technology to create graphs of functions and to observe the effect on the graph of a function when parameters are changed.

Suitable graphing software (eg WinPlot) should be used by the teacher in investigating the similarities and differences between the graphs of a variety of linear relationships.

Students could use a spreadsheet to generate a table of values and the associated graph.

Area of Study notes

Algebraic skills should be developed through practical and vocational contexts.

Application of the skills developed in this Area of Study is consolidated further in Focus Studies in this (Preliminary) course and the HSC courses for Mathematics General 1 and Mathematics General 2.

Students should develop an understanding of a function as: input, processing, output. (It is not intended that students learn a formal definition of a function.)

PMG12: Basic algebraic skills

The principal focus of this topic is to provide a foundation in basic algebraic skills used to solve real and abstract problems. The topic develops techniques used in work-related and everyday contexts.

Outcomes addressed

PG1, PG2, PG3, PG9, PG10

Students develop the following knowledge, skills and understanding

- identification and generalisation of simple linear number patterns
- addition and subtraction of like terms
- addition and subtraction of simple algebraic fractions
eg $\frac{x}{6} + \frac{x}{4}, \frac{7x}{10} - \frac{x}{3}$
- expansion and simplification of algebraic expressions, such as
 $-(x-4), 2x(x+3)+x(x-1), 2(3x+y)+3(x-y), 4(3x-5)-3(x-9)$
- factorisation of algebraic expressions, such as $3p-6, p^2+7p, -2x-14$
- multiplication of algebraic terms, such as $3M^2 \times 5M, \frac{1}{5}r \times 20r$
- division of single algebraic terms (linear, quadratic, cubic), such as $\frac{9L^3}{3L^2}, 34z^2 \div 18z$
- simplification of algebraic terms involving multiplication and division, such as
 $\frac{9y}{4} \times 5y, \frac{28L^3}{3L^2} \times \frac{15}{7L}, \frac{3(x-5)}{4x^5} \times \frac{x^3}{11(x-5)}$
- evaluation of the subject of a formula by substitution of numerical values, using a wide variety of mathematical formulae, such as
 $R = R_1 + R_2 + R_3, W = Fs$
- solving linear equations involving two steps, such as
 $5x+12=22, \frac{4x}{10}=3, x-\frac{1}{6}=15\frac{2}{7}$
- solving equations following substitution and evaluation,
eg find a , given that $v = 10, u = 5, s = 8$ and $v^2 = u^2 + 2as$
- changing the subject of formulae involving linear terms eg make t the subject of $v = u + at$.

Applications and considerations

- ‘Practical’ formulae for evaluation

eg

$$C = \frac{5}{9}(F - 32)$$

$$V = \frac{4}{3}\pi r^3$$

$$S = \frac{D}{T}$$

$$v^2 = u^2 + 2as$$

$$S = V_0 - Dn$$

$$V = IR$$

$$A = P(1 + r)^n$$

$$h^2 = a^2 + b^2$$

$$v = u + at$$

$$BAC_{Female} = \frac{(10N - 7.5H)}{5.5M}$$

$$W = Fs$$

$$E = kL + b$$

$$S = V_0(1 - r)^n$$

(BAC: Blood Alcohol Content)

- Teachers may decide to introduce some of the formulae from the Financial Mathematics Area of Study to emphasise how new variable names may be used,

eg $FV = PV(1 + r)^n$. Note that FV is a dollar amount, the ‘Future Value’ of an investment, and PV is a dollar amount, the ‘Present Value’.

- Emphasis should be placed on formulae from vocational and other practical contexts including, but not limited to, formulae students will encounter in other topics in this syllabus.
- Examples of changing the subject of formulae and evaluating by substitution could include
 - make F the subject of the formula $C = \frac{5}{9}(F - 32)$, and evaluate F given that $C = 100$
 - make C the subject of $X = \frac{1}{2\pi f C}$, and evaluate C to 3 significant figures given that $X = 0.02$ and $f = 1.85$.
- For students intending to proceed to the Mathematics General 2 HSC course, teachers could introduce the following:
 - solving linear equations involving up to three steps (fractions with numerical denominators only)

$$\text{eg } 5x - 12 = 58, \frac{T}{15} - 50 = 175, 3(a + 7) = 28, \frac{5x + 8}{6} = -3,$$

$$\frac{C - 4}{5} - 7.33 = 2.51$$

- evaluating the subject of a formula following substitution of numerical values, using a wide variety of formulae, such as

$$X = \frac{1}{2\pi f C}, e = iR + \frac{Q}{C}, D = \frac{yA}{(y + 12)}, B = \frac{m}{h^2}, D = \frac{mA}{150}$$

PMG13: Modelling linear relationships

The principal focus of this topic is the graphing and interpretation of linear relationships in practical contexts.

Outcomes addressed

PG1, PG2, PG3, PG9, PG10

Students develop the following knowledge, skills and understanding

- sketching graphical representations of quantities that vary over a period of time or in relation to each other
- identifying independent and dependent variables in practical contexts
- graphing of linear functions that are derived from everyday situations (eg cost of an excursion = fixed cost + cost per student \times number of students) by plotting ordered pairs from tables of values
- calculating the gradients of such graphs with ruler and pencil
- establishing a meaning for the gradient in the given context
- establishing a meaning for the intercept on the vertical axis in the given context
- sketching graphs of linear functions expressed in the form $y = mx + b$
- development of a linear graph of the form $y = mx$ from a description of a situation in which one quantity varies directly with another, given one ordered pair
- using the above graph to establish the value of m (the gradient) and to solve problems related to the given variation context
- finding the solution of simultaneous linear equations from a given graph
- solving simultaneous linear equations given a graph of a practical situation
- sketching the graphs of a pair of linear equations to find the point of intersection
- interpreting linear functions as models of physical phenomena
- using stepwise linear functions to model and interpret practical situations, eg parking charges, taxi fares, tax payments, freight charges
- using graphs to make conversions, eg Australian dollars to euros
- drawing a line of fit on a graphed set of ordered pairs with a ruler and pencil
- using linear equations to model practical situations, eg simple interest
- using technology to construct graphical representations of algebraic expressions
- recognising the limitations of linear models.

Applications and considerations

- Functions and associated mathematical concepts to be introduced via everyday examples, such as
 - distance travelled as a function of time elapsed
 - cost of postage as a function of mass (a step function)
 - amount of fine for speeding as a function of amount by which speed limit is exceeded (a step function)
 - a person's height as a function of age (not strictly linear but can be approximated as a linear function)
 - cost of a mobile phone plan as a function of time and/or as a function of the number of calls
 - income tax rate as a function of taxable income (a piecewise function).
- Practical examples of linear functions include:
 - printing costs, which involve an initial setup cost and a dollar rate per item (or hundreds of items ...) printed
 - cost of taxi fare, which may be approximated by hiring charge + (dollar rate per kilometre) \times (number of kilometres travelled). Vertical intercept is the hiring charge and gradient is the dollar rate per kilometre
 - catering costs, which may involve a base amount for a set number of people plus a rate for extra guests
 - drawing conversion graphs (usually lines through the origin).
- Comparison of different mobile phone plans (graphically) to determine the most economical plan for a person's likely pattern of use.
- Students should recognise the limitations of linear models in practical contexts, eg a person's height as a function of age may be approximated by a straight line for a limited number of years. Students should be aware that models may apply only over a particular domain.

Focus Study 1: Mathematics and Communication

In this Focus Study, students apply, and develop further, knowledge and skills in Financial Mathematics, Measurement, and Algebra and Modelling, to practical communication contexts involving mobile phone plans, internet access plans, digital download, and file storage.

Prerequisite topics

PMG1, PMG2, PMG5, PMG6, PMG7, PMG8, PMG11, PMG12, PMG13

Outcomes addressed

A student:

- PG1 uses Mathematics and statistics to compare alternative solutions to contextual problems
- PG2 represents information in symbolic, graphical and tabular forms
- PG3 represents the relationships between changing quantities in algebraic and graphical form
- PG5 demonstrates awareness of issues in practical measurement, including accuracy, and the choice of relevant units
- PG6 models financial situations relevant to the student's current life using appropriate tools
- PG7 determines an appropriate form of organisation and representation of collected data
- PG9 uses appropriate technology to organise information from a limited range of practical and everyday contexts
- PG10 justifies his/her response to a given problem using appropriate mathematical terminology

Content summary

- FS1.1 Mobile phone plans, internet access plans
- FS1.2 Digital download and file storage

Terminology

bits per second (bps)
contract plan
download

internet service provider (ISP)
pre-paid plan
kilobits per second (Kbps)

Use of technology

Spreadsheets should be used to create graphs and calculate summary statistics.

Access to current charges for mobile phone plans and internet plans are available online.

Focus Study notes

Learning and assessment should draw upon and integrate the Mathematics in other Areas of Study within the course, eg Data Analysis, Measurement, Probability, and Algebra and Modelling (Preliminary course).

Teaching, learning and assessment materials should use current information from a range of sources. Students require access to mobile phone plans, mobile phone bills, internet access plans, and internet bills.

Students should develop the ability to extract and evaluate the underlying Mathematics in a range of texts.

FS1.1: Mobile phone plans, internet access plans

The principal focus of this topic is the interpretation and comparison of mobile phone and internet plans and usage, and the calculation of related costs.

Students develop the following knowledge, skills and understanding

Mobile phone plans

- reading and interpreting a mobile phone bill
- calculation of call and messaging charges on a mobile phone bill
- construction and interpretation of tables and graphs of phone usage and the cost of making phone calls. (Graphs should include step-function and linear piece-wise graphs.)
- identification of any patterns of usage for a given mobile phone bill
- determining a suitable mobile phone plan using calculations based on a user's typical usage pattern.

Internet access plans

- calculation and then comparison of the cost of different internet access plans
- construction and interpretation of tables and graphs of internet usage and related costs. (Graphs should include step-function and linear piece-wise graphs.)
- determining a suitable internet access plan based on a user's typical usage pattern
- construction of an accurate graph of the cost of an internet access plan based on a written description.

Applications and considerations

Mobile phone plans

- Students require access to sample mobile phone bills and current mobile phone plans from telecommunications providers.
- Students calculate excess usage charges and consider initial setup costs and fees and/or the costs of switching from one provider to another.
- Students are to calculate the charges on a mobile phone bill given the call rates, SMS rates, times, and duration of calls, and the terms and conditions of the mobile phone plan.
- Students could analyse their own mobile phone plans and usage, or find examples of mobile phone plans, which include details of the rates and charges.
- Students describe the difference between a pre-paid plan and a contract.
- Students should construct a variety of graph types including those prescribed in PMG5, step-function graphs and linear piece-wise graphs. Line and radar charts are useful displays for data that is time-based.
- Graphs should be constructed using both pen and paper methods and technology.
- To identify usage patterns, students create a frequency table of the number of mobile phone calls made at different times of the day for a two-monthly bill.

| CALL TIME | Tally | Frequency |
|------------------|--------------|------------------|
| 6 am – < 7 am | | 4 |
| 7 am – < 8 am | | 3 |
| 8 am – < 9 am | | 4 |
| etc | | |

A line chart or radar chart could be constructed from this table.

- This exercise should be further extended by tabulating and graphing such variables as call duration, type of call (eg to a mobile, to a landline).
- Students evaluate a person's typical daily mobile phone usage against several alternative plans in order to determine which of the plans is the most cost effective. This should include analysis of both messaging and calling.
- With teacher direction, students could brainstorm to create a list of the attributes of a mobile phone user who is making effective use of a mobile phone plan. Some of these attributes are
 - knowing and understanding the mobile phone plan
 - maximising use of free-time and off-peak rates
 - keeping usage within the allowances of the plan.

Internet access plans

- Students should create an accurate graph of the cost versus usage for an internet access plan based on a written description of the plan, eg the Optra Light Plan allows for 200 Mb usage per month for \$29.95 with additional usage charged at the rate \$0.65 per Mb.
- Current plans offered by Internet Service Providers (ISPs) should be used. These can be found on the internet.
- As an extension activity, students could consider costs and calculations of integrated plans that bundle landline services, mobile phones, and internet access.

FS1.2: Digital download and file storage

In this topic, students perform a range of calculations, and collect, display and interpret statistics, in relation to the download and storage of electronic files.

Students develop the following knowledge, skills and understanding

- calculation of the time to download a file, given a download speed in bits per second, or kilobits per second, where a kilobit is defined as 1000 bits
- conversion between units for measuring secondary storage, such as flash drives and hard disks. Units of storage include bytes, kilobytes, megabytes and gigabytes. The following conversion factors apply:

$$1 \text{ kilobyte} = 2^{10} \text{ bytes} = 1024 \text{ bytes}$$

$$1 \text{ megabyte} = 2^{20} \text{ bytes} = 1\,048\,576 \text{ bytes}$$

$$1 \text{ gigabyte} = 2^{30} \text{ bytes} = 1\,073\,741\,824 \text{ bytes}$$

- conversion of units of storage from bits to bytes, and vice versa
Note: 1 byte = 8 bits
- determining the number of files of a particular format that can be stored on storage devices such as mp3 players, USB flash drives, and hard disk drives, eg calculate the number of mp3 files of average size 4.1 Mb that can be stored on a 4Gb mp3 player
- collecting, displaying and analysing statistics on the downloading of music and videos
- interpretation of statistics related to the effect of downloading audio files and video files on the sales of media companies. This should include data on different distribution systems (traditional music stores and licit online music stores versus illicit downloads).

Applications and considerations

- Students should use the internet to collect the download sizes of files and make statistical calculations, eg a box-and-whisker plot could be constructed to represent the file size of each song on an album. Comparisons could be made between different genres of music, eg classical music versus dance music.
- A manufacturer claims that a 4Gb mp3 player holds 1000 songs.
 - i) Calculate the average file size of a typical song.
 - ii) Use calculations to either support or discount the manufacturer's claim regarding the number of songs that the mp3 player will hold, eg collect file sizes for at least 50 songs and use a spreadsheet to make calculations about the average file size of a particular song.
- It should be noted that for downloading, transfer rates are expressed in bits per second, or kilobits per second.
- Students should also be able to solve some more difficult questions, eg How many seconds would it take to download an 8.3 Mb file if the transfer rate is 5Kbps? Round your answer to the nearest second.
- Statistics on the downloading of music and videos may, for example, include data on the quantity of downloads or downloads categorised by age, gender, region, and musical genre. See PMG5, PMG6 for mathematical skills that can be integrated.
- Apply reasoning based on mathematical calculations to support a position on an ethical issue related to the illicit downloading of music and videos.

Focus Study 2: Mathematics and Driving

In this Focus Study, students apply, and develop further, knowledge and skills in Financial Mathematics, Data Analysis, Measurement, Probability, and Algebra and Modelling, to practical driving contexts involving costs of purchase, running costs, and safety, of motor vehicles.

Prerequisite topics

PMG1, PMG2, PMG3, PMG5, PMG6, PMG7, PMG8, PMG12, PMG13

Outcomes addressed

A student:

- PG1 uses Mathematics and statistics to compare alternative solutions to contextual problems
- PG2 represents information in symbolic, graphical and tabular forms
- PG3 represents the relationships between changing quantities in algebraic and graphical form
- PG5 demonstrates awareness of issues in practical measurement, including accuracy, and the choice of relevant units
- PG6 models financial situations relevant to the student's current life using appropriate tools
- PG7 determines an appropriate form of organisation and representation of collected data
- PG8 performs simple calculations in relation to the likelihood of familiar events
- PG9 uses appropriate technology to organise information from a limited range of practical and everyday contexts
- PG10 justifies his/her response to a given problem using appropriate mathematical terminology

Content summary

- FS2.1 Costs of purchase
- FS2.2 Running costs
- FS2.3 Safety

Terminology

| | |
|-----------------------------------|------------------------|
| alcohol consumption | fuel consumption |
| blood alcohol concentration (BAC) | reaction-time distance |
| braking distance | stamp duty |
| comprehensive insurance | stopping distance |
| depreciation | third-party insurance |
| fatigue | vehicle registration |

Use of technology

Students should use spreadsheets to create graphical representations of collected data.

The internet should be used as a source of up-to-date information including, eg current values of vehicles, stamp-duty rates, fuel prices, and personal loan interest rates.

Online calculators can be used to investigate loan repayments.

Focus Study notes

Learning and assessment should draw upon and integrate the Mathematics in other Areas of Study within the course, eg Data Analysis, Measurement, Probability, and Algebra and Modelling (Preliminary course).

Teaching, learning and assessment materials should use current information from a range of sources including, but not limited to, newspapers, journals, magazines, real bills and receipts, and the internet.

Students should develop the ability to extract and evaluate the underlying Mathematics in a range of texts.

FS2.1: Costs of purchase

The principal focus of this topic is the calculation of motor vehicle purchase and insurance costs, and the interpretation of related tables and graphs.

Students develop the following knowledge, skills and understanding

- calculation of the percentage decrease in the price of a new vehicle after one year
- describing the different types of insurance available, including 'Green Slip' insurance, third-party insurance, and comprehensive insurance
- interpretation of theft and accident statistics in relation to insurance costs
- calculation of the cost of stamp duty payable using current rates. Note: this is a piecewise function
- creating a line graph showing the stamp duty payable on vehicles of various prices, eg create a line graph for vehicles ranging from \$1000 to \$80 000

Note: this is a piecewise function

- calculation of the cost of purchase of vehicles, including finance, transfer of registration, and insurance (emphasis on cars and motorcycles)
- use of tables or an online calculator to determine the monthly repayments on a reducing balance personal loan
- calculation of the total amount repaid over the full period of a loan
- calculation of the total number of possible car number plates available, given all of the different styles of number plates available. See support material.

Applications and considerations

- Students should make decisions about the most appropriate way to display data comparing new models, and one-year-old models, of cars.

Further discussions and investigations could include

- are the results similar for all makes of vehicle?
- are the results similar for motorcycles?

Students should pose and investigate questions generated in classroom discussions.

- Current interest rates for various lending institutions should be compared.
- Given a set amount of money, students investigate the purchase of a vehicle and write a report. Research must include selecting and describing a vehicle, making calculations if additional funding is required, the type of lending institution and lending rate, amount payable in stamp duty, cost of transfer of registration fees, and insurance costs.
- Stamp duty is levied by the Office of State Revenue when a vehicle is registered to a new owner. Stamp duty is paid on the market value of the vehicle or the price actually paid, whichever is greater.

For example, stamp duty is calculated at 3% of the market value of a vehicle up to \$45 000, plus 5% of the value of the vehicle over \$45 000, eg the stamp duty for a vehicle with a market value of \$50 000 is calculated as follows:

Stamp duty on \$45 000 at 3% = \$1350
 Stamp duty on \$5000 at 5% = \$250
 Total stamp duty = \$1600

- Compare the costs of insurance, including 'Green Slip' insurance, third-party insurance, and comprehensive insurance.
- Compare the cost of insurance for different makes of cars of the same size, or for the same make of vehicle with different-aged drivers, or for the same make of vehicle garaged at different addresses.
- Students should investigate and write a report on the factors that affect insurance premiums, eg the type of vehicle, age of the driver, and where the vehicle is to be garaged. Statistics related to vehicle theft, and the gender and age of drivers in accidents, should also be examined when investigating the cost of insurance and making comparisons.
- Students could construct a map to display the cost of insurance in different locations. Theft statistics could also be gathered and a comparison made between the cost of insurance and theft statistics.
- Students could investigate the purchase of a vehicle and write a report. Research could include selecting and describing a vehicle, calculating the amount payable for stamp duty, cost of transfer of registration fees, and cost of insurance.

FS2.2: Running costs

In this topic, students perform calculations in relation to motor vehicle fuel consumption and depreciation, and create and interpret related tables and graphs.

Students develop the following knowledge, skills and understanding

- identifying fuel consumption measures as rates
- calculation of the amount of fuel used on a trip
- calculation of fuel consumption statistics for various vehicles
- calculation of the amount of fuel needed and costs associated for various sizes, makes and models of vehicles, over various distances
- collecting and presenting data on the price of fuel over time to identify trends
- calculation of the depreciation of a vehicle using the straight-line method and the declining-balance method:
 - a) the straight line method
 $S = V_0 - Dn$, where S = salvage (current) value of asset, D = amount of depreciation apportioned per period, V_0 = purchase price of the asset, and n = total number of periods
 - b) the declining balance method
 $S = V_0(1 - r)^n$, where S is the salvage value after n periods, V_0 is the purchase price of the asset and r is the percentage interest rate per period, expressed as a decimal
- creating a depreciation graph based on the straight-line method of depreciation. (Graphs to be produced from formulae and tables.)
- use of prepared graphs and tables of straight-line depreciation and declining balance depreciation to solve problems
- comparison of costs of car sharing schemes and car pools.

Applications and considerations

- Use online calculators to estimate the total weekly running costs for different makes and models of cars and motorcycles.
- Costs should be calculated in order to compare costs for similar vehicles using different types of fuel, eg calculate and compare the costs of a particular vehicle using petrol, diesel, or liquefied petroleum gas (LPG).
- Students could investigate the breakeven cost of installing LPG. A graph would be an appropriate method for displaying the results.
- Trends in fuel prices could be compared for different types of fuel and different locations.
- Students should make calculations and comparisons of running costs using information from a table or graph.
- Calculate the yearly fuel consumption and the yearly cost of petrol for various classes of vehicle, for example, a car with a 4 cylinder, 1.6 L engine to one with a 6 cylinder, 4 L engine given their fuel consumption rate in litres per kilometre.
- Investigate cycles in the price of unleaded petrol (ULP) and calculate savings over time. See support material.
- Compare country fuel prices to metropolitan fuel prices over time and compare different fuel-type prices over time. Students can obtain data from the internet and present the data graphically.
- Calculate and compare the amount of depreciation of motor vehicles for different ages of the vehicles. (The depreciation in the first year of a new car can exceed 35%. For many vehicles, depreciation levels out to between 7% and 10% per annum after the first three years.)
- Students could compare the cost of an individual driving to and from a workplace to car pooling or car sharing schemes for the same purpose.

FS2.3: Safety

In this topic, students construct and interpret tables and graphs and solve a range of problems related to the safe operation of motor vehicles.

Students develop the following knowledge, skills and understanding

- calculation and interpretation of blood alcohol level based on drink consumption and body mass
 - use of formulae, both in words or algebraic, to calculate an estimate for blood alcohol content (BAC)
 - use of tables and graphs to estimate blood alcohol content
 - determination of the number of hours required for a person who stops consuming alcohol to reach zero BAC
 - describing limitations of methods of estimating BAC
- construction and interpretation of graphs that illustrate the level of blood alcohol over time. See support material.
- collection, presentation and interpretation of road accident, drink driving, and fatigue statistics, and calculation of summary statistics (mean, median, mode, range and interquartile range) based on data.
- calculation of distance, speed and time, given two of the three quantities, with change of units of measurement as required

$$D = ST \quad S = \frac{D}{T} \quad T = \frac{D}{S}$$

- calculation of the number of revolutions per second a wheel of a given diameter must make to maintain a certain speed
- calculation of stopping distance (stopping distance = reaction-time distance + braking distance)
- investigation of stopping distances for different speed, road conditions, and weather conditions
- calculation of stopping distance by substitution of values into suitable quadratic formulae
- collection, presentation and interpretation of tables and graphs relating to motor vehicles and motor vehicle accidents
- solving problems based on methods of speed detection, eg fixed speed cameras, hand-held radar etc.

Applications and considerations

Blood alcohol content (BAC)

- Students should discuss why BAC is a function of body weight.
- Formulae used for estimating BAC include:

$$BAC_{Male} = \frac{(10N - 7.5H)}{6.8M} \quad \text{and} \quad BAC_{Female} = \frac{(10N - 7.5H)}{5.5M}$$

where N is the number of standard drinks consumed, H is the number of hours of drinking, and M is the person's mass in kg.

Note: The following are limitations to the estimation of BAC:

- formulae and tables are based on average values and will not apply equally to everyone
- factors (variables) that affect BAC include gender, weight, fitness, health, and liver function.
- Students should discuss the limitations stated above, including why BAC differs for males and females. See support material for table.
- The following formula can be used to calculate the numbers of hours to wait before driving

$$\text{Number of hours} = \frac{BAC}{0.015}$$

eg If Jai's BAC is 0.05 then Jai must wait $3.\dot{3}$ hours = 3 hours and 20 minutes.

- Zero BAC is an important consideration for young drivers as NSW laws require a zero BAC limit for all learner and provisional drivers.
- The average speed of a journey is calculated using the formula

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

- Reaction time is the distance travelled from the time a driver decides to stop to the time the driver first commences braking.
- Students could calculate the error in a vehicle's speedometer readings when the wheel-rim size or tyre profile is changed, ie changing the circumference used to calculate the speed.