

**MATHEMATICS  
STAGE 6**

**DRAFT SYLLABUS FOR  
CONSULTATION**

**20 JULY – 31 AUGUST 2016**

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Phone: (02) 9367 8289

Fax: (02) 9279 1482

Email: [mila.buraga@bostes.nsw.edu.au](mailto:mila.buraga@bostes.nsw.edu.au)

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# CONTENTS

[THE BOSTES SYLLABUS DEVELOPMENT PROCESS](#)

[INTRODUCTION](#)

[MATHEMATICS KEY](#)

[RATIONALE](#)

[THE PLACE OF THE MATHEMATICS STAGE 6 DRAFT SYLLABUS IN THE K–12 CURRICULUM](#)

[AIM](#)

[OBJECTIVES](#)

[OUTCOMES](#)

[COURSE STRUCTURE AND REQUIREMENTS](#)

[ASSESSMENT](#)

[CONTENT](#)

[MATHEMATICS YEAR 11 COURSE CONTENT](#)

[MATHEMATICS YEAR 12 COURSE CONTENT](#)

[GLOSSARY](#)

# THE BOSTES SYLLABUS DEVELOPMENT PROCESS

BOSTES began its syllabus development process for Stage 6 English, Mathematics, Science and History in 2014. This followed state and territory Education Ministers' endorsement of senior secondary Australian curriculum.

The development of the Stage 6 syllabuses involved expert writers and opportunities for consultation with teachers and other interest groups across NSW in order to receive the highest-quality advice across the education community.



A number of key matters at consultations were raised, including the need for the curriculum to cater for the diversity of learners, the broad range of students undertaking Stage 6 study in NSW, development of skills and capabilities for the future, school-based assessment and providing opportunities for assessing and reporting student achievement relevant for post-school pathways.

There was broad support that changes to curriculum and assessment would contribute to the reduction of student stress. BOSTES will continue to use NSW credentialling processes aligned with Stage 6 assessment and HSC examination structures.

A summary of the BOSTES syllabus development process is available at <http://www.boardofstudies.nsw.edu.au/syllabuses/syllabus-development>.

## ASSISTING RESPONDENTS

The following icons are used to assist respondents:

 for your information	This icon indicates general information that assists in reading or understanding the information contained in the document. Text introduced by this icon will not appear in the final syllabus.
 consult	This icon indicates material on which responses and views are sought through consultation.

## CONSULTATION

The *Mathematics Stage 6 Draft Syllabus* is accompanied by an online consultation [survey](#) on the BOSTES website. The purpose of the survey is to obtain detailed comments from individuals and systems/organisations on the syllabus. Please comment on both the strengths and the weaknesses of the draft syllabus. Feedback will be considered when the draft syllabus is revised.

The consultation period is from 20 July to 31 August 2016.

Written responses may be forwarded to:

Louise Brierty  
Senior Project Officer, Curriculum Projects  
GPO Box 5300  
Sydney NSW 2001

Or emailed to: [louise.brierty@bostes.nsw.edu.au](mailto:louise.brierty@bostes.nsw.edu.au)

Or faxed to: (02) 9367 8476

# INTRODUCTION

## STAGE 6 CURRICULUM

Board of Studies, Teaching and Educational Standards NSW (BOSTES) Stage 6 syllabuses have been developed to provide students with opportunities to further develop skills which will assist in the next stage of their lives, whether that is academic study, vocational education or employment.

The purpose of the Higher School Certificate program of study is to:

- provide a curriculum structure which encourages students to complete secondary education
- foster the intellectual, social and moral development of students, in particular developing their:
  - knowledge, skills, understanding, values and attitudes in the fields of study they choose
  - capacity to manage their own learning
  - desire to continue learning in formal or informal settings after school
  - capacity to work together with others
  - respect for the cultural diversity of Australian society
- provide a flexible structure within which students can prepare for:
  - further education and training
  - employment
  - full and active participation as citizens
- provide formal assessment and certification of students' achievements
- provide a context within which schools also have the opportunity to foster students' physical and spiritual development.

The Stage 6 syllabuses reflect the principles of the BOSTES *K–10 Curriculum Framework* and *Statement of Equity Principles*, and the *Melbourne Declaration on Educational Goals for Young Australians* (December 2008). The syllabuses build on the continuum of learning developed in the K–10 syllabuses.

The Stage 6 syllabuses provide a set of broad learning outcomes that summarise the knowledge, understanding, skills, values and attitudes essential for students to succeed in and beyond their schooling. In particular, the literacy and numeracy skills needed for future study, employment and life are provided in Stage 6 syllabuses in alignment with the *Australian Core Skills Framework (ACSF)*.

The syllabuses have considered agreed Australian curriculum content and included content that clarifies the scope and depth of learning in each subject.

Stage 6 syllabuses support a standards-referenced approach to assessment by detailing the essential knowledge, understanding, skills, values and attitudes students will develop and outlining clear standards of what students are expected to know and be able to do. In accordance with the *Statement of Equity Principles*, Stage 6 syllabuses take into account the diverse needs of all students. The syllabuses provide structures and processes by which teachers can provide continuity of study for all students.

## DIVERSITY OF LEARNERS

NSW Stage 6 syllabuses are inclusive of the learning needs of all students. Syllabuses accommodate teaching approaches that support student diversity including Students with special education needs, Gifted and talented students and Students learning English as an additional language or dialect (EAL/D).

## STUDENTS WITH SPECIAL EDUCATION NEEDS

All students are entitled to participate in and progress through the curriculum. Schools are required to provide additional support or adjustments to teaching, learning and assessment activities for some students. Adjustments are measures or actions taken in relation to teaching, learning and assessment that enable a student to access syllabus outcomes and content and demonstrate achievement of outcomes.

Students with special education needs can access the Stage 6 outcomes and content in a range of ways. Students may engage with:

- syllabus outcomes and content with adjustments to teaching, learning and/or assessment activities
- selected outcomes and content appropriate to their learning needs
- selected Stage 6 Life Skills outcomes and content appropriate to their learning needs.

Decisions regarding adjustments should be made in the context of collaborative curriculum planning with the student, parent/carer and other significant individuals to ensure that syllabus outcomes and content reflect the learning needs and priorities of individual students.

Further information can be found in support materials for:

- Mathematics
- Special education needs
- Life Skills.

## GIFTED AND TALENTED STUDENTS

Gifted students have specific learning needs that may require adjustments to the pace, level and content of the curriculum. Differentiated educational opportunities assist in meeting the needs of gifted students.

Generally, gifted students demonstrate the following characteristics:

- the capacity to learn at faster rates
- the capacity to find and solve problems
- the capacity to make connections and manipulate abstract ideas.

There are different kinds and levels of giftedness. Gifted and talented students may also possess learning difficulties and/or disabilities that should be addressed when planning appropriate teaching, learning and assessment activities.

Curriculum strategies for gifted and talented students may include:

- differentiation: modifying the pace, level and content of teaching, learning and assessment activities
- acceleration: promoting a student to a level of study beyond their age group
- curriculum compacting: assessing a student's current level of learning and addressing aspects of the curriculum that have not yet been mastered.

School decisions about appropriate strategies are generally collaborative and involve teachers, parents and students with reference to documents and advice available from BOSTES and the education sectors.

Gifted and talented students may also benefit from individual planning to determine the curriculum options, as well as teaching, learning and assessment strategies, most suited to their needs and abilities.

## STUDENTS LEARNING ENGLISH AS AN ADDITIONAL LANGUAGE OR DIALECT (EAL/D)

Many students in Australian schools are learning English as an additional language or dialect (EAL/D). EAL/D students are those whose first language is a language or dialect other than Standard Australian English and who require additional support to assist them to develop English language proficiency.

EAL/D students come from diverse backgrounds and may include:

- overseas and Australian-born students whose first language is a language other than English, including creoles and related varieties
- Aboriginal and Torres Strait Islander students whose first language is Aboriginal English, including Kriol and related varieties.

EAL/D students enter Australian schools at different ages and stages of schooling and at different stages of English language learning. They have diverse talents and capabilities and a range of prior learning experiences and levels of literacy in their first language and in English. EAL/D students represent a significant and growing percentage of learners in NSW schools. For some, school is the only place they use English.

EAL/D students are simultaneously learning a new language and the knowledge, understanding and skills of the Mathematics Stage 6 syllabus through that new language. They require additional time and support, along with informed teaching that explicitly addresses their language needs, and assessments that take into account their developing language proficiency.



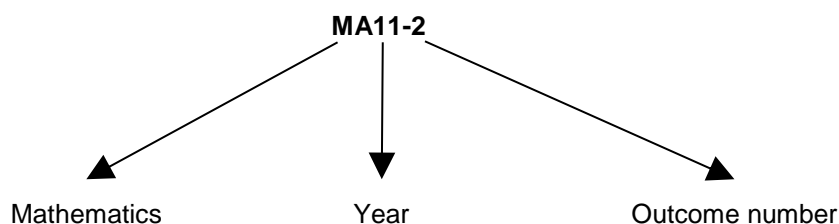
# MATHEMATICS KEY

The following codes and icons are used in the *Mathematics Stage 6 Draft Syllabus*.

## OUTCOME CODING

Syllabus outcomes have been coded in a consistent way. The code identifies the subject, Year and outcome number.

In the *Mathematics Stage 6 Draft Syllabus*, outcome codes indicate the subject, Year and outcome number. For example:

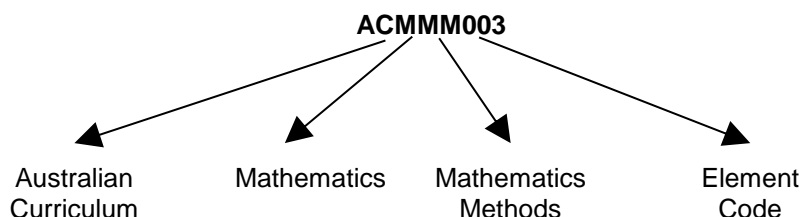


Outcome code	Interpretation
MA11-1	Mathematics, Year 11 – Outcome number 1
MA12-4	Mathematics, Year 12 – Outcome number 4

## CODING OF AUSTRALIAN CURRICULUM CONTENT

Australian curriculum content descriptions included in the syllabus are identified by an Australian curriculum code which appears in brackets at the end of each content description, for example:

Recognise features of the graph of  $y = mx + c$ , including its linear nature, its intercepts and its slope or gradient. (ACMMM003).



Where a number of content descriptions are jointly represented, all description codes are included, eg (ACMGM001, ACMMM002, ACMSM003).

## CODING OF LEARNING OPPORTUNITIES

The syllabus provides opportunities for modelling applications and exploratory work. These should enable candidates to make connections and appreciate the use of mathematics and appropriate digital technology.














<b>M</b>	This identifies an opportunity for explicit modelling applications or investigations that may or may not involve real-life applications or cross-strand integration.
<b>E</b>	This identifies opportunities for extended exploratory work. Such opportunities allow students to investigate in a rich way the evolution of mathematics or mathematics in practice. Opportunities such as this could form the basis of an internal, non-examination based assessment and are outside the scope of the HSC examination.

As these opportunities are both an integral part of each strand and merge strands together, they are identified by the letter at the end of the relevant content description.

For example: Identify contexts suitable for modelling by trigonometric functions and use the to solve practical problems (ACMMM042) **M**

## LEARNING ACROSS THE CURRICULUM ICONS

Learning across the curriculum content, including cross-curriculum priorities, general capabilities and other areas identified as important learning for all students, is incorporated and identified by icons in the *Mathematics Stage 6 Draft Syllabus*.

<b>Cross-curriculum priorities</b>	
	Aboriginal and Torres Strait Islander histories and cultures
	Asia and Australia's engagement with Asia
	Sustainability
<b>General capabilities</b>	
	Critical and creative thinking
	Ethical understanding
	Information and communication technology capability
	Intercultural understanding
	Literacy
	Numeracy
	Personal and social capability
<b>Other learning across the curriculum areas</b>	
	Civics and citizenship
	Difference and diversity
	Work and enterprise

# RATIONALE



for your information

The rationale describes the distinctive nature of the subject and outlines its relationship to the contemporary world and current practice. It explains the place and purpose of the subject in the curriculum, including:

- why the subject exists
- the theoretical underpinnings
- what makes the subject distinctive
- why students would study the subject
- how it contributes to the purpose of the Stage 6 curriculum
- how it prepares students for post-school pathways.



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Mathematics is the study of order, relation, pattern, uncertainty and generality and is underpinned by observation, logical reasoning and deduction. From its origin in counting and measuring, its development throughout history has been catalysed by its utility in explaining real-world phenomena and its inherent beauty. It has evolved in highly sophisticated ways to become the language now used to describe many aspects of the modern world.

Mathematics is an interconnected subject that involves understanding and reasoning about concepts and the relationships between those concepts. It provides a framework for thinking and a means of communication that is powerful, logical, concise and precise.

The Stage 6 Mathematics syllabuses are designed to offer opportunities for students to think mathematically. Mathematical thinking is supported by an atmosphere of questioning, communicating, reasoning and reflecting and is engendered by opportunities to generalise, challenge, find connections and to think critically and creatively.

All Stage 6 Mathematics syllabuses provide opportunities to develop students' 21st-century knowledge, skills, understanding, values and attitudes. As part of this, in all courses students are encouraged to learn with the use of appropriate technology and make appropriate choices when selecting technologies as a support for mathematical activity.

The Mathematics course is focused on enabling students to appreciate that mathematics is a unique and powerful way of viewing the world to investigate patterns, order, generality and uncertainty. The course provides students with the opportunity to develop ways of thinking in which problems are explored through observation, reflection and reasoning.

The Mathematics Course provides a basis for further studies in disciplines in which mathematics and the skills that constitute thinking mathematically have an important role. It is designed for those students whose future pathways may involve mathematics and their applications in a range of disciplines at the tertiary level.

# THE PLACE OF THE MATHEMATICS STAGE 6 DRAFT SYLLABUS IN THE K–12 CURRICULUM



for your information

NSW syllabuses include a diagram that illustrates how the syllabus relates to the learning pathways in K–12. This section places the Mathematics Stage 6 syllabus in the K–12 curriculum as a whole.



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# AIM



for your information

In NSW syllabuses, the aim provides a succinct statement of the overall purpose of the syllabus. It indicates the general educational benefits for students from programs based on the syllabus.

The aim, objectives, outcomes and content of a syllabus are clearly linked and sequentially amplify details of the intention of the syllabus.



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The study of Mathematics in Stage 6 enables students to enhance their knowledge and understanding of what it means to work mathematically, develop their understanding of the relationship between 'real world' problems and mathematical models and extend their skills of concise and systematic communication.

## OBJECTIVES



for your information

In NSW syllabuses, objectives provide specific statements of the intention of a syllabus. They amplify the aim and provide direction to teachers on the teaching and learning process emerging from the syllabus. They define, in broad terms, the knowledge, understanding, skills, values and attitudes to be developed through study in the subject. They act as organisers for the intended outcomes.



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## KNOWLEDGE, UNDERSTANDING AND SKILLS

Students:

- develop knowledge, understanding and skills about efficient strategies for pattern recognition, generalisation and modelling techniques
- develop the ability to use mathematical concepts and skills and apply complex techniques to the solution of problems in algebra and functions, measurement, financial mathematics, calculus, data and statistics, probability and matrices
- develop the ability to use advanced mathematical models and techniques, aided by appropriate technology, to organise information, investigate and solve problems and interpret complex practical situations
- develop the ability to interpret and communicate mathematics logically and concisely in a variety of forms

## VALUES AND ATTITUDES

Students will value and appreciate:

- mathematics as an essential and relevant part of life, recognising that its development and use has been largely in response to human needs by societies all around the globe
- the importance of resilience and self-motivation in undertaking mathematical challenges and the importance of taking responsibility for their own learning and evaluation of their mathematical development

# OUTCOMES



for your information

In NSW syllabuses, outcomes provide detail about what students are expected to achieve at the end of each Year in relation to the objectives. They indicate the knowledge, understanding and skills expected to be gained by most students as a result of effective teaching and learning. They are derived from the objectives of the syllabus.



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## TABLE OF OBJECTIVES AND OUTCOMES – CONTINUUM OF LEARNING

<b>Objectives</b> Students: <ul style="list-style-type: none"> <li>develop knowledge, understanding and skills about efficient strategies for pattern recognition, generalisation and modelling techniques</li> </ul>	
<b>Year 11 outcomes</b> A student:	<b>Year 12 outcomes</b> A student:
<b>MA11-1</b> uses algebraic and graphical techniques to compare alternative solutions to contextual problems	<b>MA12-1</b> uses detailed algebraic and graphical techniques to critically evaluate and construct arguments in a range of familiar and unfamiliar contexts
	<b>MA12-2</b> makes informed decisions about financial situations using mathematical reasoning and techniques
	<b>MA12-3</b> applies calculus techniques to solve complex problems

<b>Objectives</b> Students: <ul style="list-style-type: none"> <li>develop the ability to use mathematical concepts and skills and apply complex techniques to the solution of problems in algebra and functions, measurement, financial mathematics, calculus, data and statistics, probability and matrices</li> </ul>	
<b>Year 11 outcomes</b> A student:	<b>Year 12 outcomes</b> A student:
<b>MA11-2</b> uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes or trigonometric graphs	<b>MA12-4</b> uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities
<b>MA11-3</b> manipulates and solves expressions using the logarithmic and indicial laws, and uses logarithms and exponential functions to solve practical problems	
<b>MA11-4</b> uses general results for arithmetic and geometric series, and applies the results in the solution of problems	
<b>MA11-5</b> interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems	<b>MA12-5</b> applies appropriate differentiation methods to solve problems, including using the derivative to determine the features of the graph of a function
	<b>MA12-6</b> uses techniques for the determination of indefinite and definite integrals and areas under curves
<b>MA11-6</b> uses concepts and techniques from statistics and probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions	<b>MA12-7</b> solves problems using appropriate statistical processes, including the use of the normal distribution and the correlation of bivariate data
<b>MA11-7</b> performs matrix arithmetic including addition, subtraction, scalar and matrix multiplication and finding inverse matrices in a variety of contexts and to solve problems	<b>MA12-8</b> solves matrix equations and uses matrices to model and solve practical problems



<b>Objectives</b> Students: <ul style="list-style-type: none"> <li>develop the ability to use advanced mathematical models and techniques, aided by appropriate technology, to organise information, investigate and solve problems and interpret complex practical situations</li> </ul>	
<b>Year 11 outcomes</b> A student:	<b>Year 12 outcomes</b> A student:
<b>MA11-8</b> uses appropriate technology to investigate, organise and interpret information in a range of contexts	<b>MA12-9</b> chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use

<b>Objectives</b> Students: <ul style="list-style-type: none"> <li>develop the ability to communicate and interpret mathematics logically and concisely in a variety of forms</li> </ul>	
<b>Year 11 outcomes</b> A student:	<b>Year 12 outcomes</b> A student:
<b>MA11-9</b> provides reasoning to support conclusions which are appropriate to the context	<b>MA12-10</b> constructs arguments to prove and justify results

# COURSE STRUCTURE AND REQUIREMENTS



for your information

The following provides an outline of the Year 11 and Year 12 course structure and requirements for the *Mathematics Stage 6 Draft Syllabus* with indicative hours, arrangement of content, and an overview of course content.



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<b>Year 11 course (120 hours)</b>	<b>Mathematics</b>	<b>Indicative hours</b>
	The course is organised into strands, with the strands divided into topics.	
	Trigonometric Functions	20
	Functions	40
	Calculus	30
	Statistical Analysis	15
	Matrices	15
	Modelling and applications are an integral part of each strand and merge strands together, enabling students to make connections and appreciate the use of mathematics and appropriate technology.	
<b>Year 12 course (120 hours)</b>	<b>Mathematics</b>	<b>Indicative hours</b>
	The course is organised into strands, with the strands divided into topics.	
	Trigonometric Functions	15
	Functions	15
	Calculus	45
	Financial Mathematics	15
	Statistical Analysis	15
	Matrices	15

	Modelling and applications are an integral part of each strand and merge strands together, enabling students to make connections and appreciate the use of mathematics and appropriate technology.
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# ASSESSMENT



for your information

The key purpose of assessment is to gather valid and useful information about student learning and achievement. It is an essential component of the teaching and learning cycle. School-based assessment provides opportunities to measure student achievement of outcomes in a more diverse way than the HSC examination.

BOSTES continues to promote a standards-referenced approach to assessing and reporting student achievement. Assessment for, as and of learning are important to guide future teaching and learning opportunities and to give students ongoing feedback. These approaches are used individually or together, formally or informally, to gather evidence of student achievement against standards. Assessment provides teachers with the information needed to make judgements about students' achievement of outcomes.

Ongoing stakeholder feedback, analysis of BOSTES examination data and information gathered about assessment practices in schools has indicated that school-based and external assessment requirements require review and clarification. The HSC Reforms outline changes to school-based and HSC assessment practices to:

- make assessment more manageable for students, teachers and schools
- maintain rigorous standards
- strengthen opportunities for deeper learning
- provide opportunities for students to respond to unseen questions, and apply knowledge, understanding and skills to encourage in-depth analysis
- support teachers to make consistent judgements about student achievement.

## **Students with special education needs**

Some students with special education needs will require adjustments to assessment practices in order to demonstrate what they know and can do in relation to syllabus outcomes and content. The type of adjustments and support will vary according to the particular needs of the student and the requirements of the assessment activity. Schools can make decisions to offer adjustments to coursework and school-based assessment.

## **Life Skills**

Students undertaking Years 11–12 Life Skills courses will study selected outcomes and content. Assessment activities should provide opportunities for students to demonstrate achievement in relation to the outcomes, and to apply their knowledge, understanding and skills to a range of situations or environments.

The following general descriptions have been provided for consistency. Further advice about assessment, including in support materials, will provide greater detail.

Assessment for Learning	<ul style="list-style-type: none"><li>• enables teachers to use formal and informal assessment activities to gather evidence of how well students are learning</li><li>• teachers provide feedback to students to improve their learning</li><li>• evidence gathered can inform the directions for teaching and learning programs.</li></ul>
Assessment as Learning	<ul style="list-style-type: none"><li>• occurs when students use self-assessment, peer-assessment and formal and informal teacher feedback to monitor and reflect on their own learning, consolidate their understanding and work towards learning goals.</li></ul>
Assessment of Learning	<ul style="list-style-type: none"><li>• assists teachers to use evidence of student learning to assess student achievement against syllabus outcomes and standards at defined key points within a Year or Stage of learning.</li></ul>
Formal assessment	<ul style="list-style-type: none"><li>• tasks which students undertake as part of the internal assessment program, for example a written examination, research task, oral presentation, performance or other practical task</li><li>• tasks appear in an assessment schedule and students are provided with sufficient written notification</li><li>• evidence is gathered by teachers to report on student achievement in relation to syllabus outcomes and standards, and may also be used for grading or ranking purposes.</li></ul>
Informal assessment	<ul style="list-style-type: none"><li>• activities undertaken and anecdotal evidence gathered by the teacher throughout the learning process in a less prescribed manner, for example class discussion, questioning and observation</li><li>• used as part of the ongoing teaching and learning process to gather evidence and provide feedback to students</li><li>• can identify student strengths and areas for improvement.</li></ul>
Written examination	<ul style="list-style-type: none"><li>• a task undertaken individually, under formal supervised conditions to gather evidence about student achievement in relation to knowledge, understanding and skills at a point in time, for example a half-yearly, yearly or trial HSC examination</li><li>• a task which may include one or more unseen questions or items, assessing a range of outcomes and content.</li></ul>



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### Mathematics Draft Assessment Requirements

The draft guidelines for school-based assessment provide specific advice about the number of formal assessment tasks, course components and weightings, and the nature of task types to be administered in Year 11 and Year 12.

The components and weightings for Year 11 and Year 12 are mandatory.

#### Year 11

- There will be 3 formal assessment tasks
- The maximum weighting for each formal assessment task is 40%
- One task must be an assignment or investigation-style task with a weighting of 20–30%.

Component	Weighting %
Knowledge, understanding and communication	50
Problem solving, reasoning and justification	50
	<b>100</b>

#### Year 12

- There will be no more than 4 formal assessment tasks
- The maximum weighting for each formal assessment task is 40%
- One task may be a formal written examination, eg a trial HSC, with a maximum weighting of 25%
- One task must be an assignment or investigation-style task with a weighting of 20–30%.

Component	Weighting %
Knowledge, understanding and communication	50
Problem solving, reasoning and justification	50
	<b>100</b>

## Mathematics Draft Examination Specifications

### Option 1

Sections
<b>Section – Written responses</b> A variety of short answer questions to extended responses, with the possibility of a number of parts

#### Changes from current examination specifications

The objective response section will be removed. This approach provides opportunity for inclusion of cross-strand application questions, and modelling and problem solving questions to enable students to demonstrate deep understanding, conceptual knowledge, higher-order thinking and reasoning.

Questions or parts of questions may be drawn from a range of syllabus outcomes and content.

30% of marks will be common with the Mathematics General 2 examination. Preliminary/Year 11 material will not be assessed in the HSC examination.

### Option 2

Sections
<b>Section I – Objective responses</b>
<b>Section II – Written responses</b> A variety of short answer questions to extended responses, with the possibility of a number of parts

#### Changes from current examination specifications

Questions or parts of questions may be drawn from a range of syllabus outcomes and content.

30% of marks will be common with the Mathematics General 2 examination. Preliminary/Year 11 material will not be assessed in the HSC examination.

HSC examination specifications will be reviewed following finalisation of the syllabuses.

Updated assessment and reporting advice will be provided when syllabuses are released.

The Assessment Certification Examination website will be updated to align with the syllabus implementation timeline.

# CONTENT

For Kindergarten to Year 12 courses of study and educational programs are based on the outcomes and content of syllabuses. The content describes in more detail how the outcomes are to be interpreted and used, and the intended learning appropriate for each Year. In considering the intended learning, teachers will make decisions about the emphasis to be given to particular areas of content, and any adjustments required based on the needs, interests and abilities of their students.

The knowledge, understanding and skills described in the outcomes and content provide a sound basis for students to successfully transition to their selected post-school pathway.

## LEARNING ACROSS THE CURRICULUM



for your information

NSW syllabuses provide a context within which to develop core skills, knowledge and understanding considered essential for the acquisition of effective, higher-order thinking skills that underpin successful participation in further education, work and everyday life including problem-solving, collaboration, self-management, communication and information technology skills.

BOSTES has described learning across the curriculum areas that are to be included in syllabuses. In Stage 6 syllabuses, the identified areas will be embedded in the descriptions of content and identified by icons. Learning across the curriculum content, including the cross-curriculum priorities and general capabilities, assists students to achieve the broad learning outcomes defined in the BOSTES *Statement of Equity Principles*, the *Melbourne Declaration on Educational Goals for Young Australians* (December 2008) and in the Australian Government's *Core Skills for Work Developmental Framework* (2013).

Knowledge, understanding, skills, values and attitudes derived from the learning across the curriculum areas will be included in BOSTES syllabuses, while ensuring that subject integrity is maintained.

Cross-curriculum priorities enable students to develop understanding about and address the contemporary issues they face.

The cross-curriculum priorities are:

- Aboriginal and Torres Strait Islander histories and cultures 🇺🇸
- Asia and Australia's engagement with Asia 🌏
- Sustainability ♻️

General capabilities encompass the knowledge, skills, attitudes and behaviours to assist students to live and work successfully in the 21st century.

The general capabilities are:

- Critical and creative thinking 🧠
- Ethical understanding ⚖️
- Information and communication technology capability 💻
- Intercultural understanding 🌐
- Literacy 📖
- Numeracy 📊
- Personal and social capability 🧑



BOSTES syllabuses include other areas identified as important learning for all students:

- Civics and citizenship 🇦🇺
- Difference and diversity 🌍
- Work and enterprise ⭐



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## Aboriginal and Torres Strait Islander histories and cultures 🖐️

Through application and modelling across the strands of the syllabus, students can experience the relevance of Mathematics in Aboriginal and Torres Strait Islander histories and cultures. Opportunities are provided to connect Mathematics with Aboriginal and Torres Strait Islander peoples' cultural, linguistic and historical experiences. The narrative of the development of Mathematics and its integration with cultural development can be explored in the context of some topics. Through the evaluation of statistical data where appropriate, students can deepen their understanding of the lives of Aboriginal and Torres Strait Islander peoples.

When planning and programming content relating to Aboriginal and Torres Strait Islander histories and cultures teachers are encouraged to consider involving local Aboriginal communities and/or appropriate knowledge holders in determining suitable resources, or to use Aboriginal or Torres Strait Islander authored or endorsed publications.

## Asia and Australia's engagement with Asia 🌐

Students have the opportunity to learn about the understandings and application of mathematics in Asia and the way mathematicians from Asia continue to contribute to the ongoing development of mathematics. By drawing on knowledge of and examples from the Asia region, such as calculation, money, art, architecture, design and travel, students can develop mathematical understanding in fields such as number, patterns, measurement, symmetry and statistics. Through the evaluation of statistical data, students can examine issues pertinent to the Asia region.

## Sustainability 🌱

Mathematics provides a foundation for the exploration of issues of sustainability. Students have the opportunity to learn about the mathematics underlying topics in sustainability such as energy use and how to reduce it, alternative energy with solar cells and wind turbines, climate change and mathematical modelling. Through measurement and the reasoned use of data, students can measure and evaluate sustainability changes over time and develop a deeper appreciation of the world around them. Mathematical knowledge, understanding and skills are necessary to monitor and quantify both the impact of human activity on ecosystems and changes to conditions in the biosphere.

## Critical and creative thinking

Critical and creative thinking are key to the development of mathematical understanding. Mathematical reasoning and logical thought are fundamental elements of critical and creative thinking. Students are encouraged to be critical thinkers when justifying their choice of a calculation strategy or identifying relevant questions during an investigation. They are encouraged to look for alternative ways to approach mathematical problems; for example, identifying when a problem is similar to a previous one, drawing diagrams or simplifying a problem to control some variables. Students are encouraged to be creative in their approach to solving new problems, combining the skills and knowledge they have acquired in their study of a number of different topics in a new context.

## Ethical understanding

Mathematics makes a clear distinction between basic principles and the deductions made from them or their consequences in different circumstances. Students have opportunities to explore, develop and apply ethical understanding to mathematics in a range of contexts. Examples include: collecting, displaying and interpreting data; examining selective use of data by individuals and organisations; detecting and eliminating bias in the reporting of information; exploring the importance of fair comparison and interrogating financial claims and sources.

## Information and communication technology capability

Mathematics provides opportunities for students to develop ICT capacity when students investigate; create and communicate mathematical ideas and concepts using fast, automated, interactive and multimodal technologies. Students can use their ICT capability to perform calculations; draw graphs; collect, manage, analyse and interpret data; share and exchange information and ideas; and investigate and model concepts and relationships. Digital technologies, such as calculators, spreadsheets, dynamic geometry software, graphing software and computer algebra software, can engage students and promote understanding of key concepts.

## Intercultural understanding

Students have opportunities to understand that mathematical expressions use universal symbols, while mathematical knowledge has its origin in many cultures. Students realise that proficiencies such as understanding, fluency, reasoning and problem-solving are not culture- or language-specific, but that mathematical reasoning and understanding can find different expression in different cultures and languages. The curriculum provides contexts for exploring mathematical problems from a range of cultural perspectives and within diverse cultural perspectives. Students can apply mathematical thinking to identify and resolve issues related to living with diversity.

## Literacy

Literacy is used throughout mathematics to understand and interpret word problems and instructions that contain the particular language features of mathematics. Students learn the vocabulary associated with mathematics, including synonyms, technical terminology, passive voice and common words with specific meanings in a mathematical context. Literacy is used to pose and answer questions, engage in mathematical problem-solving and to discuss, produce and explain solutions. There are opportunities for students to develop the ability to create and interpret a range of texts typical of mathematics ranging from graphs to complex data displays.

## Numeracy

Mathematics has a central role in the development of numeracy in a manner that is more explicit and foregrounded than is the case in other learning areas. It is related to a high proportion of the content. Consequently, this particular general capability is not tagged in the syllabus.

Numeracy involves drawing on knowledge of particular contexts and circumstances in deciding when to use mathematics, choosing the mathematics to use and critically evaluating its use. To be numerate is to use mathematics effectively to meet the general demands of life at home, in work, and for participation in community and civic life. It is therefore important that the Mathematics curriculum provides the opportunity to apply mathematical understanding and skills in context, in other learning areas and in real-world contexts.

## Personal and social capability

Students develop personal and social competence as they learn to understand and manage themselves, their relationships and their lives more effectively. Mathematics enhances the development of students' personal and social capabilities by providing opportunities for initiative-taking, decision-making, communicating their processes and findings, and working independently and collaboratively in the mathematics classroom. Students have the opportunity to apply mathematical skills in a range of personal and social contexts. This may be through activities that relate learning to their own lives and communities, such as time-management, budgeting and financial management, and understanding statistics in everyday contexts.

## Civics and citizenship

Mathematics has an important role in civics and citizenship education because it has the potential to help us understand our society and our role in shaping it. The role of mathematics in society has expanded significantly in recent decades as almost all aspects of modern-day life are quantified. Through modelling reality with mathematics and then manipulating the mathematics in order to understand and/or predict reality, students have the opportunity to learn mathematical knowledge, skills and understanding that are essential for active participation in the world in which we live.

## Difference and diversity

Students make sense of and construct mathematical ideas in different ways, drawing upon their own unique experiences in life and mathematics learning. By valuing students' diversity of ideas, teachers foster students' efficacy in learning mathematics.

## Work and enterprise

Students develop work and enterprise knowledge, understanding and skills through their study of mathematics in a work-related context. Students are encouraged to select and apply appropriate mathematical techniques and problem-solving strategies through work-related experiences in the financial mathematics and statistical analysis strands. This allows them to make informed financial decisions by selecting and analysing relevant information.

## ORGANISATION OF CONTENT



for your information

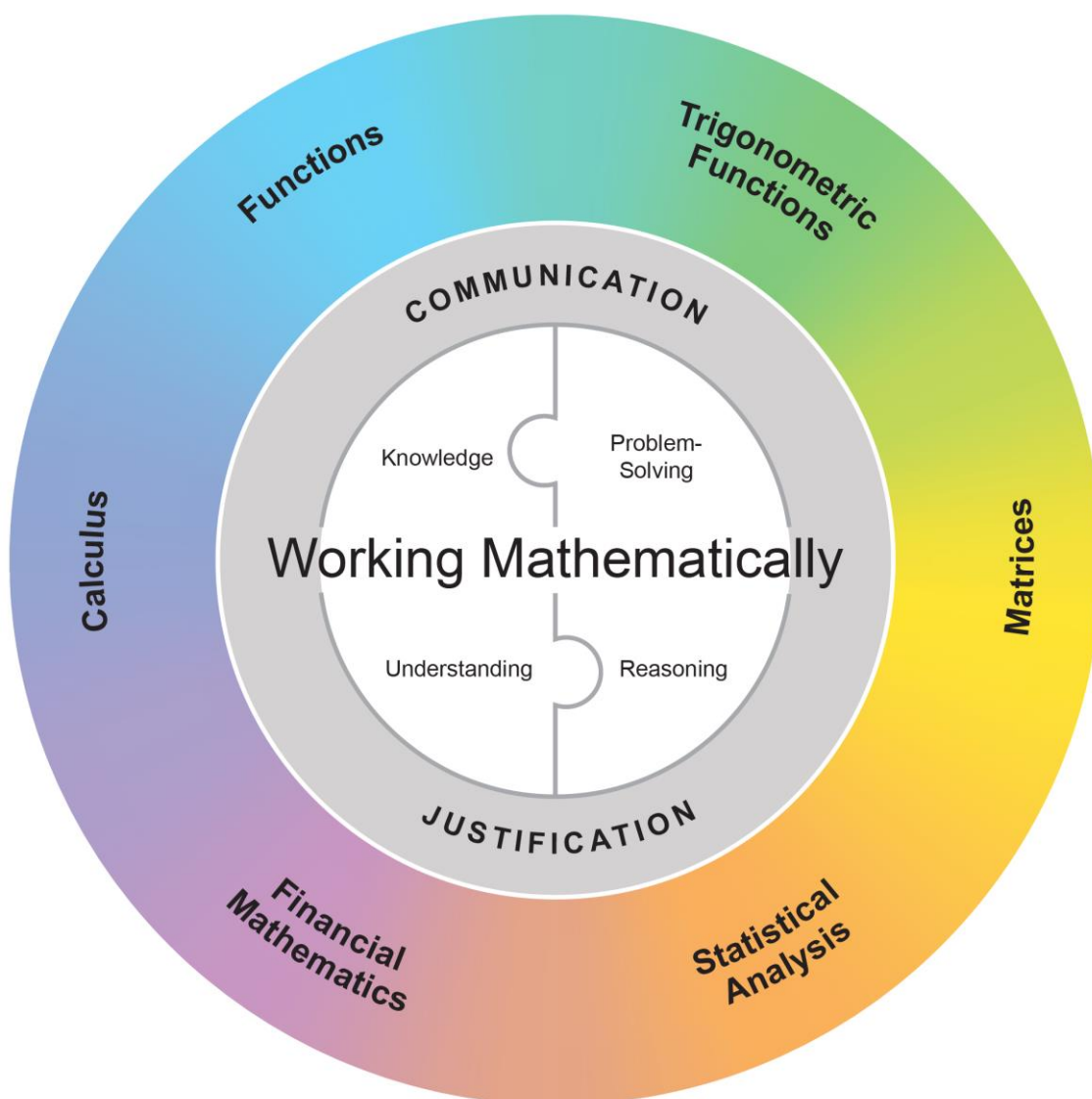
For Kindergarten to Year 12, courses of study and educational programs are based on the outcomes of syllabuses. The content describes in more detail how the outcomes are to be interpreted and used, and the intended learning appropriate for the stage. In considering the intended learning, teachers will make decisions about the sequence, the emphasis to be given to particular areas of content, and any adjustments required based on the needs, interests and abilities of their students.

The knowledge, understanding and skills described in the outcomes and content provide a sound basis for students to successfully transition to their selected post school pathways.

The following provides a diagrammatic representation of the relationships between syllabus content.



consult



## WORKING MATHEMATICALLY

Working Mathematically is integral to the learning process in mathematics. It provides students with the opportunity to engage in genuine mathematical activity and develop the skills to become flexible, critical and creative users of mathematics. In this syllabus, Working Mathematically is represented through two key components: *Knowledge, Understanding and Communication* and *Problem-Solving, Reasoning and Justification*. Together these form the focus of the syllabus, and the components of assessment.

### **Knowledge, Understanding and Communication**

Students make connections between related concepts and progressively apply familiar mathematical concepts and experiences to develop new ideas. They develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students communicate their chosen methods and efficiently calculated solutions, and develop an understanding of the relationship between the 'why' and the 'how' of mathematics. They represent concepts in different ways, identify commonalities and differences between aspects of content, communicate their thinking mathematically, and interpret mathematical information.

### **Problem-Solving, Reasoning and Justification**

Students develop their ability to interpret, formulate, model and analyse identified problems and challenging situations. They describe, represent and explain mathematical situations, concepts, methods and solutions to problems, using appropriate language, terminology, tables, diagrams, graphs, symbols, notation and conventions. They apply mathematical reasoning when they explain their thinking, deduce and justify strategies used and conclusions reached, adapt the known to the unknown, transfer learning from one context to another, prove that something is true or false, and compare and contrast related ideas and explain choices. Their communication is powerful, logical, concise and precise.

Both components and hence, Working Mathematically, are evident across the range of syllabus strands, objectives and outcomes. Teachers extend students' level of proficiency in working mathematically by creating opportunities for development through the learning experiences that they design.

# MATHEMATICS YEAR 11 COURSE CONTENT



consult

## STRAND: TRIGONOMETRIC FUNCTIONS

### OUTCOMES

#### A student:

- > uses algebraic and graphical techniques to compare alternative solutions to contextual problems MA11-1
- > uses the concepts and techniques of trigonometry in the solution of problems involving geometric shapes or trigonometric graphs MA11-2
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9

### STRAND FOCUS

*Trigonometric Functions* involves the study of periodic functions in relation to geometric, algebraic, numerical and graphical representations.

Knowledge of trigonometric functions enables the solving of practical problems involving triangles or periodic graphs.

Study of trigonometric functions is important in developing students' understanding of the links between algebraic and graphical representations and how this can be applied to solve practical problems.

### TOPICS

M-T1 Trigonometry and Radians

M-T2 Trigonometric Functions and Graphs

# TRIGONOMETRIC FUNCTIONS

## M-T1 TRIGONOMETRY AND RADIANS

### OUTCOMES

**A student:**

- > uses the concepts and techniques of trigonometry in the solution of problems involving geometric shapes or trigonometric graphs MA11-2
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9

### TOPIC FOCUS




The principal focus of this topic is to solve practical problems involving the trigonometry of triangles, and to understand and use angular measure expressed in either radians or degrees.

Students develop appreciation of the connection between circles and angle measurement and develop techniques to solve problems involving triangles.

### CONTENT



#### T1.1 Trigonometry

Students:

- use sin, cos, tan, the sine rule, cosine rule and area of a triangle formula ( $Area = \frac{1}{2}ab\sin C$ ) or Heron's Rule for solving problems in right-angled and non-right-angled triangles where angles are measured in degrees, or degrees and minutes (ACMMM028, ACMMM031, ACMMM035) 
- find angles and sides involving the ambiguous case of the sine rule
  - using digital technology and/or geometric construction to investigate the ambiguous case of the sine rule when finding an angle and the condition for it to arise (ie given sides  $a$  and  $b$  and acute angle  $A$ , ambiguous when  $b \sin A < a < b$ ) 
- solve practical problems involving Pythagoras' Theorem and the trigonometry of triangles, such as finding angles of elevation and depression and the use of true bearings and compass bearings in navigation, and may involve the ambiguous case (ACMGM037) 

#### T1.2 Radians

Students:

- develop and use the relationship between angular measurement in degrees and in radians (ACMMM032)
  - understand the unit circle definition of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  to define angles of any magnitude, and the resulting periodicity of these functions using radians and degrees (ACMMM029, ACMMM034)
  - determine that all circles are similar, and define one radian as the angle subtended by an arc whose length is equal to the radius of its circle 
  - develop the formula for the circular measure of an angle in radians (ie  $l = r\theta$ ) 

- make connections between fractions of circles and lengths of arcs or areas of sectors using radians (ACMMM033)
- recognise and use the exact values of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  at integer multiples of  $\frac{\pi}{6}$ ,  $\frac{\pi}{3}$ ,  $\frac{\pi}{4}$  and  $\frac{\pi}{2}$  (ACMMM035)



# TRIGONOMETRIC FUNCTIONS

## M-T2 TRIGONOMETRIC FUNCTIONS AND GRAPHS

### OUTCOMES

#### A student:

- > uses algebraic and graphical techniques to compare alternative solutions to contextual problems MA11-1
- > uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes or trigonometric graphs MA11-2
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9





### TOPIC FOCUS

The principal focus of this topic is to explore key features of graphs of trigonometric functions and understand and use basic transformations to solve trigonometric equations.

Students develop appreciation of how an equation can be manipulated to create various types of transformational behaviour.

### CONTENT

#### Students:

- utilise the transformational form  $y = af(b(x + c)) + k$  in a variety of contexts, where  $f(x)$  is one of  $\sin x$ ,  $\cos x$  or  $\tan x$ , stating the domain and range when appropriate (ACMSM043)
  - recognise the shape of the graphs of  $y = \sin x$ ,  $y = \cos x$  and  $y = \tan x$  (ACMMM036)
  - use digital technology or otherwise to examine the effect on the graphs of changing the amplitude,  $y = af(x)$ , the period,  $y = f(bx)$ , the phase,  $y = f(x + c)$ , and the centre of motion,  $y = f(x) + k$  (ACMMM037, ACMMM038, ACMMM039) 
  - use  $k, a, b, c$  to describe transformational shifts and sketch graphs 
- solve trigonometric equations involving functions of the form  $af(b(x + c)) + k$ , using digital technology or otherwise, within a specified domain (ACMMM043) 
- apply the use of trigonometric functions and periodic phenomenon involving functions of the form  $k + af(b(x + c))$ , to model and/or solve practical problems (ACMMM042, ACMSM050) **M** 
- students investigate and hence identify periodic functions and their associated properties and apply their knowledge to model and predict future results and conditions in a variety of practical contexts where data approximates a periodic function **E**

## STRAND: FUNCTIONS

### OUTCOMES

**A student:**

- > uses algebraic and graphical techniques to compare alternative solutions to contextual problems MA11-1
- > manipulates and solves expressions using the logarithmic and indicial laws, and uses logarithms and exponential functions to solve practical problems MA11-3
- > uses general results for arithmetic and geometric series, and applies the results in the solution of problems MA11-4
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9

### STRAND FOCUS

*Functions* involves the use of algebraic concepts and techniques to describe and interpret relationships between changing quantities.

Knowledge of functions enables students to discover connections between abstract algebra and its various graphical representations.

Study of functions is important in developing students' ability to find connections, communicate concisely, use algebraic techniques and manipulations to describe and solve problems, and to predict future outcomes.

### TOPICS

M-A1 Working with Functions

M-A2 Logarithms and Exponentials

M-A3 Sequences and Series

# FUNCTIONS

## M-A1 WORKING WITH FUNCTIONS

### OUTCOMES

**A student:**

- > uses algebraic and graphical techniques to compare alternative solutions to contextual problems MA11-1
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9

### TOPIC FOCUS

The principal focus of this topic is to introduce students to the concept of a function and develop their knowledge of functions and the associated graphs.

Students develop their use of mathematical language to describe functions and their properties and graphs.

### CONTENT


#### A1.1: Algebraic Techniques

Students:

- review and use algebraic techniques, including but not limited to the use of the index laws, solving quadratic equations by factorising, completing the square or using the quadratic formula, working with single and double linear inequalities and working with algebraic fractions to simplify, add, subtract, multiply and divide expressions (ACMMM008, ACMMM016)




#### A1.2: Introduction to Functions








Students:

- determine and use the concept of functions and relations as mappings between sets, and as a rule or a formula that defines one variable quantity in terms of another (ACMMM022)
  - identify types of functions and relations, using a variety of methods, such as the vertical line test, horizontal line test or considering mapping relationships ( $1:1$ ,  $1:n$ ,  $n:1$ ,  $n:n$ ) (ACMMM027, ACMSM094)
  - use function notation and properties of functions and graphs including domain and range, independent and dependent variables, and continuity (ACMMM023) 
  - understand and use interval notation as representative of domain and range

#### A1.3: Polynomials up to degree 3






Students:

- model, analyse and solve linear relationships, including constructing a straight-line graph and interpreting features of a straight-line graph such as the gradient and intercepts, and identifying its equation in both practical and abstract situations (ACMMM003, ACMMM005) **M**  
  - recognise that a direct variation relationship produces a straight-line graph that passes through (0,0) or has y-intercept 0
  - find the equations of straight lines given sufficient information, including parallel and perpendicular lines (ACMMM004) 

- model, analyse and solve quadratic relationships, including constructing a quadratic graph and interpreting any features of a quadratic graph such as its parabolic nature, turning points, axis of symmetry and intercepts in both practical and abstract situations (ACMMM006, ACMMM007, ACMMM008, ACMMM011) **M**   
- find turning points and zeros of quadratics, by either solving the quadratic equation, or rearranging the quadratic expression, and understand the role of the discriminant (ACMMM010) 
- find the equation of a quadratic polynomial given sufficient information (ACMMM009)
- solve practical problems involving a pair of simultaneous linear and/or quadratic functions algebraically or graphically, with or without the aid of digital technology; for example, determining and interpreting the break-even point of a simple business problem (ACMGM045) **M**  
- work with cubic equations of the form  $y = a(x - h)^3 + k$  and their graphs, including shape, turning points, intercepts and behaviour (ACMMM017) 
- factorise expressions involving the sum and difference of two cubes
- find the equation of a cubic polynomial given sufficient information
- students investigate alternate approaches to solving equations that cannot be solved using algebraic methods currently at their disposal, in particular they explore both graphical and numerical approaches such as the iterative method **E**

#### A1.4: Other function types

Students:

- recognise expressions that represent polynomial functions, identify the shape and features of their graphs, and the coefficients and degree of a polynomial both algebraically and graphically (ACMMM014, ACMMM015)  
- recognise expressions that represent inverse proportion, such as  $y = \frac{1}{x}$  and  $y = \frac{a}{x-b}$ , identify the shape and features of their graphs, including their hyperbolic shapes, and their asymptotes (ACMMM012, ACMMM013)
- Interpret, create and use piece-wise linear and step graphs used to model practical situations (ACMGM047) **M**
- solve simple absolute value equations of the form  $|ax \pm b| = k$  both algebraically and graphically (ACMSM098) 
- define the absolute value and simplify absolute value expressions, using the notation  $|x|$  for the absolute value of the real number  $x$
- sketch, interpret and determine the equation of graphs of circles
- determine that for the relation to become a function, the domain must be restricted
- recognise features of the graphs of  $x^2 + y^2 = r^2$  and  $(x - a)^2 + (y - b)^2 = r^2$ , including their circular shapes, their centres and their radii (ACMMM020)  
- find the equation of a circle given sufficient information

# FUNCTIONS

## M-A2 LOGARITHMS AND EXPONENTIALS

### OUTCOMES

**A student:**

- > manipulates and solves expressions using the logarithmic and indicial laws, and uses logarithms and exponential functions to solve practical problems MA11-3
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9

### TOPIC FOCUS


The principal focus of this topic is for students to become fluent in manipulating logarithms and exponentials and use knowledge, understanding and skills to solve problems.

Students develop appreciation of numbering systems, their representations and links to observable phenomena.

### CONTENT

Students:

- work with logarithms in a variety of practical and abstract contexts
  - determine logarithms using the relationship: if  $y = a^x$  then  $x = \log_a y$ , i.e.  $a^{\log_a b} = b$  (ACMMM151)
  - use the logarithmic laws to simplify and evaluate logarithmic expressions:  
 $\log_a m + \log_a n = \log_a (m \times n)$  and  $\log_a m - \log_a n = \log_a \left(\frac{m}{n}\right)$  and  $\log_a (m^n) = n \log_a m$ ,  
 $\log_a a^x = x$ ,  $\log_a a = 1$ ,  $\log_a 1 = 0$ ,  $\log_a \frac{1}{x} = -\log_a x$  (ACMMM152)
  - recognise and use the inverse relationship of the functions  $y = a^x$  and  $y = \log_a x$  (ACMMM153)
  - investigate the development of different number bases and their history and uses in different cultures around the world, including but not limited to those of Aboriginal and Torres Strait Islander peoples 🗺️ (ACMMM154)
  - establish and use the change of base law,  $\log_a x = \frac{\log_b x}{\log_b a}$
- work with natural logarithms in a variety of practical and abstract contexts
  - determine the natural logarithm  $\ln x = \log_e x$ , by defining  $e$  as Euler's number,  $e \approx 2.718281828$ , and use a calculator to evaluate natural logarithms (ACMMM159)
  - recognise and use the inverse relationship of the functions  $y = e^x$  and  $y = \ln x$  (ACMMM160)
  - use the logarithmic laws to simplify and evaluate natural logarithmic expressions
- use logarithms to solve equations involving indices (ACMMM155)
- graph an exponential function of the form  $y = a^x$  for  $a > 0$  and its transformations, using digital technology or otherwise (ACMMM065) 🖥️
  - interpret the meaning of the intercepts of an exponential graph and explain the circumstances in which these do not exist ⚙️
  - solve equations involving exponential functions using digital technology, and algebraically in simple cases (ACMMM067) 🖥️

- graph a logarithmic function  $y = \log_a x$  for  $a > 1$  and its transformations, using digital technology or otherwise (ACMMM156) 
- recognise that the graphs of  $y = a^x$  and  $y = \log_a x$  are reflections in the line  $y = x$
- solve simple equations involving logarithmic or exponential functions algebraically and graphically (ACMMM157)
  - identify contexts suitable for modelling by logarithmic functions, for example the use of compound interest or the growth of algae in a pond, and hence solve practical problems (ACMMM158) **M**

# FUNCTIONS

## M-A3 SEQUENCES AND SERIES

### OUTCOMES

**A student:**

- > uses general results for arithmetic and geometric series, and applies the results in the solution of problems MA11-4
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9

### TOPIC FOCUS




The principal focus of this topic is to identify, understand and apply arithmetic and geometric series in a variety of situations.

Students develop appreciation of patterns and symmetry and techniques to make generalisations to model relationships and predict values.

### CONTENT




#### A3.1: Arithmetic Sequences and Series

Students:

- recognise and use the recursive definition of an arithmetic sequence:  $T_n = T_{n-1} + d$  (ACMMM068) 
- establish and use the formula for the  $n$ th term (for  $n$  a positive integer) of an arithmetic sequence:  $T_n = a + (n - 1)d$ , where  $a$  is the first term and  $d$  is the common difference, and recognise its linear nature (ACMMM069) 
- establish and use the formulae for the sum of the first  $n$  terms of an arithmetic series:  $S_n = \frac{n}{2}\{2a + (n - 1)d\}$  and  $S_n = \frac{n}{2}(a + l)$  (ACMMM071) 
- identify and use arithmetic sequences and series in contexts involving discrete linear growth or decay (ACMMM070) **M**

#### A3.2: Geometric Sequences and Series

Students:

- recognise and use the recursive definition of a geometric sequence:  $T_n = rT_{n-1}$  (ACMMM072)
- establish and use the formula for the  $n$ th term of a geometric sequences:  $T_n = ar^{n-1}$ , where  $a$  is the first term and  $r$  is the common ratio, and recognise its exponential nature (ACMMM073)
- establish and use the formula for the sum of the first  $n$  terms of a geometric series:  $S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}$  (ACMMM075) 
  - understand the limiting behaviour as  $n \rightarrow \infty$  and its application to a geometric series as a limiting sum (ACMMM074)
  - use the notation  $\lim_{n \rightarrow \infty} r^n = 0$  for  $|r| < 1$
  - derive and use the formula for the limiting sum of a geometric series with  $|r| < 1$ :  $S = \frac{a}{1-r}$  
- identify and use geometric sequences in contexts involving exponential growth or decay (ACMMM076) **M** 

- students investigate a variety of scenarios involving the identification of types of sequences and series and generate models to explore and extend each scenario. For example, the temperature of a hot drink as it cools, the Sierpinski triangle or defining the exact value of a recurring decimal  
**E**



## STRAND: CALCULUS

### OUTCOMES

**A student:**

- > interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems MA11-5
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9

### STRAND FOCUS

*Calculus* involves rates of change and the use of differential functions to represent change-scenarios. Average and instantaneous rates of change are considered, with the derivative being defined as an instantaneous rate of change.

Knowledge of calculus enables an understanding of the concept of derivative as a function that illustrates the rate of change of the original function.

Study of calculus is important in developing students' ability to solve problems involving algebraic and graphical representations of rates of change of a function.

### TOPICS

M-C1 Introduction to Differentiation

# CALCULUS

## M-C1 INTRODUCTION TO DIFFERENTIATION

### OUTCOMES

#### A student:

- > interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems MA11-5
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9

### TOPIC FOCUS








The principal focus of this topic is for students to develop an understanding of the concept of a derivative as a function that illustrates the rate of change. This concept is reinforced numerically, by calculating difference quotients geometrically, as slopes of secants and tangents, and algebraically. The derivatives of power functions are found and used to solve simple problems including calculating slopes and equations of tangents.

Students develop appreciation of differential functions as representations of rates of change.

### CONTENT





#### C1.1: Average and Instantaneous Rates of Change





Students:

- describe the behaviour of a function using language such as: increasing, decreasing, steady, stationary, increasing at an increasing rate, decreasing at an increasing rate etc.  
  - interpret the meaning of the gradient of a function in a variety of contexts, for example – derive the formula for motion with uniform acceleration,  $v = u + at$ , by considering a velocity-time graph with initial speed  $u$  and final speed  $v$  (ACMMM094) **M**  
- interpret and use the difference quotient  $\frac{f(x+h)-f(x)}{h}$  as the average rate of change of a function  $f(x)$ , or the gradient of a chord or secant of the graph  $y = f(x)$  (ACMMM077, ACMMM080, ACMMM081)  
- interpret and use the derivative as the instantaneous rate of change of a function
  - understand that the derivative is a function itself (ACMMM084, ACMMM089) 

#### C1.2: The Derivative Function and its graph





Students:

- define the derivative  $f'(x)$  from first principles, as  $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$  and evaluate in simple contexts of polynomials up to and including degree 2 (ACMMM082)
  - use digital technologies or otherwise to illustrate the changing gradient of secant  $PQ$  on a curve as  $Q$  approaches  $P$   
  - examine the behaviour of the difference quotient  $\frac{f(x+h)-f(x)}{h}$  as  $h \rightarrow 0$  as an informal introduction to the concept of a limit (ACMMM081)  
  - use the notation for the derivative:  $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$  and  $\frac{dy}{dx} = f'(x)$  where  $y = f(x)$  (ACMMM083)

- interpret the derivative as the gradient of a particular tangent line of the graph of  $y = f(x)$  (ACMMM085)
  - sketch the derivative function (gradient function) for a given graph of a function without the use of algebraic techniques in a variety of contexts   
- identify families of curves with the same derivative function (ACMMM121) 

### C1.3: Calculating with Derivatives

Students:

- use the formula  $\frac{d}{dx} x^n = nx^{n-1}$  (ACMMM088)  
  - calculate derivatives of power functions to solve problems, such as finding an instantaneous rate of change of a function in both real life and abstract situations (ACMMM091, ACMMM092)
  - recognise and use the linearity properties of the derivative (ACMMM090)  
- use the derivative in a variety of contexts, including but not limited to finding the equation of a tangent or normal to a function or graph at a given point (ACMMM093)

## STRAND: STATISTICAL ANALYSIS

### OUTCOMES

**A student:**

- > uses concepts and techniques from statistics and probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions MA11-6
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9

### STRAND FOCUS

*Statistical Analysis* involves the relationships between two variables and the analysing of the distribution within a single data set.

Knowledge of statistical analysis enables careful interpretation of situations and an awareness of all the contributing factors when presented with information by third parties, including the possible misrepresentation of information.

Study of statistical analysis is important in developing students' ability to recognise, describe and apply statistical relationships in order to predict future outcomes, and an appreciation of how conclusions drawn from data can be used to inform decisions made by groups such as scientific investigators, business people and policy-makers.

### TOPICS

M-S1 Descriptive Statistics

M-S2 Probability

M-S3 Probability Distributions

# STATISTICAL ANALYSIS

## M-S1 DESCRIPTIVE STATISTICS

### OUTCOMES

**A student:**

- > uses concepts and techniques from statistics and probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions MA11-6
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9

### TOPIC FOCUS



The principal focus of this topic is to review descriptive statistics, graphical representations and groupings of data. It introduces students to cumulative frequency graphs, frequency density and quantiles.

Students develop their knowledge and skills in data description and graphical representation.

### CONTENT



#### S1.1: Classifying and Representing Data

Students:

- classify data as categorical, numerical, nominal, ordinal, discrete or continuous as appropriate (ACMGM027, ACMGM028)
- organise and display data (grouped and ungrouped) into appropriate tabular and/or tabular representations (including but not limited to dot plots, frequency tables, column graphs, stem and leaf plots, back-to-back stem-and-leaf plots and histograms and cumulative frequency graphs) (ACMEM045, ACMEM046, ACMEM057)  

#### S1.2: Univariate Data Analysis

Students:

- calculate the measures of central tendency for grouped and ungrouped data including the mean, median and mode (ACMEM049, ACMEM050)
  - investigate the suitability of measures of central tendency in real-world contexts (ACMEM051) 
- calculate measures of spread of grouped and ungrouped data including the range, interquartile range, quantiles, standard deviation and variance and understand what those measures indicate (ACMEM055, ACMGM030) **M** 
- identify outliers including the use of  $Q1 - 1.5 \times IQR$  and  $Q3 + 1.5 \times IQR$  criteria, and investigate their effect on the mean and the median (ACMEM047, ACMEM052)
- investigate real-world examples from the media illustrating inappropriate use or misuse of measures of central tendency and spread (ACMEM056) **M**
- construct and interpret a box and whisker plot or construct, interpret and compare parallel box and whisker plots (ACMEM053, ACMEM058, ACMEM059)
- describe and compare data sets in terms of shape (positive or negative skewness and symmetry)
- compare the suitability of different methods of data presentation in real-world contexts (ACMEM048) **M**

- students apply a range of techniques from within this topic to analyse a given set of data and present findings. For example—students analyse performance trends of a variety of school subjects **E**

# STATISTICAL ANALYSIS

## M-S2 PROBABILITY

### OUTCOMES

**A student:**

- > uses concepts and techniques from statistics and probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions MA11-6
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9


### TOPIC FOCUS

The principal focus of this topic is a review of fundamentals of probability and introduction of concepts of conditional probability and independence.

Students develop skills related to probability, its language and visual representations and use this to solve practical problems.

### CONTENT

Students:

- understand the difference between an outcome, an event and an experiment, use set language and notation and are able to calculate the relative frequency or probability of an event from the probabilities of the outcomes in the sample space (ACMMM049, ACMMM055) 
  - use everyday occurrences to illustrate set descriptions and representations of events and set operations (ACMMM051)
  - determine relative frequency as probability (ACMEM152)
  - use probability tree diagrams to illustrate and solve simple probability situations (ACMEM156)
- use Venn diagrams to represent or interpret simple probability situations
  - use Venn diagrams to illustrate where appropriate, including  $\bar{A}$  (or  $A^c$ ) for the complement of an event  $A$ ,  $A \cap B$  for the intersection of events  $A$  and  $B$ , and  $A \cup B$  for the union, and recognise mutually exclusive events (ACMMM050)
- use the rules:  $P(\bar{A}) = 1 - P(A)$  and  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  (ACMMM054)
- use the notation  $P(A|B)$  and the formula  $P(A \cap B) = P(A|B)P(B)$  (ACMMM057)
  - understand the notion of a conditional probability and recognise and use language that indicates conditionality (ACMMM056)
  - understand the notion of independence of an event  $A$  from an event  $B$ , as defined by  $P(A|B) = P(A)$  (ACMMM058)
- use the multiplication law  $P(A \cap B) = P(A)P(B)$  for independent events  $A$  and  $B$  and recognise the symmetry of independence in simple probability situations (ACMMM059)

# STATISTICAL ANALYSIS

## M-S3 PROBABILITY DISTRIBUTIONS

### OUTCOMES

**A student:**

- > uses concepts and techniques from statistics and probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions MA11-6
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9


### TOPIC FOCUS

This principal focus of this topic is the understanding of discrete random variables and their uses in modelling random processes involving chance and variation.


Students develop understanding and appreciation of probability distributions and associated statistical analysis methods and their use in modelling binomial events.

### CONTENT

Students:

- use discrete random variables and associated probabilities to solve practical problems (ACMMM142) **M**
  - use relative frequencies obtained from data to obtain estimates of probabilities associated with a discrete random variable (ACMMM137)
  - recognise uniform random variables and use them to model random phenomena with equally likely outcomes (ACMMM138) **M**
  - examine examples of discrete random variables, and construct probability distribution tables, recognising that for any random variable,  $X$ , the sum of the probabilities is 1 (ACMMM139)
  - calculate the mean or expectation,  $E(X)$ , of a discrete random variable,  $X$  (ACMMM140)
  - evaluate the variance,  $Var(X)$ , and standard deviation of a discrete random variable,  $X$  (ACMMM141)
- use a Bernoulli random variable as a model for two-outcome situations and use Bernoulli random variables and their associated probabilities to solve practical problems (ACMMM143, ACMMM146) **M**
  - identify contexts suitable for modelling by Bernoulli random variables (ACMMM144)
  - recognise and use the mean,  $p$ , and variance,  $p(1 - p)$ , of the Bernoulli distribution with parameter  $p$  (ACMMM145).
- use binomial distributions and their associated probabilities to solve practical problems (ACMMM150) **M** 
  - use the expansion of  $(a + b)^n$  where  $n$  is a positive integer, including the recognition and use of the notations  $\binom{n}{r}$  and  $n!$  (finding a general term is not included)
  - understand the concept of a binomial random variable as the number of 'successes' in  $n$  independent Bernoulli trials, with the same probability of success,  $p$ , in each trial (ACMMM147)
  - identify contexts suitable for modelling by binomial random variables (ACMMM148)



- apply the formulae for probabilities  $P(X = r) = {}^nC_r \times p^r(1 - p)^{n-r}$  associated with the binomial distribution with parameters  $n$  and  $p$  understanding the meaning of  ${}^nC_r$  as the number of ways in which an outcome can occur (ACMMM149) 
- recognise and use the mean,  $np$ , and variance,  $np(1 - p)$ , of a binomial distribution with parameters  $n$  and  $p$  (ACMMM149)

## STRAND: MATRICES

### OUTCOMES

**A student:**

- > performs matrix arithmetic including addition, subtraction, scalar and matrix multiplication and finding inverse matrices in a variety of contexts and to solve problems MA11-7
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9

### STRAND FOCUS

*Matrices* involves a different method for representing, storing and working with data.

Knowledge of matrices enables the concise representation of data and thus for complex problems to be modelled and solved.

Study of matrices is important in developing students' appreciation of the interconnectedness of mathematics and allows for complex situations and computation to be performed easily.

### TOPICS

M-M1 Matrix Operations

# MATRICES

## M-M1 MATRIX OPERATIONS

### OUTCOMES

#### A student:

- > performs matrix arithmetic including addition, subtraction, scalar and matrix multiplication and finding inverse matrices in a variety of contexts and to solve problems MA11-7
- > uses appropriate technology to investigate, organise and interpret information in a range of contexts MA11-8
- > provides reasoning to support conclusions which are appropriate to the context MA11-9

### TOPIC FOCUS

The principal focus of this topic is to enable students to become familiar with the algebraic structure of matrices including addition, subtraction, scalar multiplication and inverse matrices.

Students develop understanding of the structure of matrices and apply matrices in a variety of situations, accessing technology where appropriate to facilitate the solutions.

### CONTENT

#### M1.1: Matrix Arithmetic

Students:

- use the matrix definition and notation, and understand that matrices can be used for storing and displaying information that can be presented in rows and columns; such as databases or links in social or road networks (ACMGM013) **M** 🖨
- perform matrix addition, subtraction, multiplication by a scalar and matrix multiplication including determining the power of a matrix using digital technology or otherwise when appropriate (ACMGM015)
  - recognise different types of matrices (row, column, square, zero, identity) and determine their size (ACMGM014) 🖨
  - determine the power of a matrix as  $A^n = A^{n-1}A$  where  $n$  is a positive integer
  - apply matrices and matrix arithmetic to a variety of situations using contexts that are relevant to students' everyday lives using digital technology or otherwise, including but not limited to the representation of a road or social network as a matrix (ACMGM016) **M** ⚙️ 🖨

#### M1.2: Inverse Matrices

Students:

- calculate the determinant and inverse of  $2 \times 2$  matrices (ACMSM053)
  - define the multiplicative inverse of  $A$  as  $A^{-1}$  where  $AA^{-1} = A^{-1}A = I$ , and  $I$  is the identity matrix 🖨
  - determine and find the inverse of a matrix  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  as  $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$  where the number  $ad - bc$  is called the determinant of the matrix  $A$  and is written as  $\det A$  or  $|A|$  🖨
  - utilise the determinant to identify whether a matrix is invertible or non-invertible ⚙️ 🖨
- calculate the determinant of  $2 \times 2$  and  $3 \times 3$  matrices (ACMSM053)

# MATHEMATICS YEAR 12 COURSE CONTENT



consult

## STRAND: TRIGONOMETRIC FUNCTIONS

### OUTCOMES

#### A student:

- > uses detailed algebraic and graphical techniques to critically evaluate and construct arguments in a range of familiar and unfamiliar contexts MA12-1
- > uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities MA12-4
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10

### STRAND FOCUS

*Trigonometric Functions* involves the study of periodic functions in relation to geometric, algebraic, numerical and graphical representations.

Knowledge of trigonometric functions enables the solving of practical problems involving the manipulation of trigonometric expressions to prove identities and solve equations.

Study of trigonometric functions is important in developing students' understanding of the links between algebraic and graphical representations and how this can be applied to solve practical problems.

### TOPICS

M-T3 Trigonometric Identities and Proofs

# TRIGONOMETRIC FUNCTIONS

## M-T3 TRIGONOMETRIC IDENTITIES AND PROOFS

### OUTCOMES

**A student:**

- > uses detailed algebraic and graphical techniques to critically evaluate and construct arguments in a range of familiar and unfamiliar contexts MA12-1
- > uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities MA12-4
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10



### TOPIC FOCUS

The principal focus of this topic is to use trigonometric identities and reciprocal functions to simplify expressions, prove equivalences and solve equations.

Students develop their ability to communicate mathematical reasoning and justification in the proof of identities and simplification and solving of trigonometric expressions and equations.

### CONTENT

Students:

- define the reciprocal trigonometric functions,  $y = \operatorname{cosec} x$ ,  $y = \sec x$  and  $y = \cot x$ , sketch their graphs and simple transformations of those graphs (ACMSM045)
- derive and use the identities  $\cos^2 x + \sin^2 x = 1$  and  $\tan x = \frac{\sin x}{\cos x}$  (ACMSM046)
- use the identity  $\cos^2 x + \sin^2 x = 1$  to derive  $1 + \tan^2 x = \sec^2 x$  and  $\cot^2 x + 1 = \operatorname{cosec}^2 x$
- simplify trigonometric expressions, prove trigonometric equivalences and solve trigonometric equations using the above identities  

## STRAND: FUNCTIONS

### OUTCOMES

**A student:**

- > uses detailed algebraic and graphical techniques to critically evaluate and construct arguments in a range of familiar and unfamiliar contexts MA12-1
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10

### STRAND FOCUS

*Functions* involves the use of algebraic concepts and techniques to describe and interpret relationships of and between changing quantities.

Knowledge of functions enables students to discover connections between abstract algebra and its various graphical representations.

Study of functions is important in developing students' ability to find connections, communicate concisely, use algebraic techniques and manipulations to describe and solve problems, and to predict future outcomes.

### TOPICS

M-A4 Graphing Techniques

# FUNCTIONS

## M-A4 GRAPHING TECHNIQUES

### OUTCOMES

**A student:**

- > uses detailed algebraic and graphical techniques to critically evaluate and construct arguments in a range of familiar and unfamiliar contexts MA12-1
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10









### TOPIC FOCUS

The principal focus of this topic is to become more familiar with key features of graphs of functions, and understand and use the effect of basic transformations of these graphs to explain graphical behaviour.

Students develop understanding of transformational behaviour from a graphical and calculus-based approach.

### CONTENT

Students:

- apply transformations to sketch functions of the form  $y = cf(k(x + b)) + a$ , where  $f(x)$  is any function within the scope of this syllabus
  - examine translations and the graphs of  $y = f(x) + a$  and  $y = f(x + b)$  using digital technology or otherwise (ACMMM025) 
  - examine dilations and the graphs of  $y = cf(x)$  and  $y = f(kx)$  using digital technology or otherwise (ACMMM026)
- use graphical methods with supporting algebraic working to solve a variety of practical problems involving any of the functions within the scope of this syllabus, in both real life and abstract contexts **M**  
  - select and use an appropriate method to graph a given function, including calculus techniques where appropriate **M** 
  - determine asymptotes and discontinuities where appropriate 
  - determine the number of solutions of an equation by considering appropriate graphs 
  - solve an inequality by sketching appropriate graphs  
- students investigate the various factors influencing transformations in addition to behaviour around stationary points for a variety of functions to generate an understanding of graphing techniques and equations **E**

## STRAND: CALCULUS

### OUTCOMES

**A student:**

- > applies calculus techniques to solve complex problems MA12-3
- > applies appropriate differentiation methods to solve problems, including the derivative to determine the features of the graph of a functions MA12-5
- > uses techniques for the determination of indefinite and definite integrals and areas under curves MA12-6
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10

### STRAND FOCUS

*Calculus* involves two distinct processes: differentiation and integration.

Knowledge of calculus enables an understanding of the concept of a derivative or the anti-derivative as a function which can be used to gain more information about the original function.

Study of calculus is important in developing students' knowledge and understanding of differentiating and integrating a variety of functions, and the ability to apply this to the graphical behaviour of functions and calculating areas under curves.

### TOPICS

M-C2 Differential Calculus

M-C3 Integral Calculus



# CALCULUS

## M-C2 DIFFERENTIAL CALCULUS

### OUTCOMES

**A student:**

- > applies appropriate differentiation methods to solve problems, including the derivative to determine the features of the graph of a functions MA12-5
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10

### TOPIC FOCUS






The principal focus of this topic is to develop and apply rules for differentiation, introduce the second derivative, its meanings and applications to behaviour of graphs and functions, such as stationary points, concavity and extremities of the graph.

Students develop an appreciation of the interconnectedness of strands from across the syllabus and the utilisation of calculus to help solve problems.

### CONTENT



#### C2.1: Rules of Differentiation


Students:

- apply the product, quotient and chain rules to differentiate functions of the form  $f(x) \cdot g(x)$ ,  $\frac{f(x)}{g(x)}$  and  $f(g(x))$  where  $f(x)$  and  $g(x)$  are any of the functions covered in the scope of this syllabus, including cases such as the reciprocal trigonometric functions (ACMMM106) 
  - understand and use the product and quotient rules (ACMMM104)  
  - understand the notion of the composition of functions, and use the chain rule,  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ , for determining the derivatives of composite functions (ACMMM105)  

#### C2.2: Differentiation of non-power functions


















Students:

- calculate derivatives of trigonometric functions by rule (ACMMM102)
  - sketch the gradient function graph,  $y = f'(x)$ , for a given trigonometric function,  $y = f(x)$ , and hence determine the relationship between trigonometric functions and their derivatives 
- calculate the derivative of the exponential function,  $y = ke^x$ ,  $\frac{dy}{dx} = ke^x$  by rule, where  $k$  is a constant
  - using graphing software or otherwise, sketch and explore the gradient function of exponential functions and determine that there is a unique number  $e$ , between 2.7 and 2.8, for which  $\frac{d(e^x)}{dx} = e^x$  where  $e$  is Euler's number (ACMMM099, ACMMM100) 
  - use the composite function rule (chain rule) to establish that  $\frac{d(e^{f(x)})}{dx} = f'(x)e^{f(x)}$
- calculate the derivative of natural logarithm functions (ACMMM161)
  - establish and use the formula  $\frac{d}{dx} \{\ln f(x)\} = \frac{f'(x)}{f(x)}$
  - use the logarithmic laws to simplify an expression before differentiating

- calculate derivatives of exponential functions,  $y = ka^x$  by rule
  - using graphing software or otherwise, sketch and explore the gradient function graph for a given exponential function, recognise it as another exponential function and hence determine the relationship between exponential functions and their derivatives 
- use functions and their derivative to solve practical and abstract problems (ACMMM101, ACMMM103, ACMMM163)

### C2.3: The Second Derivative

Students:

- determine and interpret the concept of the second derivative as the rate of change of the first derivative function in a variety of contexts, including but not limited to acceleration as the second derivative of position with respect to time (ACMMM108, ACMMM109)   
- sketch the graph of a function using algebraic methods to determine the first and second derivatives to locate stationary points, local and global maxima and minima and points of inflection; and examine behaviour as  $x \rightarrow \infty$  and  $x \rightarrow -\infty$  (ACMMM095, ACMMM112)   
  - understand the concepts of concavity and points of inflection and their relationship with the second derivative (ACMMM110)   
  - understand the notion of the differentiability of a function at any point and its relationship to the continuity of the function  
  - understand and use the second derivative test for finding local maxima and minima (ACMMM111)  
- use derivatives to solve optimisation or other practical problems from a wide variety of fields using first and second derivatives for any of the functions covered in the scope of this syllabus (ACMMM096, ACMMM113) **M**    
- investigate derivatives and their relationships to graphs of functions and other basic properties and behaviours. They use both manual and digital technology to create functions over given domains and investigate the behaviour of tangents to the curve at stationary points introducing concepts of increasing and decreasing gradients, concavity and the change in the first derivative around stationary points **E**

# CALCULUS

## M-C3 INTEGRAL CALCULUS

### OUTCOMES

#### A student:

- > applies calculus techniques to solve complex problems MA12-3
- > uses techniques for the determination of indefinite and definite integrals and areas under curves MA12-6
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10

### TOPIC FOCUS

The principal focus of this topic is to develop and apply methods for finding the area under a curve, including the trapezoidal rule and the definite integral, for a range of functions in a variety of contexts.

Students develop their understanding of how integral calculus relates to area under curves.

### CONTENT

#### C3.1: Approximate Areas

Students:

- interpret the meaning of the 'area under a curve' between two values of the independent variable in a variety of contexts, including but not limited to deriving the formulae for motion with uniform acceleration,  $s = ut + \frac{1}{2}at^2$ , by considering the area under a velocity–time graph with initial speed  $u$  and final speed  $v$  **M** ⚙️ 🖨️
- use the trapezoidal rule to estimate areas under curves
  - using digital technology or otherwise, consider inner and outer rectangles of equal widths to approximate the area under simple curves, including those which cannot be integrated in the scope of this syllabus. e.g.  $f(x) = \ln x$  and consider the effect of increasing the number of rectangles used ⚙️ 🖨️
  - interpret the notation of the definite integral,  $\int_a^b f(x) dx$  as the area under the curve  $y = f(x)$  from  $x = a$  to  $x = b$  if  $f(x) \geq 0$  (ACMMM125)
  - approximate the area under a curve with a trapezium to develop the trapezoidal rule:  $\int_a^b f(x) dx = \frac{b-a}{2} [f(a) + f(b)]$  ⚙️
  - divide the area under a curve into a given number of trapezia with equal widths to develop the rule:  $\int_a^b f(x) dx = \frac{b-a}{2n} [f(a) + 2\{f(a + \frac{b-a}{n}) + f(a + \frac{2(b-a)}{n}) + \dots + f(a + \frac{(n-1)(b-a)}{n})\} + f(b)]$  where  $n$  is the number of trapezia ⚙️
- use geometrical calculations and/or approximate methods to solve practical problems involving the area under a curve **M** ⚙️ 🖨️

### C3.2: The Anti-Derivative

Students:

- determine  $f(x)$ , given  $f'(x)$  and an initial condition  $f(a) = b$  (ACMMM122)
  - recognise anti-differentiation as the reverse of differentiation, and understand the linearity of anti-differentiation (ACMMM114, ACMMM119)
  - establish and use the formula  $\int ax^n dx = \frac{a}{n+1} x^{n+1} + c$  where  $n \neq -1$  (ACMMM116)
  - establish and use the formula  $\int f'(x)[f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + C$  where  $n \neq -1$  (the reverse chain rule) (ACMSM117)
  - establish and use the formulae for the anti-derivatives of  $\sin(ax + b)$ ,  $\cos(ax + b)$  and  $\sec^2(ax + b)$  (ACMMM118)
  - establish and use the formulae  $\int e^x dx = e^x + C$  and  $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$  (ACMMM117)
  - establish and use the formulae  $\int \frac{1}{x} dx = \ln x + C$  for  $x > 0$  and  $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$  (ACMMM162, ACMSM118)
  - establish and use the formulae  $\frac{d(a^x)}{dx} = \ln a \cdot a^x$  and  $\int a^x dx = \frac{a^x}{\ln a} + C$

### C3.3: The Definite Integral

Students:

- Use the formula  $\int_a^b f(x) dx = F(b) - F(a)$ , where  $F(x)$  is the anti-derivative of  $f(x)$ , to calculate definite integrals (ACMMM131)
  - using digital technology or otherwise, establish the link between the anti-derivative and the area under a curve
  - interpret  $\int_a^b f(x) dx$  as a sum of signed areas (ACMMM127)
  - recognise and use the additivity and linearity of definite integrals. (ACMMM128)
- calculate areas between curves determined by any functions within the scope of this syllabus (ACMMM134)
  - use a definite integral to evaluate an area bounded by a function, the  $x$ -axis, and two vertical lines (including a logarithmic function by subtracting the area to the  $y$ -axis from the surrounding rectangle) (ACMMM132)
  - use a definite integral to evaluate an area bounded by a function, the  $y$ -axis, and two horizontal lines (including a logarithmic function by using the inverse relationship between logarithmic and exponential functions) (ACMMM132)
- calculate anti-derivatives or definite integrals and apply them to solving practical problems (ACMMM097, ACMMM135) **M**
- students investigate a variety of methods to find an area under a curve such as the trapezoidal rule and the Monte Carlo method **E**

## STRAND: FINANCIAL MATHEMATICS

### OUTCOMES

**A student:**

- > makes informed decisions about financial situations using mathematical reasoning and techniques MA12-2
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10

### STRAND FOCUS

*Financial Mathematics* involves sequences and series and their application to financial situations.

Knowledge of financial mathematics enables analysis of different financial situations and the calculation of the best options for the circumstances, as well as solving financial problems.

Study of financial mathematics is important in developing students' ability to make informed financial decisions, to be aware of the consequences of such decisions, and to manage personal financial resources effectively.

### TOPICS

M-F1 Annuities

# FINANCIAL MATHEMATICS

## M-F1 ANNUITIES

### OUTCOMES

**A student:**

- > makes informed decisions about financial situations using mathematical reasoning and techniques MA12-2
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10

### TOPIC FOCUS

The principal focus of this topic is the nature and mathematics of annuities, the processes by which they accrue, and ways of maximising their value as an investment.

Students develop appreciation for the use of annuities in their lives, for example superannuation and home loans.

### CONTENT

Students:

- use geometric sequences to model and analyse practical problems involving exponential growth and decay (ACMGM074) **M** ⚙️
  - solve problems involving compound interest loans or investments; for example, determining the future value of a loan, the number of compounding periods for an investment to exceed a given value and/or the interest rate needed for an investment to exceed a given value (ACMGM096) **M** ⚙️ 🖨️
  - recognise a reducing balance loan as a compound interest loan with periodic repayments, and solve problems such as the amount owing on a reducing balance loan after each payment is made (ACMGM077) **M** 🖨️ 🧮
- solve problems involving financial decisions such as a home loan, a savings account, a car loan or superannuation **M** ⚙️ 🖨️ 🧮
  - identify an annuity as an investment account with regular, equal contributions and interest compounding at the end of each period, or a single sum investment from which regular, equal withdrawals are made 🧮
  - using digital technology or otherwise, model an annuity as a recurrence relation, and investigate (numerically or graphically) the effect of varying the amount and frequency of each contribution and the interest rate or payment amount on the duration and/or future value of the annuity (ACMGM099) **M** 🖨️
  - calculate the future value of an annuity by developing an expression for the sum of the calculated compounded values of each contribution and using the formula for the sum of the first  $n$  terms of a geometric series 🖨️ 🧮
  - solve problems involving annuities (including perpetuities as a special case); for example, determining the amount to be invested in an annuity to provide a regular monthly income of a certain amount (ACMGM100)

- students are given a personal context surrounding their income and capacity to put a deposit on a house and investigate the effects of changing the size of repayments, time, etc. from the perspective of reducing balance loans **E**

## STRAND: STATISTICAL ANALYSIS

### OUTCOMES

**A student:**

- > solves problems using appropriate statistical processes, including the use of the normal distribution and the correlation of bivariate data MA12-7
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10

### STRAND FOCUS

*Statistical Analysis* involves the relationships between two variables and analysis of correlation in bivariate data sets.

Knowledge of statistical analysis enables careful interpretation of situations and an awareness of the contributing factors when presented with information by third parties, including the possible misrepresentation of information.

Study of statistical analysis is important in developing students' ability to recognise, describe and apply statistical relationships in order to predict future outcomes, and an appreciation of how conclusions drawn from data can be used to inform decisions made by groups such as scientific investigators, business people and policy-makers.

### TOPICS

M-S4 Bivariate Data Analysis  
M-S5 The Normal Distribution



# STATISTICAL ANALYSIS

## M-S4 BIVARIATE DATA ANALYSIS

### OUTCOMES

#### A student:

- > solves problems using appropriate statistical processes, including the use of the normal distribution and the correlation of bivariate data MA12-7
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10

























### TOPIC FOCUS

The principal focus of this topic is to introduce students to some methods for identifying, analysing and describing associations between pairs of variables.

Students develop the ability to display, interpret and analyse statistical relationships.

### CONTENT

Students:

- describe an association in terms of differences observed in percentages across categories in a systematic and concise manner and interpret this in the context of the data (ACMGM051)
  - construct contingency tables, two-way frequency tables and calculate the row and column sums and percentages (ACMGM049)
  - use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association (ACMGM050)
- construct, interpret and analyse scatterplots for bivariate data in practical contexts, while demonstrating awareness of issues of privacy and bias, ethics, and responsiveness to diverse groups and cultures (ACMEM133, ACMGM052) 

# STATISTICAL ANALYSIS

## M-S5 THE NORMAL DISTRIBUTION

### OUTCOMES

#### A student:

- > solves problems using appropriate statistical processes, including the use of the normal distribution and the correlation of bivariate data MA12-7
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10








### TOPIC FOCUS

The principal focus of this topic is to introduce students to normal distribution and its use in a variety of contexts.

Students develop understanding of the probability density function, its application to integration and the normal distribution and use the normal distribution to produce measures of spread and location to analyse given datasets.

### CONTENT

Students:

- use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable (ACMMM164)
- understand and use the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals (ACMMM165)  

  - examine simple types of continuous random variables and use them in appropriate contexts **M** 
- recognise the mean or expected value, median, variance and standard deviation of a continuous random variable and evaluate them in simple cases (ACMMM166) **M** 
  - understand the effects of linear changes of scale and origin on the mean and the standard deviation (ACMMM167) 
- using digital technology or otherwise, calculate probabilities and quantiles associated with a given normal distribution, and use these to solve practical problems (ACMMM170) **M**  
  - identify contexts such as naturally occurring variations that are suitable for modelling by normal random variables (ACMMM168)
  - recognise features of the graph of the probability density function of the normal distribution with mean  $\mu$  and standard deviation  $\sigma$ , and the use of the standard normal distribution (ACMMM169)
- determine the z-score (standardised score) corresponding to a particular score in a set of scores as  $z = \frac{x - \bar{x}}{s}$  where  $s$  is the standard deviation, and use calculated z-scores to compare scores from different data sets 
- solve problems using the normal approximation, with a continuity correction
  - use simple conditions to test whether a binomial distribution can be approximated reasonably by a normal distribution (ie if  $X \sim B(n, p)$ , and if  $np > 5$  and  $n(1 - p) > 5$ )

## STRAND: MATRICES

### OUTCOMES

**A student:**

- > solves matrix equations and uses matrices to model and solve practical problems MA12-8
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10

### STRAND FOCUS

*Matrices* involves a different method for representing, storing and working with data.

Knowledge of matrices enables concise representation of data and complex problems to be modelled and solved.

Study of matrices is important in developing students' appreciation of the interconnectedness of mathematics and allows for complex situations and computation to be performed easily.

### TOPICS

M-M2 Matrix Applications

# MATRICES

## M-M2 MATRIX APPLICATIONS

### OUTCOMES

#### A student:

- > solves matrix equations and uses matrices to model and solve practical problems MA12-8
- > chooses and uses appropriate technology effectively in a range of contexts, and applies critical thinking to recognise appropriate times for such use MA12-9
- > constructs arguments to prove and justify results MA12-10

### TOPIC FOCUS

The principal focus of this topic is to apply matrices in a variety of situations to facilitate the solution of problems.

Students develop appreciation of alternate problem-solving methods such as finding simultaneous solutions to equations.

### CONTENT

#### M2.1: Solving Matrix Equations

Students:

- understand matrix definition and notation and use appropriate matrix arithmetic (ACMSM051, ACMSM052)
- solve matrix equations of the form  $AX = B$ , where  $A$  is a  $2 \times 2$  matrix and  $X$  and  $B$  are column vectors (ACMSM053) 🗨️
- use matrices to solve linear simultaneous equations  $ax + by = e$  and  $cx + dy = f$  (where  $a, b, c, d, e, f$  are constants) by expressing as the matrix equation  $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$  which is of the form  $AX = B$  and then solving that equation to find  $x$  and  $y$  🗨️
- use Cramer's rule to solve one variable from a system of equations without needing to solve for all variables
  - understand and use subscript notation, such as  $n_1, n_2, n_k, n_{\{k+1\}}, a_{21}$ , etc.

#### M2.2: Other Matrix Applications

Students:

- use matrices, including matrix products and powers of matrices, to model and solve problems; for example, costing or pricing problems or squaring a matrix to determine the number of ways pairs of people in a communication network can communicate with each other via a third person (ACMGM016) ⚙️
- use a Leslie matrix to predict populations broken down into age groups, using the probability of survival and birth rates of subsequent generations **M**
  - use a matrix as data storage and explore applications of matrices in both life-related and purely mathematical situations **M** ⚙️
- students investigate more complex situations involving the use of Markov chains or Leontief matrices **E**

# GLOSSARY



for your information

The glossary explains terms that will assist teachers in the interpretation of the subject. The glossary will be based on the NSW Mathematics K–10 glossary and the Australian curriculum senior secondary years Mathematics Methods glossary.



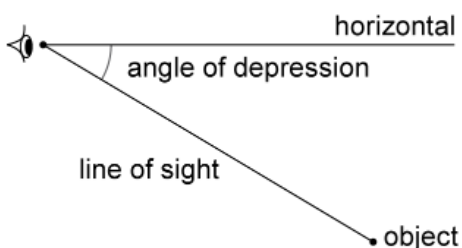
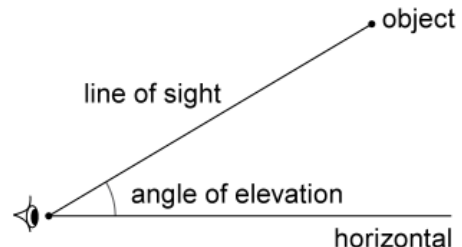
consult

\* Indicates new glossary terms

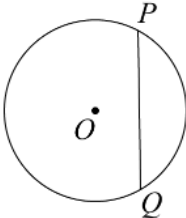
\*\* Indicates updated glossary terms

\*\*\* Indicated glossary terms also in Mathematics General

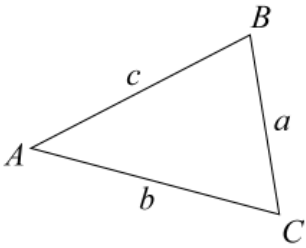
Glossary term	Definition
<b>absolute value*</b>	The absolute value or modulus $ x $ of a real number $x$ is the non-negative value of $x$ without regard to its sign. I.e. $ x  = x$ if $x \geq 0$ , $ x  = -x$ if $x < 0$ .
<b>acceleration*</b>	Acceleration is the rate of change of velocity with respect to time. It is measured in terms of the length units and time units used for the velocity. For example, $\text{km/h}^2$ or $\text{m/s}^2$ .
<b>ambiguous case***</b>	The ambiguous case refers to using the sine rule to find an angle in the case where there are two possibilities for the angle: one obtuse and one acute.
<b>amplitude*</b>	For functions that are periodic waves, such as $y = \sin x$ and $y = \cos x$ , the amplitude is the distance from the centre (of motion) to the peak or trough (maximum or minimum value). For example, the functions $y = A \sin x$ and $y = A \cos x$ have amplitude $A$ units.

Glossary term	Definition
<b>angles of elevation and depression</b>	<p>When an observer looks at an object that is lower than 'the eye of the observer', the angle between the line of sight and the horizontal is called the angle of depression.</p>  <p>When an observer looks at an object that is higher than 'the eye of the observer', the angle between the line of sight and the horizontal is called the angle of elevation.</p> 
<b>annuity***</b>	A specified income payable at stated intervals for a fixed or a contingent period.
<b>anti-differentiation*</b>	<p>An anti-derivative, primitive or indefinite integral of a function <math>f(x)</math> is a function <math>F(x)</math> whose derivative is <math>f(x)</math>, i.e. <math>F'(x) = f(x)</math>.</p> <p>The process of solving for anti-derivatives is called anti-differentiation.</p> <p>Anti-derivatives are not unique. If <math>F(x)</math> is an anti-derivative of <math>f(x)</math>, then so too is the function <math>F(x) + c</math> where <math>c</math> is any number. We write <math>\int f(x)dx = F(x) + c</math> to denote the set of all anti-derivatives of <math>f(x)</math>. The number <math>c</math> is called the constant of integration. For example, since <math>\frac{d}{dx}(x^3) = 3x^2</math>, we can write <math>\int 3x^2 dx = x^3 + c</math></p>
<b>arithmetic sequence*</b>	An arithmetic sequence is a sequence of numbers such that the difference of any two successive members of the sequence is a constant. For instance, the sequence 2, 5, 8, 11, 14, 17, ... is an arithmetic sequence with common difference 3.
<b>arithmetic series*</b>	An arithmetic series is a sum of the terms of an arithmetic sequence.
<b>asymptote*</b>	<p>A line is an asymptote to a curve if the distance between the line and the curve approaches zero as they tend to infinity.</p> <p>For example, the line with equation <math>x = \frac{\pi}{2}</math> is a vertical asymptote to the graph of <math>y = \tan x</math>, and the line with equation <math>y = 0</math> is a horizontal asymptote to the graph of <math>y = \frac{1}{x}</math>.</p>

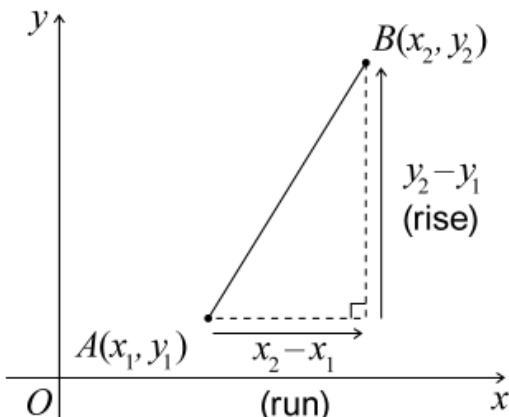
Glossary term	Definition
<b>average rate of change*</b>	An average rate of change is the change in the value of a quantity divided by the elapsed time over which the average is being calculated.
<b>bearing***</b>	A direction from one point on Earth's surface to another, measured in degrees, as a deviation from north in a clockwise direction.
<b>Bernoulli distribution*</b>	The Bernoulli distribution is the probability distribution of a random variable which takes the value 1 with probability of $p$ and the value 0 with probability $q = 1 - p$ . The Bernoulli distribution is a special case of the binomial distribution, where $n = 1$ .
<b>Bernoulli random variable*</b>	A Bernoulli random variable has two possible values, namely 0 and 1. The parameter associated with such a random variable is the probability $p$ of obtaining a 1.
<b>Bernoulli trial*</b>	A Bernoulli trial is a chance experiment with possible outcomes, typically labeled 'success' and 'failure'.
<b>binomial distribution*</b>	The binomial distribution with parameters $n$ and $p$ is the discrete probability distribution of the number of successes in a sequence of $n$ independent yes/no experiments, each of which yields success with probability $p$ .
<b>binomial random variable*</b>	A binomial random variable $X$ represents the number of successes in $n$ trials with two possible outcomes, called success and failure. The probability of success is $p$ and the probability of failure is $q = 1 - p$ .
<b>box and whisker plot***</b>	Also referred to as a box plot, a box and whisker plot is a graphical display of a five-number summary.  In a box and whisker plot, the 'box' covers the interquartile range (IQR), with 'whiskers' reaching out from each end of the box to indicate maximum and minimum values in the data set. A vertical line in the box is used to indicate the location of the median.
<b>break-even point*</b>	Break-even (or break even) is the point of balance where a business is making neither a profit nor a loss.
<b>categorical data***</b>	A categorical variable is a variable whose values are categories.  Examples: <i>blood group</i> is a categorical variable; its values are: A, B, AB or O. So too is <i>construction type</i> of a house; its values might be brick, concrete, timber or steel.  Categories may have numerical labels, for example, for the variable <i>postcode</i> the category labels would be numbers like 3787, 5623, 2016, etc, but these labels have no numerical significance. For example, it makes no sense to use these numerical labels to calculate the average postcode in Australia.  Categorical data is sometimes also called qualitative data.

Glossary term	Definition
<b>centre of motion (of trigonometric graph)</b>	The mean value is the value that represents the centre of the wave of the graph. For example, the mean value of $y = \cos x + k$ is $k$ .
<b>chain rule*</b>	<p>The chain rule relates the derivative of the composite of two functions to the functions and their derivatives.</p> <p>If <math>h(x) = f(g(x))</math> then <math>h'(x) = f'(g(x))g'(x)</math></p> <p>In other notation, if <math>h(x) = (f \circ g)(x)</math> then <math>(f \circ g)'(x) = (f' \circ g)(x)g'(x)</math></p> <p>Or <math>\frac{du}{dx} = \frac{du}{dy} \times \frac{dy}{dx}</math></p>
<b>chord</b>	<p>A chord is a line segment (interval) joining two points on a circle.</p> 
<b>coefficient of determination*</b>	The coefficient of determination is $r^2$ , where $r$ is Pearson's Correlation coefficient. It is a number that indicates the proportion of the variance in the dependent variable that is predictable from the independent variable.
<b>compass bearing***</b>	Compass bearings are specified as angles either side of north or south. For example, a compass bearing of N50° E is found by facing north and moving through an angle of 50° to the East.
<b>complement*</b>	The complement of an event is when the event does <u>not</u> occur. For example, if $A$ is the event of throwing a 5 on a dice, then the complement of $A$ , denoted by $\bar{A}$ or $A^c$ , is <u>not</u> throwing a 5 on a dice.
<b>composition of functions*</b>	<p>When two functions are composed, they are performed one after the other.</p> <p>For example, the composite of <math>f</math> and <math>g</math>, acting on <math>x</math>, is written as <math>(f \circ g)(x)</math> and means <math>f(g(x))</math>, with <math>g(x)</math> being performed first.</p> <p>If <math>y = g(x)</math> and <math>z = f(y)</math> for functions <math>f</math> and <math>g</math>, then <math>z</math> is a composite function of <math>x</math>. We write <math>z = (f \circ g)(x) = f(g(x))</math>. For example, <math>z = \sqrt{x^2 + 3}</math> expresses <math>z</math> as a composite of the functions <math>f(y) = \sqrt{y}</math> and <math>g(x) = x^2 + 3</math>.</p>
<b>compound interest (and formula)</b>	<p>The interest earned by investing a sum of money (the principal) is compound interest if each successive interest payment is added to the principal for the purpose of calculating the next interest payment.</p> <p>For example, if the principal <math>\\$P</math> earns compound interest at the rate of <math>r</math> per period, then after <math>n</math> periods the principal plus interest is <math>\\$P(1 + r)^n</math>.</p>

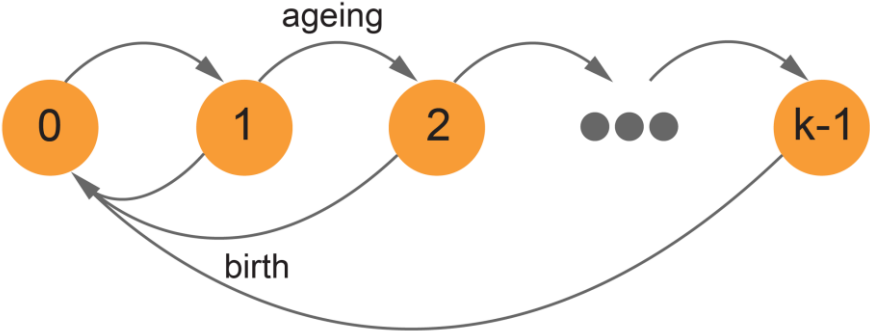


Glossary term	Definition
<b>concavity*</b>	<p>A function can be described as concave up or concave down.</p> <p>If it is concave up, then the second derivative is positive, and the rate of change (slope) of the graph is increasing.</p> <p>If it is concave down, then the second derivative is negative and the rate of change (slope) of the graph is decreasing.</p> <p>The concavity of a graph changes at a point of inflection.</p>
<b>conditional probability*</b>	<p>The probability that an event <math>A</math> occurs can change if it becomes known that another event <math>B</math> occurs. The new probability is known as conditional probability and is written as <math>P(A B)</math>. If <math>B</math> has occurred, the sample space is reduced by discarding all outcomes that are not in the event <math>B</math>.</p> <p>The conditional probability of event <math>A</math> is given by <math>P(A B) = \frac{P(A \cap B)}{P(B)}</math>.</p>
<b>continuity*</b>	A function is continuous when its graph is a single unbroken curve.
<b>continuity correction*</b>	A continuity correction is an adjustment that is made when a discrete distribution is approximated by a continuous distribution.
<b>continuous variable</b>	A continuous variable is a numerical variable that can take any value that lies within an interval. Examples include height, reaction time to a stimulus and systolic blood pressure.
<b>cosine rule</b>	<p>In any triangle <math>ABC</math>,</p> $c^2 = a^2 + b^2 - 2ab \cos C.$ 
<b>Cramer's rule*</b>	Cramer's Rule is used to solve for just one of the variables without having to solve a whole system of equations.
<b>cubic equation*</b>	The general cubic equation for one variable is $ax^3 + bx^2 + cx + d = 0, a \neq 0$ .
<b>cumulative frequency*</b>	The total of a frequency and all frequencies so far in a frequency distribution.
<b>decile*</b>	A decile is any of the nine values that divide the sorted data into ten equal parts, so that each part represents 1/10 of the sample or population.
<b>degree of a polynomial*</b>	The degree of a polynomial $p(x)$ is the highest power of $x$ occurring in the polynomial.

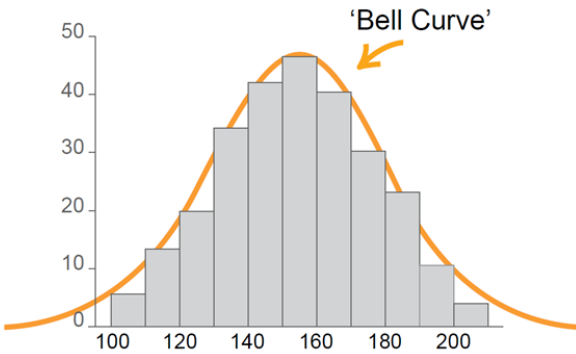
Glossary term	Definition
<b>derivative*</b>	The derivative $\frac{dy}{dx} = f'(x)$ of a function $f(x)$ is defined as $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
<b>dependent and independent variable*</b>	An independent variable stands alone and isn't affected by the other variables you are considering. For example, someone's age might be an independent variable. A dependent variable is something that depends on other factors.
<b>differentiability*</b>	A function is differentiable wherever its gradient is defined.
<b>dilations*</b>	A dilation makes an object (or the graph of a function) enlarge or reduce in size. This could happen either in the $x$ or $y$ direction, or both.
<b>direct variation/proportion***</b>	One variable "varies directly" as the other.  The formula is $y = kx$ where $k$ is the constant of variation. It is a linear formula and produces a linear graph.
<b>discrete variable***</b>	A discrete variable is a variable (numerical or otherwise) that has a finite number of possible values.
<b>discriminant*</b>	The discriminant of a quadratic expression identifies the number of roots of $ax^2 + bx + c$ and is calculated using $b^2 - 4ac$ .
<b>domain*</b>	the domain of a function is the set of "input" or argument values for which the function is defined.
<b>Euler's number*</b>	The number $e$ is an important mathematical constant that is the base of the natural logarithm. It is approximately equal to 2.71828.
<b>event</b>	An event is a subset of the sample space for a random experiment.
<b>expectation</b>	The expectation, or expected value of a random variable, is the average value.
<b>exponential function***</b>	An exponential function is a function in which the independent variable occurs as an exponent (power/index).
<b>exponential graph***</b>	An exponential graph is a graph drawn to represent an exponential function.
<b>exponential growth and decay*</b>	Exponential growth occurs when the growth rate of a mathematical function is proportional to the function's current value, resulting in its growth with time being an exponential function. Exponential decay occurs in the same way when the growth rate is negative.
<b>exponential model***</b>	Using a model that involves fitting an exponential graph to the situation.

Glossary term	Definition
<b>function</b>	<p>A function <math>f</math> assigns to each element of one set <math>S</math> precisely one element of a second set <math>T</math>.</p> <p>The functions most commonly encountered in elementary mathematics are real functions of real variables. For such functions, the domain and codomain are sets of real numbers.</p> <p>Functions are usually defined by a formula for <math>f(x)</math> in terms of <math>x</math>.</p> <p>For example, the formula <math>f(x) = x^2</math> defines the 'squaring function' that maps each real number <math>x</math> to its square <math>x^2</math>.</p>
<b>geometric sequence*</b>	A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed number called the common ratio.
<b>geometric series*</b>	A geometric series is a sum of the terms of a geometric sequence.
<b>gradient</b>	<p>If <math>A(x_1, y_1)</math> and <math>B(x_2, y_2)</math> are points in the plane, <math>x_2 - x_1 \neq 0</math>, the gradient of the line segment (interval) <math>AB = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}</math>.</p>  <p>The gradient of a line is the gradient of any line segment (interval) within the line.</p>
<b>Heron's rule</b>	<p>Heron's rule is a rule for determining the area of a triangle given the lengths of its sides.</p> <p>The area of a triangle with sides <math>a, b, c</math> is given by</p> $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{a+b+c}{2}$
<b>horizontal line test*</b>	If a horizontal line intersects a functions' graph more than once at any point, then the function is not one-to-one.

Glossary term	Definition
<b>identity matrix (multiplicative identity matrix)*</b>	<p>A (multiplicative) identity matrix is a square matrix in which all the elements in the leading diagonal are 1s and the remaining elements are 0s. Identity matrices are designated by the letter <math>I</math>.</p> <p><math>\begin{bmatrix} 1 &amp; 0 \\ 0 &amp; 1 \end{bmatrix}</math> and <math>\begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \\ 0 &amp; 1 &amp; 0 &amp; 0 \\ 0 &amp; 0 &amp; 1 &amp; 0 \\ 0 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix}</math> are both identity matrices.</p> <p>There is an identity matrix for each order of square matrix. When clarity is needed, the order is written with a subscript: <math>I_n</math></p>
<b>independent event</b>	Two events are independent if knowing the outcome of one event tells us nothing about the outcome of the other event.
<b>interquartile range (IQR)</b>	The interquartile range ( $IQR$ ) is a measure of the spread within a numerical data set. It is equal to the upper quartile ( $Q_3$ ) minus the lower quartile ( $Q_1$ ); that is, $IQR = Q_3 - Q_1$ .
<b>interval notation</b>	Interval Notation is a notation for representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or brackets are used to show whether the endpoints are excluded or included. For example, $[3, 8)$ is the interval of real numbers between 3 and 8, including 3 and excluding 8.
<b>instantaneous rate of change*</b>	The instantaneous rate of change is the rate of change at a particular moment. For a function, the instantaneous rate of change at a point is the same as the slope of the tangent line at that point.
<b>inverse of a matrix</b>	<p>The inverse (also called multiplicative inverse) of a square matrix <math>A</math> is written as <math>A^{-1}</math> and has the property that <math>AA^{-1} = A^{-1}A = I</math></p> <p>Not every square matrix has an inverse. A matrix that has an inverse is said to be invertible.</p> <p>The inverse of the matrix <math>A = \begin{bmatrix} a &amp; b \\ c &amp; d \end{bmatrix}</math> is <math>A^{-1} = \frac{1}{\det A} \begin{bmatrix} d &amp; -b \\ -c &amp; a \end{bmatrix}</math>, when <math>\det A \neq 0</math></p>
<b>inverse variation/ proportion/ relationship***</b>	As one variable increases, the other decreases in proportion. The formula is $y = \frac{k}{x}$ where $k$ is the constant of variation.

Glossary term	Definition
<b>Leslie matrix*</b>	<p>The Leslie Matrix is used to calculate the predicted population broken down into age groups, from the current population broken down into age groups, based on generational birth and survival rates.</p> <p>The equation for one time step of the model as</p> $\begin{bmatrix} n_{t+1}(0) \\ n_{t+1}(1) \\ . \\ . \\ n_{t+1}(k-1) \end{bmatrix} = \begin{bmatrix} f_0 & f_1 & f_2 & f_3 & \dots & f_{k-1} \\ s_0 & 0 & 0 & 0 & \dots & 0 \\ 0 & s_1 & 0 & 0 & \dots & 0 \\ 0 & 0 & s_2 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & \dots & 0 \\ 0 & 0 & 0 & \dots & s_{k-2} & 0 \end{bmatrix} \begin{bmatrix} n_t(0) \\ n_t(1) \\ . \\ . \\ n_t(k-1) \end{bmatrix}$ <p>or more succinctly as <math>n_{t+1} = Ln_t</math> where <math>L</math> is called the Leslie Matrix.</p> <p>The relationships can be illustrated by the diagram below.</p> <p style="text-align: center;"><b>Age Classes</b></p>  <p>Age specific survival rate governs ageing, from class <math>i</math> to <math>i + 1</math>. Age specific fecundity (per capita birth rate) governs births, but all births start in age category 0.</p>
<b>limit*</b>	<p>A limit is the value that a function approaches as the input or independent variable approaches some value.</p> <p>The limit of a sequence is the value a term approaches as the term number increases or approaches some value.</p>
<b>limiting sum*</b>	The limiting sum of a geometric series, is the sum of the terms in the geometric sequence as it approaches its limit (the number of terms approaches infinity).
<b>linear formula or function*** or relationship*</b>	A formula or function defines one variable in terms of other variables or constants. A linear formula or function will only involve linear terms, that is, polynomials of degree 1.

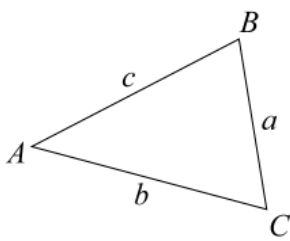
Glossary term	Definition
<b>linear graph***</b>	When graphed, a linear function forms a straight line, or linear graph. It can be expressed in the form $y=mx+c$ where $y$ and $x$ are the two variables, $m$ is the gradient, and $c$ is the intercept with the vertical axis.
<b>linear model***</b>	Using a model that involves fitting a straight line graph to the situation.
<b>local and global maximum and minimum*</b>	<p>We say that <math>f(x_0)</math> is a local maximum of the function <math>f(x)</math> if <math>f(x) \leq f(x_0)</math> for all values of <math>x</math> near <math>x_0</math>. We say that <math>f(x_0)</math> is a global maximum of the function <math>f(x)</math> if <math>f(x) \leq f(x_0)</math> for all values of <math>x</math> in the domain of <math>f</math>.</p> <p>We say that <math>f(x_0)</math> is a local minimum of the function <math>f(x)</math> if <math>f(x) \geq f(x_0)</math> for all values of <math>x</math> near <math>x_0</math>. We say that <math>f(x_0)</math> is a global minimum of the function <math>f(x)</math> if <math>f(x) \geq f(x_0)</math> for all values of <math>x</math> in the domain of <math>f</math>.</p>
<b>logarithm</b>	<p>The logarithm of a positive number <math>x</math> is the power to which a given number <math>b</math>, called the base, must be raised in order to produce the number <math>x</math>. The logarithm of <math>x</math>, to the base <math>b</math> is denoted by <math>\log_b x</math>.</p> <p>Algebraically: <math>\log_b x = y \leftrightarrow b^y = x</math>.</p>
<b>logarithmic function*</b>	A logarithmic function defined by $y = \log_b x$ , including when the base, $b$ , is equal to $e$ , the base of the natural logarithm.
<b>matrix (matrices)*</b>	<p>A matrix is a rectangular array of elements or entries displayed in rows and columns.</p> <p>For example,</p> $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \text{ are both matrices.}$ <p>Matrix <math>A</math> is said to be a <math>3 \times 2</math> matrix (three rows and two columns) while <math>B</math> is said to be a <math>2 \times 3</math> matrix (two rows and three columns).</p>
<b>matrix multiplication*</b>	Matrix multiplication is the process of multiplying a matrix by another matrix. The product $AB$ of two matrices $A$ and $B$ with dimensions $m \times n$ and $p \times q$ is defined only if $n = p$ . If it is defined, the product $AB$ is an $m \times q$ matrix.
<b>mean</b>	The arithmetic mean of a list of numbers is the sum of the data values divided by the number of numbers in the list. In everyday language, the arithmetic mean is commonly called the average.
<b>measures of central tendency***</b>	Measures of location summarise a list of numbers by a "typical" value. The three most common measures of location are the mean, the median, and the mode.
<b>measures of spread***</b>	Measures of spread describe how similar or varied the set of observed values are for a particular variable. Measures of spread include the range, quartiles and the interquartile range, variance and standard deviation.

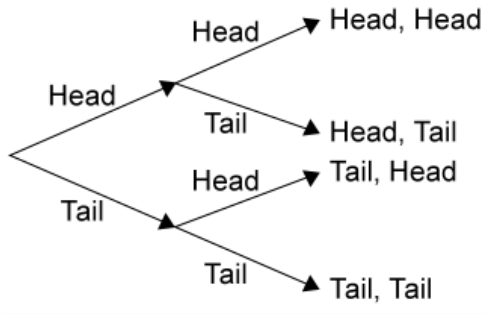
Glossary term	Definition
<b>median</b>	<p>The median is the value in a set of ordered data that divides the data into two parts. It is frequently called the 'middle value'. Where the number of observations is odd, the median is the middle value. Where the number of observations is even, the median is calculated as the mean of the two central values.</p> <p>The median provides a measure of location of a data set that is suitable for both symmetric and skewed distributions and is also relatively insensitive to outliers.</p>
<b>mode***</b>	<p>The mode is the most frequently occurring value in a set of data. There can be more than one mode. When a dataset has one mode, it is called unimodal, and when it has more than one, it is called multi-modal. This way, the modality of the dataset can be discussed.</p>
<b>nominal data*</b>	<p>Data that is listed by name (categorical) in which the order of the categories does not matter.</p>
<b>non-invertible matrix</b>	<p>A matrix is non-invertible if <math>\det A = 0</math>. A non-invertible matrix does not have a multiplicative inverse.</p>
<b>normal*</b>	<p>The normal line (or 'the normal') to a curve at a given point <math>P</math> is the line that is perpendicular to the tangent line at that point.</p>
<b>normally distributed*</b>	<p>In cases where data tends to be around a central value with no bias left or right, it is said to have a "Normal Distribution" like this:</p>  <p>The Bell Curve is a Normal Distribution. The Normal Distribution has:</p> <ul style="list-style-type: none"> <li>• mean = median = mode</li> <li>• symmetry about the centre</li> <li>• 50% of values less than the mean and 50% greater than the mean</li> </ul>
<b>normal random variable*</b>	<p>A variable which varies according to the Normal Distribution and produces a bell curve, is said to be a normal random variable.</p>

Glossary term	Definition
<b>numerical data***</b>	<p>Numerical data is data associated with a numerical variable.</p> <p>Numerical variables are variables whose values are numbers, and for which arithmetic processes such as adding and subtracting, or calculating an average, make sense.</p> <p>A discrete numerical variable is a numerical variable, each of whose possible values is separated from the next by a definite 'gap'. The most common numerical variables have the counting numbers 0, 1, 2, 3, ... as possible values. Others are prices, measured in dollars and cents.</p> <p>Examples include the number of children in a family or the number of days in a month.</p>
<b>ordinal data*</b>	Data that is named, rather than numbered (categorical), but in which the order of the categories matters. For example, very happy, happy, neutral, unhappy, very unhappy.
<b>outcome*</b>	An outcome is a possible result of an experiment
<b>outlier</b>	An outlier is a data value that appears to stand out from the other members of the data set by being unusually high or low.
<b>Pearson's correlation coefficient***</b>	Also referred to as a correlation coefficient, or a coefficient of determination, Pearson's correlation coefficient is a measure of the linear relationship between two variables in a sample and is used as an estimate of the correlation in the whole population.
<b>percentile*</b>	A percentile (or a centile) is a measure used in statistics indicating the value below which a given percentage of observations in a group of observations fall. For example, the 20th percentile is the value (or score) below which 20% of the observations may be found
<b>period*</b>	A function that repeats on intervals, is said to be periodic, and the length of the interval is called the period.
<b>phase*</b>	<p>The phase is the amount by which the zero value has been shifted horizontally.</p> <p>For example, the phase of <math>y = \sin(x - c)</math> is <math>c</math>.</p>
<b>point of inflection*</b>	A point of inflection is a point on a curve at which the curve changes from being concave down to concave up, or vice versa.
<b>polynomial functions*</b>	A polynomial function is a function in one or more terms where the independent variable has only non-negative integer indices and there is a finite number of terms.
<b>probability density function*</b>	A probability density function (PDF), is a function that describes the relative likelihood for this random variable to take on a given value.



Glossary term	Definition
<b>product rule*</b>	<p>The product rule relates to the derivative of the product of two functions to the functions and their derivatives.</p> <p>If <math>h(x) = f(x)g(x)</math> then <math>h'(x) = f(x)g'(x) + f'(x)g(x)</math>,</p> <p>Or in other notation, <math>\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}</math></p>
<b>quadratic equations</b>	A quadratic equation is a polynomial equation of degree 2.
<b>quadratic function** or relationship*</b>	A quadratic expression or function contains one or more of the terms in which the variable is raised to the second power, but no variable is raised to a higher power. Examples of quadratic expressions include $3x^2 + 7$ and $x^2 + 2xy + y^2 - 2x + y + 5$ .
<b>quadratic graph***</b>	A quadratic graph is a graph drawn to illustrate a quadratic function. Its shape is called a parabola.
<b>quadratic model***</b>	Using a model that involves fitting a quadratic graph to the situation.
<b>quantiles*</b>	<p>The term quantiles includes deciles, percentiles and quartiles.</p> <p>Quantiles are cutpoints dividing the range of a probability distribution into contiguous intervals with equal probabilities, or dividing the observations in a sample in the same way. There are one fewer quantiles than the number of groups created.</p>
<b>quartiles</b>	<p>Quartiles are the values that divide an ordered data set into four (approximately) equal parts. It is only possible to divide a data set into exactly four equal parts when the number of data of values is a multiple of four.</p> <p>There are three quartiles. The first, the lower quartile (<math>Q_1</math>), divides off (approximately) the lower 25% of data values. The second quartile (<math>Q_2</math>) is the median. The third quartile, the upper quartile (<math>Q_3</math>), divides off (approximately) the upper 25% of data values.</p>
<b>quotient rule*</b>	<p>The quotient rule relates the derivative of the quotient of two functions to the functions and their derivatives.</p> <p>If <math>h(x) = \frac{f(x)}{g(x)}</math> then <math>h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}</math></p> <p>Or in other notation, <math>\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}</math></p>
<b>radian*</b>	A radian is a unit of angular measure defined such that an angle of one radian subtended from the centre of a unit circle produces an arc with arc length 1.
<b>range (of data)</b>	The range is the difference between the largest and smallest observations in a data set. It is sensitive to outliers.

Glossary term	Definition
<b>range (of function)*</b>	The range of a function is the set of "output" values for which the function is defined. In a graph, it is the $y$ values.
<b>reciprocal trigonometric functions*</b>	$\sec A = \frac{1}{\cos A}, \cos A \neq 0$ $\operatorname{cosec} A = \frac{1}{\sin A}, \sin A \neq 0$ $\cot A = \frac{\cos A}{\sin A}, \sin A \neq 0$
<b>reducing balance loan***</b>	A reducing balance loan calculates the interest owed from the balance at the time of each payment rather than on the original loan amount, and therefore that amount reduces with each loan repayment.
<b>relative frequency</b>	Relative frequency is given by the ratio $\frac{f}{n}$ , where $f$ is the frequency of occurrence of a particular data value or group of data values in a data set and $n$ is the number of data values in the data set.
<b>sample space</b>	The sample space of an experiment is the set of all possible outcome for that experiment.
<b>scalar*</b>	A scalar is a constant that represents magnitude.
<b>secant (line)*</b>	A straight line passing through two points on the graph of a function is a secant.
<b>second derivative*</b>	<p>The second derivative is the derivative of the first derivative.</p> <p>It is notated as <math>f''(x)</math> or <math>\frac{d^2y}{dx^2}</math></p>
<b>second derivative test*</b>	According to the second derivative test, if $f'(x) = 0$ , then $f(x)$ is a local maximum of $f$ if $f''(x) < 0$ and $f(x)$ is a local minimum if $f''(x) > 0$
<b>sine rule</b>	<p>In any triangle <math>ABC</math>,</p> $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ 
<b>sketch*</b>	A sketch is a clear representation of a graph, including labelled axes, intercepts and other important points where relevant. Compared to a graph, a sketch should approximate scale but does not need to be exact.
<b>straight-line graph***</b>	See linear graphs

Glossary term	Definition
<b>standard deviation**</b>	Standard deviation is a measure of the variability or spread of a data set. It gives an indication of the degree to which the individual data values are spread around their mean. It is also the square root of the variance.
<b>stationary points*</b>	<p>A stationary point on the graph <math>y = f(x)</math> of a differentiable function is a point where <math>f'(x) = 0</math>.</p> <p>A stationary point could be a local or global maximum or minimum or a point of inflection.</p>
<b>subtended*</b>	The angle that an interval or arc with endpoints $A$ and $B$ subtends at a point $P$ is angle $APB$ .
<b>tangent line*</b>	The tangent line (the tangent) to a curve at a given point $P$ can be described intuitively as the straight line that “just touches” the curve at that point. At $P$ the curve has the same direction as the tangent line. In this sense it is the best straight-line approximation to the curve at point $P$ .
<b>translations**</b>	<p>Shifting a figure in the plane without turning it is called translation. To describe a translation, it is enough to say how far left or right and how far up or down the figure is moved.</p> <p>A translation is a transformation that moves each point to its translation image.</p> <p>This can also be applied to graphs of functions.</p>
<b>trapezoidal rule*</b>	A technique for approximating the area under a curve, or the definite integral.
<b>tree diagram</b>	<p>A tree diagram is a diagram that can be used to enumerate the outcomes of a multistep random experiment.</p> <p>The diagram below shows a tree diagram that has been used to enumerate all of the possible outcomes when a coin is tossed twice. This is an example of a two-step random experiment.</p> 
<b>true bearing***</b>	True (or 3 figure) bearings are measured in degrees from the north line. Three figures are used to specify the direction. Thus the direction of north is specified $000^0$ , east is specified as $090^0$ , south is specified as $180^0$ and north-west is specified as $315^0$ .

Glossary term	Definition
<b>uniform acceleration*</b>	Uniform or constant acceleration is a type of motion in which the velocity of an object changes by an equal amount in every equal time period.
<b>unit circle*</b>	A unit circle is a circle with radius 1, centred at the origin (0,0).
<b>variance*</b>	Variance is the expectation of the squared deviation of a random variable from its mean. It measures how far a set of (random) numbers spread out from their mean. It is the square of the standard deviation.
<b>vertical line test*</b>	If a vertical line intersects a graph more than once then the graph is not a function (as it is not single-valued).
<b>z score*</b>	<p>A z-score is a statistical measurement of a value's relationship to the mean of the values in a dataset.</p> <p>A z-score can be positive or negative, indicating whether it is above or below the mean and by how many standard deviations.</p> <p>A z-score of 0 means the value is the same as the mean.</p>