Mathematics General 2

General Instructions
• Reading time – 5 minutes
• Working time – 2 1/2 hours
• Write using black or blue pen
  Black pen is preferred
• Board-approved calculators may
  be used
• A formulae and data sheet is
  provided at the back of this paper
• In Questions 26–30, show
  relevant mathematical reasoning
  and/or calculations

Total marks – 100

Section I Pages 2–12
25 marks
• Attempt Questions 1–25
• Allow about 35 minutes for this section

Section II Pages 13–34
75 marks
• Attempt Questions 26–30
• Allow about 1 hour and 55 minutes for this section
1. The step graph shows the cost of telephone calls to China.

Bella made a telephone call at 9.50 pm to a friend in China and spoke to him until 10.07 pm.

How much did this telephone call cost?

(A) $4
(B) $8
(C) $12
(D) $16

2. A measurement of 72 cm is increased by 20% and then the result is decreased by 20%.

What is the new measurement, correct to the nearest centimetre?

(A) 46 cm
(B) 69 cm
(C) 72 cm
(D) 104 cm
The diagram shows the graph of an equation.

Which of the following equations does the graph best represent?

(A) \( y = \frac{3}{x} + 1 \)
(B) \( y = 3^x + 1 \)
(C) \( y = 3x^2 + 1 \)
(D) \( y = 3x^3 + 1 \)

Young’s formula below is used to calculate the required dosages of medicine for children aged 1–12 years.

\[
\text{Dosage} = \frac{\text{age of child (in years)} \times \text{adult dosage}}{\text{age of child (in years)} + 12}
\]

How much of the medicine should be given to an 18-month-old child in a 24-hour period if each adult dosage is 45 mL? The medicine is to be taken every 6 hours by both adults and children.

(A) 5 mL
(B) 20 mL
(C) 27 mL
(D) 30 mL
5 How many kilobytes are there in 2 gigabytes?

(A) $2^{20}$
(B) $2^{21}$
(C) $2^{30}$
(D) $2^{31}$

6 A cafe menu has 3 entrees, 5 main courses and 2 desserts. Ariana is choosing a three-course meal consisting of an entree, a main course and a dessert.

How many different three-course meals can Ariana choose?

(A) 3
(B) 10
(C) 15
(D) 30

7 Which of the following is the graph of $y = 2x - 2$?

(A) 

(B) 

(C) 

(D)
A group of 150 people was surveyed and the results recorded.

<table>
<thead>
<tr>
<th></th>
<th>Owns a mobile</th>
<th>Does not own a mobile</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>42</td>
<td>28</td>
<td>70</td>
</tr>
<tr>
<td>Female</td>
<td>63</td>
<td>17</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>105</td>
<td>45</td>
<td>150</td>
</tr>
</tbody>
</table>

A person is selected at random from the surveyed group.

What is the probability that the person selected is a male who does not own a mobile?

(A) \( \frac{28}{150} \)

(B) \( \frac{45}{150} \)

(C) \( \frac{28}{70} \)

(D) \( \frac{45}{70} \)

A car is bought for $19,990. It will depreciate at 18% per annum.

Using the declining balance method, what will be the salvage value of the car after 3 years, to the nearest dollar?

(A) $8968

(B) $9195

(C) $11,022

(D) $16,392

The top of the Sydney Harbour Bridge is measured to be 138.4 m above sea level.

What is the percentage error in this measurement?

(A) 0.036%

(B) 0.050%

(C) 0.072%

(D) 0.289%
11 Simplify $6w^4 \times \frac{1}{3}w^2$.

(A) $2w^6$
(B) $2w^8$
(C) $18w^6$
(D) $18w^8$

12 A path 1.5 metres wide surrounds a circular lawn of radius 3 metres.

What is the approximate area of the path?

(A) 7.1 m$^2$
(B) 21.2 m$^2$
(C) 35.3 m$^2$
(D) 56.5 m$^2$

13 Jane sells jewellery. Her commission is based on a sliding scale of 6% on the first $2000 of her sales, 3.5% on the next $1000, and 2% thereafter.

What is Jane’s commission when her total sales are $5670?

(A) $188.40$
(B) $208.40$
(C) $321.85$
(D) $652.05
Twenty Year 12 students were surveyed. These students were asked how many hours of sport they play per week, to the nearest hour.

The results are shown in the frequency table.

<table>
<thead>
<tr>
<th>Hours per week</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–2</td>
<td>5</td>
</tr>
<tr>
<td>3–5</td>
<td>10</td>
</tr>
<tr>
<td>6–8</td>
<td>3</td>
</tr>
<tr>
<td>9–11</td>
<td>2</td>
</tr>
</tbody>
</table>

What is the mean number of hours of sport played by the students per week?

(A) 3.3  
(B) 4.3  
(C) 5.0  
(D) 5.3

Which expression will give the shortest distance, in kilometres, between Mount Isa (20°S 140°E) and Tokyo (35°N 140°E)?

(A) \( \frac{15}{360} \times 2 \times \pi \times 6400 \)  
(B) \( \frac{55}{360} \times 2 \times \pi \times 6400 \)  
(C) \( \frac{140}{360} \times 2 \times \pi \times 6400 \)  
(D) \( \frac{305}{360} \times 2 \times \pi \times 6400 \)
16  In Mathsville, there are on average eight rainy days in October.

Which expression could be used to find a value for the probability that it will rain on two consecutive days in October in Mathsville?

(A) \( \frac{8}{31} \times \frac{7}{30} \)

(B) \( \frac{8}{31} \times \frac{7}{31} \)

(C) \( \frac{8}{31} \times \frac{8}{30} \)

(D) \( \frac{8}{31} \times \frac{8}{31} \)

17  A child who weighs 14 kg needs to be given 15 mg of paracetamol for every 2 kg of body weight. Every 10 mL of a particular medicine contains 120 mg of paracetamol.

What is the correct dosage of this medicine for the child?

(A) 5.6 mL

(B) 8.75 mL

(C) 11.43 mL

(D) 17.5 mL

18  The average NSW annual water consumption from the residential sector is equal to 90 340 litres per person per year. The Building Sustainability Index (BASIX) uses this as the benchmark to set a target for reducing water consumption by up to 40%.

A new building, planned to house 50 people, has been designed to meet a 25% reduction on this water consumption benchmark.

How much water per year, to the nearest kilolitre, is this building designed to save when fully occupied?

(A) 1129

(B) 1807

(C) 2710

(D) 3388
Jaz has 2 bags of apples.

Bag A contains 4 red apples and 3 green apples.

Bag B contains 3 red apples and 1 green apple.

Jaz chooses an apple from one of the bags.

Which tree diagram could be used to determine the probability that Jaz chooses a red apple?

(A) 
\[
\begin{array}{c}
\frac{1}{2} \\
\text{Bag A} \\
\frac{4}{7} \text{ Red} \\
\frac{3}{7} \text{ Green} \\
\frac{1}{2} \\
\text{Bag B} \\
\frac{3}{4} \text{ Red} \\
\frac{1}{4} \text{ Green}
\end{array}
\]

(B) 
\[
\begin{array}{c}
\frac{1}{2} \\
\text{Bag A} \\
\frac{7}{11} \text{ Red} \\
\frac{4}{11} \text{ Green} \\
\frac{1}{2} \\
\text{Bag B} \\
\frac{7}{11} \text{ Red} \\
\frac{4}{11} \text{ Green}
\end{array}
\]

(C) 
\[
\begin{array}{c}
\frac{4}{7} \text{ Red} \\
\frac{3}{4} \text{ Green} \\
\frac{3}{7} \text{ Green} \\
\frac{3}{4} \text{ Red} \\
\frac{1}{4} \text{ Green}
\end{array}
\]

(D) 
\[
\begin{array}{c}
\frac{4}{11} \text{ Red} \\
\frac{7}{10} \text{ Green} \\
\frac{7}{11} \text{ Green} \\
\frac{4}{10} \text{ Red} \\
\frac{6}{10} \text{ Green}
\end{array}
\]
20 In a household of 4, each member uses an average of 13 minutes of hot water per day.

The household uses a 9 kW hot water unit.

Electricity is charged at 11.97c/kWh when the hot water unit is being used.

What is the electricity cost for the hot water used by this household in one week?

(A) $1.63
(B) $6.54
(C) $392.14
(D) $653.56

21 A table of future value interest factors is shown.

<table>
<thead>
<tr>
<th>Period</th>
<th>Interest rate per period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1% 2% 3% 4% 5%</td>
</tr>
<tr>
<td>1</td>
<td>1.0000 1.0000 1.0000 1.0000 1.0000</td>
</tr>
<tr>
<td>2</td>
<td>2.0100 2.0200 2.0300 2.0400 2.0500</td>
</tr>
<tr>
<td>3</td>
<td>3.0301 3.0604 3.0909 3.1216 3.1525</td>
</tr>
<tr>
<td>4</td>
<td>4.0604 4.1216 4.1836 4.2465 4.3101</td>
</tr>
</tbody>
</table>

A certain annuity involves making equal contributions of $25 000 into an account every 6 months for 2 years at an interest rate of 4% per annum.

Based on the information provided, what is the future value of this annuity?

(A) $50 500
(B) $51 000
(C) $103 040
(D) $106 162
22 Heather’s car uses fuel at the rate of 6.6 L per 100 km for long-distance driving and 8.9 L per 100 km for short-distance driving.

She used the car to make a journey of 560 km, which included 65 km of short-distance driving.

Approximately how much fuel did Heather’s car use on the journey?

(A) 37 L
(B) 38 L
(C) 48 L
(D) 50 L

23 The following information is given about the locations of three towns X, Y and Z:

- X is due east of Z
- X is on a bearing of 145° from Y
- Y is on a bearing of 060° from Z.

Which diagram best represents this information?

(A)  
(B)  
(C)  
(D)  

NOT TO SCALE
The weights of 10,000 newborn babies in NSW are normally distributed. These weights have a mean of 3.1 kg and a standard deviation of 0.35 kg.

How many of these newborn babies have a weight between 2.75 kg and 4.15 kg?

(A) 4985
(B) 6570
(C) 8370
(D) 8385

A grain silo is made up of a cylinder with a hemisphere (half a sphere) on top. The outside of the silo is to be painted.

What is the area to be painted?

(A) 8143 m²
(B) 11762 m²
(C) 12667 m²
(D) 23524 m²
Section II

75 marks
Attempt Questions 26–30
Allow about 1 hour and 55 minutes for this section

Answer the questions in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided on pages 33 and 34. If you use this space, clearly indicate which question you are answering.

Write your Centre Number and Student Number at the top of this page.

Please turn over
Question 26 (15 marks)

(a) Expand $4x \left( 7x^4 - x^2 \right)$.

(b) Calculate the value of $h$ correct to two decimal places.

(c) Solve the equation $\frac{5x + 1}{3} - 4 = 5 - 7x$.
(d) Solve these simultaneous equations to find the values of $x$ and $y$.

\begin{align*}
y &= 2x + 1 \\
x - 2y &= 4
\end{align*}

(e) The times taken for 160 music downloads were recorded, grouped into classes and then displayed using the cumulative frequency histogram shown.

On the diagram, draw the lines that are needed to find the median download time.
Question 26 (continued)

(f) The weight of an object on the moon varies directly with its weight on Earth. An astronaut who weighs 84 kg on Earth weighs only 14 kg on the moon.

A lunar landing craft weighs 2449 kg when on the moon. Calculate the weight of this landing craft when on Earth.

........................................................................................................................................................
........................................................................................................................................................
........................................................................................................................................................
........................................................................................................................................................
........................................................................................................................................................
........................................................................................................................................................
........................................................................................................................................................
........................................................................................................................................................
........................................................................................................................................................
........................................................................................................................................................
........................................................................................................................................................
........................................................................................................................................................
........................................................................................................................................................
........................................................................................................................................................

Question 26 continues on page 17
(g) Singapore is located at 1°N 104°E and Sydney is located at 34°S 151°E.

What is the time difference between Singapore and Sydney? (Ignore daylight saving.)

End of Question 26

Please turn over
(a) Alex is buying a used car which has a sale price of $13 380. In addition to the sale price there are the following costs:

<table>
<thead>
<tr>
<th>Cost Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transfer of registration</td>
<td>$30</td>
</tr>
<tr>
<td>Stamp Duty</td>
<td>.......</td>
</tr>
</tbody>
</table>

(i) Stamp Duty for this car is calculated at $3 for every $100, or part thereof, of the sale price.

Calculate the Stamp Duty payable.

...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................

(ii) Alex borrows the total amount to be paid for the car including Stamp Duty and transfer of registration. Interest on the loan is charged at a flat rate of 7.5% per annum. The loan is to be repaid in equal monthly instalments over 3 years.

Calculate Alex’s monthly repayments.

...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
(iii) Alex wishes to take out comprehensive insurance for the car for 12 months. The cost of comprehensive insurance is calculated using the following:

- Base rate: $845
- Fire Service Levy (FSL): 1% of base rate
- Stamp Duty: 5.5% of the total of base rate and FSL
- GST: 10% of the total of base rate and FSL.

Find the total amount that Alex will need to pay for comprehensive insurance.

(iv) Alex has decided he will take out the comprehensive car insurance rather than the less expensive non-compulsory third-party car insurance.

What extra cover is provided by the comprehensive car insurance?
(b) Xuso is comparing the costs of two different ways of travelling to university.

Xuso’s motorcycle uses one litre of fuel for every 17 km travelled. The cost of fuel is $1.67/L and the distance from her home to the university car park is 34 km. The cost of travelling by bus is $36.40 for 10 single trips.

Which way of travelling is cheaper and by how much? Support your answer with calculations.

(c) The base of a water tank is in the shape of a rectangle with a semicircle at each end, as shown.

The tank is 1400 mm long, 560 mm wide, and has a height of 810 mm.

What is the capacity of the tank, to the nearest litre?
(a) James plays a game involving a spinner with sectors of equal size labelled $A$, $B$ and $C$, as shown.

He pays $2 to play the game. He wins $5 if the spinner stops in $A$ and 50 cents if it stops in $B$ or $C$.

Calculate James’s financial expectation for the game.

........................................................................................................................................
........................................................................................................................................
........................................................................................................................................
........................................................................................................................................
........................................................................................................................................
........................................................................................................................................
........................................................................................................................................

Question 28 continues on page 22
(b) A radial compass survey of a sports centre is shown in the diagram.

(i) Show that the size of angle $AOB$ is $114^\circ$.

...................................................................................................................
...................................................................................................................

Question 28 continues on page 23
Question 28 (continued)

(ii) Calculate the length of the boundary $AB$, to the nearest metre.

...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................

(iii) Find the area of triangle $AOB$ in hectares, correct to two significant figures.

...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................

(c) A fair coin is tossed three times. Using a tree diagram, or otherwise, calculate the probability of obtaining two heads and a tail in any order.

...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................

Question 28 continues on page 24
(d) An aerial diagram of a swimming pool is shown.

The swimming pool is a standard length of 50 metres but is not in the shape of a rectangle.

(i) By measuring the length $AB$, determine the scale of the diagram.

\[ 1 \text{ cm} = \boxed{\text{_______ m}} \]

(ii) Using this scale, calculate the length $XY$ of the car park, in metres.

\[ \text{_______ m} \]
(iii) In the diagram of the swimming pool, the five widths are measured to be:

\[
\begin{align*}
CD &= 21.88\, \text{m} \\
EF &= 25.63\, \text{m} \\
GH &= 31.88\, \text{m} \\
IJ &= 36.25\, \text{m} \\
KL &= 21.88\, \text{m}
\end{align*}
\]

The average depth of the pool is 1.2 m.

Calculate the approximate volume of the swimming pool, in cubic metres. In your calculations, use TWO applications of Simpson’s Rule.

...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................

End of Question 28
Question 29 (15 marks)

(a) The cost of hiring an open space for a music festival is $120,000. The cost will be shared equally by the people attending the festival, so that $C$ (in dollars) is the cost per person when $n$ people attend the festival.

(i) Complete the table below by filling in the THREE missing values.

<table>
<thead>
<tr>
<th>Number of people $(n)$</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
<th>2500</th>
<th>3000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost per person $(C)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>48</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(ii) Using the values from the table, draw the graph showing the relationship between $n$ and $C$.

(iii) What equation represents the relationship between $n$ and $C$?

...................................................................................................................
...................................................................................................................
...................................................................................................................

Question 29 continues on page 27
Question 29 (continued)

(iv) Give ONE limitation of this equation in relation to this context.

...................................................................................................................
...................................................................................................................
...................................................................................................................

(v) Is it possible for the cost per person to be $94? Support your answer with appropriate calculations.
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................

(b) What is the maximum number of standard drinks that a male weighing 84 kg can consume over 4 hours in order to maintain a blood alcohol content (BAC) of less than 0.05?
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................

Question 29 continues on page 28
Question 29 (continued)

(c) Terry and Kim each sat twenty class tests. Terry’s results on the tests are displayed in the box-and-whisker plot shown in part (i).

(i) Kim’s 5-number summary for the tests is 67, 69, 71, 73, 75.

Draw a box-and-whisker plot to display Kim’s results below that of Terry’s results.

Terry

Kim

65 66 67 68 69 70 71 72 73 74 75 76 77

(ii) What percentage of Terry’s results were below 69?

(iii) Terry claims that his results were better than Kim’s. Is he correct? Justify your answer by referring to the summary statistics and the skewness of the distributions.
Question 30 (15 marks)

(a) Chandra and Sascha plan to have $20,000 in an investment account in 15 years time for their grandchild’s university fees.

The interest rate for the investment account will be fixed at 3% per annum compounded monthly.

Calculate the amount that they will need to deposit into the account now in order to achieve their plan.

...............................................................................................................................
...............................................................................................................................
...............................................................................................................................
...............................................................................................................................
...............................................................................................................................
...............................................................................................................................
...............................................................................................................................
...............................................................................................................................
...............................................................................................................................
...............................................................................................................................
...............................................................................................................................
...............................................................................................................................

Question 30 continues on page 30
(b) The scatterplot shows the relationship between expenditure per primary school student, as a percentage of a country’s Gross Domestic Product (GDP), and the life expectancy in years for 15 countries.

(i) For the given data, the correlation coefficient, $r$, is 0.83. What does this indicate about the relationship between expenditure per primary school student and life expectancy for the 15 countries?

................................................................................................................................................
................................................................................................................................................
................................................................................................................................................

Question 30 continues on page 31
Question 30 (continued)

(ii) For the data representing expenditure per primary school student, $Q_L$ is 8.4 and $Q_U$ is 22.5.

What is the interquartile range?

(iii) Another country has an expenditure per primary school student of 47.6% of its GDP. Would this country be an outlier for this set of data? Justify your answer with calculations.

(iv) The expenditures per primary school student for the 15 countries in the scatterplot are:

5.9, 7, 7.6, 8.4, 11.2, 11.2, 13.7, 17.1, 18.7, 21.1, 22, 22.5, 23.2, 24.9, 27.6

Complete the table below by calculating the mean, $\bar{x}$, and the standard deviation, $\sigma_x$, of these data. Calculate both values to two decimal places.

The table also shows the mean, $\bar{y}$, and the standard deviation, $\sigma_y$, of life expectancy for the same 15 countries.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expenditure per primary school student</td>
<td>$\bar{x} =$</td>
<td>$\sigma_x =$</td>
</tr>
<tr>
<td>Life expectancy</td>
<td>$\bar{y} = 70.73$</td>
<td>$\sigma_y = 10.94$</td>
</tr>
</tbody>
</table>

Question 30 continues on page 32
Question 30 (continued)

(v) Using the values from the table in part (iv), show that the equation of the least-squares line of best fit is

\[ y = 1.29x + 49.9. \]

(vi) On the scatterplot on page 30, draw the least-squares line of best fit, \( y = 1.29x + 49.9. \)

(vii) Using this line, or otherwise, estimate the life expectancy in a country which has an expenditure per primary school student of 18% of its GDP.

(viii) Why is this line NOT useful for predicting life expectancy in a country which has expenditure per primary school student of 60% of its GDP?
Section II extra writing space

If you use this space, clearly indicate which question you are answering.
Section II extra writing space

If you use this space, clearly indicate which question you are answering.
FORMULAE AND DATA SHEET

### Financial Mathematics

#### Simple interest

\[ I = Prn \]

- \( P \) is initial amount
- \( r \) is interest rate per period, expressed as a decimal
- \( n \) is number of periods

#### Compound interest

\[ A = P(1 + r)^n \]

- \( A \) is final amount
- \( P \) is initial amount
- \( r \) is interest rate per period, expressed as a decimal
- \( n \) is number of compounding periods

#### Present value and future value

\[ PV = \frac{FV}{(1 + r)^n}, \quad FV = PV(1 + r)^n \]

- \( r \) is interest rate per period, expressed as a decimal
- \( n \) is number of compounding periods

#### Straight-line method of depreciation

\[ S = V_0 - Dn \]

- \( S \) is salvage value of asset after \( n \) periods
- \( V_0 \) is initial value of asset
- \( D \) is amount of depreciation per period
- \( n \) is number of periods

#### Declining-balance method of depreciation

\[ S = V_0(1 - r)^n \]

- \( S \) is salvage value of asset after \( n \) periods
- \( V_0 \) is initial value of asset
- \( r \) is depreciation rate per period, expressed as a decimal
- \( n \) is number of periods

### Data Analysis

#### Mean of a sample

\[ \bar{x} = \frac{\text{sum of scores}}{\text{number of scores}} \]

#### z-score

For any score \( x \),

\[ z = \frac{x - \bar{x}}{s} \]

- \( \bar{x} \) is mean
- \( s \) is standard deviation

#### Outlier(s)

- score(s) less than \( Q_L - 1.5 \times IQR \)
- score(s) more than \( Q_U + 1.5 \times IQR \)

- \( Q_L \) is lower quartile
- \( Q_U \) is upper quartile
- \( IQR \) is interquartile range

#### Least-squares line of best fit

\[ y = \text{gradient} \times x + y\text{-intercept} \]

\[ \text{gradient} = r \times \frac{\text{standard deviation of } y \text{ scores}}{\text{standard deviation of } x \text{ scores}} \]

\[ y\text{-intercept} = \bar{y} - (\text{gradient} \times \bar{x}) \]

- \( r \) is correlation coefficient
- \( \bar{x} \) is mean of \( x \) scores
- \( \bar{y} \) is mean of \( y \) scores

#### Normal distribution

- approximately 68% of scores have \( z \)-scores between \(-1 \) and \( 1 \)
- approximately 95% of scores have \( z \)-scores between \(-2 \) and \( 2 \)
- approximately 99.7% of scores have \( z \)-scores between \(-3 \) and \( 3 \)


**Spherical Geometry**

**Circumference of a circle**

\[ C = 2\pi r \quad \text{or} \quad C = \pi D \]

- \( r \) is radius
- \( D \) is diameter

**Arc length of a circle**

\[ l = \frac{\theta}{360} 2\pi r \]

- \( r \) is radius
- \( \theta \) is number of degrees in central angle

**Radius of Earth**

(taken as) 6400 km

**Time differences**

For calculation of time differences using longitude:

\[ 15^\circ = 1 \text{ hour time difference} \]

---

**Area**

**Circle**

\[ A = \pi r^2 \]

- \( r \) is radius

**Sector**

\[ A = \frac{\theta}{360} \pi r^2 \]

- \( r \) is radius
- \( \theta \) is number of degrees in central angle

**Annulus**

\[ A = \pi \left( R^2 - r^2 \right) \]

- \( R \) is radius of outer circle
- \( r \) is radius of inner circle

**Trapezium**

\[ A = \frac{h}{2} (a + b) \]

- \( h \) is perpendicular height
- \( a \) and \( b \) are the lengths of the parallel sides

---

**Surface Area**

**Sphere**

\[ A = 4\pi r^2 \]

- \( r \) is radius

**Closed cylinder**

\[ A = 2\pi r^2 + 2\pi rh \]

- \( r \) is radius
- \( h \) is perpendicular height

---

**Volume**

**Prism or cylinder**

\[ V = Ah \]

- \( A \) is area of base
- \( h \) is perpendicular height

**Pyramid or cone**

\[ V = \frac{1}{3}Ah \]

- \( A \) is area of base
- \( h \) is perpendicular height

---

**Volume and capacity**

unit conversion: 1 m³ = 1000 L

---

**Approximation Using Simpson’s Rule**

**Area**

\[ A \approx \frac{h}{3} \left( d_f + 4d_m + d_l \right) \]

- \( h \) is distance between successive measurements
- \( d_f \) is first measurement
- \( d_m \) is middle measurement
- \( d_l \) is last measurement

**Volume**

\[ V \approx \frac{h}{3} \left( A_L + 4A_M + A_R \right) \]

- \( h \) is distance between successive measurements
- \( A_L \) is area of left end
- \( A_M \) is area of middle
- \( A_R \) is area of right end
Trigonometric Ratios

\[ \begin{align*}
\sin \theta &= \frac{\text{opposite side}}{\text{hypotenuse}} \\
\cos \theta &= \frac{\text{adjacent side}}{\text{hypotenuse}} \\
\tan \theta &= \frac{\text{opposite side}}{\text{adjacent side}}
\end{align*} \]

Sine rule

In \( \triangle ABC \),
\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

Area of a triangle

In \( \triangle ABC \),
\[ A = \frac{1}{2}ab \sin C \]

Cosine rule

In \( \triangle ABC \),
\[ c^2 = a^2 + b^2 - 2ab \cos C \]

or
\[ \cos C = \frac{a^2 + b^2 - c^2}{2ab} \]

Blood Alcohol Content Estimates

\[ BAC_{\text{male}} = \frac{10N - 7.5H}{6.8M} \]

or
\[ BAC_{\text{female}} = \frac{10N - 7.5H}{5.5M} \]

\( N \) is number of standard drinks consumed
\( H \) is number of hours of drinking
\( M \) is person's mass in kilograms

Distance, Speed and Time

\[ D = ST, \quad S = \frac{D}{T}, \quad T = \frac{D}{S} \]

average speed = \( \frac{\text{total distance travelled}}{\text{total time taken}} \)

stopping distance = \( \left\{ \text{reaction-time} \right\} + \left\{ \text{braking} \right\} \)

Probability of an Event

The probability of an event where outcomes are equally likely is given by:
\[ P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} \]

Straight Lines

Gradient

\[ m = \frac{\text{vertical change in position}}{\text{horizontal change in position}} \]

Gradient–intercept form

\[ y = mx + b \]

\( m \) is gradient
\( b \) is \( y \)-intercept