2014 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics General 2

Section II

75 marks
Attempt Questions 26–30
Allow about 1 hour and 55 minutes for this section

Answer the questions in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided on pages 33 and 34. If you use this space, clearly indicate which question you are answering.

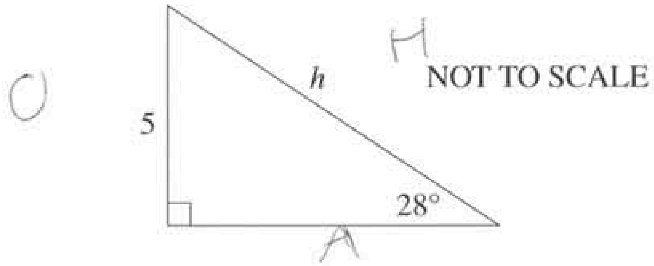
Please turn over

Question 26 (15 marks)

(a) Expand $4x\left(7x^4 - x^2\right)$. **1**

$28x^5 - 4x^3$

(b) Calculate the value of h correct to two decimal places. **2**



$\sin 28 = \frac{5}{h}$

$5/\sin 28 = 10.6502\ldots$

$= 10.65$ units (2dp)

(c) Solve the equation $\frac{5x+1}{3} - 4 = 5 - 7x$. **3**

$+4 \quad +4$

$\frac{5x+1}{3} = 9 - 7x$

$\times 3 \quad \times 3$

$5x + 1 = 3(9 - 7x)$

$5x + 1 = 27 - 21x$

-1

$5x = 26 - 21x$

$\frac{26x}{26} = \frac{26}{26}$

$x = 1$

Question 26 continues on page 15

Question 26 (continued)

(d) Solve these simultaneous equations to find the values of x and y. **3**

$$y = 2x + 1$$
$$x - 2y - 4 = 0$$

$x - 2(2x+1) - 4 = 0$

$\cancel{x-2}\ x - 4x - 2 - 4 = 0$

$+6$

$-3x = \frac{6}{-3}$

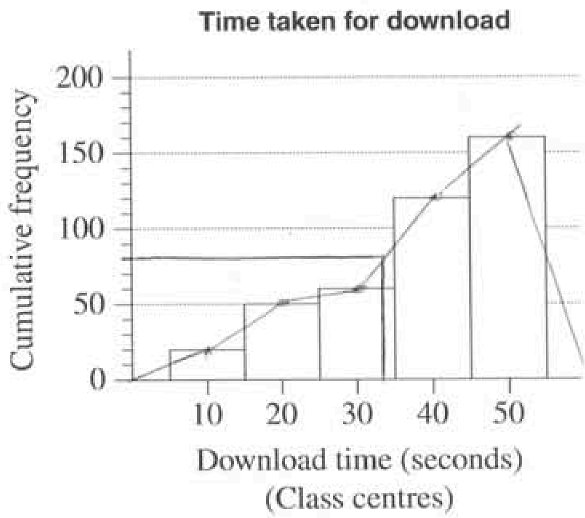
$x = -2$

$y = 2(-2) + 1$

$y = -3$

$\therefore y = -3,\ x = -2$

(e) The times taken for 160 music downloads were recorded, grouped into classes and then displayed using the cumulative frequency histogram shown. **2**



On the diagram, draw the lines that are needed to find the median download time.

Question 26 continues on page 16

Question 26 (continued)

(f) The weight of an object on the moon varies directly with its weight on Earth. An astronaut who weighs 84 kg on Earth weighs only 14 kg on the moon. **2**

A lunar landing craft weighs 2449 kg when on the moon. Calculate the weight of this landing craft when on Earth.

~~84/x~~ $84 \div 14$

$= 6$

$2449 \div 6 = 408.16$ kg

Question 26 continues on page 17

Question 26 (continued)

Si Sy

(g) Singapore is located at 1°N 104°E and Sydney is located at 34°S 151°E. **2**

What is the time difference between Singapore and Sydney? (Ignore daylight saving.)

$151 - 104 = 47 \div 15$

$= 3.13 = 3$ hrs and 8 mins

End of Question 26

Please turn over

Question 27 (15 marks)

(a) Alex is buying a used car which has a sale price of $13 380. In addition to the sale price there are the following costs:

Transfer of registration	$30
Stamp Duty	....

(i) Stamp Duty for this car is calculated at $3 for every $100, or part thereof, of the sale price. **1**

Calculate the Stamp Duty payable.

$13380 \div 100$

$= 133.8 = 134 \times 3$

$= \$402$

(ii) Alex borrows the total amount to be paid for the car including Stamp Duty and transfer of registration. Interest on the loan is charged at a flat rate of 7.5% per annum. The loan is to be repaid in equal monthly instalments over 3 years. **4**

$7.5 \div 12 = 0.625$

Calculate Alex's monthly repayments.

$13380 + 30 + 402 = 13812$

$(13812 \times 0.625\% +) 13812 \div 36$

$= \$469.99$

Question 27 continues on page 19

Question 27 (continued)

(iii) Alex wishes to take out comprehensive insurance for the car for 12 months. The cost of comprehensive insurance is calculated using the following: **3**

Base rate	\$845
Fire Service Levy (FSL)	1% of base rate
Stamp Duty	5.5% of the total of base rate and FSL
GST	10% of the total of base rate and FSL.

Find the total amount that Alex will need to pay for comprehensive insurance.

845 +

FSL = 8.45 +

SD = 46.93975 +

GST = 85.345

= \$985.73

(iv) Alex has decided he will take out the comprehensive car insurance rather than the less expensive non-compulsory third-party car insurance. **1**

What extra cover is provided by the comprehensive car insurance?

Damage done to other vehicles.

Question 27 continues on page 20

Question 27 (continued)

(b) Xuso is comparing the costs of two different ways of travelling to university. **2**

Xuso's motorcycle uses one litre of fuel for every 17 km travelled. The cost of fuel is $1.67/L and the distance from her home to the university car park is 34 km. The cost of travelling by bus is $36.40 for 10 single trips.

Which way of travelling is cheaper and by how much? Support your answer with calculations.

Motorcycle | Bus

~~2~~ 2×1.67

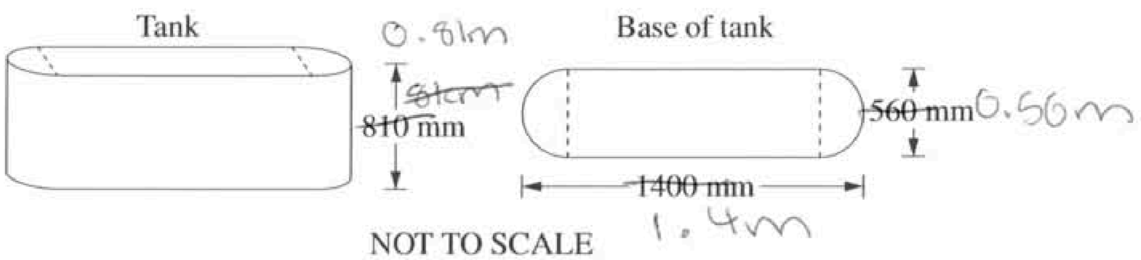
$= \$3.34$ | $\$36.40 -$

$\times 10 = \$33.4$

∴ The motorcycle is cheaper by $3 for 10 single trips

(c) The base of a water tank is in the shape of a rectangle with a semicircle at each end, as shown. **4**

The tank is 1400 mm long, 560 mm wide, and has a height of 810 mm.



NOT TO SCALE

What is the capacity of the tank, to the nearest litre?

$\pi \times 0.28^2 = \text{—} \times 0.81 = 0.1995036999 +$

$0.84 \times 0.56 = 0.4704 \times 0.81 = 0.381024$

$= 0.5805276999\,m^3 \times 1000$

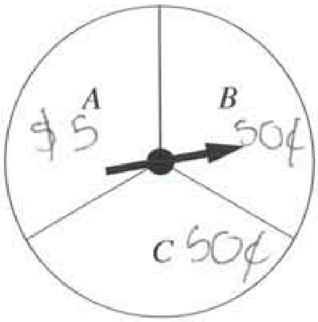
$= 580.5276999\,L$

$= 581\,L$ (nearest L)

End of Question 27

Question 28 (15 marks)

(a) James plays a game involving a spinner with sectors of equal size labelled A, B and C, as shown. **2**



He pays \$2 to play the game. He wins \$5 if the spinner stops in A and 50 cents if it stops in B or C.

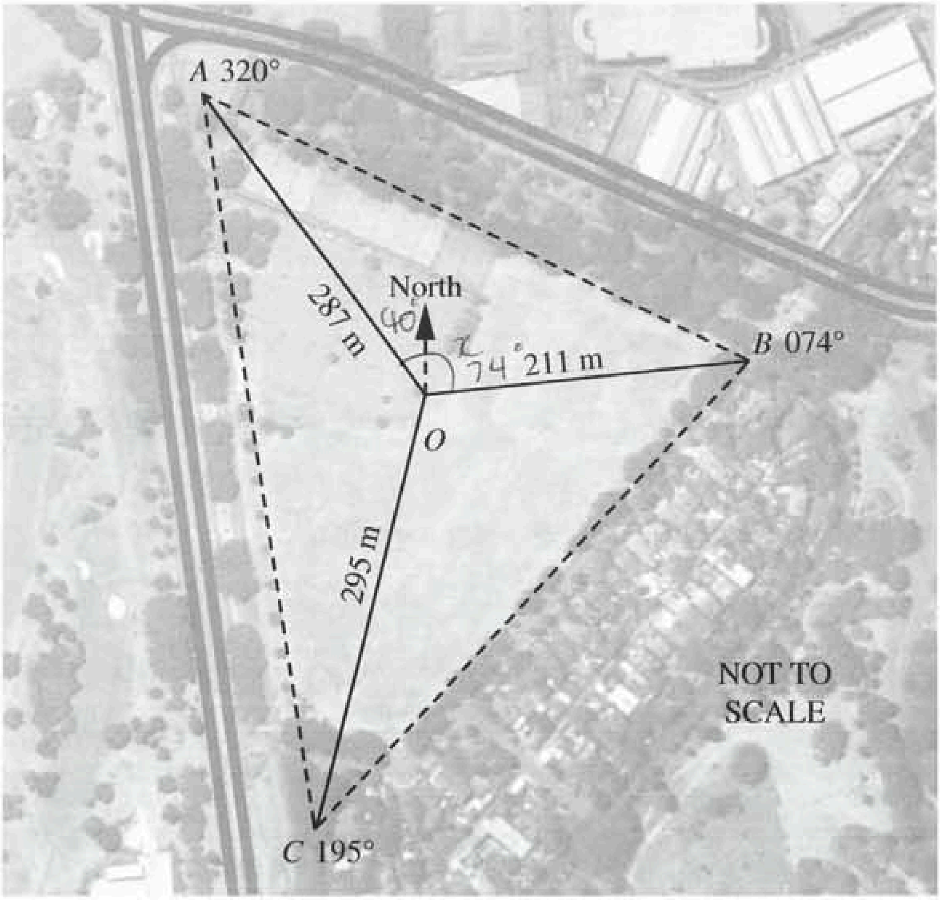
Calculate James's financial expectation for the game.

5 + 0.5 + 0.5 = 6 ÷ 3
= \$2 − \$2
= \$0

Question 28 continues on page 22

Question 28 (continued)

(b) A radial compass survey of a sports centre is shown in the diagram.



(i) Show that the size of angle AOB is 114°. **1**

40 + 74 = 114°

Question 28 continues on page 23

Question 28 (continued)

(ii) Calculate the length of the boundary AB, to the nearest metre. **2**

$c^2 = 211^2 + 287^2 - 2 \times 211 \times 287 \times \cos 114$

$c^2 = 176151.5018$

$\sqrt{\quad}$

$= 419.704\ldots$ m

$AB = 420$ m (nearest m.)

(iii) Find the area of triangle AOB in hectares, correct to two significant figures. **3**

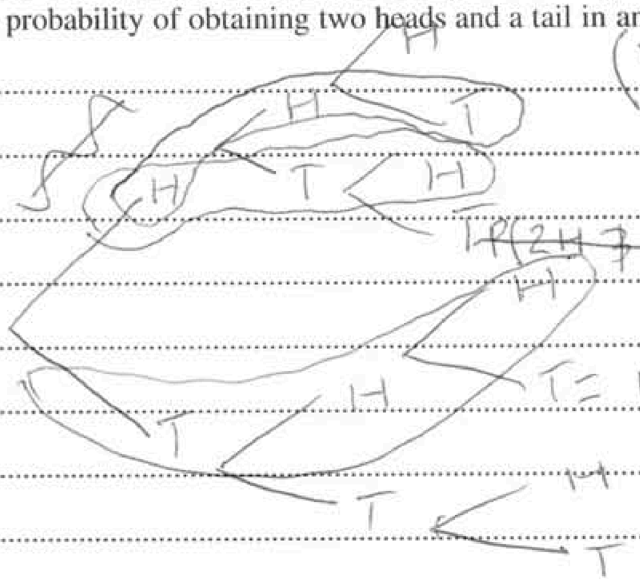
$\cancel{\tfrac{1}{2}}$ $\frac{1}{2} \times 211 \times 287 \times \sin 114$

$= 27660.78614 \text{ m}^2 \div 10000$

$= 2.76607\ldots$ ha

$= 2.8$ ha (2 s.f)

(c) A fair coin is tossed three times. Using a tree diagram, or otherwise, calculate the probability of obtaining two heads and a tail in any order. **2**



$(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) +$

$(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) +$

$(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$

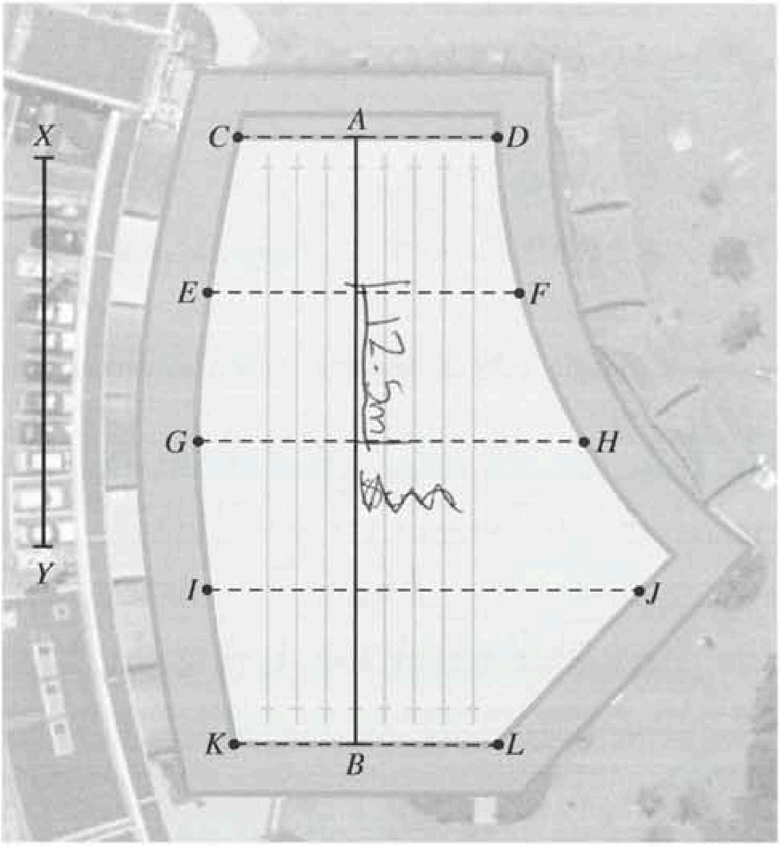
~~P(2H & 1T) = 3/8~~

$= P(2H \& 1T) = 3/8$

Question 28 continues on page 24

Question 28 (continued)

(d) An aerial diagram of a swimming pool is shown.



The swimming pool is a standard length of 50 metres but is not in the shape of a rectangle.

(i) By measuring the length AB, determine the scale of the diagram. **1**

$50 \div 8 = 6.25$

1 cm = 6.25 m

(ii) Using this scale, calculate the length XY of the car park, in metres. **1**

5.1×6.25

$= 31.875$ m

Question 28 continues on page 25

Question 28 (continued)

(iii) In the diagram of the swimming pool, the five widths are measured to be: **3**

$CD = 21.88$ m
$EF = 25.63$ m
$GH = 31.88$ m
$IJ = 36.25$ m
$KL = 21.88$ m

The average depth of the pool is 1.2 m.

Calculate the approximate volume of the swimming pool, in cubic metres. In your calculations, use TWO applications of Simpson's Rule.

$\frac{12.5}{3}(26.256 + 4 \times 30.756 + 38.256) + \frac{12.5}{3}(38.256 + 4 \times 43.5 + 26.256)$

$AL = 21.88 \times 1.2 = 26.256$
$AM = 25.63 \times 1.2 = 30.756$
$AR = 31.88 \times 1.2 = 38.256$

$AL = 31.88 \times 1.2 = 38.256$
$AM = 36.25 \times 1.2 = 43.5$
$AR = 21.88 \times 1.2 = 26.256$

$= 1775.2 \text{ m}^3$

End of Question 28

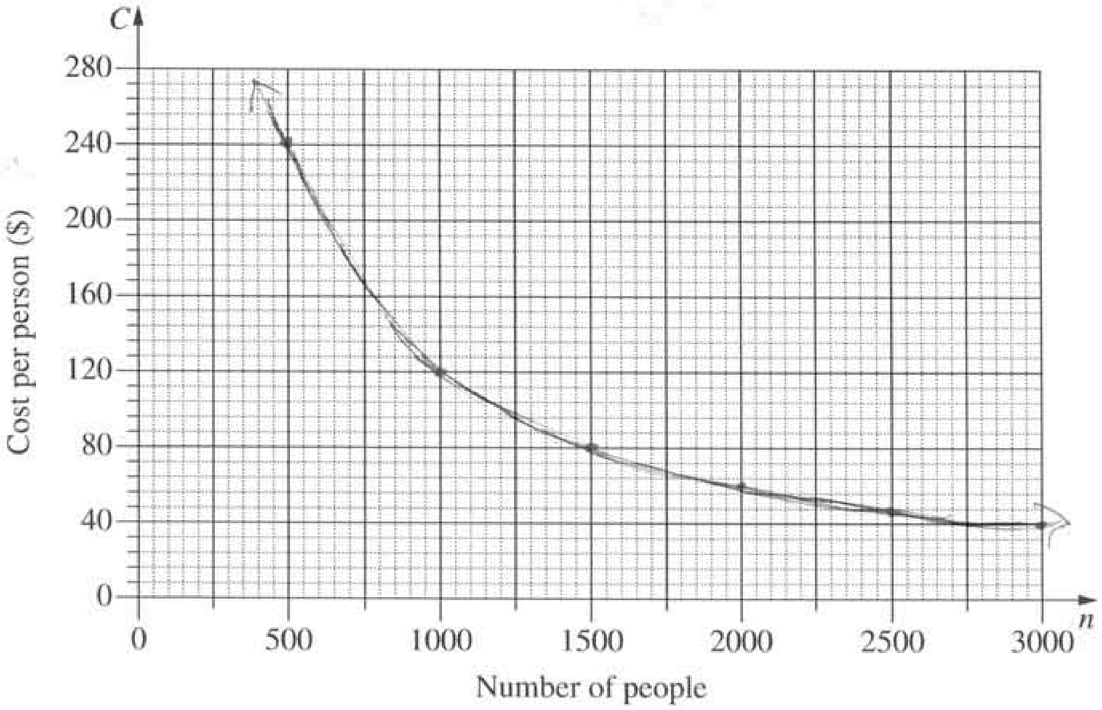
Question 29 (15 marks)

(a) The cost of hiring an open space for a music festival is \$120 000. The cost will be shared equally by the people attending the festival, so that C (in dollars) is the cost per person when n people attend the festival.

(i) Complete the table below by filling in the THREE missing values. **1**

Number of people (n)	500	1000	1500	2000	2500	3000
Cost per person (C)	240	120	80	60	48	40

(ii) Using the values from the table, draw the graph showing the relationship between n and C. **2**



(iii) What equation represents the relationship between n and C? **1**

$C = \dfrac{120000}{n}$

Question 29 continues on page 27

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Question 29 (continued)

(iv) Give ONE limitation of this equation in relation to this context. **1**

It doesn't explain what happens when they reach a large number of attendants, the price could reach as low as $10.

(v) Is it possible for the cost per person to be $94? Support your answer with appropriate calculations. **1**

$94 = \frac{120000}{n} \times n$ $\quad 94n = 120000$

$= 1276.595745$

∴ it is not possible because the amount of people must be a whole number not a decimal

(b) What is the maximum number of standard drinks that a male weighing 84 kg can consume over 4 hours in order to maintain a blood alcohol content (BAC) of less than 0.05? **3**

$0.05 = \frac{10N - 7.5(4)}{6.8(84)} = \frac{10N - 30}{571.2} = 0.05$ $\times 571.2$

$28.56 = 10N - 30$
$+30$

$\frac{10N}{10} = \frac{58.56}{10}$

$N = 5.856$

∴ he can consume a maximum of 5 standard drinks in order to maintain a BAC of less than 0.05.

Question 29 continues on page 28

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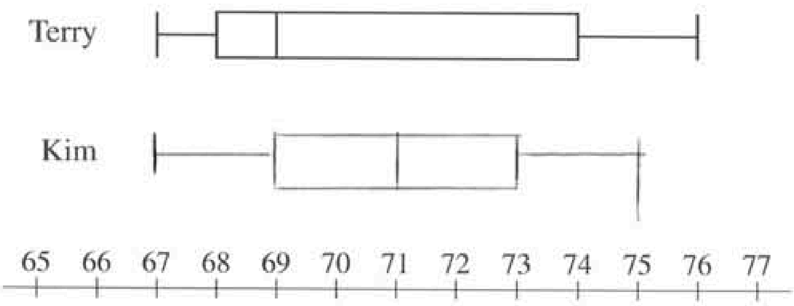
Question 29 (continued)

(c) Terry and Kim each sat twenty class tests. Terry's results on the tests are displayed in the box-and-whisker plot shown in part (i).

(i) Kim's 5-number summary for the tests is 67, 69, 71, 73, 75. **1**

Draw a box-and-whisker plot to display Kim's results below that of Terry's results.



(ii) What percentage of Terry's results were below 69? **1**

50 %

(iii) Terry claims that his results were better than Kim's. Is he correct? Justify your answer by referring to the summary statistics and the skewness of the distributions. **4**

He is incorrect because his median of 69 is lower than Kim's (71), indicating the majority of his scores being lower than hers despite him having the highest score. Kim's interquartile range (4) is much better than Terry's (6) displaying her more consistent marks while Kim's symmetrical skewness supports her consistency as opposed to Terry's positive skewness, indicating the majority of his scores were lower than hers.

End of Question 29

Question 30 (15 marks)

FV

(a) Chandra and Sascha plan to have \$20 000 in an investment account in 15 years time for their grandchild's university fees. **3**

The interest rate for the investment account will be fixed at 3% per annum compounded monthly.

Calculate the amount that they will need to deposit into the account now in order to achieve their plan.

$$PV = \frac{20000}{(1 + 0.25\%)^{180}}$$

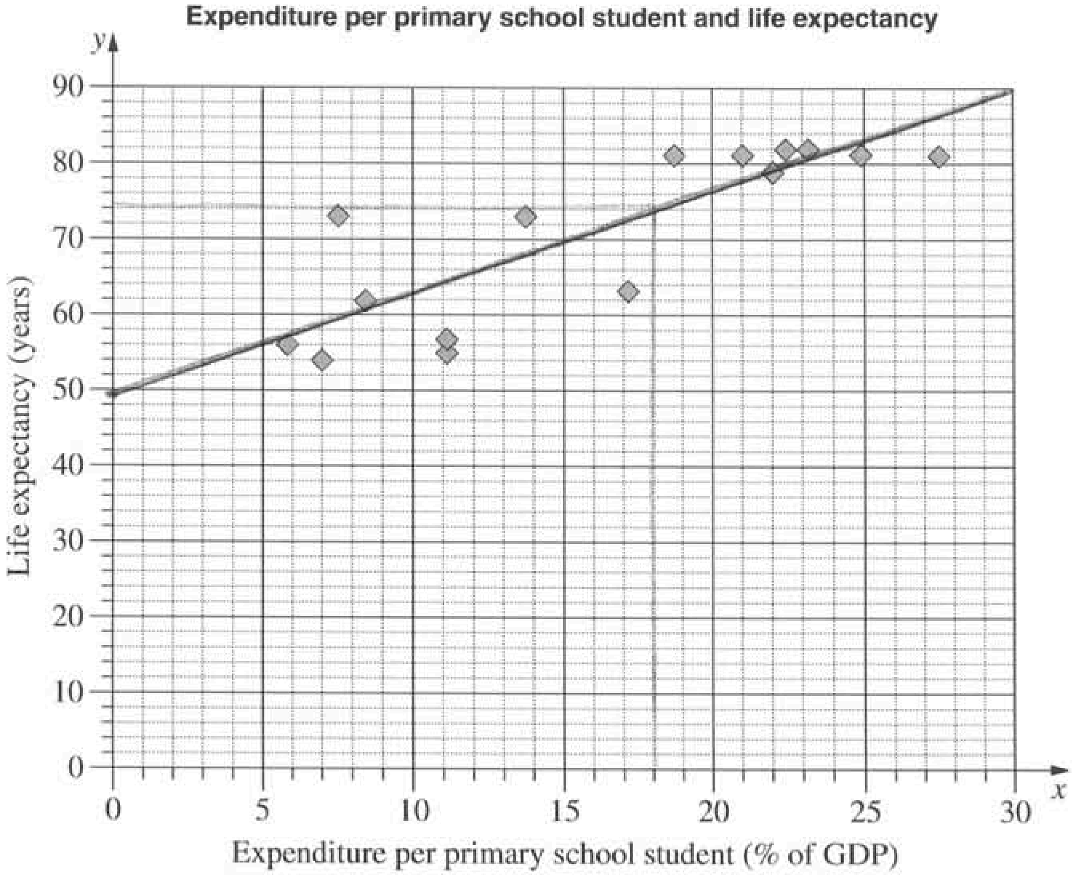
$= \$12759.72643$

$= \$12759.73$

Question 30 continues on page 30

Question 30 (continued)

(b) The scatterplot shows the relationship between expenditure per primary school student, as a percentage of a country's Gross Domestic Product (GDP), and the life expectancy in years for 15 countries.



(i) For the given data, the correlation coefficient, r, is 0.83. What does this indicate about the relationship between expenditure per primary school student and life expectancy for the 15 countries? **1**

This indicates a strong positive relationship between the expenditure per primary school student and the life expectancy for the 15 countries.

Question 30 continues on page 31

Question 30 (continued)

(ii) For the data representing expenditure per primary school student, Q_L is 8.4 and Q_U is 22.5. **1**

What is the interquartile range?

22.5 − 8.5 = 14

(iii) Another country has an expenditure per primary school student of 47.6% of its GDP. Would this country be an outlier for this set of data? Justify your answer with calculations. **2**

47.6 − 22.5 = 25.1 + 1.5 × 14

= 46.1

∴ the score would not be an outlier.

(iv) The expenditures per primary school student for the 15 countries in the scatterplot are: **2**

5.9, 7, 7.6, 8.4, 11.2, 11.2, 13.7, 17.1, 18.7, 21.1, 22, 22.5, 23.2, 24.9, 27.6

Complete the table below by calculating the mean, $\bar{x}$, and the standard deviation, σ_x, of these data. Calculate both values to two decimal places.

The table also shows the mean, $\bar{y}$, and the standard deviation, σ_y, of life expectancy for the same 15 countries.

	Mean	*Standard deviation*
Expenditure per primary school student	$\bar{x}$ = 16.14	σ_x = 7.03
Life expectancy	$\bar{y} = 70.73$	$\sigma_y = 10.94$

Question 30 continues on page 32

Question 30 (continued)

(v) Using the values from the table in part (iv), show that the equation of the least-squares line of best fit is **2**

$$y = 1.29x + 49.9.$$

gradient $= 0.83 \times \frac{10.94}{7.03} = 1.291635846$
$= 1.29$ (2dp)

y int. $= 70.73 - (1.291635846 \times 16.14)$
$= 49.88299745 \approx 49.9$ (1dp)

$\therefore y = 1.29x + 49.9$

(vi) On the scatterplot on page 30, draw the least-squares line of best fit, $y = 1.29x + 49.9$. **2**

(vii) Using this line, or otherwise, estimate the life expectancy in a country which has an expenditure per primary school student of 18% of its GDP. **1**

$\approx$ 74 years.

(viii) Why is this line NOT useful for predicting life expectancy in a country which has expenditure per primary school student of 60% of its GDP? **1**

Because the line would reach unrealistic ages of ~~over~~ up to 150 years old which can't be reached.

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Section II extra writing space

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