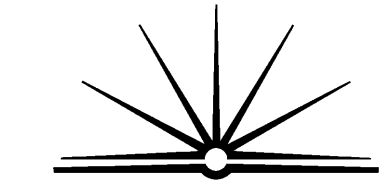

Mathematics
Years 7–10 Syllabus

Advice on Programming and Assessment

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Contents

1 Introduction

This support document has been designed to help teachers understand key aspects of the new *Mathematics Years 7–10 Syllabus* and to provide guidance for implementation. The document shows how these aspects can be incorporated into teaching and learning programs, and how these programs are underpinned by the principles of *assessment for learning* (*Mathematics Years 7–10 Syllabus* p 168) that aim to support students in their learning.

Advice is provided about constructing a program that will cover the scope and sequence of Mathematics for a Stage. The document sets out a process of planning and sequencing units of work and developing teaching and learning activities.

Five sample units of work have been prepared to assist teachers in initial planning for implementation of the new syllabus. They are:

- a Stage 3 unit of work on Two-dimensional Space to support Year 7 students who have not demonstrated achievement of Stage 3 outcomes in this area
- a Stage 4 unit of work on Fractions to assist in the teaching of this topic area to a Year 7 class
- a Stage 5.1 unit of work on Trigonometry to demonstrate an introduction to a traditional topic of Mathematics that was not taught at this level in the previous Years 9–10 syllabus
- a Stage 5.2 unit of work on Graphs of Physical Phenomena that demonstrates teaching and learning of a newer area
- a Stage 5.3 unit of work on Graphs of Physical Phenomena that builds on the Stage 5.2 unit.

These sample units can be used as a model for planning units of work. They include relevant outcomes and content, assessment activities that have been designed and integrated into the units of work, different types of possible feedback, a variety of teaching and learning experiences, and opportunities for student reflection.

These sample units may be modified or amended to suit the needs, interests and abilities of students.

For a small percentage of students with special education needs who are undertaking Life Skills outcomes and content, support materials will be provided which will assist in the development of a meaningful and relevant program of study related to the Mathematics Years 7–10 syllabus.

One assessment activity and a sample of student work from each unit have been chosen to show how the activity fits into the teaching and learning sequence. The activities and work samples show how teachers can monitor student performance and provide students with feedback on their learning, including what they will need to do next to achieve the identified learning goals.

2 Advice on programming

2.1 Assessment for learning

The Board's revised syllabuses advocate *assessment for learning,* not just assessment for accountability. Assessment that enhances learning recognises that learners use their current understanding to discover, develop and incorporate new knowledge, skills and understanding. Assessment for learning helps teachers and students to know the breadth and depth of the students' understanding.

Assessment occurs as a regular part of teaching and learning. Teacher instruction and assessment influence student learning and learning processes. This involves using assessment activities to clarify student understanding of concepts, and planning ways to remedy misconceptions and promote deeper understanding.

Assessment for learning encourages self-assessment and peer assessment. Students can develop and use a range of strategies to actively monitor and evaluate their own learning and the learning strategies they use.

The feedback that students receive from completing assessment activities will help teachers and students decide whether they are ready for the next phase of learning or whether they need further learning experiences to consolidate their knowledge, skills and understanding. Teachers should consider the impact assessment and feedback have on student motivation and self-esteem, and the importance of active involvement of students in their own learning.

By integrating learning and assessment, the teacher can choose which aspects of a student's performance to record. These records can be used to consider students' progress, what to teach next and at what level of detail. At key points, such as the end of the year, this information is also available for the teacher to use to form a snapshot of a student's performance against levels of achievement. This snapshot can be used to inform the parent, the next teacher and especially the student of where they are up to. Consequently, teachers using their professional judgement in a standards-referenced framework are able to extend the process of assessment *for* learning into their assessment *of* learning.

2.2 Establishing a scope and sequence

Five sample units of work and the sample program overviews in Section 3 have been developed to demonstrate ways in which teachers can build a teaching/learning program and units of work to ensure coverage of the syllabus. The process outlined in Section 2.3 is one way to realise the aim and objectives of the syllabus.

The *Mathematics Years 7–10 Syllabus* allows teachers a degree of flexibility in the way that they sequence their teaching and learning program, allowing the development of a program of study that best meets students' needs.

However, the scope and sequence must reflect the syllabus requirements. This syllabus has been designed so that students would typically achieve the standards described in 400 hours. These indicative hours will provide the basis for timetabling and programming decisions. In any particular Stage, some students may achieve the standard sooner than the majority of their cohort, while others may require additional time. Students entering Year 7 who have not achieved Stage 3 outcomes may need more time.

By incorporating *assessment for learning* activities, these sample units show ways teachers can address the needs, interests and abilities of their students, while assessing their progress towards the demonstration of outcomes. The sample units also illustrate ways in which assessment activities may be integrated into the teaching and learning sequence.

The sample units will assist teachers to:

- be explicit about the outcomes and content they are addressing
- be explicit about the evidence required to demonstrate student learning
- give meaningful feedback to students
- adapt teaching and learning programs to students' demonstrated needs
- have a sound basis for modification of future teaching and learning programs (in the light of students' demonstrated needs).

The opportunities provided for questioning and dialogue, for self-assessment and peer assessment, and for reflection, enable students to be clear about their own learning, understanding and needs.

2.3 Planning units of work for effective learning and assessment

A program overview should be constructed for each Stage to ensure that the outcomes and content from the syllabus are covered.

The syllabus recognises the hierarchical nature of most learning in mathematics. New learning typically builds upon earlier learning in the same, or a related, topic area from the syllabus. In this way, it is possible that students may have met content related to the unit of work at an earlier time. If the outcomes for the topic have been partially achieved, the unit should focus on consolidating and extending the students' knowledge, skills and understanding.

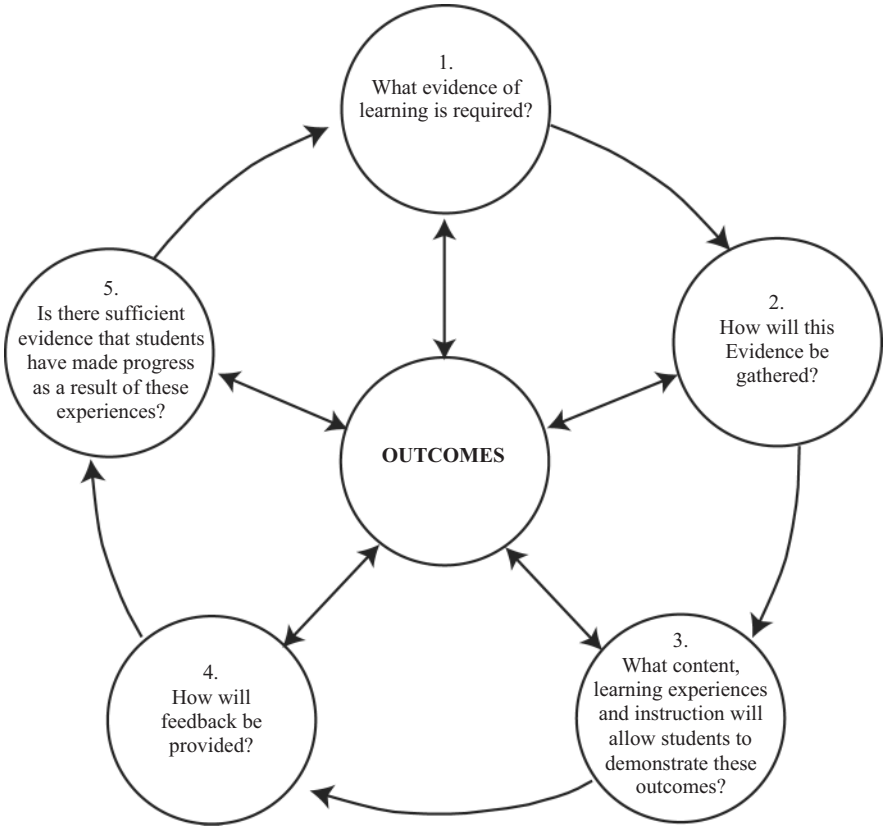
This syllabus acknowledges that students learn in different ways and at different rates. This has major implications for programming and designing units of work. For example, teachers may need to incorporate a range of activities to accommodate the different ways students learn. Decisions will have to be made as to how best (within the context of available resources) to cater for the range of levels of students' current knowledge, skills and understanding in mathematics. In Year 7, for example, while there may be a majority of students working within Stage 4 content, other students may be working within Stage 3, or perhaps within Stage 2 or Stage 5.

The sample units of work have been developed using the following process:

- identify the outcomes that will be highlighted in the unit of work
- decide on the subject matter or focus of the unit
- decide on the evidence of learning required and how students will demonstrate this in relation to the outcomes, and how this evidence will be gathered and recorded
- select the relevant syllabus content for the identified outcomes relating to the knowledge, skills and understanding that students will develop
- plan learning experiences identifying those that will provide evidence of learning
- ensure a range of assessment strategies is used and meaningful feedback is given to students
- provide opportunities for the teacher to reflect on student progress and modify future learning experiences accordingly.

The following diagram summarises a model for developing integrated assessment activities. It emphasises that outcomes are central to the decisions teachers make about the learning to be undertaken and the evidence of learning that needs to be collected. This evidence enables teachers to provide students with feedback on their learning and to determine how well students are achieving in relation to these outcomes.

Evidence of learning will assist teachers and students to decide whether they are ready for the next phase of learning or whether further learning experiences are needed to consolidate students' knowledge, skills and understanding.



2.4 Designing effective learning and assessment

When designing effective learning experiences, the type of activity selected should allow evidence of learning to be gathered. Methods of gathering evidence could include teacher observation, questioning, self-assessment and peer assessment as well as more formalised assessment activities. Assessment should be an integral part of the unit of work and support student learning.

Teachers should consider whether the assessment:

- has explicitly stated purposes and addresses the outcomes
- is integral to the teaching and learning program
- shows a clear relationship between the outcomes and content being assessed
- allows students to demonstrate the extent of their knowledge, skills and understanding
- focuses on what was taught in class and what students were informed would be assessed
- provides opportunities to gather information about what further teaching and learning is required for students to succeed
- provides valid and reliable evidence of student learning
- is fair.

2.5 Sharing learning and assessment intentions

Students need to be aware of what they need to do to demonstrate evidence of learning. This information could be conveyed to them informally or formally, for particular learning activities. Students should be informed of the criteria that will be used to assess their learning. They should be clear about the meaning of the language used, the subject-specific terminology and glossary terms. They also need to be clear about any sources or stimulus material that are appropriate to the activity.

It may be helpful to give students models of good responses, and templates or procedures to help them demonstrate the extent of their knowledge, skills and understanding.

2.6 Effective feedback to students

The aim of feedback is to communicate to students how well their knowledge, skills and understanding are developing in relation to the outcomes. Students are then given opportunities to improve and further develop their knowledge, skills and understanding. Feedback enables students to recognise their strengths and their areas for development, and to plan with their teacher the next steps in their learning.

Students should be provided with regular opportunities to reflect on their learning. Teacher feedback about student work in relation to outcomes is essential for students and is integral to the teaching and learning process. Student self-reflection and peer evaluation will also provide valuable feedback to students.

Feedback should:

- focus on the activity and what was expected
- be constructive
- provide meaningful information to students about their learning
- correct misunderstanding
- identify and reinforce students' strengths
- state clearly how students can improve.

Forms of feedback include:

- discussion with the class, groups or individual students
- written annotations
- general comments to the class about those aspects of the activity in which students excelled and those aspects that still need addressing
- examples of good responses
- peer and self-evaluation.

2.7 Recording evidence for assessment

Recording student performance needs to be manageable. It need not occur after each *assessment for learning* activity. Teachers should make decisions about when student performance on an assessment activity should be recorded, which aspects to record and in what format. The teacher can use this information to ascertain where students are up to, what to teach next and at what level of detail, and to form a snapshot of student achievement at key points.

Record keeping should reflect the reporting processes of the school and may take the form of individual comments/notations for the tasks, marks, grades or visual representations such as that provided below.

A scale may be a useful way to summarise the extent of students' learning. This example shows individual students' performances on the same assessment activity.

Student	Assessment Activity – Operations with Fractions
1	x
2	x
3	x
4	x
5	x
6	x
7	x
8	x
	Developing **Sound** **Thorough**

This method can be adapted to capture evidence of an individual student's strengths and weaknesses on various elements of one activity, or the performance of a particular student, class, group or cohort of students, across a range of assessment activities.

3 Practical programming and assessment

3.1 Sample program overviews

The sample program overviews contained in this section demonstrate the flexibility available to teachers in implementing this syllabus. Consideration will need to be given to the knowledge, skills and understanding of students as they enter secondary school in order to plan units of work that meet their needs. While decisions may be made early in Year 7 about possible overviews, they will need to be reviewed and revised as students' needs change, particularly for those students who experience considerable growth in understanding during the early years of secondary school.

Teachers in schools will manage this in ways appropriate for their students. They will need to consider how to most effectively meet the needs of different groups of students over the four years from Year 7 to Year 10. Some students who enter Year 7 may not have demonstrated the mathematics outcomes for Stage 3, or perhaps even for Stage 2. Other students may have commenced work on Stage 4 topics while still in primary school.

For those students who require more time to learn mathematical concepts, it is possible to include the mathematical content outlined in Stage 3, Stage 4 and Stage 5.1 in programs covering the first four years of secondary school. It may be that the three years from Year 7 to Year 9 are needed to achieve the outcomes up to and including those in Stage 4, with students commencing work on Stage 5.1 outcomes from the beginning of, or during, Year 10.

For those students who require less time to learn mathematical concepts, it may be possible to achieve Stage 4 outcomes during Year 8 or earlier, and then to complete work in Stages 5.1 and 5.2 during Years 9 and 10. Other students may be able to complete all of Stages 5.1, 5.2 and 5.3 by the end of Year 10.

Teachers can program for related topics from different Stages to be taught sequentially or concurrently in a particular year. For example, students who are able to work with abstractions and generalisations could cover the work on Properties of Geometrical Figures in Stages 4 and 5.2 concurrently as a lead-in to Deductive Geometry in Stage 5.3. Topics from different strands may be integrated, for example, teachers may extend work on positive integral indices (Stage 4) to cover negative indices (Stage 5.1) and fractional indices (Stage 5.3) within both the Number and Patterns and Algebra strands.

Sample program overviews, or 'scope and sequences', for different groups of students are outlined below. The overviews have been designed for:

- students who have not completed Stage 3 by the end of Year 6 and who may achieve Stage 5.1 outcomes by the end of Year 10
- students who have not completed Stage 3 by the end of Year 6 but who can be expected to achieve Stage 5.2 outcomes by the end of Year 10
- students who have completed Stage 3 in Year 6 and who will aim to achieve Stage 5.2
- students who complete Stage 4 in Year 8 and who wish to study the Stage 6 Mathematics course (Stage 5.2 outcomes plus additional outcomes from Stage 5.3)
- students who commence Stage 4 in Year 7 and who will achieve all Stage 5.3 outcomes including the optional outcomes
- students who have commenced Stage 4 in Year 6 and who will achieve all 5.3 outcomes including the optional outcomes.

Each of these overviews contains a list of the outcome codes and topic headings that could be treated in each Year from 7 to 10. Note that the topics are listed in the sequence in which they appear in the syllabus rather than in any recommended teaching sequence.

Program overview for students who have not completed Stage 3 by the end of Year 6 and who may achieve Stage 5.1 outcomes by the end of Year 10

Year 7	Year 8	Year 9	Year 10
NS3.1 Whole Numbers NS3.2 Addition and Subtraction NS3.3 Multiplication and Division NS3.4 Fractions and Decimals PAS3.1a, PAS3.1b Patterns and Algebra DS3.1 Data MS3.1 Length MS3.2 Area MS3.4 Mass MS3.5, MS4.3 Time SGS3.1 3D Space SGS3.2a, SGS3.2b 2D Space SGS3.3 Position	NS3.5 Chance NS4.1 Operations with Whole Numbers NS4.2 Integers NS4.3 Fractions, Decimals and Percentages PAS4.1 Algebraic Techniques PAS4.2 Number Patterns DS4.1 Data Representation MS4.1 Perimeter and Area SGS4.2 Angles SGS4.3 Properties of Geometrical Figures	NS4.4 Probability PAS4.3, PAS4.4 Algebraic Techniques DS4.2 Data Analysis and Evaluation MS3.3 Volume and Capacity MS4.2 Surface Area and Volume SGS4.1 Properties of Solids SGS4.4 Properties of Geometrical Figures	NS5.1.1 Rational Numbers NS5.1.2 Consumer Arithmetic NS5.1.3 Probability PAS4.5 Linear Relationships PAS5.1.1 Algebraic Techniques PAS5.1.2 Coordinate Geometry DS5.1.1 Data Representation and Analysis MS5.1.1 Perimeter and Area MS5.1.2 Trigonometry

Program overview for students who have not completed Stage 3 by the end of Year 6 but who can be expected to achieve Stage 5.2 outcomes by the end of Year 10

Year 7	Year 8	Year 9	Year 10
NS3.1 Whole Numbers NS3.2 Addition and Subtraction NS3.3 Multiplication and Division NS3.4 Fractions and Decimals NS3.5 Chance NS4.1 Operations with Whole Numbers NS4.2 Integers PAS3.1a, PAS3.1b Patterns and Algebra PAS4.1 Algebraic Techniques PAS4.2 Number Patterns DS3.1 Data MS3.1 Length MS3.2 Area MS3.4 Mass MS3.5, MS4.3 Time SGS3.1 3D Space SGS3.2a, SGS3.2b 2D Space SGS3.3 Position SGS4.1 Properties of Solids SGS4.2 Angles	NS4.3 Fractions, Decimals and Percentages NS4.4 Probability PAS4.3, PAS4.4 Algebraic Techniques PAS4.5 Linear Relationships DS4.1 Data Representation DS4.2 Data Analysis and Evaluation MS3.3 Volume and Capacity MS4.1 Perimeter and Area MS4.2 Surface Area and Volume SGS4.3, SGS4.4 Properties of Geometrical Figures	NS5.1.1 Rational Numbers NS5.2.1 Rational Numbers NS5.1.2 Consumer Arithmetic NS5.1.3 Probability PAS5.1.1 Algebraic Techniques PAS5.1.2 Coordinate Geometry PAS5.2.1, PAS5.2.2 Algebraic Techniques (part in Year 9 and part in Year 10) DS5.1.1 Data Representation and Analysis MS5.1.1 Perimeter and Area MS5.2.1 Perimeter and Area MS5.1.2 Trigonometry SGS5.2.1 Properties of Geometrical Figures	NS5.2.2 Consumer Arithmetic PAS5.2.1, PAS5.2.2 Algebraic Techniques (part in Year 9 and part in Year 10) PAS5.2.3, PAS5.2.4 Coordinate Geometry PAS5.2.5 Graphs of Physical Phenomena DS5.2.1 Data Analysis and Evaluation MS5.2.2 Surface Area and Volume MS5.2.3 Trigonometry SGS5.2.2 Properties of Geometrical Figures

Program overview for students who have completed Stage 3 in Year 6 and who will aim to achieve Stage 5.2

Year 7	Year 8	Year 9	Year 10
Review and Consolidation of Stage 3 NS4.1 Operations with Whole Numbers NS4.2 Integers NS4.3 Fractions, Decimals and Percentages NS4.4 Probability PAS4.1 Algebraic Techniques PAS4.2 Number Patterns PAS4.3 Algebraic Techniques MS4.1 Perimeter and Area MS4.3 Time SGS4.1 Properties of Solids SGS4.2 Angles	NS5.1.1 Rational Numbers PAS4.4 Algebraic Techniques PAS4.5 Linear Relationships DS4.1 Data Representation DS4.2 Data Analysis and Evaluation MS4.2 Surface Area and Volume SGS4.3, SGS4.4 Properties of Geometrical Figures	NS5.2.1 Rational Numbers NS5.1.2 Consumer Arithmetic NS5.1.3 Probability PAS5.1.1 Algebraic Techniques PAS5.1.2 Coordinate Geometry PAS5.2.1, PAS5.2.2 Algebraic Techniques (part in Year 9 and part in Year 10) DS5.1.1 Data Representation and Analysis MS5.1.1 Perimeter and Area MS5.2.1 Perimeter and Area MS5.1.2 Trigonometry SGS5.2.1 Properties of Geometrical Figures	NS5.2.2 Consumer Arithmetic PAS5.2.1, PAS5.2.2 Algebraic Techniques (part in Year 9 and part in Year 10) PAS5.2.3, PAS5.2.4 Coordinate Geometry PAS5.2.5 Graphs of Physical Phenomena DS5.2.1 Data Analysis and Evaluation MS5.2.2 Surface Area and Volume MS5.2.3 Trigonometry SGS5.2.2 Properties of Geometrical Figures

Program overview for students who complete Stage 4 in Year 8 and who wish to study the Stage 6 Mathematics course (Stage 5.2 outcomes plus additional outcomes from Stage 5.3)

Year 7	Year 8	Year 9	Year 10
Review and Consolidation of Stage 3 NS4.1 Operations with Whole Numbers NS4.2 Integers NS4.3 Fractions, Decimals and Percentages NS4.4 Probability PAS4.1 Algebraic Techniques PAS4.2 Number Patterns PAS4.3 Algebraic Techniques MS4.1 Perimeter and Area MS4.3 Time SGS4.1 Properties of Solids SGS4.2 Angles	NS5.1.1 Rational Numbers NS5.2.1 Rational Numbers NS5.1.2 Consumer Arithmetic PAS4.4 Algebraic Techniques PAS4.5 Linear Relationships DS4.1 Data Representation DS4.2 Data Analysis and Evaluation MS4.2 Surface Area and Volume SGS4.3, SGS4.4 Properties of Geometrical Figures	NS5.1.3 Probability NS5.2.2 Consumer Arithmetic PAS5.1.1 Algebraic Techniques PAS5.2.1, PAS5.2.2 Algebraic Techniques PAS5.1.2 Coordinate Geometry PAS5.2.3 Coordinate Geometry DS5.1.1 Data Representation and Analysis DS5.2.1 Data Analysis and Evaluation MS5.1.1, MS5.2.1 Perimeter and Area MS5.2.2 Surface Area and Volume MS5.1.2, MS5.2.3 Trigonometry SGS5.2.1, SGS 5.2.2 Properties of Geometrical Figures	NS5.3.1 Real Numbers PAS5.2.4 Coordinate Geometry PAS5.3.1, PAS5.3.2 Algebraic Techniques PAS5.3.3, PAS5.3.4 Coordinate Geometry PAS5.2.5 Graphs of Physical Phenomena MS5.3.2 Trigonometry SGS5.3.1 Deductive Geometry SGS5.3.2 Deductive Geometry SGS5.3.3 Deductive Geometry

Program overview for students who commence Stage 4 in Year 7 and who will achieve all Stage 5.3 outcomes including the optional outcomes

Year 7	Year 8	Year 9	Year 10
NS4.1 Operations with Whole Numbers NS4.2 Integers NS4.3 Fractions, Decimals and Percentages NS4.4 Probability PAS4.1 Algebraic Techniques PAS4.2 Number Patterns PAS4.3, PAS4.4 Algebraic Techniques DS4.1 Data Representation MS4.1 Perimeter and Area MS4.2 Surface Area and Volume MS4.3 Time SGS4.1 Properties of Solids SGS4.2 Angles SGS4.3 Properties of Geometrical Figures	NS5.1.1 Rational Numbers NS5.2.1 Rational Numbers NS5.1.2 Consumer Arithmetic NS5.1.3 Probability PAS4.5 Linear Relationships PAS5.1.1 Algebraic Techniques PAS5.1.2 Coordinate Geometry DS4.2 Data Analysis and Evaluation DS5.1.1 Data Representation and Analysis MS5.1.1, MS5.2.1 Perimeter and Area SGS4.4. SGS5.2.1, SGS5.2.2 Properties of Geometrical Figures	NS5.2.2 Consumer Arithmetic NS5.3.1 Real Numbers PAS5.2.1, PAS5.2.2 Algebraic Techniques PAS5.2.3, PAS5.2.4 Coordinate Geometry DS5.2.1 Data Analysis and Evaluation MS5.2.2 Surface Area and Volume MS5.3.1 Surface Area and Volume MS5.1.2 Trigonometry MS5.2.3 Trigonometry SGS5.3.1 Deductive Geometry SGS5.3.2 Deductive Geometry SGS5.3.3 Deductive Geometry	NS5.3.2 Probability PAS5.3.1, PAS5.3.2 Algebraic Techniques PAS5.3.3, PAS5.3.4 Coordinate Geometry PAS5.2.5, PAS5.3.5 Graphs of Physical Phenomena #PAS5.3.6, #PAS5.3.7 Curve Sketching and Polynomials #PAS5.3.8 Functions and Logarithms MS5.3.2 Trigonometry #SGS5.3.4 Circle Geometry

Program overview for students who have commenced Stage 4 in Year 6 and who will achieve all Stage 5.3 outcomes including the optional outcomes

Year 6	Year 7	Year 8	Year 9	Year 10
Stage 3 content as well as some Stage 4 topics that might include: NS4.1 Operations with Whole Numbers DS4.1 Data Representation MS4.3 Time	NS4.2 Integers NS4.3 Fractions, Decimals and Percentages NS4.4 Probability PAS4.1 Algebraic Techniques PAS4.2 Number Patterns PAS4.3, PAS4.4 Algebraic Techniques MS4.1 Perimeter and Area MS5.1.1, MS5.2.1 Perimeter and Area MS4.2 Surface Area and Volume SGS4.1 Properties of Solids SGS4.2 Angles SGS4.3 Properties of Geometrical Figures	NS5.1.1 Rational Numbers NS5.2.1 Rational Numbers NS5.1.2 Consumer Arithmetic NS5.1.3 Probability PAS4.5 Linear Relationships PAS5.1.1 Algebraic Techniques PAS5.1.2 Coordinate Geometry DS4.2 Data Analysis and Evaluation DS5.1.1 Data Representation and Analysis SGS4.4. SGS5.2.1, SGS5.2.2 Properties of Geometrical Figures SGS5.3.1 Deductive Geometry	NS5.2.2 Consumer Arithmetic NS5.3.1 Real Numbers NS5.3.2 Probability PAS5.2.1, PAS5.2.2 Algebraic Techniques PAS5.2.3, PAS5.2.4 Coordinate Geometry DS5.2.1 Data Analysis and Evaluation MS5.2.2 Surface Area and Volume MS5.3.1 Surface Area and Volume MS5.1.2 Trigonometry MS5.2.3 Trigonometry SGS5.3.2 Deductive Geometry SGS5.3.3 Deductive Geometry	PAS5.3.1, PAS5.3.2 Algebraic Techniques PAS5.3.3, PAS5.3.4 Coordinate Geometry PAS5.2.5, PAS5.3.5 Graphs of Physical Phenomena #PAS5.3.6, #PAS5.3.7 Curve Sketching and Polynomials #PAS5.3.8 Functions and Logarithms MS5.3.2 Trigonometry #SGS5.3.4 Circle Geometry

3.2 Programmed units of work

These sample units of work are designed to assist teachers as they plan for the implementation of the *Mathematics Years 7–10 Syllabus*. The units provide initial programming ideas for selected syllabus content. Each unit of work relates to a particular topic from the syllabus.

The sample units of work include:

- a Stage 3 unit of work on Two-dimensional Space to support Year 7 students who have not demonstrated achievement of Stage 3 outcomes in this area
- a Stage 4 unit of work on Fractions to assist in the teaching of this topic area to a Year 7 class
- a Stage 5.1 unit of work on Trigonometry to demonstrate an introduction to a traditional topic of Mathematics that was not taught at this level in the previous Years 9–10 syllabus
- a Stage 5.2 unit of work on Graphs of Physical Phenomena that demonstrates teaching and learning in a newer area
- a Stage 5.3 unit of work on Graphs of Physical Phenomena that builds on the Stage 5.2 unit.

Each of these units includes the specific content outcome(s) that provide the main focus of the unit as well as the key ideas, knowledge and skills statements, and Working Mathematically statements from the syllabus. The page reference to the relevant content page in the syllabus is also provided.

> **WM** Working Mathematically
>
> The Working Mathematically outcomes are listed so that teachers consider the development and assessment of these outcomes as well as the content outcomes. Assessment activities that provide opportunities for students to demonstrate one or more of the five Working Mathematically processes (*Questioning, Applying Strategies, Communicating, Reasoning, Reflecting*) have been labelled with WM.

Advice regarding the appropriate use of technology, links to other substrands and strands, suitable resources, and language and literacy considerations are also provided. The resources include suggested references to support some of the Learning Experiences. A reference list for the units of work included can be found at the end of this section (p 48).

The Learning Experiences provide suggestions to support teaching and learning for this topic. Teachers are encouraged to choose experiences appropriate for their students, and to develop additional learning experiences when necessary. The assessment activities also provide suggestions for teachers to choose where appropriate. Each unit will need to be adapted to meet the needs of particular students.

The assessment activity and annotated work sample included at the end of each unit demonstrate how teachers can monitor student performance and provide feedback to support further learning.

3.3 Stage 3 Unit: Two-dimensional Space

3.3.1 Outcomes, Key Ideas and Content from the Syllabus

Space and Geometry - Two-dimensional Space	**Syllabus Content pp 151-152**
SGS3.2a Manipulates, classifies and draws two-dimensional shapes and describes side and angle properties **SGS3.2b** Measures, constructs and classifies angles	**Key Ideas** Identify right-angled, isosceles, equilateral and scalene triangles Identify and draw regular and irregular two-dimensional shapes Identify and name parts of a circle Enlarge and reduce shapes, pictures and maps Identify shapes that have rotational symmetry Classify angles as right, acute, obtuse, reflex, straight or a revolution Measure in degrees and construct angles using a protractor

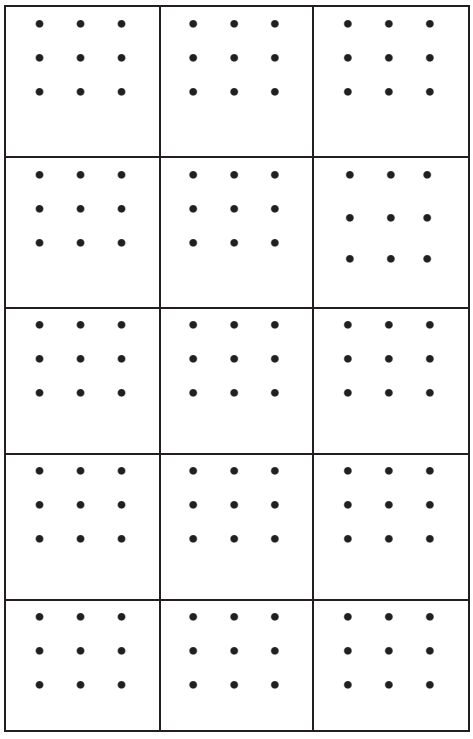
Working Mathematically Outcomes

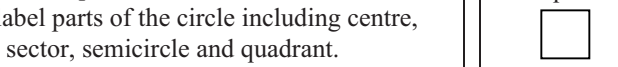
Questioning	Applying Strategies	Communicating	Reasoning	Reflecting
Asks questions that could be explored using mathematics in relation to Stage 3 content	Selects and applies appropriate problem-solving strategies, including technological applications, in undertaking investigations	Describes and represents a mathematical situation in a variety of ways using mathematical terminology and some conventions	Gives a valid reason for supporting one possible solution over another	Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 3 content

Knowledge and Skills	**Working Mathematically**
Two-dimensional Shapes • identifying and naming right-angled triangles • manipulating, identifying and naming isosceles, equilateral and scalene triangles • comparing and describing side properties of isosceles, equilateral and scalene triangles • exploring by measurement angle properties of isosceles, equilateral and scalene triangles • exploring by measurement angle properties of squares, rectangles, parallelograms and rhombuses • identifying and drawing regular and irregular two-dimensional shapes from descriptions of their side and angle properties • using templates, rulers, set squares and protractors to draw regular and irregular two-dimensional shapes • identifying and drawing diagonals on two-dimensional shapes • comparing and describing diagonals of different two-dimensional shapes • creating circles by finding points that are equidistant from a fixed point (the centre) • identifying and naming parts of a circle, including the centre, radius, diameter, circumference, sector, semi-circle and quadrant • identifying shapes that have rotational symmetry, determining the order of rotational symmetry • making enlargements and reductions of two-dimensional shapes, pictures and maps • comparing and discussing representations of the same object or scene in different sizes eg student drawings enlarged or reduced on a photocopier	• select a shape from a description of its features *(Applying Strategies, Communicating)* • describe side and angle properties of two-dimensional shapes *(Communicating)* • construct a shape using computer drawing tools, from a description of its side and angle properties *(Applying Strategies)* • explain classifications of two-dimensional shapes *(Communicating)* • inscribe squares, equilateral triangles, regular hexagons and regular octagons in circles *(Applying Strategies)* • explain the difference between regular and irregular shapes *(Communicating)* • construct designs with rotational symmetry, including using computer drawing tools *(Applying Strategies)* • enlarge or reduce a graphic or photograph using computer software *(Applying Strategies)* • use computer drawing tools to manipulate shapes in order to investigate rotational symmetry *(Applying Strategies)*

Knowledge and Skills	Working Mathematically
Angles • identifying the arms and vertex of an angle where both arms are invisible, such as rotations and rebounds • recognising the need for a formal unit for the measurement of angles • using the symbol for degrees (º) • using a protractor to construct an angle of a given size and to measure angles • estimating and measuring angles in degrees • classifying angles as right, acute, obtuse, reflex, straight or a revolution • identifying angle types at intersecting lines	• describe angles found in their environment *(Communicating, Reflecting)* • compare angles in different two-dimensional shapes *(Applying Strategies)* • explain how an angle was measured *(Communicating)* • rotate a graphic or object through a specified angle about a particular point, including using the rotate function in a computer drawing program *(Applying Strategies)*
Technology Dynamic geometry software and computer drawing tools could be used to manipulate shapes, investigate their properties, and construct designs.	**Links** Shapes can be used in designs created in Visual Arts. Reduction of shapes enables scale drawings to be created and described using scales. This links to Measurement.
Resources Chalk, ropes, string, stakes, tape measure, protractor (180º and 360º), compass, templates, paper circles, pattern blocks, geoboards, elastic bands, rulers, grid paper. List of References (p 48) – No.s 4, 9, 11, 13	**Language** Circle, circumference, compass, radius, diameter, centre, triangle, equilateral, scalene, isosceles, right-angle triangle, rotation, symmetry, polygon, angle, degree.

3.3.2 Learning experiences and assessment activities

Learning Experiences	Assessment Activities
Review of Two-dimensional Space from earlier Stages To review and consolidate the learning of Two-dimensional Space from Stage 2, students could: • sort and classify a collection of two-dimensional shapes • sort a collection of quadrilaterals and describe the strategies used • describe a particular shape in their own language • identify shapes in the environment • select a shape from a description of its features • use a shape to make a tessellating pattern • draw all lines of symmetry on each of a collection of shapes • identify examples of angles in the environment • describe angles using everyday language • identify right angles in drawings. **Stage 3 Content:** **Triangles – Side Properties** The teacher provides students with a variety of equilateral, scalene and isosceles triangles including some right-angled triangles. In small groups, students sort the triangles and discuss reasons for the selection of criteria for sorting. **The students share and compare sorting procedures.** The teacher prompts students to reflect on all information generated and devise generalisations about the side properties of the types of triangles they have identified. **Protractors** Students are introduced to the formal unit for measuring angles and the symbol for degrees (°), and shown how to use protractors to measure angles. Circular protractors are effective tools for measuring reflex angles. In pairs, students estimate the size of various angles and check their partner's estimates. Students replicate various angles in the room using geostrips, copy these onto paper, and measure the angles. Students use protractors to construct angles of various sizes. **Classifying Angles (right, acute, obtuse, reflex, straight, revolution)** Students are introduced to the classification of angles and practise measuring a selection of each of these angles. Students identify, record and classify as many angles in the environment as they can. In pairs, students create a table for each type of angle they have found and write a description for each type eg 'These angles are all obtuse because they are greater than 90° but smaller than 180°.' **Triangles – Angle Properties** The teacher provides students with a variety of scalene, isosceles and equilateral triangles, including a variety of right-angled, acute-angled and obtuse-angled triangles. In small groups, students sort the triangles and investigate the sizes of the angles for each type of triangle. They then write descriptions of their observations of the angle	**Open-ended Questions** WM Open-ended questions can be used to find out what students know and can do from earlier Stages. Students could work on such questions in pairs to encourage discussion of ideas. Listening to students' conversations provides valuable insights into their knowledge, skills and understanding. For example: • tell me everything you know about a particular shape • draw a shape with one line of symmetry (repeat for two, three and four lines of symmetry). **Card-matching Activity** Students match the description of a two-dimensional shape to a diagram of the shape, and to the name of the shape. **Angle Search** WM **Students are given a worksheet like the following.**  Students are required to draw angles by joining dots, creating as many different angles as possible. Students measure and classify the angles, and are encouraged to find a pattern in the answers. Students could investigate angles on different-sized grids such as 4×4 and 5×5.

Learning Experiences	Assessment Activities
properties for each of the types of triangles. The teacher prompts students to reflect on all information generated and devise generalisations about the angle properties of the types of triangles they have identified. **Measuring Angles in Quadrilaterals** Students are provided with a variety of quadrilaterals. Using a protractor they measure the angles. Possible questions include: • what do you expect to find out in your investigations? • how can you record your investigations? • can you classify the angles you have found? • can you classify the quadrilaterals according to the angles they have? • can you compare the quadrilaterals by the angles they have? How? **Investigating Diagonals** Students investigate the number of diagonals that can be drawn for four, five, six, seven, … sided figures. They record solutions and predict the number of diagonals of figures with more sides. Students examine the diagonals drawn in squares, rectangles, parallelograms, trapeziums, kites, and rhombuses and describe their observations. Possible questions include: • for which quadrilaterals are the diagonals also axes of symmetry? • for which quadrilaterals are the diagonals the same length? **Circles** Students draw circles using chalk and string to determine a possible definition of a circle. Then pairs of compasses can be used to draw circles on paper. Students identify and label parts of the circle including centre, radius, diameter, circumference, sector, semicircle and quadrant. Possible questions include: • which parts of the circle can you find a label for? • do these labels apply to all circles? • is there a relationship between some parts of the circle? Students practise using a pair of compasses to make designs using circles. These designs could include inscribing squares, equilateral triangles, regular hexagons and regular octagons in the circles. **Rotational Symmetry** Students investigate the rotational symmetry of a variety of polygons , determining the order of rotational symmetry **Enlargements and Reductions of Shapes** Students create enlargements and reductions of various shapes using grid paper and use measurements to make conclusions about the sizes of angles and the relationship between the lengths of the sides.	**Assessment Activity and Work Sample (see page 18)** **Triangles** WM Students are given a protractor and ruler, and asked to: 1. Use these instruments to draw three different types of triangle. 2. Name the types of triangle you have drawn. 3. Explain how the triangles are different from each other. Use appropriate mathematical terms when naming and explaining. **Quadrilaterals** WM Students are asked to list the similarities and differences between pairs of shapes, with diagrams supplied to aid observation. They are prompted to think about sides, angles, diagonals and symmetry. **Shapes** / **Same** / **Different** Rectangle  Square  **Circles** WM Draw a circle with a pair of compasses. (i) Label the centre O. (ii) Place a point on the circumference, and label it A. (iii) Place another point on the circumference, and label it B. (iv) Join OA, AB and BO. What kind of triangle have you drawn? Justify your answer. **Odd One Out** WM Students select the shape that does not belong to a set of given shapes (diagrams supplied). They explain their choice. **What am I?** WM Students select a shape and construct questions about the side and angle properties of that shape. In turn, students share questions with the class, who attempt to identify the shape eg 'My shape has four sides and four equal angles. Two of the sides are the same length. What am I?'

3.3.3 Assessment activity and work sample

Focus: Space and Geometry, Measurement and Working Mathematically

Triangles

Students are given a protractor and ruler, and asked to:

1. Use these instruments to draw three different types of triangles.
2. Name the types of triangle you have drawn.
3. Explain how the triangles are different from each other. Use appropriate mathematical terms when naming and explaining.

Adapted from *SNAP 2000*, NSW Department of Education and Training


Individual


30 minutes

End

Possible prompts to assist student engagement

- How did you use the instruments to draw different triangles?
- Can you describe one of your triangles?
- Why did you choose that particular name for your triangle?
- Can you think of any other names for your triangles?
- Do any of your triangles look the same? If so, does this mean that they would have the same name?

Suggested Materials

protractors, rulers, pencils, blank paper

Outcomes

Length (MS3.1)

Selects and uses the appropriate unit and device to measure lengths, distances and perimeters

Two-dimensional Space (SGS3.2a)

Manipulates, classifies and draws two-dimensional shapes and describes side and angle properties

Two-dimensional Space (SGS3.2b)

Measures, constructs and classifies angles

Applying Strategies (WMS3.2)

Selects and applies appropriate problem-solving strategies, including technological applications, in undertaking investigations

Communicating (WMS3.3)

Describes and represents a mathematical situation in a variety of ways using mathematical terminology and some conventions

Criteria for judging quality of performance

The student may demonstrate the following:

- draws three triangles that are not similar to each other
- uses the instruments appropriately
- describes triangles using everyday language and some mathematical terminology
- describes triangles using terms such as ‘equilateral’, ‘isosceles’, ‘scalene’, ‘acute angle’, ‘right-angled’ and ‘obtuse angle’
- covers comprehensively the range of possible triangles.

Feedback

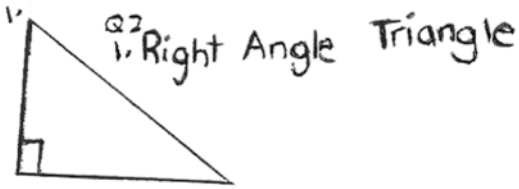
Students will receive:

- possible prompt questions related to the assessment activity to assist their engagement
- written feedback on individual responses to indicate correct naming of triangles, use of appropriate language, and accuracy of measurement
- specific advice about strategies to further consolidate knowledge, skills and understanding, and suggestions for completing related additional tasks
- oral feedback (to the class and individuals) related to the overall understanding of the concepts and misconceptions about the mathematical ideas.

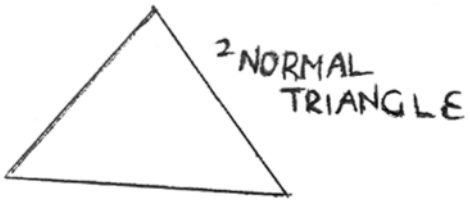
Triangles Work Sample

Overall Comment

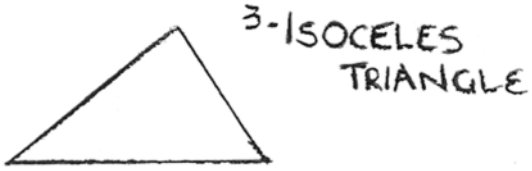
This student can draw three triangles that are not similar but is unable to correctly name the triangles.
The student has used the two ways in which triangles are classified, side properties (scalene, isosceles and equilateral) and angle properties (right-, acute- and obtuse-angled).



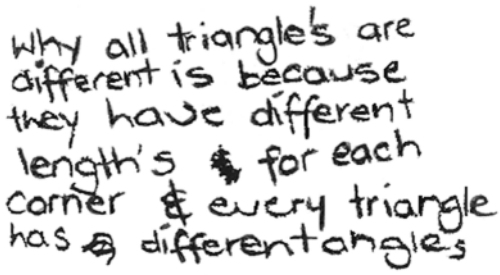
1. A right-angled triangle is correctly drawn and named with the right angle correctly indicated.



2. The triangle drawn with two equal sides is called 'normal', perhaps because this is the shape of the triangle *normally* drawn on the board.



3. The scalene triangle drawn has been mislabelled as isosceles.



4. The explanation of how the triangles are different shows little use of appropriate mathematical terms eg *'different length's* (sic) *for each corner'.*

Follow Up

This student demonstrates a need for further language development. More practice is needed in using appropriate mathematical terms in oral and written form. Spelling and use of apostrophes also need to be addressed.
Further activities to promote understanding of classification and naming of triangles according to both side and angle properties need to be provided.
The teacher may need to draw scalene triangles as well as either equilateral or isosceles triangles to ensure that students do not think that the last two are *normal*. Triangles used in demonstrations should be of different shapes, sizes and orientations.

3.4 Stage 4 Unit: Fractions

3.4.1 Outcomes, Key Ideas and Content from the Syllabus

Number - Fractions, Decimals and Percentages — **Syllabus Content p 63**

NS4.3	**Key Ideas**
Operates with fractions, decimals, percentages, ratios and rates	Perform operations with fractions and mixed numerals

Working Mathematically Outcomes

Questioning	Applying Strategies	Communicating	Reasoning	Reflecting
Asks questions that could be explored using mathematics in relation to Stage 4 content	Analyses a mathematical or real-life situation, solving problems using technology where appropriate	Uses mathematical terminology and notation, algebraic symbols, diagrams, text and tables to communicate mathematical ideas	Identifies relationships and the strengths and weaknesses of different strategies and solutions, giving reasons	Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 4 content

Knowledge and Skills

- finding highest common factors and lowest common multiples
- finding equivalent fractions
- reducing a fraction to its lowest equivalent form
- adding and subtracting fractions using written methods
- expressing improper fractions as mixed numerals and vice versa
- adding mixed numerals
- subtracting a fraction from a whole number
  eg $3-\frac{2}{3}=2+1-\frac{2}{3}=2\frac{1}{3}$
- multiplying and dividing fractions and mixed numerals

This is a subset of the content for this outcome since the focus in this unit is on operations with fractions. It does not include decimals, percentages, ratios or rates.

Working Mathematically

- explain multiplication of a fraction by a fraction using a diagram to illustrate the process *(Reasoning, Communicating)*
- explain why division by a fraction is equivalent to multiplication by its reciprocal *(Reasoning, Communicating)*
- recognise and explain incorrect operations with fractions eg explain why $\frac{2}{3}+\frac{1}{4}\neq\frac{3}{7}$ *(Applying Strategies, Reasoning, Communicating)*
- question the reasonableness of statements in the media that quote fractions, decimals or percentages eg 'the number of children in the average family is 2.3' *(Questioning)*
- solve a variety of real-life problems involving fractions, decimals and percentages *(Applying Strategies)*
- use a number of strategies to solve unfamiliar problems, including:
  - using a table
  - looking for patterns
  - simplifying the problem
  - drawing a diagram
  - working backwards
  - guess and refine

  (Applying Strategies, Communicating)

Technology

Electronic calculators can be used to investigate patterns, check solutions, and convert between fractions, decimals and percentages.

Links

Fraction concepts are applied in other areas of mathematics eg simplifying algebraic expressions, Probability, Trigonometry and Measurement.

Resources

Fraction circles, pattern blocks

Squares of paper, strips of paper

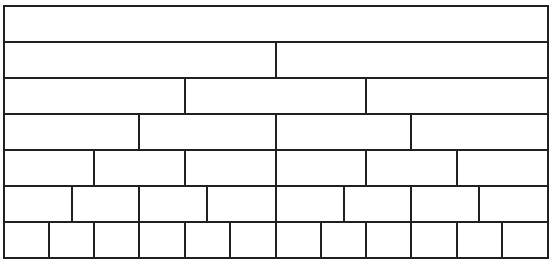
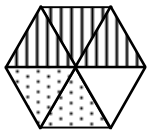
Fractions mats

List of References (p 48) – No.s 4, 9, 10, 11, 12, 13

Language

Students may need assistance with the subtleties of the English language when solving word problems. Consider for example the phrases 'one third *of*', 'how many thirds *in*', and 'one third of the part *remaining*'.

3.4.2 Learning experiences and assessment activites

Learning Experiences	Assessment Activities
Introduction Activities require students to be flexible in their thinking about fractions. They need to be able to visualise and interpret a fraction represented as part of a whole using a linear model or a number line, and also to recognise a fraction as part of a whole using an area model. **Review of Fractions** To review and consolidate the learning of fractions from Stages 2 and 3, students could: • use fraction circles, fraction strips or pattern blocks to model halves, quarters and eighths; thirds, sixths and twelfths; fifths and tenths; and record equivalence statements • use concrete models of simple fractions to aid addition of fractions where one denominator is a multiple of the other • use a fractions mat like the one below to determine equivalent fractions or to perform addition and subtraction of simple fractions  • order fractions that have related denominators • change improper fractions to mixed numerals and mixed numerals to improper fractions (with denominators 2, 3, 4, 5, 6, 8, 10, 12) **Introduction to Stage 4 Content** To extend students experiences of fractions beyond denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100, students could: • assign fractional values to each of the pieces in a tangram puzzle • find a fraction that is equidistant from two others on the number line eg $\frac{5}{8}$ and $\frac{3}{4}$. **Addition and Subtraction of Fractions** Students could: • model addition and subtraction of fractions using concrete materials for those with co-prime denominators. This should include the development of understanding of lowest common multiple (LCM) eg $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$  • describe a method for finding the LCM of two numbers to a partner. Choose any two numbers less than 13 and ask the partner to find the LCM. Repeat this several times, with students swapping notes • use a diagram to illustrate how $\frac{3}{4}+\frac{5}{6}$ can be added by first finding the LCM, shading equivalent areas, and then adding • model addition and subtraction of mixed numerals using concrete materials. In pairs, students discuss a method that could be used without the use of concrete materials and share this with another pair • develop and use written methods for adding and subtracting fractions	**Open-ended Questions** WM Open-ended questions can be used to find out what students know and can do from earlier Stages. Students could work on such questions in pairs to encourage discussion of ideas. Listening to students' conversations provides valuable insights into their knowledge, skills and understanding. For example: a write down everything you know about one half b draw a shape and shade one quarter of it. Can you shade it in a different way? c i) place the following fractions in order: $\frac{1}{5}$, $\frac{3}{10}$, and $\frac{7}{100}$ ii) record three different fractions like these and place them in order d i) record a fraction that lies between $\frac{1}{4}$ and $\frac{1}{2}$ ii) find another fraction iii) how did you get your answers? e write down 5 fractions that are equivalent to $\frac{2}{3}$ and explain how you determined these answers to another student. **What's the Problem?** WM The teacher poses the question: 'The answer to a problem is one and a half, what might the question be?' Students record a variety of questions, including word problems, number sentences and questions that involve more than one operation. They are encouraged to include the four operations in their questions. Possible prompts: • What are you trying to find out? • How could you change your question to give you the right answer? • What if the answer was one half? Could you think of a question? • What could you use to help you? • Can you think of another question? • What do you notice about your questions? • Can you use different denominators?

Learning Experiences

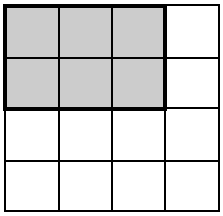
Before beginning multiplication of fractions, it will be necessary to determine students' knowledge, skills and understanding of multiplying a fraction by a whole number from Stage 3. The open-ended questions from the assessment activities provide a good starting point.

Multiplication of Fractions

1. Fold a square of paper in half along a vertical axis and in half again. Unfold, and repeat folds horizontally.

Students could:

- determine that if the side of the square is 1 unit, the area of the whole square is 1×1 square unit ie 1 square unit
- determine that the area of each square defined by the fold lines is $\frac{1}{16}$ of the total square ie $\frac{1}{16}$ square unit
- count in quarters along the side of the square
- trace a rectangle, defined by the fold lines having a length of $\frac{3}{4}$ units and a breadth of $\frac{2}{4}$ units
- express the area of the rectangle as a fraction of the total square
- use the formula for finding the area of a rectangle to write a number sentence linking the length and breadth of the rectangle to its area ie $\frac{3}{4} \times \frac{2}{4} = \frac{6}{16}$
- trace out other rectangles using the fold lines as guides, and record the fraction multiplication facts found by considering the area of each rectangle formed.



2. Students repeat the above, folding the paper square in thirds vertically and in quarters horizontally.

3. Students repeat the above, folding the paper square in thirds vertically and in thirds horizontally.

4. Students pool their results and record in a table:

Length of rectangle	Breadth of rectangle	Area of rectangle	Fraction multiplication L x B = A
$\frac{3}{4}$	$\frac{2}{4}$	$\frac{6}{16}$	$\frac{3}{4} \times \frac{2}{4} = \frac{6}{16}$

Students could:

- discuss patterns or short cuts for multiplying fractions, without folding a square sheet
- write a rule for multiplying fractions using words
- develop and use written methods for multiplying fractions.

Assessment Activities

Open-ended Questions WM

For example:

a if a student wrote the solution to a word problem as $3 \times \frac{1}{2}$, what might the word problem have been?

b make up a word problem that involves multiplication of a whole number by a fraction and calculate the answer

c draw a diagram to illustrate $5 \times \frac{1}{4}$

d describe a method that can be used to multiply a fraction by a whole number

e when you multiply a whole number by a fraction less than one, is the answer bigger or smaller than the original number? Give three examples to justify your answer.

f describe what error has been made in the following calculation and correct the solution

$$4 \times \frac{2}{3} = \frac{4 \times 2}{4 \times 3} = \frac{8}{12} = \frac{2}{3}$$

Assessment Activity and Work Sample (see page 24) WM

Operations with Fractions

The assessment tasks included on the attached worksheet can be used to assess addition as well as multiplication of fractions.

Possible prompts:

- Is there another denominator you could use to add these two fractions?
- If you make a mistake in shading, draw another grid like the one given, and try again.
- Have you checked your calculations?
- Have you shown all necessary working?
- Can you do the calculation another way? Do you get the same answer?

Learning Experiences

Note that the Stage 3 content does not include division of fractions.

Division of Fractions

Students could:

- discuss the relationship between multiplication and division leading to the conclusion that for each multiplication statement, two division statements may be written eg for $3 \times 4 = 12$, the corresponding division statements are $12 \div 4 = 3$ and $12 \div 3 = 4$
- use the columns in the table below to record corresponding fraction division statements (fraction division L = A ÷ B and B = A ÷ L)

Area of Rectangle L x B = A	Length of Rectangle A ÷ B = L	Breadth of Rectangle A ÷ L = B
$\frac{3}{4} \times \frac{2}{4} = \frac{6}{16}$	$\frac{6}{16} \div \frac{2}{4} = \frac{3}{4}$	$\frac{6}{16} \div \frac{3}{4} = \frac{2}{4}$

- discuss if they can find a pattern or short cut for dividing fractions
- write a rule for dividing fractions using words
- experiment with practice exercises to see if their rule can be applied in all instances.

Students will most likely conclude that their rule for dividing fractions is to divide the first numerator by the second numerator, and place over the first denominator divided by the second denominator

eg $\frac{6}{16} \div \frac{3}{4} = \frac{6 \div 3}{16 \div 4} = \frac{2}{4} = \frac{1}{2}$.

They may not be able to apply this rule in instances where the first numerator or denominator is not a multiple of the second eg $\frac{1}{3} \div \frac{2}{5}$.

They should be encouraged to experiment to find a method to use. Either of the following approaches may assist students.

Method 1

To assist with the development of the rule, students could investigate dividing whole numbers by a fraction eg $3 \div \frac{1}{2}$, $4 \div \frac{1}{2}$, … From here they determine a rule and test it with further examples such as $1\frac{1}{2} \div \frac{1}{2}$, $1\frac{1}{2} \div \frac{1}{4}$.

It may be helpful to use concrete materials to model these questions.

Method 2

Students could determine that $\frac{1}{3} \div \frac{2}{5} = \frac{5}{15} \div \frac{6}{15} = \frac{5 \div 6}{15 \div 15} = \frac{\frac{5}{6}}{1} = \frac{5}{6}$.

Alternatively, students may find other solutions, such as

$\frac{1}{3} \div \frac{2}{5} = \frac{\frac{1}{3}}{\frac{2}{5}} = \frac{\frac{1}{3} \times \frac{5}{2}}{\frac{2}{5} \times \frac{5}{2}} = \frac{\frac{1}{3} \times \frac{5}{2}}{1} = \frac{5}{6}$.

Finally, students

- share their rules, identifying strengths and weaknesses of different methods
- refine and record the rule of 'multiplying by the reciprocal' for division of fractions
- develop and use written methods for dividing fractions.

Students need to solve a variety of problems involving fractions that may combine several operations.

Assessment Activities

Open-ended and Closed Questions

For example:

a draw a diagram to illustrate $4 \div \frac{1}{3}$

b which of the following expressions will help solve the word problem 'How many quarters can be cut using 5 oranges?':

$\frac{1}{4} \times 5$, $5 \div \frac{1}{4}$, $5 \div 4$, $\frac{1}{4} \div 5$

c when you divide a fraction by a whole number, is the answer bigger or smaller than the original fraction? Give 3 examples to justify your answer. WM

d if a student wrote the solution to a word problem as $6 \div \frac{1}{2}$, what might the word problem have been? WM

e write the missing fractions in the boxes:

$\frac{2}{3} \div \square = 1$

$\frac{3}{5} \div \frac{1}{3} = \frac{3}{5} \times \square$

$\frac{3}{4} \div \square = \frac{1}{2}$

f match the statements in List A with a numerical expression from List B:

List A	List B
How many thirds in 4?	$\frac{1}{3} \times 4$
Share 3 pizzas among 4 people	$4 \div \frac{1}{3}$
How many threes in 4?	$\frac{1}{4} \times 3$
One third of 4	$3 \div 4$
	$\frac{1}{3} \div \frac{1}{4}$
	$4 \div 3$

g given $\frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$, write two corresponding division statements

h investigate and write a report on multiplying and dividing with mixed numbers. WM

3.4.3 Assessment activity and work sample

Focus: Number and Working Mathematically

Operations with Fractions

See worksheet on page 25.


Individual


30 minutes

End

Possible prompts to assist student engagement

- Is there another denominator you could use to add these two fractions?
- If you make a mistake in shading, draw another grid, and try again.
- Have you checked your calculations?
- Have you shown all necessary working?
- Can you do the calculation another way? Do you get the same answer?

Suggested Materials

pen and paper
worksheet

Outcomes

Fractions, Decimals and Percentages (NS4.3)
Operates with fractions, decimals, percentages, ratios and rates
Applying Strategies (WMS4.2)
Analyses a mathematical or real-life situation, solving problems using technology where appropriate
Communicating (WMS4.3)
Uses mathematical terminology and notation, algebraic symbols, diagrams, text and tables to communicate mathematical ideas
Reasoning (WMS4.4)
Identifies relationships and the strengths and weaknesses of different strategies and solutions, giving reasons

Criteria for judging quality of performance

The student may demonstrate the following:

- determines the lowest common denominator of the two fractions
- expresses two fractions with the same denominator to enable addition and subtraction
- reduces a fraction to its simplest equivalent form at the completion of calculations
- expresses two fractions with the lowest common denominator to enable addition and subtraction
- converts between improper fraction and mixed numeral form
- cancels common factors in multiplying fractions to increase efficiency
- performs calculations accurately.

Feedback

Students will receive:

- possible prompt questions related to the assessment activity to assist their engagement
- written feedback on individual responses to indicate evidence of understanding of the concepts related to addition and subtraction of fractions
- specific advice about strategies to further consolidate knowledge, skills and understanding, and suggestions for completing related additional tasks
- oral feedback (to the class and individuals) related to the overall understanding of the concepts and misconceptions about the mathematical ideas.

Student Worksheet for the Assessment Task

Operations with Fractions **Name: _____**

Part 1

A student was asked to evaluate $\frac{2}{3}+\frac{5}{6}$.

Here is her working:

$$\frac{2}{3}+\frac{5}{6}$$
$$=\frac{12}{18}+\frac{15}{18}$$
$$=\frac{27}{18}$$
$$=\frac{3}{2}$$
$$=1\frac{1}{2}$$

1. This student chose 18 as the common denominator. Is this the *lowest* common denominator? Show how you can evaluate $\frac{2}{3}+\frac{5}{6}$ using the lowest common denominator.

2. Evaluate $\frac{7}{10}+\frac{3}{4}$.

Part 2

A student in answering a word problem wrote $\frac{1}{2}\times\frac{3}{4}$.

3. What might the word problem have been?

4. Draw a diagram to illustrate $\frac{1}{2}\times\frac{3}{4}$.

5. Find $\frac{1}{2}\times\frac{3}{4}$.

6. Evaluate $\frac{5}{6}\times\frac{7}{10}$, expressing your answer in simplest form.

7. Evaluate $3\frac{1}{5}\times 2\frac{3}{8}$.

Operations with Fractions Work Sample

1. This student chose 18 as the common denominator. Is this the *lowest* common denominator? Show how you can evaluate $\frac{2}{3}+\frac{5}{6}$ using the lowest common denominator.

$\frac{2+5}{3\times6} = \frac{4}{6}+\frac{5}{6} = \frac{9}{6}\ (\div3)\ \frac{3}{2} = 1\frac{1}{2}$

2. Evaluate $\frac{7}{10}+\frac{3}{4}$

$\frac{7\times4}{10\times4}\ \frac{3}{4} = \frac{14}{20}+\frac{3\times5}{4\times5} = \frac{14}{20}+\frac{15}{20} = \frac{29}{20} = 1\frac{9}{20}$

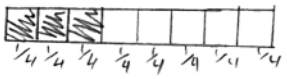
Part 2

A student in answering a word problem wrote $\frac{1}{2}\times\frac{3}{4}$

3. What might the word problem have been?

Michael had 60 pens. Nick had 100. Michael lost 30 & Nick lost 75. write down the remaining fraction & multiply it.

4. Draw a diagram to illustrate $\frac{1}{2}\times\frac{3}{4}$



5. Find $\frac{1}{2}\times\frac{3}{4}$

$\frac{1}{2}\times\frac{3}{4} = \frac{3}{8}$

6. Evaluate $\frac{5}{6}\times\frac{7}{10}$, expressing your answer in simplest form.

$\frac{5\times7}{6\ 10} = \frac{35}{60} = \frac{7}{12}$

7. Evaluate $3\frac{1}{5}\times2\frac{3}{8}$

$3\frac{1}{5}\times2\frac{3}{8} = 6\frac{3}{40}$

Q1 and 2.
Identifies lowest common denominator.
Shows detailed working when finding equivalent fractions.
Adds fractions correctly.
Correctly reduces answer $\frac{9}{6}$ (improper fraction) to lowest terms.
Converts improper fraction to mixed numeral correctly.

Q3.
Shows no understanding of the significance of multiplying two fractions, however correctly expresses $\frac{1}{2}$ as $\frac{30}{60}$ and $\frac{3}{4}$ as $\frac{75}{100}$.

Q4 and 5.
Correctly applies multiplication process in Q5. Probably used this answer to draw the diagram in Q4, but incorrectly labelled the parts as quarters.

Q6.
Multiplies proper fractions correctly.
Reduces answer to simplest form.
Needs to learn to cancel common factors before multiplying to increase efficiency.

Q7.
Cannot multiply mixed numerals.
By multiplying the whole numbers together and then multiplying the fractional parts, the student shows confusion with the 'rule' for addition.

Follow Up

Concrete activities that demonstrate multiplication of fractions such as $\frac{1}{2}\times\frac{3}{4}$ need to be presented.

These could include halving quantities in recipes, shading areas or paper folding.
The student needs further practice in cancelling common factors prior to multiplication.
Further review and consolidation is needed for multiplication of mixed numerals.
Further experience is required with word problems leading to the understanding that multiplication requires both fractions to be of the same quantity.

3.5 Stage 5.1 Unit: Trigonometry

3.5.1 Outcomes, Key Ideas and Content from the Syllabus

<table>
<tr><td colspan="5">Measurement - Trigonometry Syllabus Content p 139</td></tr>
<tr><td colspan="3">MS5.1.2
Applies trigonometry to solve problems (diagrams given) including those involving angles of elevation and depression</td><td colspan="2">Key Ideas
Use trigonometry to find sides and angles in right-angled triangles
Solve problems involving angles of elevation and angles of depression from diagrams</td></tr>
<tr><td colspan="5">Working Mathematically Outcomes
Questioning
Asks questions that could be explored using mathematics in relation to Stage 5.1 content
Applying Strategies
Analyses a mathematical or real-life situation, systematically applying problem-solving strategies
Communicating
Uses mathematical terminology and notation, algebraic symbols, diagrams, text and tables to explain mathematical ideas
Reasoning
Explains and verifies mathematical relationships
Reflecting
Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 5.1 content</td></tr>
<tr><td colspan="3">Knowledge and Skills
Trigonometric Ratios of Acute Angles
• identifying the hypotenuse, adjacent and opposite sides with respect to a given angle in a right-angled triangle in any orientation
• labelling the side lengths of a right-angled triangle in relation to a given angle eg the side c is opposite angle C
• recognising that the ratio of matching sides in similar right-angled triangles is constant for equal angles
• defining the sine, cosine and tangent ratios for angles in right-angled triangles
• using trigonometric notation eg sin A
• using a calculator to find approximations of the trigonometric ratios of a given angle measured in degrees
• using a calculator to find an angle correct to the nearest degree, given one of the trigonometric ratios of the angle
Trigonometry of Right-Angled Triangles
• selecting and using appropriate trigonometric ratios in right-angled triangles to find unknown sides, including the hypotenuse
• selecting and using appropriate trigonometric ratios in right-angled triangles to find unknown angles correct to the nearest degree
• identifying angles of elevation and depression
• solving problems involving angles of elevation and depression when given a diagram</td><td colspan="2">Working Mathematically
• label sides of right-angled triangles in different orientations in relation to a given angle (Applying Strategies, Communicating)
• explain why the ratio of matching sides in similar right-angle triangles is constant for equal angles (Communicating, Reasoning)
• solve problems in practical situations involving right-angled triangles eg finding the pitch of a roof (Applying Strategies)
• interpret diagrams in questions involving angles of elevation and depression (Communicating)
• relate the tangent ratio to gradient of a line (Reflecting)</td></tr>
<tr><td colspan="3">Technology
Cabri Geometry software can be used to explore similar triangles and ratios of sides. Spreadsheets can be used to generate tables of values for the trigonometric ratios.</td><td colspan="2">Links
Geometry – similar triangles
Measurement – Pythagoras' theorem, measuring lengths
Number – ratio, rounding, average</td></tr>
<tr><td colspan="3">Resources
Clinometer, measuring tape
List of References (p 48) – No.s 3, 4, 7</td><td colspan="2">Language
The word trigonometry is derived from two Greek words meaning 'triangle' and 'measurement'. Students need to understand the meaning of 'opposite' and 'adjacent'.</td></tr>
</table>

3.5.2 Learning experiences and assessment activities

Learning Experiences	Assessment Activities
Review of Related Topics For this unit of work, students will need to review their knowledge and skills in: • Number – rounding a number to a stated number of decimal places • Ratio including writing ratios and finding the missing value, x, given $\frac{x}{5}=\frac{3}{2}$. • Geometry including angle sum of a triangle, complementary angles and similar figures • Algebra including solving equations of the form $5=\frac{x}{3}$ and perhaps $4=\frac{3}{x}$ • Measurement – Pythagoras' theorem **Introduction to Trigonometry** Using measurements and calculation of the ratios opposite/hypotenuse, adjacent/hypotenuse and opposite/adjacent, students learn that in sets of similar right-angled triangles the ratios of pairs of sides are constant AND the ratio depends on the angles in the triangle, not the size of the triangle. 1. Students identify and label the sides of right-angled triangles, in various orientations, as being opposite, adjacent or hypotenuse in relation to a nominated angle, and use abbreviations for these sides eg opp, adj, hyp. Students should relate the hypotenuse to work completed in Pythagoras' theorem. 2. Groups of students are given a set of similar right-angled triangles printed on a worksheet eg Group 1 has a set of 20º right-angled triangles (all marked with a 20° angle and a right angle), Group 2 has 30º right-angled triangles etc. Students are asked to describe what they notice about the triangles in their set, concluding that the triangles are equiangular ie similar. 3. Students label the sides of the triangles as opp, adj and hyp in relation to the marked angle and measure each side to the nearest mm. 4. For each triangle in the set, the ratio opp/hyp is written as a fraction and converted to a decimal correct to 3 decimal places. 5. Each group reports on their findings. They should discover that triangles with the same angles have approximately equal ratios. They also should be able to see that the ratio opp/hyp increases as the angle increases. A table is created on the board as follows: Ratio Values by Measurement | Angle | Opp/Hyp | Adj/Hyp | Opp/Adj | | 20° | | | | | 30° | | | | 6. Class discussion should emphasise that triangles that have the same ratios also have the same angles. The concept that for each similar right-angled triangle, the ratio of matching sides is constant should be reinforced. 7. The teacher introduces the names of the trigonometric ratios. 8. Students use their measurements to calculate the values for the second and third columns of the table. Patterns of values in the table can be discussed.	**Open-ended Questions** WM Open-ended questions can be used to find out what students know and can do from earlier Stages. Students could work on such questions in pairs to encourage discussion of ideas. Listening to students' conversations provides valuable insights into their knowledge, skills and understanding. For example: a write down everything you know about two similar figures b describe how you would solve equations like $\frac{a}{2}=10$ and how you would determine if the solution is correct c list 3 ratios that are equivalent to 3:4. Record a method for finding equivalent ratios. d explain why the ratio of matching sides in similar right-angled triangles is constant for equal angles. **Writing Activities to Show Reflection** WM Students' responses could assist teacher assessment of the Working Mathematically outcomes. Group activities will promote students' development and use of terminology and give them the opportunity to consolidate understanding through discussion, reporting and questioning. Students can be encouraged to think about their learning by writing about the activity. For example: a write about the group activity in which you compared side lengths of similar triangles. Make sure you include what you have learnt from participating in this activity. b write down everything you know about the sine, cosine and tangent ratios c draw a triangle with an unknown side length, place relevant information on the diagram, and describe how trigonometry can be used to find the value of the pronumeral.

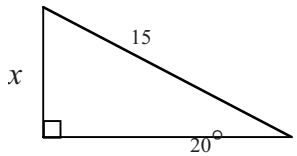
Learning Experiences

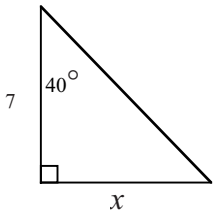
9. Students compare their results to answers obtained using a calculator to find the ratios for each angle. A new table is written on the board, containing trigonometric ratios from the calculator, as follows:

Ratio Values from the Calculator

Angle θ	$\sin\theta = \frac{Opp}{Hyp}$	$\cos\theta = \frac{Adj}{Hyp}$	$\tan\theta = \frac{Opp}{Adj}$
20°			
30°			

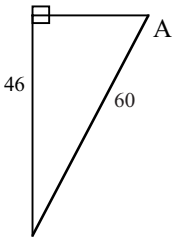
10. At this stage students could try to find the missing sides in the following triangles, without the teacher formalising a method.

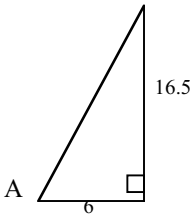




Students could demonstrate their method of solution to the class. The aim here is to show the immediate application of the trigonometric ratios that have been developed in this lesson. Formal methods of setting out can be discussed in the next lesson.

11. If time allows, students could also use the table of values from the board to establish the value of angle A in the following triangles. (Note sides have been carefully chosen so that A is one of 20°, 30°…80°)





12. Students complete exercises reinforcing the skills of naming sides of right-angled triangles, writing sin, cos and tan ratios, and selecting the correct ratio.

Development and Application of Trigonometric Concepts

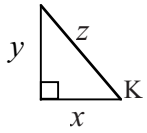
In the lessons that follow this introduction to Trigonometry, the teacher will need to guide the students to record their working clearly. A variety of exercises will reinforce knowledge, skills and understanding.

Students could:

- match a series of right-angled triangles to a set of written problems
- match cloze statements to diagrams to emphasise use of 'adjacent' and 'opposite' and the correct ratios to solve the unknown
- decide which trigonometric ratio is needed for a particular problem.

Assessment Activities

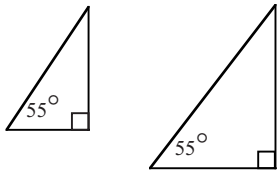
1. Sketch a right-angled triangle that has an angle of sixty degrees and a hypotenuse of 8 metres.
2. Complete these statements:
   a) The side labelled ... is adjacent to angle K.



   b) To solve the equation $0.765 = \frac{x}{9}$ I must

   ……..

3.



   Explain why these two triangles are similar. WM

   What else will be the same in each triangle?

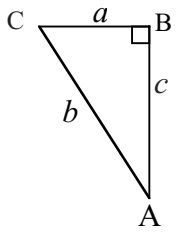
4. Which of these statements is true about ΔABC?

$\sin C = \frac{a}{b}$

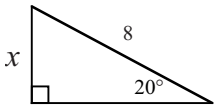
$\tan B = \frac{c}{a}$

$\cos A = \frac{c}{b}$

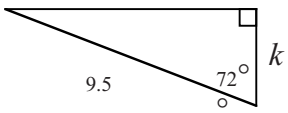
$\sin A = \frac{a}{c}$



5. Given $\sin 20^\circ = 0.342$ find the value of x in the diagram, correct to 1 decimal place.



6. Find the value of k, correct to 1 decimal place. Show all working.

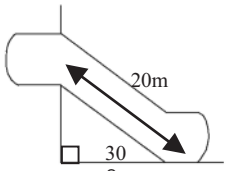


Learning Experiences

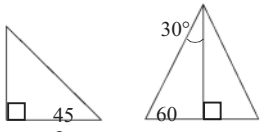
Development and Application of Trigonometric Concepts (continued)

Students could:

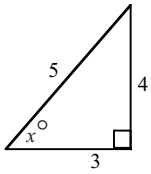
- solve problems like: an escalator at an airport slopes at an angle of 30° and is 20 m long. Through what height would a person be lifted by travelling on the escalator?



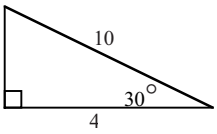
- investigate the lengths of the sides of the triangles illustrated

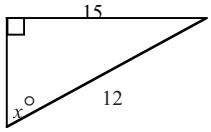


- find x in the diagram in three different ways



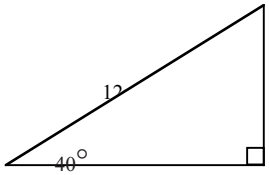
- explain what is wrong with these diagrams:



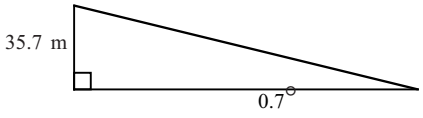


- use a clinometer and a measuring tape to investigate the height of trees and buildings in the school grounds
- solve problems involving angles of elevation and depression, with diagrams given, like

a) The top of a flagpole is connected to the ground by a cable 12 metres long. The angle that the cable makes with the ground is 40°.
Find the height of the flagpole.



b) A ship's navigator observes a lighthouse on a cliff. She knows from a chart that the top of the lighthouse is 35.7 metres above sea level. She measures the angle of elevation of the top of the lighthouse to be 0.7°.



The coast is very dangerous in this area and ships have been advised to keep at least 4 km from this cliff to be safe.
Is the ship safe?

Assessment Activities

7. I used a calculator and found the tan ratio of a certain angle to be 1.234. What could the size of the angle be?

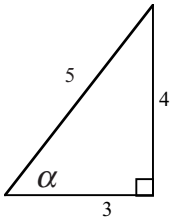
8. Toni wants to find the value of α in the triangle below. Which statement is a correct line of working?

$\sin\alpha = \frac{4}{5}$

$\cos\left(\frac{3}{5}\right) = \alpha$

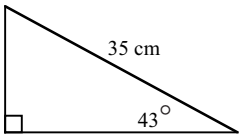
$\tan\alpha = \frac{3}{4}$

$\sin 0.8 = \alpha$



9. Find out everything you can about the right-angled triangle below.

WM



Assessment Activity and Work Sample (see page 31) WM

Can Do Trigonometry

Students are asked to demonstrate what they understand and can do in relation to their work in trigonometry.

Students complete the attached worksheet.

Possible prompts:

- Why don't you draw a right-angled triangle to get started?
- Do you remember the names given to each side in relation to a marked angle?
- Can you write down the relationship for sin A in your diagram?
- Can you remember some types of trigonometry questions you did in class exercises?
- Can you use trigonometry to find an unknown side? An unknown angle?
- In part 2, what do you think is meant by the words 'practical problems'?
- Can you extend your response by creating and solving more difficult questions in part1? part 2?

3.5.3 Assessment activity and work sample

Focus: Measurement and Working Mathematically

Can Do Trigonometry

Students are asked to demonstrate what they understand and can do in relation to their work in trigonometry.

Students complete the worksheet on page 32.

From *Securing Their Future: Mathematics*


Individual


35 minutes

End

Possible prompts to assist student engagement

- Why don't you draw a right-angled triangle to get started?
- Do you remember the names given to each side in relation to a marked angle?
- Can you write down the relationship for sin *A* in your diagram?
- Can you remember some types of trigonometry questions you did in class exercises?
- Can you use trigonometry to find an unknown side? an unknown angle?
- In part 2, what do you think is meant by the words 'practical problems'?
- Can you extend your response by creating and solving more difficult questions in part1? part 2?

Suggested Materials

pen and paper
calculator
worksheet

Outcomes

Trigonometry (MS5.1.2)

Applies trigonometry to solve problems (diagrams given) including those involving angles of elevation and depression

Applying Strategies (WMS5.1.2)

Analyses a mathematical or real-life situation, systematically applying problem-solving strategies

Communicating (WMS5.1.3)

Uses mathematical terminology and notation, algebraic symbols, diagrams, text and tables to explain mathematical ideas

Reasoning (WMS5.1.4)

Explains and verifies mathematical relationships

Criteria for judging quality of performance

The student may demonstrate the following:

- expresses the three ratios in terms of sides eg $\sin = \frac{\text{opposite}}{\text{hypotenuse}}$
- creates and solves routine trigonometry questions that involve finding an unknown side
- creates and solves routine trigonometry questions that involve finding unknown sides and angles
- creates and solves practical problems which involve finding unknown sides
- creates and solves problems involving angles of elevation or depression given a diagram
- creates problems including those involving angles of elevation and depression

Feedback

Students will receive:

- possible prompt questions related to the assessment activity to assist their engagement
- written feedback on individual responses to indicate evidence of understanding of each of the trigonometric ratios and their use to find sides and angles in right-angled triangles
- specific advice about strategies to further consolidate knowledge, skills and understanding, and suggestions for completing related additional tasks
- oral feedback (to the class and individuals) related to the overall understanding of the concepts and misconceptions about the mathematical ideas.

Can Do Trigonometry

In the examples below, you are asked to demonstrate that you can do something, **by doing it**.

In each case, **you** get to pick the examples that you think will show that you can do what has been asked.

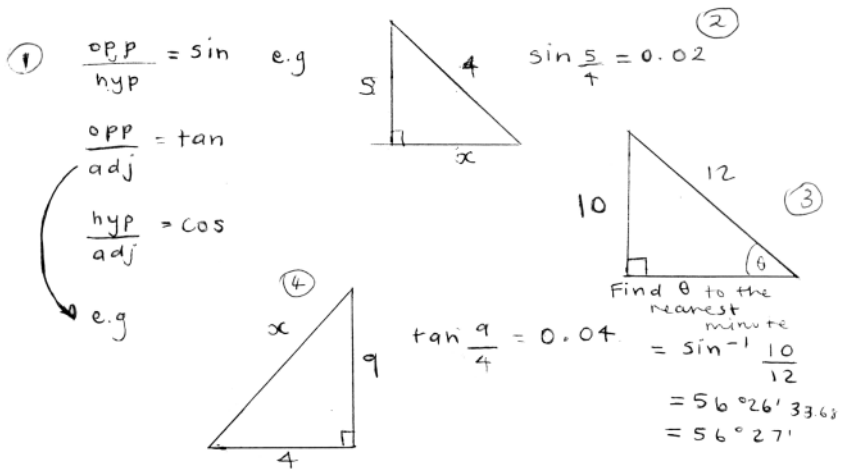
1. **I can** understand and use the trigonometric ratios (sine, cosine, tangent) in right-angled triangles:

2. **I can** use trigonometry to solve practical problems involving right-angled triangles:

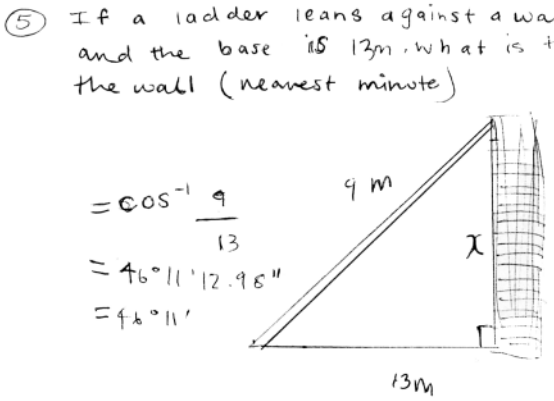
Can Do Trigonometry Work Sample

Overall Comment:
In this activity, students should give examples of finding the lengths of sides of a triangle as well as finding the sizes of angles. This student has made no attempt to use an angle in any diagram to find a side length.
The student has demonstrated a lack of understanding of basic trigonometry. In attempting to give a numerical answer to each question he/she appears to have used any ratio that does not give an error message on the calculator.
An attempt to use inverse trigonometric notation ($\sin^{-1}$) has been made but without real understanding.
The student may have been more successful if labelled diagrams had been supplied as is usual in Stage 5.1. Triangles in which the student did not give the hypotenuse as the longest side caused calculation difficulty.

1. **I can** understand and use the trigonometric ratios (sine, cosine, tangent) in right-angled triangles:



2. **I can** use trigonometry to solve practical problems involving right-angled triangles:



1. Has correctly learnt the definitions for the sine and tangent ratios, but has confused the cosine definition.

2. This triangle and the one drawn in Question 5 show that the student does not understand that the hypotenuse is always the longest side in a right-angled triangle.
In Question 2, because $\sin^{-1}\left(\frac{5}{4}\right)$ gives an error message, the numerical answer was probably obtained from $\tan\frac{5}{4}$ ($\cong 0.02$).
In Questions 2, 4 and 5 Pythagoras' theorem would enable the unknown side lengths to be calculated.

3. This example shows correct application of trigonometry and is the only example where an angle was marked and found.

4. A side was marked x but the student attempted to find an angle (using tan).
$\text{Tan}\ \frac{9}{4} = 0.04$ shows correct use of a calculator.

5. The situation represented in the diagram is physically impossible, as the length given for the ladder is shorter than the distance of the foot of the ladder from the wall. The question asks for the height of the wall, but to the nearest minute.
An attempt was made to find the angle using the cosine ratio because of the 'known' sides. The solution uses the incorrect definition for the cosine ratio.

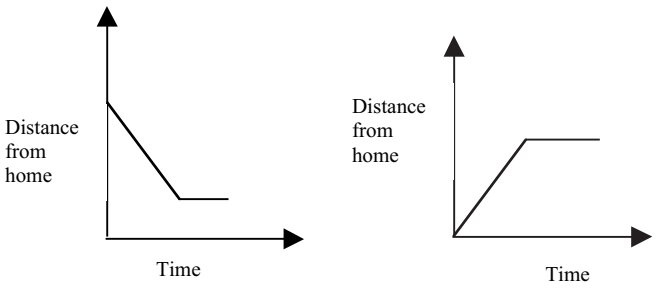
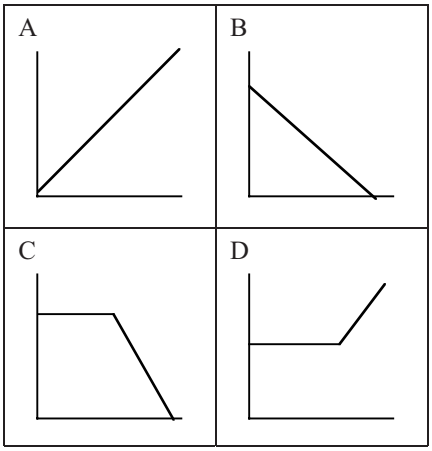
Follow Up
Review and consolidation needs to include:
- labelling angles and corresponding sides
- determining that the hypotenuse is the longest side of any right-angled triangle
- recognising when Pythagoras' theorem can be used to find a side length
- labelling sides as 'opp', 'adj' and 'hyp'
- identifying and correctly using a trigonometric ratio to find a side or angle.

3.6 Stage 5.2 Unit: Graphs of Physical Phenomena

3.6.1 Outcomes, Key Ideas and Content from the Syllabus

<table>
<tr><td colspan="5">Patterns and Algebra - Graphs of Physical Phenomena Syllabus Content p 105</td></tr>
<tr><td colspan="2">PAS5.2.5
Draws and interprets graphs of physical phenomena</td><td colspan="3">Key Ideas
Draw and interpret graphs of physical phenomena</td></tr>
<tr><td colspan="5">Working Mathematically Outcomes</td></tr>
<tr><td>Questioning
Asks questions that could be explored using mathematics in relation to Stage 5.2 content</td><td>Applying Strategies
Selects and uses appropriate problem-solving strategies that include selecting and organising key information and identifying and working on related problems</td><td>Communicating
Uses appropriate mathematical language and algebraic, statistical and other notations and conventions in written, oral or graphical form</td><td>Reasoning
Uses mathematical arguments to reach and justify conclusions</td><td>Reflecting
Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 5.2 content</td></tr>
<tr><td colspan="2">Knowledge and Skills
• interpreting distance/time graphs made up of straight line segments
• determining which variable should be placed on the horizontal axis
• drawing distance/time graphs
• telling a story shown by a graph by describing how one quantity varies with the other
eg number of cars at a red light, the temperature of water in a storage heater
• sketching informal graphs to model familiar events
eg noise level within the classroom during the lesson
• using the relative positions of two points on a graph, rather than a detailed scale, to interpret information</td><td colspan="3">Working Mathematically
• describe the meaning of different gradients for the graph of a particular event (Communicating)
• distinguish between positive and negative gradients from a graph (Communicating)
• match a graph to a description of a particular event and explain reasons for the choice (Reasoning, Communicating)
• compare graphs of the same simple situation, decide which one is the most appropriate and explain why (Applying Strategies, Reasoning, Communicating)
• use spreadsheets to generate examples of everyday graphs (Applying Strategies)
• model, record data and sketch graphs to investigate the distance of a moving object from a fixed point in relation to time
eg move along a measuring tape for 30 seconds using a variety of activities that involve a constant rate such as:
– walking slowly
– walking for 10 seconds, stopping for 10 seconds and continuing at the same rate for the remaining 10 seconds to the end of the tape
– walking for 10 seconds, stopping for 10 seconds and then turning around and walking back to the beginning of the tape for 10 seconds
– starting at the other end of the line and walking back towards the beginning at a constant speed and record the distance at fixed time intervals so that a graph can be drawn to represent each situation (Applying Strategies, Communicating)
• use technology such as data loggers to collect data for constant speeds and graph the data to compare and contrast graphs (Applying Strategies, Reasoning)</td></tr>
<tr><td colspan="2">Technology
Data loggers are used in Science for the collection of data and should be readily available in schools.</td><td colspan="3">Links
Data – Travel Graphs
Coordinate Geometry – Gradient</td></tr>
<tr><td colspan="2">Resources
Measuring tapes, stop watches
List of References (p 48) – No.s 1, 2, 5, 6, 8, 14, 15</td><td colspan="3">Language
This unit is language-based in that students need to develop skills in describing change.</td></tr>
</table>

3.6.2 Learning experiences and assessment activities

Learning Experiences	Assessment Activities
Review of Related Concepts Note: at this Stage, the focus is on examining situations where the data yields a constant rate of change. It is possible that some practical situations may yield a variable rate of change. This is the focus at the next Stage in PAS5.3.5. For this unit of work, students will need to have a sound understanding of reading, interpreting, and drawing line graphs. In particular, from Stage 4 they will need to be able to draw and interpret travel graphs, recognising concepts such as change of speed and change of direction. **Language Development** Throughout this unit, students need to develop language to describe the graphs. Terms such as the following should be used when appropriate: gradient, positive, negative, zero gradient, increasing, decreasing, constant, rate of change, steady, linear, variable, independent variable, dependent variable, maximum value, minimum value, relationship, range. This can be achieved by providing opportunities for discussion in small groups eg card-matching activities and finding errors in descriptions of graphs. Cloze passages give students practice in using the correct mathematical language when interpreting and describing graphs. Passages can be completed as a whole class activity using an overhead projector, by pairs of students, or by individual students. **Travel Graphs** Students describe the features of graphs, making comparisons. For example, students could describe each of the following, highlighting similarities and differences.  **Drawing Graphs** In pairs or small groups, students could: • discuss and determine the variables in a particular situation and justify their placement on the horizontal or vertical axis eg the noise level in the classroom during a period, the temperature of the water in a water heater during the day • draw/create a travel graph to represent their trip to and from school • draw qualitative graphs of mood swings during a grand final football or netball match from different points of view.	**Open-ended Questions** WM Open-ended questions can be used to determine students' knowledge and understanding of related concepts from earlier Stages. These questions could be attempted in small groups so that the teacher can listen to the discussions and determine the level of understanding. For example: • write a story that matches a particular travel graph • draw a graph to represent your height at your birthday for each year you have been at school. Use estimates of how tall you think you were each year. Include all relevant information on the axes of the graph. **Card-matching Activities** WM One set of cards contains graphs and a second set of cards contains descriptions of the graphs. In groups, students match a graph to a description. After completion, one group compares its solution to that of another group. Examples of graphs:  Examples of descriptions: After leaving home, I travelled at a constant speed for the entire journey I returned home travelling at a constant speed I waited for my friend to arrive then we travelled home at a constant speed I waited for my friend to arrive and then we completed the journey at a constant speed

Learning Experiences

Distance/Time Graph Practical Activity

In this activity, students model and record data, then sketch graphs, to investigate the distance of a moving object from a fixed point in relation to time. Students work in small groups with one moving along the measuring tape, another keeping the time, and two others recording distances.

The distance is recorded at fixed time intervals so that a graph can be drawn to represent each situation. This activity could be done using data loggers.

Students could change roles after each activity.

Students move along a measuring tape for 30 seconds using a variety of activities that involve a constant rate such as:

- walking slowly
- walking for 10 seconds, stopping for 10 seconds, and continuing at the same rate for the remaining 10 seconds to the end of the tape
- walking for 10 seconds, stopping for 10 seconds, and then turning around and walking back to the beginning of the tape for 10 seconds
- starting at the other end of the tape and walking back towards the beginning at a constant speed.

Students could be given a graph representing the movement of an object from a fixed point. They could model the graph by moving along the tape.

Describing a Line Graph

1. Students are shown a graph and asked to write a story to describe what has occurred.

For example:

The plug is put in the bath.

The tap is turned on, then off.

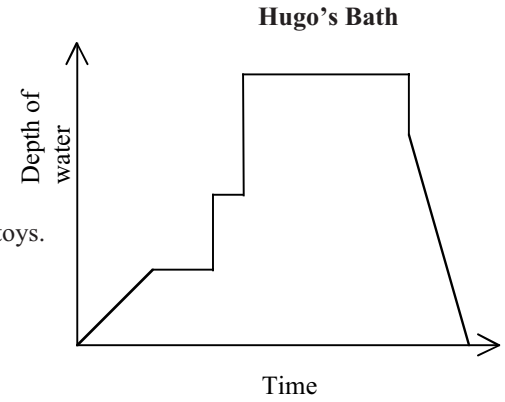
Hugo gets in.

Hugo's brother, Tom, gets in.

Hugo and Tom play with their toys.

Hugo gets out.

The plug is pulled out but Tom keeps playing.



2. Groups of students are given graphs of similar situations and a number of statements on slips of cardboard. They arrange the statements in the correct order to match the graphs.

3. Students each draw a graph to represent a similar situation and compare their graph with that of another student.

Graphs and Consumer Arithmetic

Students could draw graphs of these situations:

- total wage against time, given an hourly rate of pay
- total wage against time, where overtime is paid after a certain number of hours are worked
- the wage of a salesperson who is paid a commission on sales (with no retainer)
- the wage of a salesperson who is paid a retainer plus commission on sales
- a goods and services tax (GST) levied on goods and services
- the cost of production of items in a business.

Assessment Activities

Class Display WM

Groups of students label the parts of a large given graph with information eg 'this line is steeper so the car is travelling faster here'. The labelled graphs are displayed and discussed.

Assessment Activity and Work Sample (see page 38) WM

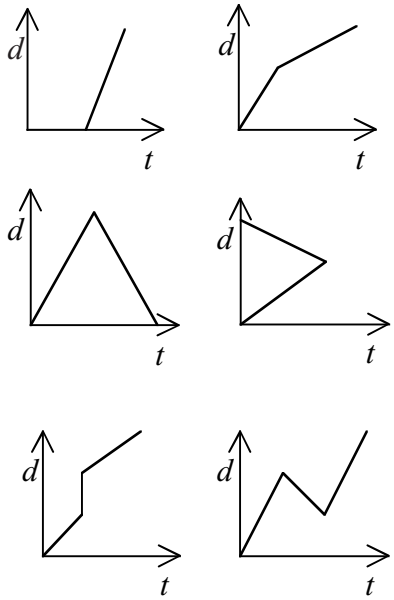
Describing Travel Graphs

Students are presented with a set of graphs on a worksheet (see below) with the following instructions:

Consider each of the distance/time graphs below.

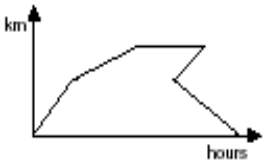
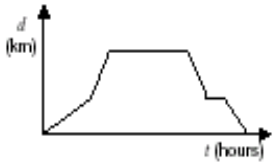
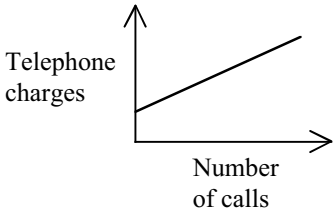
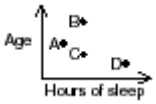
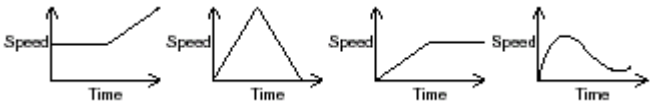
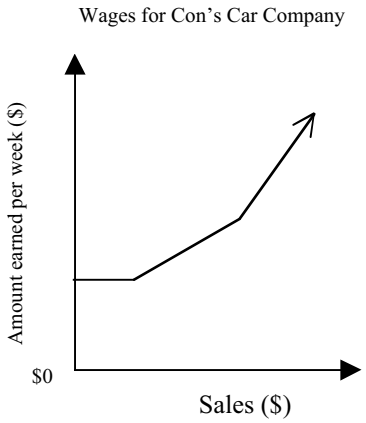
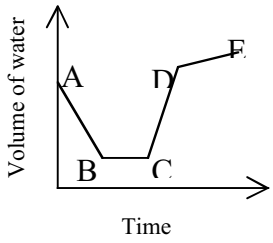
Write a brief description of the journey that the graph might represent.

Some graphs do not represent possible journeys. For these, explain why not.



Possible prompts:

- Where does the graph begin? What does this mean?
- Consider some specific examples.
- Can you explain why you think the graph has that shape?
- How would you describe the slope of the line? What does this mean?
- What occurs when the graph changes slope? How would you describe this in terms of a journey?
- Is it possible to be in two places at the same time? Does this occur in any of the graphs?

Learning Experiences	Assessment Activities
Interpreting Graphs Students could: • explain why this graph could not be a travel graph showing distance in kilometres and time in hours  • explain what is happening to the speed of the car in the graph  • explain why the graph does not pass through the origin.  Students could consider questions like: • the graph shows the hours of sleep and ages of four students. Who is the oldest? Who is the youngest? Who gets the most sleep? Who gets the least sleep? Which children get the same amount of sleep?  • a child climbs a mountain at a steady speed and then starts to run down the mountain. Choose the graph that matches this situation from the graphs below.  Students could: • interpret travel graphs eg compare two sketch graphs (which do not have detailed scales on the axes) each of which purport to show how far a person is from home at various times after they leave school and walk up the hill towards home.	*Independent and Dependent Variables* For each of these statements, decide which is the independent variable to be graphed on the horizontal axis: 1. 'the bigger the hall, the more people can attend a concert' 2. 'the height of a child increases as the child gets older' 3. 'during cold weather the speed of the wind increases the wind-chill factor' 4. 'the speed of a car on a race track changes with the time taken for a lap'. **Interpreting Graphs** WM Write a short description of how the weekly wage is calculated for salespeople working for this company.  **Describing graphs** WM  Students are asked to describe the graph and its features using as many of the following terms as possible: gradient, positive, negative, zero gradient, increasing, decreasing, constant, maximum value, minimum value, stationary, variable, linear, rate of change.

3.6.3 Assessment activity and work sample

Focus: Patterns and Algebra and Working Mathematically

Describing Travel Graphs

The students are presented with a set of six travel graphs and are asked to write a brief description of the journey that the graph might represent (see worksheet on page 40).


Pairs


20 minutes

Middle or End

Possible prompts to assist student engagement

- Where does the graph begin? What does this mean?
- Consider some specific examples.
- Can you explain why you think the graph has that shape?
- How would you describe the slope of the line? What does this mean?
- What occurs when the graph changes slope? How would you describe this in terms of a journey?
- Is it possible to be in two places at the same time? Does this occur in any of the graphs?

Suggested Materials

worksheet
pen and paper

Outcomes

Graphs of Physical Phenomena (PAS5.2.5)
Draws and interprets graphs of physical phenomena
Communicating (WMS5.2.3)
Uses appropriate mathematical language and algebraic, statistical and other notations and conventions in written, oral or graphical form
Reasoning (WMS5.2.4)
Uses mathematical arguments to reach and justify conclusions

Criteria for judging quality of performance

The student may demonstrate the following:

- explains the graphs clearly
- identifies which graphs do not represent distance/time graphs
- explains why particular graphs are not possible
- uses a range of terms to describe the graphs.

Feedback

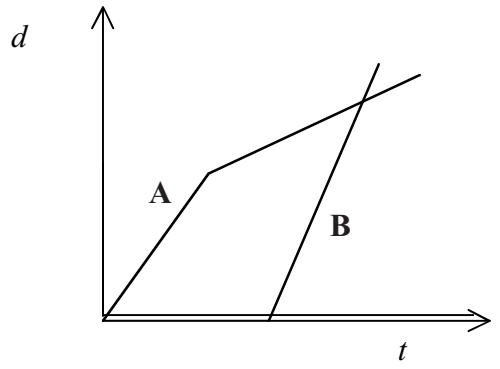
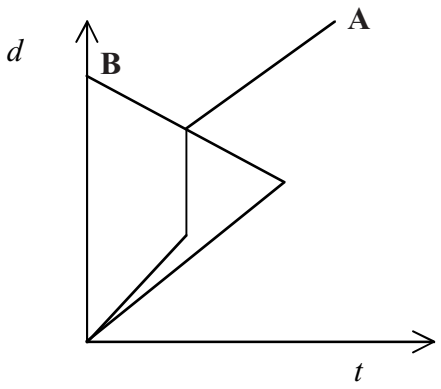
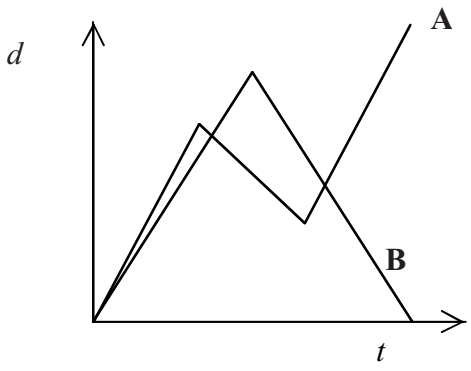
Students will receive:

- possible prompt questions related to the assessment activity to assist their engagement
- written feedback on individual responses to indicate evidence of understanding of the relationship between distance and time in each of the graphs, and their explanations about graphs that are not possible
- specific advice about strategies to further consolidate knowledge, skills and understanding, and suggestions for completing related additional tasks
- oral feedback (to the class and individuals) related to the overall understanding of the concepts and misconceptions about the mathematical ideas.

Describing Travel Graphs Worksheet

Consider the distance/time graphs below. For each pair of graphs, write a brief description of the journey that graph A and graph B might represent.

Some graphs do not represent possible journeys. For these, explain why not.

Describing Travel Graphs Work Sample

Overall Comment
When given two graphs and the same axes, students should relate these to one another. Some discussion of 'sharp points' on these travel graphs could prepare students for Stage 5.3. The everyday use of the word 'acceleration' needs careful discussion.

A- The person accelerates before reaching a point, then slows down, and accelerates at a slower rate B- can't be a journey because the line didn't start at the origin	1. This description of journey A uses the word 'accelerates' to mean travels or moves. There is no reference to *constant* speed, but the idea of slope of the line indicating speed is expressed. The student has not understood the significance of graph B not starting at the origin.
A is travelling fast then stops for a while then continues at a slower than the first speed. B is not possible. You cannot go back in time which this graph is actually representing.	2. In describing graph A, the student has confused the meaning of a horizontal line with that of a vertical line. Thinking of the meaning of each point on a graph would help. The student has correctly interpreted the slopes of graph A to indicate relative speed. Graph B has been correctly interpreted.
A is travelling at a constant speed slightly faster than B, then turns around and begins to travel back home again then turns ~~xxx~~ again and continues travelling away. B is travelling at a constant speed further than A's turning point but turns eventually and heads all the way back home	3. The description of both graphs is clear and there is an accurate use of correct language. There is a careful comparison of relative speeds. This student could have commented on the fact the two travellers twice pass each other, travelling in opposite directions.

Follow Up
Review and consolidation needs to include:

- clarification of the meaning and use of the word 'acceleration'
- practice in identifying and explaining the meaning of each point on a graph, in particular, where a graph does not start at the origin
- recognition of the relationship between two graphs drawn on the same axes, including points of intersection

3.7 Stage 5.3 Unit: Graphs of Physical Phenomena

3.7.1 Outcomes, Key Ideas and Content from the Syllabus

Patterns and Algebra - Graphs of Physical Phenomena — **Syllabus Content p 106**

PAS5.3.5	**Key Ideas**
Analyses and describes graphs of physical phenomena	Analyse and describe graphs of physical phenomena

Working Mathematically Outcomes

Questioning	Applying Strategies	Communicating	Reasoning	Reflecting
Asks questions that could be explored using mathematics in relation to Stage 5.3 content	Solve problems using a range of strategies including deductive reasoning	Uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures	Uses deductive reasoning in presenting arguments and formal proofs	Links mathematical ideas and makes connections, and generalisations about, existing knowledge and understanding in relation to Stage 5.3 content

Knowledge and Skills

- interpreting distance/time graphs when the speed is variable
- analysing the relationship between variables as they change over time

  eg draw graphs to represent the relationship between the depth of water in containers of different shapes when they are filled at a constant rate
- interpreting graphs, making sensible statements about the rate of increase or decrease, the initial and final points, constant relationships as denoted by straight lines, variable relationships as denoted by curved lines, etc
- describing qualitatively the rate of change of a graph using terms such as 'increasing at a decreasing rate'
- sketching a graph from a simple description given a variable rate of change

Working Mathematically

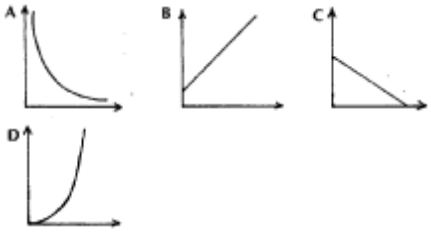
- decide whether a particular graph is a suitable representation of a given physical phenomenon (*Communicating*)
- match a set of distance/time graphs to a set of descriptions and give reasons for choices (*Applying Strategies, Reasoning, Reflecting, Communicating*)
- model, record data and sketch graphs to investigate the distance of a moving object from a fixed point in relation to time

  eg move along a measuring tape for 30 seconds in a variety of activities (including variable speeds) such as:
  - running as fast as possible
  - walking slowly
  - walking for 10 seconds, stopping for 10 seconds and walking for the remaining 10 seconds
  - starting at the other end of the line and walking back towards the beginning
  - walking slowly for 10 seconds and then speeding up until the end of the tape
  - running at a decreasing speed

  and record the distance at fixed time intervals so that a graph can be drawn to represent each situation (*Applying Strategies, Communicating*)
- match a set of distance/time graphs to situations, as in the example above, and discuss the likelihood that they are accurate, appropriate, and whether they are possible (*Applying Strategies, Communicating, Reasoning)*
- use technology such as data loggers to collect data for variable speeds and graph the data to compare and contrast the graphs (*Applying Strategies, Reasoning*)

Technology	**Links**
Data loggers are used in Science for the collection of data and should be readily available in schools.	Data – Travel Graphs Number – rates
Resources	**Language**
Measuring tapes, stop watches List of References (p 48) – No.s 1, 2, 5, 6, 8, 10, 14, 15	This unit is language-based in that students need to develop skills in describing change.

3.7.2 Learning experiences and assessment activities

Learning Experiences	Assessment Activities
Introduction This unit could be taught in conjunction with the unit for Stage 5.2 which examines data that yields a constant rate of change. At this Stage, the focus is on examining situations where the data yields constant or variable rates of change. This topic is intended to provide experiences for students that will give them an intuitive understanding of rates of change and will assist the development of appropriate vocabulary. No quantitative analysis is needed at this Stage. **Language Development** Throughout this unit, students need to develop language to describe graphs. Terms such as the following should be used where appropriate: gradient, positive, negative, zero gradient, increasing, decreasing, constant, rate of change, stationary, steady, linear, variable, independent variable, dependent variable, maximum value, minimum value, relationship, range. This can be achieved by providing opportunities for discussion in small groups eg card-matching activities and finding errors in descriptions of graphs. **Drawing Graphs** Students could: • work in small groups to draw a graph to model an event at school eg the number of students at the canteen over a given time or the number of students remaining in the classroom over a five-minute period after the bell • conduct an experiment filling jars of different shapes. A scoop can be used to add fixed amounts of liquid to the jars and the water level graphed against the number of scoops. Students could predict the graph before conducting the experiment and then test their prediction. • use technology such as data loggers to collect data and graph the data to compare and contrast the graphs. **Distance/Time Graph Practical Activity** As in Stage 5.2, students model and record data, then sketch graphs to investigate the distance of a moving object from a fixed point in relation to time; however the speed of the object is not always constant. Students move along a measuring tape for 30 seconds in a variety of activities such as: - running as fast as possible - walking slowly - walking for 10 seconds, stopping for 10 seconds and walking for the remaining 10 seconds - starting at the other end of the line and walking back towards the beginning - walking slowly for 10 seconds and then speeding up until the end of the tape - running at a decreasing speed and record the distance at fixed time intervals so that a graph can be drawn to represent each situation. Students could be given a graph so that they can model the movement of an object along the tape.	**Assessment Activity and Work Sample (see page 45)** WM ***Draw the graph to fit the story*** The questions included on the attached worksheet can be used to assess this unit. The teacher discusses the first worked example with the class or group. Students, in pairs, complete the first 3 questions on the attached worksheet. In questions 4 and 5, students are asked to make up a story and draw a graph to match. Students report back to the group on their results. Possible prompts: • What name/label would you put on the *x* axis? The *y* axis? • Consider some specific examples. • Can you explain why you think the graph has that shape? **Card-matching Activities** WM One set of cards contains graphs and a second set of cards contains descriptions of the graphs. In groups, students match a graph to a description. After completion, one group compares its solution to that of another. Examples of graphs:  Examples of descriptions: a. As one variable increases at a constant rate, the other variable increases at a constant rate. b. As one variable increases at a constant rate the other variable decreases at a constant rate. c. As one variable increases at a constant rate, the other variable increases at an increasing rate. d. As one variable increases at a constant rate, the other variable decreases at a decreasing rate.

Learning Experiences

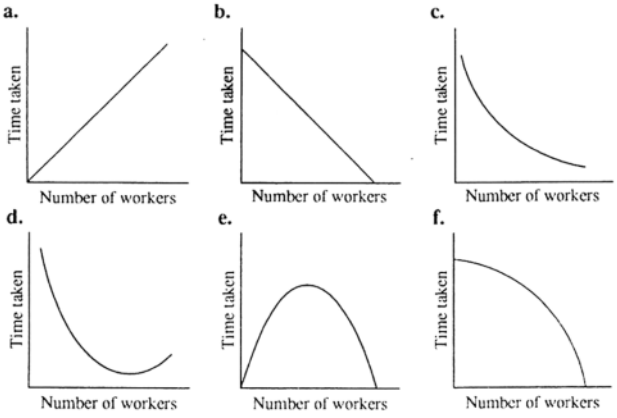
Interpreting Graphs

Students could:

- play a card-matching game, with graphs of physical phenomena to be paired with corresponding descriptions
- work in groups writing a story to match an informal graph as follows:
  - each group is given a different graph
  - a story is written to match the graph
  - the story is given to another group of students who try to sketch the graph to match the story
  - the two groups compare the graphs drawn with the original graphs. Differences should be discussed.
- complete a cloze passage to match a given graph.

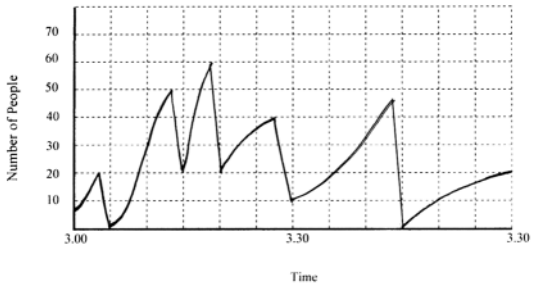
Stocktake

A hardware store is conducting a stocktake in which every item in the store must be counted. Which of the following graphs shows most realistically the relationship between the number of staff involved in the stocktake and the time it takes to complete the job?



Bus Stop

The graph shows the number of people waiting at a bus stop at particular times after school.



Number of people waiting at a bus stop

(i) What time did the buses leave?
(ii) Describe how the number of students waiting varies with time.
(iii) How many students caught the third bus?
(iv) What is the total number of students who caught buses during the first half-hour?

Show on the graph what would have happened if the fifth bus did not arrive at all.

Assessment Activities

Independent and Dependent Variables WM

Read the statements below and decide which variable is independent and which variable *depends* on the other:

1. 'the amount of fuel consumed by a vehicle varies with the distance travelled'
2. 'as the air leaked out, the balloon's diameter decreased'
3. 'the warmer the weather, the louder the crickets chirped'.

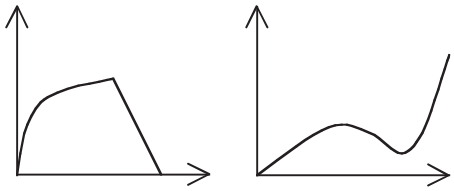
Is time always the independent variable? Consider these statements and decide if time should be placed on the horizontal axis or the vertical axis:

1. 'the temperature of a liquid placed in the refrigerator varies with time'
2. 'the job will take less time to complete if more people help'
3. 'the number of bacteria increases slowly at first but then grows exponentially'
4. 'if the speed is constant, the time taken depends on the distance'.

Interpreting Graphs WM

Students are given graphs and a series of statements that can be used to label parts of the graph. For example:

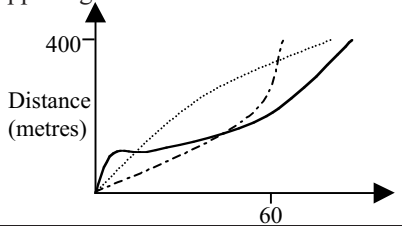
A. increasing at a decreasing rate
B. decreasing at a constant rate
C. increasing at an increasing rate
D. decreasing at an increasing rate

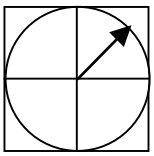
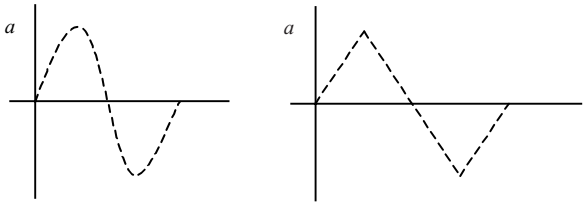
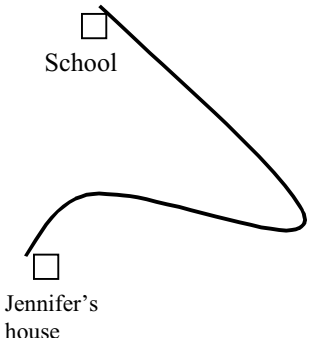
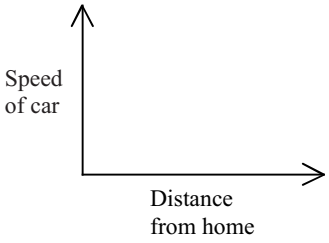


The Race WM

The graph below shows the time taken for three Year 12 students to complete the 400 metre race at the school athletics carnival.

If you were the commentator of the race, how would you describe what was happening?



Learning Experiences	Assessment Activities
Graphs Representing Motion in a Circle Using a Trigonometry Board or a teaching clock, the teacher rotates the arm in an anticlockwise direction, starting from the positive horizontal position.  1. Students track the height of the end of the arm above the starting line (the vertical displacement) as the arm is rotated a full circle. They observe that as the angle of rotation θ increases, the vertical displacement changes value from zero, max(+), zero, min(-), zero. Students could try to graph vertical displacement against θ to obtain graphs like the following, for one rotation:  Discuss the value of the maximum height, a, obtained by the rotating arm. 2. Students track the sideways movement of the end of the arm from the centre (the horizontal displacement). They observe that as the angle of rotation θ increases, the horizontal displacement changes value from max (+), zero, min(-), zero, max(+). Students make a graph of horizontal displacement against θ for one rotation. 3. Students observe possible values of the gradient of the arm, as it rotates. Discuss for what values of θ the gradient of the arm is zero, positive, infinitely steep, and negative. The graph of this could be difficult. Note: the graph of $y = \tan\theta$ is not in this syllabus. Later, when teaching the Unit Circle in Trigonometry, these graphs can be linked to, and $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$.	**Travelling to School** WM This drawing shows the country road which goes from Jennifer's house to her school.  She travels to school each day in her mother's car. They are able to drive at 60 km/h on straight sections of the road, but they slow down for the corners. Sketch a graph on the axes below to show the journey to school. 

3.7.3 Assessment activity and work sample

Focus: Patterns and Algebra and Working Mathematically

Draw the graph to fit the story

The teacher discusses the first worked example with the class or group.
Students, in pairs, complete the first 3 questions on the attached worksheet.
In questions 4 and 5, students are asked to make up a story and draw a graph to match.
Students report back to the group on their results.

From *Securing Their Future: Mathematics*


Pairs


40 minutes

Middle or End

Possible Prompts to assist student engagement

- What name/label would you put on the *x* axis? The *y* axis?
- Consider some specific examples.
- Can you explain why you think the graph has that shape?

Suggested Materials

worksheet
pen and paper

Outcomes

Graphs of Physical Phenomena (PAS5.3.5)
Analyses and describes graphs of physical phenomena
Communicating (WMS5.3.3)
Uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
Reasoning (WMS5.3.4)
Uses deductive reasoning in presenting arguments and formal proofs

Criteria for judging quality of performance

The student may demonstrate the following:

- draws a graph to match one story
- creates only one story with assistance eg height of a person against age
- draws graphs to match stories
- explains the graphs clearly with some prompting
- creates a graph and a matching story
- creates several different types of stories and matching graphs
- explains the graphs succinctly.

Feedback

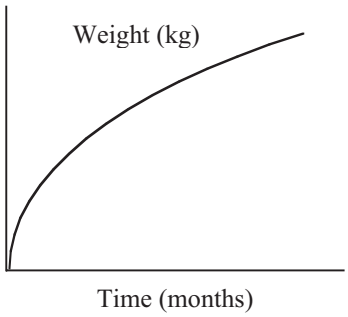
Students will receive:

- possible prompt questions related to the assessment activity to assist their engagement
- written feedback on individual responses to indicate evidence of understanding of the relationship between the two variables in each of the graphs
- specific advice about strategies to further consolidate knowledge, skills and understanding, and suggestions for completing related additional tasks
- oral feedback (to the class and individuals) related to the overall understanding of the concepts and misconceptions about the mathematical ideas.

Draw the graph to fit the story

The graph here matches the following story:

"The weight of the pumpkin increased quite quickly at first, but then the rate of growth slowed down."



Now it's your turn!
On the axes below, *label the axes and draw the graph* for each story:

1. "The temperature of my cup of tea cooled rapidly at first, then cooled more slowly, until it reached room temperature."

2. "The smaller the slices of pizza, the more pieces I can eat."

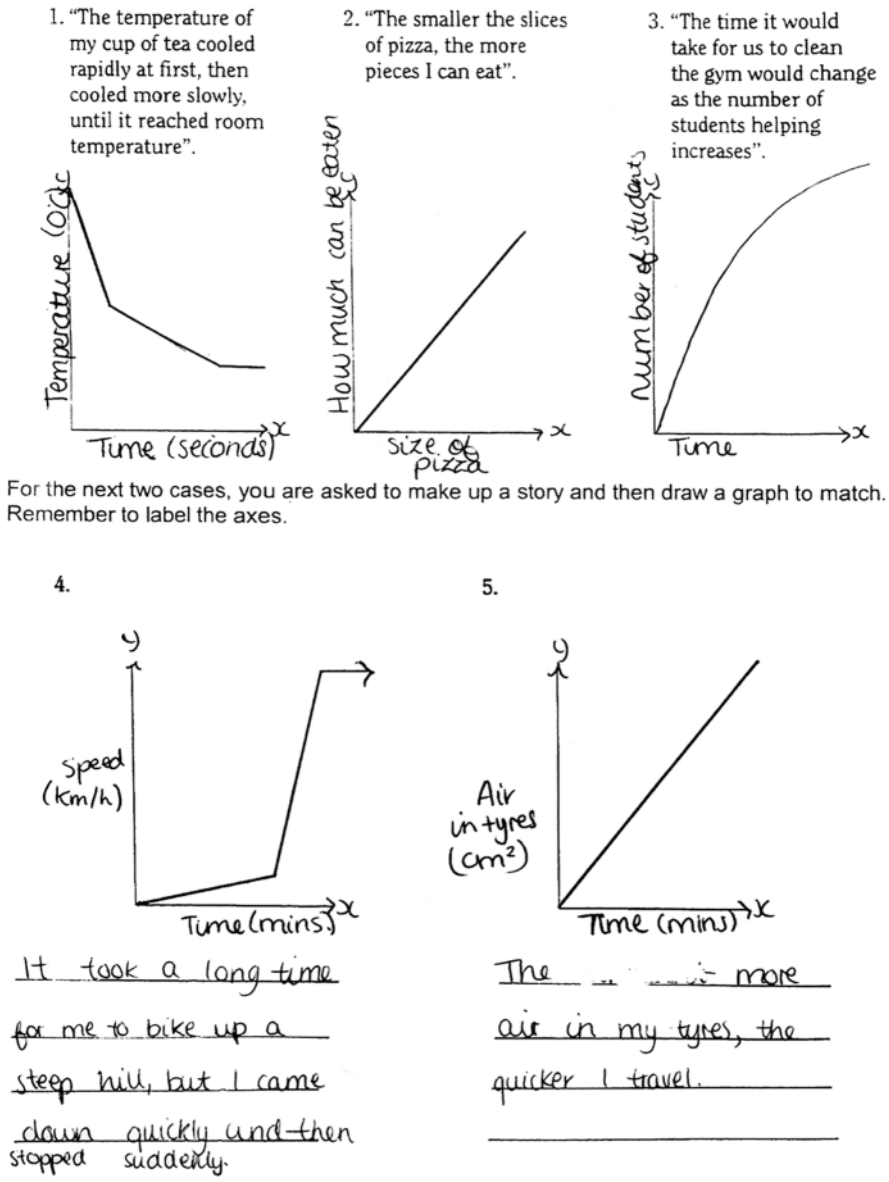
3. "The time it would take for us to clean the gym would change as the number of students helping increases."

For the next two cases, you are asked to make up a story and then draw a graph to match. Remember to label the axes.

4.

5.

Draw the graph to fit the story Work Sample



1. Correct choice of variables for the axes.
Good attempt to show different rates of cooling but the student does not use a curve.
Display of constant room temperature is correct.

2. The student has interpreted the statement as 'the larger the pizza piece, the more is eaten', which is incorrect.
The number of slices has not been graphed.

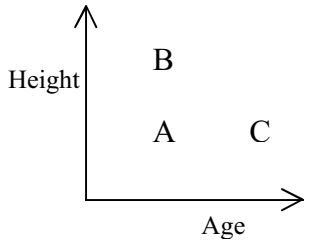
3. The student has incorrectly chosen time as the independent variable.
The teacher could prompt with the comment:
'Your graph tells me that in a small amount of time the gym will be cleaned by a small number of students. Is that what you meant to show?'

4. There is confusion about distance, speed and time. If the vertical axis represented distance, then this graph would fit the story. The student understands that the slope of the graph indicates speed.

5. The dependent variable (time) in the description has been placed on the horizontal axis. The graph should have a negative slope because the student's story implies that the time taken to travel a certain distance is reduced when there is more air in the tyres.

Follow Up
The student needs:

- further experiences in deciding which variable is dependent. *Time* is not always the independent variable.
- more experience with indirect relationships
- further practice clarifying variables from given statements
- practice in analysing information implicit in each point on the graph, as in the example below:



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