This document contains ‘sample answers’. These are developed by the examination committee for two purposes. The committee does this:

(a) as part of the development of the examination paper to ensure the questions will effectively assess students’ knowledge and skills, and

(b) in order to provide some advice to the Supervisor of Marking about the nature and scope of the responses expected of students.

The ‘sample answers’ or similar advice, are not intended to be exemplary or even complete responses. They have been reproduced in their original form as part of the examination committee’s ‘working document’.
Question 1

1(a) \[ y - 2x = 3 \]
\[ y = 2x + 3 \]

(b) \[ \frac{5x - 4}{x} = 2 \]
\[ 5x - 4 = 2x \]
\[ 3x = 4 \]
\[ x = \frac{4}{3} \]

(c) \[ x + 1 = \pm 5 \]
\[ x = 4, -6 \]

(d) \[ y = x^4 - 3x \]
\[ \frac{dy}{dx} = 4x^3 - 3 \]

When \( x = 1 \), \[ \frac{dy}{dx} = 4 - 3 = 1 \]

(e) \[ 2 \cos \theta = 1 \]
\[ \cos \theta = \frac{1}{2} \]
\[ \theta = \frac{\pi}{3} \]

(f) \[ \ln x = 2 \]
\[ x = 7.3891 \]
Question 2.

(a) (i) \( y = x \sin x \)
\[ y' = x \cos x + \sin x \]

(ii) \( y = (e^x + 1)^2 \)
\[ y' = 2(e^x + 1) \cdot e^x \]

(b) (i) \( \int 5 \, dx = 5x + c \)

(ii) \( \int \frac{3}{(x-6)^2} \, dx = \frac{-3}{x-6} + c \)

(iii) \( \int_1^4 x^2 + \sqrt{x} \, dx = \left[ \frac{x^3}{3} + \frac{2}{3} x \sqrt{x} \right]_1^4 \)
\[ = \frac{63}{3} + \frac{2}{3} \times 7 = \frac{77}{3} \]

(c) \( \sum_{k=1}^{4} (-1)^k k^2 = -1 + 4 - 9 + 16 = 10 \)
Question 3

(a) \( S = \frac{3 + 53}{2} \times 21 = \frac{56}{2} \times 21 = 588 \)

(b) (i) \( \frac{y-1}{x-2} = \frac{5-1}{5-2} = \frac{4}{3} \)

\[ 4(x-2) = 3(y-1) \]
\[ 4x - 3y - 5 = 0 \]

(ii) \( NP = \left| \frac{4 \times 1 - 3 \times 3 - 5}{\sqrt{4^2 + 3^2}} \right| = 2 \)

(iii) \( (x-1)^2 + (y-3)^2 = 2^2 \)

(c) \[ \text{includes boundary} \]

(d) \( \text{Area} = \frac{210 + 4 \times 220 + 2 \times 200 + 4 \times 190 + 2 \times 210 + 4 \times 240 + 2 \times 240 + 2 + 4 + 1}{1 + 4 + 2 + 4 + 2 + 4 + 1} \times 300 \)
\[ = 64500 \text{ m}^2 \]
(a) \( \text{eventual height } = \frac{12}{1 - \frac{9}{10}} = 12 \text{ m} \)

(b) \( x^2 - (k+4)x + (k+7) = 0 \)
\[
\Delta = (k+4)^2 - 4(k+7) = 0
\]
\[
k^2 + 4k - 12 = 0
\]
\[
k = -6 \text{ or } 2
\]

(c) (i) \( \text{PM} \perp \text{AC}, \text{BC} \perp \text{AC} \text{ so } \text{PM} \parallel \text{CB} \)
so \( \angle \text{PMA} = \angle \text{CBA} \) (corresponding angles)
\( \angle \text{APM} = \angle \text{ACB} \)
and \( \angle \text{PAM} = \angle \text{CAB} \)
so \( \triangle \text{AMP} \parallel \triangle \text{ABC} \) (First two lines are unnecessary)

(ii) \( \text{AP: AC} = \text{AM: AB} = 1:2 \)

(iii) From (ii) \( \text{AP} = \text{CP} \)
\( \text{MP} = \text{MP} \) (Common)
\( \angle \text{CPM} = \angle \text{APM} \) (= 90°)

\( \therefore \triangle \text{CPM} \equiv \triangle \text{APM} \) (SAS)

\( \therefore \text{CM} = \text{AM} \)
\( \therefore \triangle \text{AMC} \text{ is isosceles} \)

(iv) \( \text{MC = MA = MB} \) so \( \triangle \text{BMC} \text{ is isosceles} \)
so \( \triangle \text{ABC} \text{ can be subdivided into two isosceles triangles} \).
Two right-angled triangles, each of which is subdivided into 2 isosceles triangles.

End of Question 4
Question 5.

(a) (i) \( B (\sqrt{3}, 0) \)  
Slope of \( BC = -\frac{1}{\sqrt{3}} \)

Equation of \( BC \): \( y = -\frac{1}{\sqrt{3}} (x + \sqrt{3}) \)

or \( \sqrt{3} y + x = \sqrt{3} \)

(ii) \( C (0,1) \)
\( BC = 2, \ AB = 2\sqrt{3} \)
Area = \( \frac{2\sqrt{3}}{2} \) sq. units

(b) (i) \( \frac{1}{3} \)

(ii) \( \frac{2}{3} \times \frac{1}{2} \times 1 = \frac{1}{3} \)

(iii) \( \left(\frac{2}{3}\right)^5 = \frac{32}{243} \)

(c) (i) \( \frac{1}{2} \times 2^2 \sin \theta = \sqrt{3} \)
\( \sin \theta = \frac{\sqrt{3}}{2} \)
\( \theta = \frac{\pi}{3} \) or \( \frac{2\pi}{3} \)

(ii) (a) Area of sector = \( \frac{1}{2} \times 2^2 \times \frac{\pi}{3} = \frac{2\pi}{3} \)

(2) \( AB = 2 \) (equilateral \( \Delta \))
arc \( AB = \frac{1}{6} \times 2 \times \pi \times 2 = \frac{2\pi}{3} \)

Perimeter = \( \frac{2\pi}{3} + 2 \).
Mathematics

Question 6.

(a) \[ V = \int_{\pi/3}^{\pi} y^2 \, dx \]
\[ = \int_{\pi/3}^{\pi} \pi \sec^2 x \, dx \]
\[ = \pi \left[ \tan x \right]_{\pi/3}^{\pi} \]
\[ = 2\pi \sqrt{3} \]

(b) (i) \[ Q = Ae^{-kt} \]
\[ \frac{1}{2}A = Ae^{-1600k} \]
\[ e^{1600k} = 2 \]
\[ 1600k = \ln 2 \]
\[ k = \frac{\ln 2}{1600} \]

(ii) \[ A = 3S e^{-\frac{\ln 2}{1600}t} = S \quad (S = \text{safe level}) \]
\[ e^{\frac{\ln 2}{1600}} = 3 \]
\[ \frac{\ln 2}{1600} = \frac{\ln 3}{1600} \]
\[ t = \frac{1600 \ln 3}{\ln 2} \approx 2536 \text{ years} \]
(c)(i) \[ y = ax^2 + bx \]
\[ y' = 2ax + b \]

At \( x = 0 \), \( y' = b = 1.2 \)
so \[ y = ax^2 + 1.2x \]
\[ y' = 2ax + 1.2 \]

At \( x = 30 \), \( y' = 60a + 1.2 = -1.8 \)
\[ 60a = -3.0 \]
\[ a = -0.05 \]
\[ y = -0.05x^2 + 1.2x \]

(ii) \[ y' = -0.1x + 1.2 = 0 \]
\[ x = 12 \]
\[ y = -0.05 \times 12^2 + 1.2 \times 12 \]
\[ = -7.2 + 14.4 \]
\[ = 7.2 \]

\( x = 30 \), \( y = -0.05 \times 900 + 1.2 \times 30 \)
\[ = -45 + 36 \]
\[ = -9 \]

\( d = 16.2 \) (metres)

End of Question 6
(a) (i) \[ \ddot{x} = 8e^{-2t} + 3e^{-t} \]
\[ \dot{x} = -4e^{-2t} - 3e^{-t} + c \]
when \( t = 0 \), \( \dot{x} = -6 \) so \( c = 1 \)
\[ \dot{x} = -4e^{-2t} - 3e^{-t} + 1 \]
\[ x = 2e^{-2t} + 3e^{-t} + t + c \]
when \( t = 0 \), \( x = 5 \) so \( c = 0 \)
\[ x = 2e^{-2t} + 3e^{-t} + t \]

(ii) \[ \ddot{x} = -4e^{-2t} - 3e^{-t} + 1 \]
\[ \dot{x} = 0 \]
\[ 4(e^{-t})^2 + 3(e^{-t}) - 1 = 0 \]
\[ e^{-t} = \frac{-3 \pm \sqrt{9 + 16}}{8} \]
\[ e^{-t} = \frac{-3 \pm 5}{8} \]
\[ e^{-t} = \frac{1}{4} \quad (\text{since } e^{-t} > 0) \]
\[ e^t = 4 \]
\[ t = \ln 4 \]

(iii) \[ \dot{x} = 2x + 3t + \ln 4 + \ln 2 \]
\[ = \frac{1}{8} + 2 \ln 2 \]
(b) \( h = 1 + 0.7 \sin \frac{\pi}{6} t \) for \( 0 \leq t \leq 12 \)

(i) 12 hours

(ii) \( 1 - 0.7 = 0.3 \)

\[
\sin \frac{\pi}{6} t = -1 \quad \frac{\pi}{6} t = \frac{3\pi}{2} \quad t = 9
\]

Low tide - at 2 pm.

(iii) \( 1 + 0.7 \sin \frac{\pi}{6} t \geq 1.35 \)

\[
\sin \frac{\pi}{6} t \geq 0.5
\]

\[
\frac{\pi}{6} \leq \frac{\pi}{6} t \leq \frac{5\pi}{6}
\]

\( 1 \leq t \leq 5 \)

between 6 am and 10 am

End of Question 7
2009 Mathematics

Question 8

(a) (i) \( f'(x) < 0 \) for \(-1 < x < 3\)

(ii) \( f'(x) \rightarrow 0 \) as \( x \rightarrow \infty \)

(iii)

\[ f'(x) \]

(b) (i) 
\[
$350,000 \times 1.0075 - 2937$
\]
\[
= $349,688
\]

(ii) 
\[
346,095 \times 1.005^{288} - M \left( 1 + 1.005 + \ldots + 1.005^{287} \right)
\]
\[
= 0
\]

\[
346,095 \times 1.005^{288} - M \left( \frac{1.005^{288} - 1}{0.005} \right) = 0
\]

\[
M = \frac{346,095 \times 0.005 \times 1.005^{288}}{1.005^{288} - 1}
\]

\[
= \frac{346,095 \times 0.005}{1 - 1.005^{-288}}
\]

\[
= 1730.475
\]

\[
0.76222067
\]

\[
= 2270.31 \quad \text{So suppose } M = 2270
(b) (iii)

\[346095 \times 1.005^n - 2937 \left( \frac{1.005^n - 1}{0.005} \right) = 0\]

\[346095 \times 1.005^n - 587400 \times 1.005^n + 587400 = 0\]

\[1.005^n = \frac{587400}{241305} = 2.434263691\]

\[n \ln 1.005 = \ln 2.434263691\]

\[n = \frac{0.889644325}{0.004987541511}\]

\[= 178.37\]

So, about 178 payments or 14 years, 10 months.

(iv) \[288 \times \$2270 = \$653760\]

\[178 \times \$2937 = \$522786\]

\[178.37 \times \$2937 = \$523872\]

Saving \( \approx \$130000\)

**End of Question 8**
(a) \[ \text{probability} = 1 - \left( \frac{8}{9} \right)^3 \left( \frac{15}{16} \right)^3 \]
\[ = 1 - \left( \frac{5}{6} \right)^3 \]
\[ = 1 - \frac{125}{216} \]
\[ = \frac{91}{216} \]

(b) (i) \[ 5 \times $1000 + 3 \times $2600 = $12800 \]

(ii) \[ \sqrt{5^2 + 3^2} \times $2600 = $15160 \]

(iii) \[ C = 1000 \times (5-x) + 2600 \sqrt{x^2 + 9} \]
\[ = 1000 \left( 5 - x + 2.6 \sqrt{x^2 + 9} \right) \]

(iv) \[ \frac{dC}{dx} = 1000 \left( -1 + \frac{2.6x}{\sqrt{x^2 + 9}} \right) \]
\[ = 0 \]
when \[ 2.6x = \sqrt{x^2 + 9} \]

\[ 6.76x^2 = x^2 + 9 \]

\[ 5.76x^2 = 9 \]
\[ x^2 = \frac{9}{5.76} \]
\[ x = \frac{3}{2.4} = 1.25 \]

\[ C = 12200 \text{ is a minimum (below the other values)} \]
9 (b) (v) Now \( C = 1000 \left( 5 - x + 1.1 \sqrt{x^2 + 9} \right) \)

\[
C' = 1000 \left( -1 + \frac{1.1x}{\sqrt{x^2 + 9}} \right)
\]

\[= 0\]

when \( 1.1x = \sqrt{x^2 + 9} \)

\[1.21x^2 = x^2 + 9\]

\[0.21x^2 = 9\]

\[x^2 = \frac{9}{0.21} > 25\]

ie when \( x > 5 \)

Indeed \( C' < 0 \) for \( 0 < x < 5 \)

so min occurs at \( x = 5 \).

The cable should be laid straight from P to S.

**End of Question 9**
Question 10

(a) \[ f'(x) = 1 - x + x^2 = (x - \frac{1}{2})^2 + \frac{3}{4} \geq \frac{3}{4} \]

So \( f'(x) \neq 0 \), \( f(x) \) has no turning points.

(b) \[ f''(x) = -1 + 2x \text{ changes sign at } x = \frac{1}{2} \]

\[ x = \frac{1}{2}, \quad f(x) = \frac{1}{3} - \frac{1}{8} + \frac{1}{16} = \frac{5}{12} \]

Inflection is \( (\frac{1}{2}, \frac{5}{12}) \)

(c)(i) \[ 1 - x + x^2 - \frac{1}{1+x} = \frac{(1-x+x^2)(1+x)-1}{1+x} \]

\[ = \frac{1+x^3-1}{1+x} \]

\[ = \frac{x^3}{1+x} \]

(ii) \[ f(x) - g(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ln(1+x) \]

\[ f'(x) - g'(x) = 1 - x + x^2 - \frac{1}{1+x} = \frac{x^3}{1+x} \]

\[ \geq 0 \text{ for } x \geq 0 \]

\[ f'(x) \geq g'(x) \text{ for } x \geq 0 \]
Question 10 (continued)

(d) 

\[ g(x) \]

\[ 0 \quad 1 \]

(e) \[ \frac{d}{dx} \left( (x+1) \ln(1+x) - (1+x) \right) \]

\[ = (x+1) \times 1 + \ln(1+x) \times 1 - 1 \]

\[ = \ln(1+x) \]

(f) \[ \text{Area} = \int_0^1 f(x) - g(x) \, dx \]

\[ = \int_0^1 x - \frac{x^2}{2} + \frac{x^3}{3} - \ln(1+x) \, dx \]

\[ = \left[ \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{12} - (1+x) \ln(1+x) + (1+x) \right] \]

\[ = \frac{1}{2} - \frac{1}{6} + \frac{1}{12} - 2 \ln 2 + 2 - 1 \]

\[ = \frac{17}{12} - 2 \ln 2 \]

End of Question 10