Number
Guidelines N1

It is vital to recognise that place value is one of the key mathematical concepts. It cannot be assumed that all students will have a clear understanding of place value at the beginning of secondary school. The listed suggestions might be appropriate for students experiencing difficulty with this concept and for consolidation and review with other students. The emphasis is to move from a manipulative concrete representation to a standard recorded format.

Concrete manipulation should concentrate on trading to reinforce important place-value concepts.

cg

trade up

ten sticks

ten ones

trade down

one bundle of ten

ie one ten

ten rod or long

one ten

A similar approach can be used for successive powers of 10, as each new place (hundreds, thousands etc) is considered and revised. The base ten material (Base ten blocks) is most powerful for the demonstration of place value.

From Concrete Form To Recorded Form
An Example

<table>
<thead>
<tr>
<th>Place Value Headings</th>
<th>th</th>
<th>h</th>
<th>t</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>'ten cubed'</td>
<td></td>
<td></td>
<td></td>
<td>'units'</td>
</tr>
<tr>
<td>'ten squared'</td>
<td></td>
<td></td>
<td>'tens'</td>
<td></td>
</tr>
</tbody>
</table>

'Spike Abacus'

<table>
<thead>
<tr>
<th>Number Expander</th>
<th>1 thousand</th>
<th>2 hundreds</th>
<th>3 tens</th>
<th>6 units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expanded Form</td>
<td>$1 \times 10^3$</td>
<td>$2 \times 10^2$</td>
<td>$3 \times 10$</td>
<td>$6 \times 1$</td>
</tr>
<tr>
<td>Numeral</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Recording</td>
<td>1 236</td>
<td>OR</td>
<td>1236</td>
<td></td>
</tr>
</tbody>
</table>

Number Line Exploration

<table>
<thead>
<tr>
<th>What's over here?</th>
<th>What's in here?</th>
<th>How far?</th>
</tr>
</thead>
<tbody>
<tr>
<td>shorts</td>
<td>longs</td>
<td>flats</td>
</tr>
<tr>
<td>blocks</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What is zero?

Are number lines always horizontal?
### Content and Skills Objectives

#### N1.1 Overview of the history of number and the development of the Hindu-Arabic number system

**To be able to:**

- recognise and state place value
- read, write, interpret and order numbers of any magnitude
- use inequality symbols to compare whole numbers.

#### Applications

- Money applications (see N2 and N3)
- Dewey system for libraries
- House numbers
- Arranging numbers according to magnitude,
  - eg populations
  - money, including incomes and ‘Lotto’ prizes
  - distances
  - crop yields

#### Implications and Considerations

- The concept of place value is probably the most important mathematical idea encountered by students. It must be emphasised, reinforced and continually revised.
- It should prove fruitful to present a cultural and historical view of number systems based on needs of different societies, past and present, eg Egyptian, Roman, Mayan, Aboriginal, Papuan New Guinean.
- Different cultures use different symbols,
  - eg commas, gaps as ‘thousands’ markers
  - 7 or 7, 4 or 4
  - 0 or . as the place holder.
- Consider the reading, writing and speaking of numbers,
  - eg 310: ‘three hundred ten’ (USA)
  - the unusual names of the ‘teen’ numbers
  - the value of a ‘billion’.
- Symbols such as >, =, < etc are efficient ways of ordering, arranging and comparing number.
- The technical distinction between fewer/fewest (number) and less/least (measuring) might be raised with linguistically able children.
- Other bases may be explored by some students who are interested and capable.
- The words squared and cubed can be linked with materials.

### N1.2 Expanded notation

**To be able to:**

- read and write in expanded form,
  - eg $2536 = 2 \times 1000 + 5 \times 100 + 3 \times 10 + 6 \times 1$
  - $= 2 \times 10^3 + 5 \times 10^2 + 3 \times 10 + 6 \times 1$
<table>
<thead>
<tr>
<th>Content and Skills Objectives</th>
<th>Applications</th>
<th>Implications and Considerations</th>
</tr>
</thead>
</table>
| **N1.3 Nature and structure of number lines** | - Time lines  
- Temperature  
- Credit and debit  
(See N9) | - Problem solving:  
  - How can the temperature go below zero?  
  - How do you record BC time?  
  - Is there a number between 0.1 and 0.2?  
- Some reference to numbers other than rationals may be made for some students.  
- Inequality symbols should be used in conjunction with the location of numbers on a line. |

To be able to:  
- draw a number line and position whole numbers on the line  
- recognise the position of zero and the general location of negative numbers and fractions on the line.
Guidelines N2

Non-Routine Problem Solving

Problems of the following type:

\[
\begin{array}{ccc}
\square & \square & \times \\
2 & 5 & 6
\end{array}
\]

Fill in the frames with 3 different digits.

Routine Problem Solving

Problems of the following type:

- A class of 30 is going to a picnic. Eight parents have volunteered to act as drivers, using their own cars. Discuss possible travelling arrangements.
  
  30 shared among 8

  Record \(3\text{R}6\) (NB Calculator display vs this recording method)
  
  8) 30

  Interpret 3 per car, BUT in 6 cars a 4th child must be taken.

- 30 L of petrol has been donated to share equally among these 8 parents. How much petrol should each receive? NB Acceptable answers include:

  \[3 \frac{3}{4} \text{L}\] (see fractions for equivalence concepts)

  \[3 \frac{3}{4} \text{L}\]

  \[3.75 \text{ L}\]

The first problem type has discrete data and should be recorded as shown.

The second problem type has continuous data and can be recorded as a fraction or decimal (see appropriate sections).

Calculator Usage

Use a calculator constant facility to reinforce table facts.

\[
\begin{align*}
\text{eg } & 7 \times x \times 1 \times 2 = \\
& 3 = \text{What is the calculator keying sequence for } 4 + 5 \times 6 + \frac{27}{5}?
\end{align*}
\]

Also use \(K\) facility for completing tables of values (including 'function machines').
<table>
<thead>
<tr>
<th>Content and Skills Objectives</th>
<th>Applications</th>
<th>Implications and Considerations</th>
</tr>
</thead>
</table>
| **N2.1 Addition, subtraction and multiplication of whole numbers** | • Money problems using whole dollars or cents only  
• Measurement  
• Statistics (See S4) | • Alternative language for the same operation should be met,  
eg ‘Seven minus three’  
‘Seven take away three’  
‘What is three less than seven?’  
‘What is the difference between seven and three?’ |
| To be able to: |  | Words such as difference and product have everyday meaning as well as specific formal mathematical usage. |
| • Perform routine addition, subtraction and multiplication operations using written and calculator techniques |  | Appropriate number laws should be investigated only as an aid to computation,  
eg 3 x 98 is 3 lots of 100 less 3 lots of 2  
That is 300 - 6. Why?  
25 x 137 x 4 is 100 times 137. Why? |
| • Give automatic responses to single digit addition and subtraction facts (combinations) |  |  |
| • Instantly recall multiplication facts up to 10 x 10 |  |  |
| • Estimate results, being alert to unreasonable answers. |  |  |
| **N2.2 The division process as:** | • ‘Sharing’ problems from students’ lives, eg equipment, drink, food, transport, money  
• Averages |  |
| Repeated subtraction  
Sharing  
The opposite of multiplication  
Fraction operators  
\( \frac{1}{2} \) of, \( \frac{1}{3} \) x etc for whole numbers |  |  |
<p>| To be able to: |  |  |
| • divide by single digit and two digit numbers with no remainder, and remainder |  |  |
| • divide by large numbers using a calculator. |  |  |</p>
<table>
<thead>
<tr>
<th>Content and Skills Objectives</th>
<th>Applications</th>
<th>Implications and Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>N2.3 Order of operations. Use of grouping symbols.</strong></td>
<td>- Stocktaking and inventories: packets, cartons, boxes etc, with known content quantities</td>
<td>- Not all calculators have inbuilt algebraic logic.</td>
</tr>
</tbody>
</table>

**To be able to:**

- simplify expressions requiring correct order of operations including those with grouping symbols
- insert grouping symbols to change the value of an expression.
Guidelines N3

Concrete And Recorded Forms Of A Decimal Number

<table>
<thead>
<tr>
<th>Units</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

Some money activities:
- A mock shop or supermarket might be set up so that students can:
  estimate shopping bills
  check that they have sufficient cash
  calculate totals
  make change.
- A comparative shopping excursion might also be possible.
- When does an amount like $1.346 occur?
- Who gets the advantage when the value is rounded down (truncated) to $1.34?
- The class could discuss the classic computer crime of diverting parts of 1 cent
  into another account.

Money Manipulation

- Coins:
  - $1
  - 10c
  - 1c

$3.26 is represented as:

```
$1  $1  $1  10c  10c
  1c  1c  1c
  1c  1c  1c
```

3 . 2 6

- Trading $1 for 10c 10c 10c 10c 10c

Could be used to re-inforce that
10 x $0.10 = $1.00
10 x 0.1 = 1.0 etc
<table>
<thead>
<tr>
<th>Content and Skills Objectives</th>
<th>Applications</th>
<th>Implications and Considerations</th>
</tr>
</thead>
</table>
| **N3.1 Place Value**        | - Cash registers  
- Speedometers/odometers  
- Display on gas/electricity meters | - It is important, at this stage, to link decimal place value names with those established in N1.1, eg 'hundreds', 'hundredths'.  
- The 'European comma' is often used as a decimal marker, eg 3,57 to mean 'three point five seven'.  
- The number of decimal places maybe fixed on a calculator.  
- Money can be represented in different ways, eg 56¢ = $0.56.  
- The distinction between 'metre' and 'meter' should be made.  
- Many students have difficulty in comparing 7.09 and 7.1 (for example).  
- Materials and calculators can be used to strengthen comparison skills. | |
| To be able to:               |              | - Examples should be given, requiring students to line up decimal points in vertical recording form. |
| - read, write and interpret numbers written in decimal form  
- interpret calculator displays, eg 134.6 could mean $134.60 or $1.35 depending on given problem  
- compare and order decimals using inequality notation. |              | |
| **N3.2 Addition and subtraction of decimals** | - Money applications:  
  shopping bills, change costs and payments (eg mortgage), budgeting estimates  
  Consumer arithmetic applications such as comparative shopping  
  Application to perimeter (see M1.2) |              |
| To be able to:               |              | - There is potential for monetary gain by rounding down values such as $12.367.  
- Situations exist where rounding up is necessary, eg if 4.6 cans of paint are needed, then 5 must be bought.  
- Some calculators have an automatic rounding facility. |
| - use pen and paper, and calculator methods for adding and subtracting decimals. |              | |
| **N3.3 Multiplication and rounding off** | - One and two-step problem applications to reinforce skills,  
  eg 5 m @ $1.36/m  
  5.3 kg @ $2.67/kg (rounding)  
  One dozen oranges @ 18¢ each |              |
<p>| To be able to:               |              | |</p>
<table>
<thead>
<tr>
<th>Content and Skills Objectives</th>
<th>Applications</th>
<th>Implications and Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>N3.4 Division and rounding off</strong></td>
<td>- Jobs, flat rates, eg How long does a person earning $3.65/h take to earn $60?</td>
<td>- Ensure that sex and race stereotyping are avoided: both boys and girls should be portrayed in a variety of occupations and life-style situations.</td>
</tr>
<tr>
<td>To be able to:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- use pen and paper methods for simple division</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- use calculator methods for division, interpreting displays and rounding appropriately.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Guidelines N4

This section should be read in conjunction with the early part of the Algebra strand.

Figurate Numbers
(built with materials)

eg  Square Numbers


\[
\begin{array}{ccc}
1 & 4 & 9 \\
\end{array}
\]

Notation \( S_1 = 1, \ S_2 = 4, \ S_3 = 9, \ldots \)

(Can the students discover \( S_n \)?)

Odd Numbers


\[
\begin{array}{ccc}
1 & 3 & 5 \\
\end{array}
\]

\( 0_1 = 1, \ 0_2 = 3, \ 0_3 = 5, \ 0_4 = 7 \)

(Can the students discover \( O_n \)?)

Adding Square And Odd Numbers

\[
\begin{align*}
1 + 3 &= 4 \\
4 + 5 &= 9 \\
9 + 7 &= 16 \quad \text{etc}
\end{align*}
\]

These should be investigated, built, discussed and recorded. Better students may offer generalisations and reasons for the patterns, and the relationship between square and odd numbers.

Palindromic Numbers

A palindrome reads the same when reversed:

\begin{itemize}
  \item NOON, MADAM, TUMUT, GLENELG, etc.
  \item A palindromic sentence reads the same when reversed:
    \begin{itemize}
      \item A man, a plan, a canal, Panama!
      \item Able was I ere I saw Elba (who said this?)
      \item No x in Mr R M Nixon.
    \end{itemize}
  \item A palindromic number reads the same when the digits are reversed:
    \begin{itemize}
      \item 121; 13 031; 726 542 787 245 627
    \end{itemize}
\end{itemize}

A number which is not palindromic (eg 59) can be reversed and the numbers added, the process being repeated being repeated until a palindromic number is reached.

Further explorations (eg counting reversals, trying to find numbers which won't work, selecting 3 digit numbers, etc) can be motivating and rewarding.

A Problem Generating A Number Pattern

If students are allowed to take one or two steps at a time up a staircase, in how many ways can they climb up a staircase, in how many ways can they climb a flight of 10 steps?

Complete the table, seeking patterns in rows, columns, diagonals, etc.

<table>
<thead>
<tr>
<th>Number</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Square</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>Pentagonal</td>
<td>1</td>
<td>5</td>
<td>12</td>
<td>22</td>
</tr>
<tr>
<td>Hexagonal</td>
<td>1</td>
<td>6</td>
<td>15</td>
<td>28</td>
</tr>
<tr>
<td>Content and Skills Objectives</td>
<td>Applications</td>
<td>Implications and Considerations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td>--------------</td>
<td>---------------------------------</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| **N4.1 Positive whole numbers classified into special groups** | - Fibonacci numbers occur in nature  
- Triangular numbers can be represented by snooker/pool balls in the ‘triangle’; ten-pin bowling; pharmacist’s pill-tray  
- Even/odd numbers on either side of streets | - Students should be aware of some of the historical and environmental background to number patterns.  
- Palindromic numbers are closely linked to palindromes and palindromic sentences.  
- Some students might explore the more ‘exotic’ patterns such as amicable and perfect numbers.  
- Rules for number patterns can be informal, eg ‘add the previous two numbers’.  
- Number patterns extend naturally to algebraic concepts.  
- Once patterns are established the calculator can be used to explore and extend various patterns.  
- The history and achievements of Eratosthenes (including the ‘sieve’) will interest many students.  
- The sieve works best when numbers are structured in rows of 6. |
| To be able to: |  
- distinguish between odd and even numbers  
- recognise and represent figurate (triangular, square, etc); palindromic numbers and Fibonacci numbers; tetrahedral, cubic etc  
- extend number patterns such as  
  \[1, 4, 7, \ldots ; 30, 15, 7_{\frac{1}{2}}, 3_{\frac{3}{4}}, \ldots ; 1, 8, 27, 64, \ldots ; 2, 3, 5, 7, 11, \ldots \]  
- recognise rules from number patterns  
- create number patterns from rules. |  |
| **N4.2 Multiples, factors and index notation** | - Calculator facilities such as constant operators and memories could be utilised |  |
| To be able to: |  
- list multiples of a given number  
- classify numbers as composite or prime  
- determine and list factors of a number  
- use index notation to find HCF and LCM. |  |
| **N4.3 Square and cube roots** |  
- estimate and find (where possible) square roots and cube roots  
- use \(\sqrt{\cdot}\) and \(\sqrt[3]{\cdot}\) notation. |  
- The meaning (and possibly the derivation) of the ‘radical’ signs may give an interesting historical perspective to students. The link between \(2^3 = 8\) and \(\frac{1}{3} \times 8 = 2\) should be explained.  
- Students should be aware of \(\sqrt[3]{\cdot}\), \(x^\frac{1}{3}\), \(x^\frac{1}{2}\) and other relevant calculator keys.  
- The square root sign signifies a positive (or zero) number. Thus \(\sqrt{9} = 3\) (only) however, the two numbers whose square is 9 are \(\sqrt{9}\) and \(-\sqrt{9}\), ie 3 and -3. |
<table>
<thead>
<tr>
<th>Content and Skills Objectives</th>
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<th>Implications and Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td>N4.4 Divisibility tests</td>
<td>- Students should be encouraged to create patterns on a hundred square to discover simple tests of divisibility and on Pascal's triangle. Informal descriptions of the patterns (e.g. 'diagonal stripes coloured in') should be discussed and the reasons explored.</td>
<td></td>
</tr>
</tbody>
</table>
Guidelines N5

Fractions should be represented concretely and visually using examples from life and the mathematical strands:

- 'half an apple'
- Region Models (Geometry based)

Various fractions of known shapes are a good link with the geometry strand:

- 'a third of the class'
- Set Models (Number based)

This representation can cause confusion as the 'third' still has nine members. The whole (e.g., class, group, box, etc.) must be emphasised, so that part of the whole can then be investigated.

Length Models (Measurement based)

'Cuisenaire material is very useful for demonstrating equivalent fractions:'

<table>
<thead>
<tr>
<th>orange</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>halves</td>
<td></td>
</tr>
<tr>
<td>fifths</td>
<td></td>
</tr>
<tr>
<td>tenths</td>
<td></td>
</tr>
</tbody>
</table>

Tangrams And Pattern Blocks

These materials are excellent for establishing and reinforcing fraction concepts and skill with equivalence.

Pattern blocks allow the manipulation of halves, thirds, and mixed numbers.

As they also demonstrate sixths and twelfths concretely and visually, these denominators should not be totally avoided.
### Content and Skills Objectives

**N5.1 Concept and representation of fractions as parts of a whole:**
- Parts of whole real objects
- Parts of shapes or regions
- Parts of a measurement

To be able to:
- recognise fractions shaded

![Fraction Diagrams](image)

- find $\frac{1}{2}$ or $\frac{1}{3}$ of a quantity
- recognize fractional parts as being equal.

### Applications

- Division of a circle into 360 equal parts (degrees)
- Time division based on sevenths, 24ths, 60ths, etc
- Guitar fret positions
- Graduated gauges (e.g., petrol)

### Implications and Considerations

- It is important that students recognise that fractions infer the division process. Three quarters ($3 \div 4$) could, for example, be explored on a calculator.
- Students should be encouraged to use ‘conversational’ terms (such as halves, thirds) in conjunction with objects, shapes and informal measurements, e.g., ‘half a string length’, ‘an orange quarter’, ‘three-quarter time’.
- Formal and informal terms can be linked.

### N5.2 Components of a fraction

To be able to:
- distinguish between numerator and denominator.
N5.3 Equivalence of fractions

To be able to:
- generate equivalent fractions, especially those with denominators of 10 and 100
- convert fractions to decimals (include terminating and repeating decimals).

Applications
- Fractions with denominators of 100 apply directly to percentages. Those with other powers of 10 apply to decimals (See N3 and N7).

Implications and Considerations
- Students should realise that any fraction can be expressed in an infinite number of forms. Multiplication by various forms of unity (\(\frac{1}{2}, \frac{1}{3}, \frac{4}{7}\), etc) should be strongly emphasised as this is essential for operations work.

N5.4 Ordering fractions, including mixed numbers

To be able to:
- arrange fractions in order of magnitude using the number line, decimal conversions and inequalities.

Applications
- Shutter speeds in photography
- Practical measurement requiring accuracy

Implications and Considerations
- The ordinal names third, fifth, sixth etc are the same as the fractional names.
- The fact that a third is larger than a fifth therefore requires thorough development and discussion (3 > 5 but \(\frac{1}{3} < \frac{1}{5}\)).

N5.5 Use of fractions as a way of expressing the remainder in division

To be able to:
- record and interpret remainders in division.  
  eg  \(105 \div 20 = \frac{105}{20} = 5 \frac{5}{20} = 5 \frac{1}{4} = 5.25\)

Applications
- Measurement and money situations where remainders occur (See N2.2)

Implications and Considerations
- The context of problems and applications is important: measurement situations with sensible, familiar denominators (2, 3, 4, 5, 10 etc) should occur, eg \(\frac{1}{2}\) kg, \(\frac{1}{1000}\) s, \(\frac{1}{4}\) km etc.
Guidelines N6

- The models and materials used in N5 can also be used to demonstrate fraction operations.
  
  ```
  \begin{align*}
  \frac{1}{4} + \frac{1}{4} & = \frac{3}{4} \\
  \frac{1}{2} + \frac{1}{2} & = \frac{3}{2} \\
  1 - \frac{1}{2} & = \frac{1}{2} \\
  \end{align*}
  ```

  **Measurement Model (using number lines)**

  - Examples such as $13 \frac{1}{3} - 1 \frac{1}{3}$ are very cumbersome by improper fractions.
  - Another approach is $13 \frac{1}{3} - 1 \frac{1}{3} = 13 \frac{1}{3} - 1 - \frac{1}{3} = 12 \frac{1}{3} - \frac{1}{3} = 11 + 1 \frac{1}{3} - \frac{1}{3} = 11 + \frac{5}{3} - \frac{1}{3} = 11 + \frac{12}{10} - \frac{5}{10} = 11 + \frac{7}{10} = 11 \frac{7}{10}$

  (There is potential to use fewer steps).

**Pattern Blocks**

- $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

- $1\frac{1}{2} + 2\frac{1}{2} = 3\frac{3}{4}$ (can be broken down into extra steps by shorter jumps).

‘Pop stick’ calculators can assist the concept of equivalence and operation facility:

Prepare the sticks by writing numerals, spaced by 1 cm.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

1-stick

<table>
<thead>
<tr>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
</tr>
</tbody>
</table>

2-stick

<table>
<thead>
<tr>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
<th>18</th>
<th>21</th>
<th>24</th>
<th>27</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
<td>24</td>
<td>27</td>
<td>30</td>
</tr>
</tbody>
</table>

3-stick

<table>
<thead>
<tr>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
<th>28</th>
<th>32</th>
<th>36</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
<td>28</td>
<td>32</td>
<td>36</td>
<td>40</td>
</tr>
</tbody>
</table>

4-stick

This activity, in itself, is a good revision of table work, factors and measurement.

- Now make $\frac{1}{2} + \frac{3}{4}$
- $1 \frac{2}{3} + \frac{1}{2}$
- $1 \frac{3}{4}$
- $1 \frac{1}{2}$

\begin{align*}
\frac{4}{8} + \frac{6}{8} & = \frac{10}{8} \\
& = 1\frac{1}{4} \\
& = 1\frac{1}{4}
\end{align*}
Content and Skills Objectives

N6.1 Addition and subtraction of fractions

To be able to:

- perform operations using denominators 2, 3, 4, 5 and 10 such as:

\[ \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2} \]
\[ \frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4} \]
\[ \frac{3}{5} + \frac{1}{2} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10} = 1\frac{1}{10} \]
\[ \frac{4}{5} - \frac{1}{2} = \frac{8}{10} - \frac{5}{10} = \frac{3}{10} \]
\[ 13\frac{3}{5} - 1\frac{1}{2} = 11\frac{7}{10} \]

by methods appropriate to the student's ability.

N6.2 Multiplication of fractions

To be able to:

- perform operations such as:

\[ 4 \times \frac{1}{2} = 2 \]
\[ \frac{3}{5} \times 6 = \frac{18}{5} \]
\[ \frac{3}{5} \times \frac{3}{4} = \frac{9}{20} \]
\[ 2\frac{1}{3} \times 1\frac{1}{3} = 3\frac{4}{9} \]

N6.3 Division involving fractions

To be able to:

- perform operations such as:

\[ 4 + \frac{1}{2} = 8 \]
\[ \frac{1}{2} + \frac{1}{3} = 1 \]
\[ \frac{3}{4} + \frac{1}{2} = 1\frac{1}{2} \]

Applications

- Addition and subtraction of time in payroll calculations, e.g. \( 3\frac{1}{2} + 7\frac{1}{2} + 1\frac{1}{2} \) hours
- Recipes with fractions of quantities (cups, teaspoons, etc)

Implications and Considerations

- Due to metrical and calculators, operations with fractions are of less importance and significance than in the past. However, some industries still use imperial units for their machinery.
- Sex-stereotyped examples for cooking and trades should be avoided.

- Time and money in overtime situations (e.g. \( 3\frac{1}{2} \) hours at 'time and a half')
- Measurement applications such as the area of a rectangle \( 2\frac{1}{2} \) units by \( 1\frac{1}{4} \) units
- The reciprocal calculator key might be used by some students.
- The concept of reduction by a common factor in the numerator and the denominator is essential (students should not just cross out or 'cancel').
- Cooking for a number of people other than the number specified in the recipe
- The 'overdrilling' of fraction operations should be avoided. One purpose of topic N6 is to prepare students for comparable algebraic operations.
<table>
<thead>
<tr>
<th>Content and Skills Objectives</th>
<th>Applications</th>
<th>Implications and Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>N6.4 Fractions of quantities</strong></td>
<td>- Grocery shopping</td>
<td></td>
</tr>
<tr>
<td>To be able to:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• find a fraction of a quantity,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>eg $\frac{1}{2}$ of 1 kg, $\frac{7}{10}$ of 2 L etc</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{3}{5}$ of 2 km etc</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• express one quantity as a fraction of another, eg what fraction is 10 cm of 1 m?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Guidelines N7

- The emphasis of this topic should be realism from the students' environment: the home, town or city, school, family and friends, sport and leisure, the media, etc.

- Encouragement to find percentages in everyday life will lead the class, class groups and individual students to sources such as:
  - local industry
  - shops and supermarkets
  - the bank, or school banking service
  - an agent (stock and station, TAB, newsagent)
  - the council (municipal or shire)
  - containers of food and drink.

The following example types are readily available:

How The Grounds Compare

First-class matches 1984-85

<table>
<thead>
<tr>
<th></th>
<th>Runs</th>
<th>Wkts</th>
<th>Avg runs per wkt</th>
<th>Wkts by fast blrs</th>
<th>Wkts by slow blrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sydney (all matches)</td>
<td>3397</td>
<td>133</td>
<td>25.5</td>
<td>37.6%</td>
<td>60.2%</td>
</tr>
<tr>
<td>Syd (NSW batting)</td>
<td>1377</td>
<td>44</td>
<td>31.3</td>
<td>45.5%</td>
<td>47.7%</td>
</tr>
<tr>
<td>Syd (NSW bowling)</td>
<td>1133</td>
<td>60</td>
<td>18.9</td>
<td>23.3%</td>
<td>68.3%</td>
</tr>
<tr>
<td>Melbourne</td>
<td>5360</td>
<td>125</td>
<td>42.9</td>
<td>67.2%</td>
<td>29.6%</td>
</tr>
<tr>
<td>Perth</td>
<td>7894</td>
<td>245</td>
<td>32.2</td>
<td>79.2%</td>
<td>14.6%</td>
</tr>
<tr>
<td>Adelaide</td>
<td>6508</td>
<td>198</td>
<td>32.9</td>
<td>62.1%</td>
<td>30.9%</td>
</tr>
<tr>
<td>Brisbane</td>
<td>6305</td>
<td>185</td>
<td>34.1</td>
<td>80.0%</td>
<td>16.8%</td>
</tr>
</tbody>
</table>

Statistics compiled by Ross Dundas

Source: Sydney Morning Herald
<table>
<thead>
<tr>
<th>Content and Skills Objectives</th>
<th>Applications</th>
<th>Implications and Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>N7.1 Percentages - concept and notation</strong></td>
<td>• Discounts in shops and media advertisements</td>
<td>• Derivation of % symbol.</td>
</tr>
<tr>
<td>To be able to:</td>
<td>• Profit and loss (as percentages of cost or selling price)</td>
<td>• There is tremendous scope in this topic: care should be taken not to overemphasise percentage applications and examples should be relatively simple.</td>
</tr>
<tr>
<td>• express percentages as a fraction out of 100</td>
<td>• Commission, government takings from Lotto, TAB etc</td>
<td>• Pen and paper, mental and calculator methods will all be applicable (eg 10% mentally, $\frac{8}{4}$% on a calculator).</td>
</tr>
<tr>
<td>• express percentages as a decimal</td>
<td></td>
<td>• See Statistics strand for a pictorial representation of percentages.</td>
</tr>
<tr>
<td>• convert from a simple fraction or decimal to a percentage</td>
<td></td>
<td>• The calculator technique of increasing and decreasing by percentages should be considered:</td>
</tr>
<tr>
<td>eg convert $\frac{2}{5}$ to a percentage</td>
<td></td>
<td>- $1.05$ will increase by 5%</td>
</tr>
<tr>
<td>convert $0.85$ to a percentage</td>
<td></td>
<td>- $0.87$ will decrease by 13%.</td>
</tr>
<tr>
<td><strong>N7.2 Percentage composition</strong></td>
<td>• Simple interest</td>
<td></td>
</tr>
<tr>
<td>To be able to:</td>
<td>• Ingredients (as printed on containers)</td>
<td></td>
</tr>
<tr>
<td>• find a percentage of a given quantity, eg find 20% of</td>
<td>• Time and motion studies (percentage time on given tasks)</td>
<td></td>
</tr>
<tr>
<td>1 kg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• compare like quantities (money, time, length, mass,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>capacity) by converting to a percentage, eg what</td>
<td></td>
<td></td>
</tr>
<tr>
<td>percentage is $15$ of $300$?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• recognise and evaluate media reports and advertising</td>
<td></td>
<td></td>
</tr>
<tr>
<td>claims which use percentage.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>N7.3 Percentage change</strong></td>
<td>• Tare weight examples (eg truck overloaded by 20%)</td>
<td></td>
</tr>
<tr>
<td>To be able to:</td>
<td>• Wastage (eg timber off-cuts as a percentage, food wastage, etc)</td>
<td></td>
</tr>
<tr>
<td>• increase or decrease a quantity by a given percentage,</td>
<td>• Inflation and economic indicators (CPI etc)</td>
<td></td>
</tr>
<tr>
<td>eg increase $200$ by 5%</td>
<td>• Consumer decisions based on savings expressed as percentages (eg &quot;third off&quot; compared with 30% discount)</td>
<td></td>
</tr>
<tr>
<td>decrease $3t$ by 22%.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Guidelines N8

- As with topic N7, there is an abundance of real life situations which enable relevant approaches to this topic.
  
  eg A couple sharing a flat might share costs in the ratio 3 : 2 because of the relative sizes of their rooms.
  
  Rent, electricity charges and the like can be divided into this ratio.

Enlargements and scale drawings using given ratios can be a practical, enjoyable topic for students choosing a known location - home, school, town, locality, farm, etc makes these activities relevant.

Rates occur in daily life and provide opportunities for short excursions, the use of community resources and the pictorial representation of collected data.

Postal, telegram and telephone rates could be shown on 'step graphs' and the use of these graphs discussed.

Students should be given access to rate notices for land and water rates as well as to gas and electricity bills.

Get students to discuss the formula for a bowler's 'strike rate' which is shown on television.
<table>
<thead>
<tr>
<th>Content and Skills Objectives</th>
<th>Applications</th>
<th>Implications and Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>N8.1 Ratios in various notational forms</strong></td>
<td>- Scale drawing and enlargements - including maps, plans, etc</td>
<td>- The language link between ‘rational’, ‘ratio’ and ‘rate’ should be discussed.</td>
</tr>
<tr>
<td>To be able to:</td>
<td>- Mixes - fuel, concrete, fertilizer</td>
<td>- Slopes and grades are a forerunner to later work on gradients.</td>
</tr>
<tr>
<td>- recognise 6/4, 6:4, 6 to 4, 3:2:1 (extended ratio).</td>
<td>- Gear ratios</td>
<td>- Students sometimes confuse order which is vital in ratio work.</td>
</tr>
<tr>
<td><strong>N8.2 Equivalent ratios</strong></td>
<td>- Slopes of hills</td>
<td></td>
</tr>
<tr>
<td>To be able to:</td>
<td>- Railway grades</td>
<td></td>
</tr>
<tr>
<td>- write ratios in simplest form, eg 4 : 6 = 2 : 3</td>
<td>- Betting odds</td>
<td></td>
</tr>
<tr>
<td>- Unit pricing in shopping</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>N8.3 Unitary method</strong></td>
<td>- Sharing applications (money, football cards, marbles, stocks and shares)</td>
<td>- The translation from words to mathematical language is a vital part of this section.</td>
</tr>
<tr>
<td>To be able to:</td>
<td>- Dividing lines into a given ratio</td>
<td></td>
</tr>
<tr>
<td>- apply the unitary method to ratio problems.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>N8.4 Dividing quantities into given ratios</strong></td>
<td>- Land and water rates</td>
<td>- Comprehension skills in the English language need strong emphasis.</td>
</tr>
<tr>
<td>To be able to:</td>
<td>- Batting and bowling strike rates</td>
<td></td>
</tr>
<tr>
<td>- solve sharing problems for given ratios.</td>
<td>- Postal and telephone rates</td>
<td></td>
</tr>
<tr>
<td><strong>N8.5 Rates</strong></td>
<td>- Speed</td>
<td>- A clear distinction between ratio and rates needs to be made (concentration on like and unlike units).</td>
</tr>
<tr>
<td>To be able to:</td>
<td>- Consumption (eg fuel in L/100 km)</td>
<td>- At this level, it is appropriate to use constant rates only.</td>
</tr>
<tr>
<td>- calculate rates from given information, eg 150 km travelled in 2 h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- apply rates to problem solving</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- convert rates from one set of units to another (eg km/h to m/s).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mathematics Syllabus Years 7-8
Guidelines N9

- This topic attempts to tie up some "loose-ends" as well as to open up new horizons. Students should now be able to work confidently with a number line and number plane extended into negative numbers,

  eg

  ![Point plotting and joining to form shapes and drawings](image)

  locating on the number line: -1, \( \sqrt{2} \), -2.6, and other directed rational numbers.

- There are many favourite ways of modelling directed numbers with a real situation. Students should be asked to describe a story which demonstrates 3 - 4, -2 + 1, -3 -4, etc.

  eg "I walked in from lunch and went down 3 levels to my car in the underground carpark to collect some money. I then went up 7 levels in the lift. On which floor do I work?" \(-3 + 7 = ?\) This story should be demonstrated concretely and visually.

- The problem of two negatives should be modelled both in subtraction and multiplication contexts:

  eg In the mail, the postman delivers a cheque for $5 and a bill for $3, ie 5 + -3. This situation leads to the recording 5 + -3 = 5 - 3 = 2 (ie you are $2 in front). The postman however is mistaken and takes back the bill, ie 2 - -3. This situation leads to the recording 2 - (-3) = 2 + 3 = 5 (ie you are back to $5 in front).

- Multiplication might be demonstrated by a bank balance analogy:

<table>
<thead>
<tr>
<th>Situation</th>
<th>Weekly deposit</th>
<th>Time</th>
<th>Balance compared with now</th>
<th>Suggested number sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>If I deposit $2 every week, in 4 weeks time bank balance will be $8 more than now.</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>2 \times 4 = 8</td>
</tr>
<tr>
<td>The balance three weeks ago was $6 less, if I deposit $2 per week.</td>
<td>2</td>
<td>-3</td>
<td>-6</td>
<td>2 \times -3 = -6</td>
</tr>
<tr>
<td>If I withdraw $3 every week, in 4 weeks time the balance will be $12 less.</td>
<td>-3</td>
<td>4</td>
<td>-12</td>
<td>-3 \times 4 = 6</td>
</tr>
<tr>
<td>The balance two weeks ago was $6 more if I withdraw $3 per week.</td>
<td>-3</td>
<td>-2</td>
<td>6</td>
<td>-3 \times -2 = 6</td>
</tr>
<tr>
<td>Content and Skills Objectives</td>
<td>Applications</td>
<td>Implications and Considerations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td>--------------</td>
<td>---------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>N9.1 Directed number</strong></td>
<td>• East/West, North/South lines, up/down scales (e.g. lifts)</td>
<td>• Complex recording formats of directed numbers can be confusing (e.g. raised signs).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>To be able to:</td>
<td>• Money transactions (e.g. banking, gambling, etc)</td>
<td>• Formats such as</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• recognise directed number as having both direction and magnitude</td>
<td>• Time before and after an event</td>
<td>-2 - 3 = -5  <em>Students should construct stories explaining these number sentences.</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• order directed numbers on a number line</td>
<td>• Expenditure and income</td>
<td>-3 + 6 = 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• add and subtract directed numbers</td>
<td>• Temperature</td>
<td>-3 + (-4) = -3 - 4 = -7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• multiply and divide directed numbers</td>
<td>• Profit and loss</td>
<td>-2 - (-3) = -2 + 3 = 1 and</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-3.25 + 6.83 = 3.58 (e.g. money) are recommended.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>N9.2 Ordered pairs on the number plane</strong></td>
<td>• Tides: rise and fall</td>
<td>• The +/- calculator key is a good way to change signs in a directed number situation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>To be able to:</td>
<td>• Reading maps, plans, street directories, theatre seating</td>
<td>• Concepts such as -7 &lt; -6 (although 7 &gt; 6) can be demonstrated on a number line. Formal inequalities or truth/solution sets are not required.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• locate positions on grids</td>
<td>• Latitude and longitude</td>
<td>• Note different recording conventions: 25°, 15°E; (3, 5); A2; military formats, etc.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• read and plot ordered pairs on number planes.</td>
<td></td>
<td>• Whilst alternative grid systems may be used in early experiences, it is intended that the standard rectangular grid systems be established.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Algebra

The algebra sequence has been greatly influenced by recent research. This research has identified a number of different meanings that can be given to pronumerals in generalised arithmetic. Two important meanings identified are related to (i) variables, as in expressions and (ii) unknowns, as in equations.

Mathematics teaching is often seen as the initiation of students into rules and procedures. Though powerful, these rules and procedures are often meaningless to students and do not take into account their methods and levels of understanding. Students need gradual exposure to abstract ideas as they begin to relate algebraic terms to real situations. Algebra is best learned by progressing from experience with concrete materials through oral language to written language and then to symbolic representation.

To gain an understanding of algebra the student must be introduced to the concepts of patterns, relationships, variables, expressions, unknowns, equations and graphs in a wide variety of contexts. For each successive context, these ideas need to be redeveloped. As understanding increases, redevelopment should take less time.

Research supports the use of approaches of the following types over an extended period of time:

- Manipulation of concrete materials - leading to recognition of number pattern - leading to verbalisation of a rule - leading to symbolic representation of the rule.
- Number pattern - leading to modelling in concrete materials of this pattern - leading to verbalisation of a rule - leading to symbolic representation of the rule.
Guidelines A1

Generalisations from geometric and number patterns.

Repeated experiences of the following types need to be given in the early development of algebra.

(a) Use concrete materials such as matches, centicubes, pattern blocks, etc to build geometric patterns.

(b) Students identify counting patterns which give the number of objects (eg matches) in the growing geometric pattern being built. They describe these patterns in their own words,

eg if students build a line of squares starting with one match:

```
  _ _ _ _
```

they could identify the pattern as:

‘The number of matches needed is ...
one plus three times the number of squares’

OR ‘one plus the number of squares times three’

OR . . .

Note: The skill of forming generalisations can be developed by encouraging students to identify different counting patterns for the one geometric pattern,

eg for the line of squares, other possibilities (for describing the number of matches needed) are:

• ‘four plus three times one less than the number of squares’ (if starting from one complete square)

• ‘four times the number of squares minus one less than the number of squares’ (if separate squares are pushed together and unwanted matches are removed).

Extension: More than one aspect of a geometric pattern may be considered, eg perimeter, area, number of corners, etc.

(c) Students form an initial table of values, eg if a line of squares is being built,

(i) starting from one match

(ii) starting from one square,

the table could be:

<table>
<thead>
<tr>
<th>number of squares</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of matches (i)</td>
<td>$1 + 3 = 4$</td>
<td>$1 + 3 + 3 = 7$</td>
<td>$1 + 3 + 3 + 3 = 10$</td>
</tr>
<tr>
<td>or</td>
<td>$1 + 3 \times 1 = 4$</td>
<td>$1 + 3 \times 2 = 7$</td>
<td>$1 + 3 \times 3 = 10$</td>
</tr>
<tr>
<td>or</td>
<td>$1 + 1 \times 3 = 4$</td>
<td>$1 + 2 \times 3 = 7$</td>
<td>$1 + 3 \times 3 = 10$</td>
</tr>
<tr>
<td>or</td>
<td>. . .</td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of matches (ii)</td>
<td>4</td>
<td>$4 + 3 \times 1 = 7$</td>
<td>$4 + 3 \times 2 = 10$</td>
</tr>
</tbody>
</table>

(d) Students recognise relationships in the table of values and extend the table to include cases which would be impractical to build, basing their calculations on their own verbal description of the pattern,

eg for 102 squares, method (i) would lead to $1 + 3 \times 102 = 307$

OR $1 + 102 \times 3 = 307$

and method (ii) would lead to $4 + 3 \times 101 = 307$

(e) Similarly, number patterns may be used as sources for verbal generalisations. Emphasis should be given to encouraging students to describe how they can obtain one term from earlier terms,

eg $1, 3, 5, 7, 9, ...$

‘you keep adding two to get the next number’

$1, 1 + 2, 1 + 2 + 2, 1 + 2 + 2 + 2$

OR $1, 1 + 2 \times 1, 1 + 2 \times 2, 1 + 2 \times 3$

OR . . .

Build the pattern with concrete materials or in diagram form (counters, pen and paper, etc).

```
  . . . . .
```

$0 \times 2 + 1, 1 \times 2 + 1, 2 \times 2 + 1, 3 \times 2 + 1, ...$

OR $2 \times 0 + 1, 2 \times 1 + 1, 2 \times 2 + 1, 2 \times 3 + 1, ...$

OR . . .
<table>
<thead>
<tr>
<th>Content and Skills Objectives</th>
<th>Applications</th>
<th>Implications and Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A1 Algebra Without Symbols</strong></td>
<td>• Use concrete materials to lead to number patterns</td>
<td>• Learners should be given opportunities to discover and create patterns and to describe, in their own words, relationships contained in those patterns.</td>
</tr>
<tr>
<td>Generalisations from geometric patterns and number patterns.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Patterns existing in numbers and generated and observed from properties of geometric patterns.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Language to describe geometric and number patterns.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>To be able to:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• build and extend geometric patterns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• form tables of values of counting patterns based on geometric patterns</td>
<td></td>
<td></td>
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<tr>
<td>• use geometric patterns to generate number patterns</td>
<td></td>
<td></td>
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<tr>
<td>• build and extend number patterns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• use number patterns to build patterns with concrete materials</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• explain/describe patterns in language that peers and teachers could understand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• recognise that there are different ways to describe the same geometric or number pattern</td>
<td></td>
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<tr>
<td>• find different ways to describe a given number pattern or geometric pattern</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• find particular terms of a pattern given a description by a peer and vice versa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• use a written language to describe geometric and number patterns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• interpret written sentences by peers and teachers which accurately describe geometric and number patterns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• express, in different ways, written sentences which describe geometric and number patterns.</td>
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</tbody>
</table>
The Language of Algebra.

(a) Language translation: English to algebra

Early treatment of algebraic symbols could flow from language used to describe counting patterns experienced in A1, e.g. “The number of matches I need to make a line of squares is one plus y times three” (where y = number of squares).

It may be found with some students that it is better to introduce just one ‘letter’ at a time, e.g., use ‘Number of matches = 1 + y x 3’ (or 1 + 3 x y, or 1 + 3y) before using ‘n = 1 + y x 3’ (where n = number of matches)

Using generalisations about geometric patterns (as in A1) may give rise to algebraic expressions that are very complicated. Hence it may be helpful for students to experience a range of concrete representations for algebraic expressions such as those which follow.

(b) Concrete representations for linear (ie first degree) algebraic expressions.

Students could use wooden rods or centicubes to represent an expression such as \(2b + 4\) as an area:

This area of 14 cm\(^2\) represents \(2b + 4\) with \(b = 5\).

This area of 20 cm\(^2\) represents \(2b + 4\) with \(b = 8\).

Guidelines A2

Other representations could use variable numbers of centimetres of length, or variable numbers of similar objects hidden in similar containers,

eg If \(\bigcirc\) represents 1 marble and \(\mathbb{T}\) represents \(b\) marbles in a container, then

represents \(2b + 4\) marbles, or 14 marbles if 5 marbles are in each container for \(b = 5\).

represents 2 lots of \((b + 2)\) or \(2(b + 2)\), or 20 marbles for \(b = 8\).
<table>
<thead>
<tr>
<th>Content and Skills Objectives</th>
<th>Applications</th>
<th>Implications and Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A2 The Language Of Algebra</strong></td>
<td>• Use situations from other strands of the syllabus (eg measuring areas, lengths, perimeters and using ideas in section N4) &lt;br&gt;• Establish rules in words or symbols that describe practical situations (eg models of number patterns in A1)</td>
<td>• Learners should be encouraged to use language and symbols appropriate to their particular stage of development. &lt;br&gt;• Using generalisations about geometric patterns (as in A1) may give rise to algebraic expressions that are very complicated. Hence it may be helpful for students to experience a range of concrete representations for algebraic expressions. &lt;br&gt;• Algebraic rules and conventions should be discussed informally as they arise. &lt;br&gt;• It is important to develop an understanding of the use of letters as algebraic symbols for variable numbers of objects rather than for the objects themselves. The concrete aids referred to in the guidelines can help form correct thought processes. The practice of using the first letter of the name of an object as a symbol for the number of such objects (or still worse as a symbol for the object) can lead to misconceptions and should be avoided, especially in the early stages. &lt;br&gt;• Treatment of A1 and A2 can include an informal introduction to later work (A3, ...). However it is wise to take time establishing basic concepts before moving into such exercises as manipulation of abstract algebraic symbols.</td>
</tr>
<tr>
<td><strong>Introduction to the use of letters to represent numbers.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Language translation: English to algebra.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Language translation: algebra to English.</strong></td>
<td></td>
<td></td>
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<tr>
<td><strong>To be able to:</strong></td>
<td></td>
<td></td>
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<tr>
<td>• develop a more succinct language to replace written sentences describing patterns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• replace written sentences describing patterns by algebraic expressions</td>
<td></td>
<td></td>
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<tr>
<td>• form algebraic sentences to describe geometric and number patterns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• represent algebraic expressions using concrete materials</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• translate written expressions into mathematical symbols and vice versa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• extend a table of values using a rule in words or symbols.</td>
<td></td>
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</tr>
</tbody>
</table>
Guidelines A3, A4

A3 Students should now have considerable experience in seeing each algebraic symbol as possessing a number of possible values.

Conventions for operating with algebraic terms may now be introduced.

Start from arithmetic operations placing emphasis on the similarities and differences,

eg The conventions for \(4 + 3\), \(4 - 3\), and \(4 \div 3\), are the same as for \(a + b\), \(a - b\), \(a \div b\).

The convention for \(4 \times 3\) is different from that for \(a \times b\): \(a \times b\) may be written as \(ab\), i.e. the term \(ab\) has multiplication implications; \(4 \times 3\) cannot be written as 43 as this has place value implications.

Apply these conventions to previous experiences of students, eg in A2(b) the number of matches = \(1 + 3y\).

Care needs to be taken to avoid students’ interpreting that an algebraic symbol represents an object,

eg \(3a\) may represent 3 times a number of apples, not 3 times an apple, and the number of apples can be any number.

Note: Teachers may wish to proceed through Sections A3 to A7 using only linear expressions (eg \(4y\), \(5n\), ...) and later repeat the sequence for non-linear cases (eg \(3a^2\), \(ab\), ...).

A4 Students will need continued experience in generating number patterns from algebraic expressions. Where possible these number patterns could be illustrated with concrete materials.

eg (i) given the algebraic expression \(4n + 4\) generate some values
(ii) using \(4n + 4\) as the general term generate a pattern;
    starting with \(n = 7\) the pattern is \(32, 36, 40, ...\)
(iii) using \(4n + 4\) as the general term and starting with \(n = 1\),
    a) generate the sequence/pattern
    b) find the eighth term
    c) find the 126th term.

This pattern can be illustrated in concrete materials by completely enclosing 1 unit, 2 unit, 3 unit, etc square pools with unit squares as paving stones, and stating the number of paving stones required:

(4 x 1 + 4)

(4 x 2 + 4)

Students could be posed questions of the type:

How many paving stones are needed to enclose a square pool with sides 25 units in length?

The pool example illustrates alternative formulae such as \(4(n + 1)\), \((n + 2)^2 - n^2\), which can be related to the concrete example. Other strands such as measurement will provide further formulae for substitution.
**A3 Algebraic abbreviations**

To be able to:
- use and interpret algebraic equivalences
  
  \[ y + y + y + y = 4y \]
  
  \[ 5 \times n = n \times 5 = 5n \]
  
  \[ w \times w = w^2 \]
  
  \[ 3 \times a \times a = 3a^2 \]
  
  \[ a \times b = ab \]
  
  \[ a + b = \frac{a}{b} \]

- recognise the role of grouping symbols, eg 2\(a + 1\) and 2\((a + 1)\) have different meanings.

**A4 Substitution in algebraic expressions and formulae. (Related to known and discovered formulae).**

To be able to:
- generate a number pattern from an algebraic expression
- represent algebraic expressions as geometric patterns using concrete materials
- substitute into a simple formula related to experience.

**Applications**
- Writing rules using recognised conventions, eg rules as in A1 and A2
- Applying symbols to generalisations derived from concrete models (as in A1 and A2) and from real life (eg taxi fares)

**Implications and Considerations**
- Conventions for operating with numbers need to be examined before introducing the conventions for operating with algebraic terms.
- Much of the material in A3 may emerge from the practical activities in other sections of this strand.

- Generate sequences given the general term
- Conversion formulae
- Substituting in formulae from other sections of the syllabus, eg \(D = 2r\)

- The substitution should continually reinforce the conventions referred to in A3.
- Use could be made here of algebraic expressions which were given meaning in A2, A3.
- Measurement formulae provide algebraic expressions which have a meaning for arbitrary positive (or sometimes negative) values of the variables.
Guidelines A5, A6, A7

A5  Introduce grid paper, establish reference lines and hence plot points following an agreed convention. Data, rules, tables of values can all be drawn from previous experiences to give the work greater meaning to the students.

A6  To illustrate $3a + 2a = 5a$, let ‘a’ represent the number of matches required to make a regular geometrical shape.
I make three shapes, my neighbour makes two identical to these.
Write an expression that represents the number of matches needed.
Simplify this expression.
Linear expressions can be clarified using concrete representations as in A2 Guidelines (b).

\[
\begin{array}{|c|c|}
\hline
l \text{ units} & \quad a \quad b \text{ units} \\
\hline
\end{array}
\]

The number of square units in the area of this rectangle is $lb$.
Write an expression for the total area of three such rectangles.
Simplify this expression.
(ie $lb + lb + lb = 3lb$).

Discuss the meaning of an algebraic expression in practical examples,
egg If $SA$ is the cost of an adult ticket and $SC$ is the cost of a child’s ticket what is the meaning of $2A + 3C$?

A7  Begin with exercises of the type:

\[
\begin{align*}
3(2 + 4) &= (2 + 4) + (2 + 4) + (2 + 4) \\
&= 2 + 4 + 2 + 4 + 2 + 4 \\
&= 2 + 2 + 2 + 4 + 4 + 4 \\
&= 3 \times 2 + 3 \times 4
\end{align*}
\]

Leading to:

\[
\begin{align*}
3(a + 4) &= (a + 4) + (a + 4) + (a + 4) \\
&= a + 4 + a + 4 + a + 4 \\
&= a + a + a + 4 + 4 + 4 \\
&= 3a + 12
\end{align*}
\]

Similar action should follow with more difficult types. With experience students will be able to write $3(a + 4) = 3a + 12$.

Partitioning arrays of objects can help understanding here:

\[
\begin{array}{|c|c|}
\hline
\quad & \quad & \quad \quad & \quad & \quad \\
\hline
\quad & \quad & \quad \quad & \quad & \quad \\
\hline
\quad & \quad & \quad \quad & \quad & \quad \\
\hline
\quad & \quad & \quad \quad & \quad & \quad \\
\hline
\end{array}
\]

\[3(2 + 4) = \quad = 3 \times 2 + 3 \times 4\]

Concrete representations as in A2 Guidelines (b) can also help here.
<table>
<thead>
<tr>
<th>Content and Skills Objectives</th>
<th>Applications</th>
<th>Implications and Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A5 Graphing of relationships</strong></td>
<td>- Graphing of relationships such as in A1 to A4 above.</td>
<td>- Whilst alternative grid systems may be used in early experiences, it is intended that the standard rectangular grid systems be established (see N9.2).</td>
</tr>
<tr>
<td>To be able to:</td>
<td></td>
<td>- Students should generate data to be graphed (see A1 - A4).</td>
</tr>
<tr>
<td>- graph data on given axes</td>
<td></td>
<td>- Students should be aware of the need to label the axes and the graphs.</td>
</tr>
<tr>
<td>- derive a rule for a given set of points</td>
<td></td>
<td>- Calculators could be useful when plotting non-integer values for continuous variables.</td>
</tr>
<tr>
<td>- establish co-ordinate systems and plot points from a table of values</td>
<td></td>
<td>- The algebraic expression is a way of summarising a concrete situation and/or a verbally expressed statement.</td>
</tr>
<tr>
<td><strong>A6 Simplifying algebraic expressions</strong></td>
<td>- Simplifying derived formulae, eg Perimeter of a rectangle (= l + b + l + b) (= 2l + 2b)</td>
<td>- Students need to appreciate that expressions such as (2a + 3b) cannot be simplified algebraically because the variables may take differing values.</td>
</tr>
<tr>
<td>To be able to:</td>
<td>- Simplifying expressions derived from geometric patterns and number patterns as in A1 to A4</td>
<td></td>
</tr>
<tr>
<td>- recognise like terms</td>
<td></td>
<td>- It is important that students understand (3(a + b) = (a + b)3) and that they can remove grouping symbols from each expression.</td>
</tr>
<tr>
<td>- combine like terms</td>
<td></td>
<td>- Some students may proceed to examples such as ((a + b)(c + d)).</td>
</tr>
<tr>
<td>- simplify by eliminating operation signs, eg (2n + 3m + n = 3n + 3m) (ab + ab = 2ab) (4tm - tm = 3tm) (2pq + 3pq + pr = 5pq + pr)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>A7 Expansions - distributive law</strong></td>
<td>- Applications to area and perimeter problems</td>
<td></td>
</tr>
<tr>
<td>To be able to:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- remove grouping symbols and collect like terms, eg (3(a + 2) = 3a + 6) (2(a - b) = 2a - 2b) (m(n + 2) = mn + 2m) (a(a + b) = a^2 + ab)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Guidelines A8, A9

A8 Other areas of the syllabus are ideal for giving students practical experience, eg find the total surface area, A square units, of a square prism of base length a units and height h units:

\[
A = a \times a + a \times a + a \times h + a \times h + a \times h + a \times h
\]

\[
= a^2 + a^2 + ah + ah + ah + ah
\]

\[
= 2a^2 + 4ah
\]

A9 Work from the pattern obtained when finding the area of a one unit path placed around a rectangular garden of length \(l\) and breadth \(b\) units.

Area of path\(= l \times 1 + l \times 1 + b \times 1 + b \times 1 + 4\)

(in square units)\(= 2l + 2b + 4\)

\(= 2(l + b + 2)\)

through to a similar example with a path of \(a\) units in width.

Area of path\(= al + al + ab + ab + 4a^2\)

(in square units)\(= 2al + 2ab + 4a^2\)

\(= 2a(l + b + 2a)\)

An area can be used for a problem solving approach to factorising, eg ask students to form one rectangle from three \(a \times 1\) rectangles and six \(1 \times 1\) squares (maybe have these made out of cardboard).

\[
\begin{array}{ccc}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

leading to \(3a + 6 = 3(a + 2)\)

\[
\begin{array}{ccc}
3 & \text{OR} & 3 \\
\text{OR} & \text{OR} & \text{OR} \\
\end{array}
\]
<table>
<thead>
<tr>
<th>Content and Skills Objectives</th>
<th>Applications</th>
<th>Implications and Considerations</th>
</tr>
</thead>
</table>
| **A8 Index notation**  
(Integral values only for the index) | - Use in area and volume problems | - Recognition of like terms as in A6 can now be extended to include index notation, eg $t^3 + 2t^2 = 3t^2$.  
- Some students may discover meanings for zero and negative indices. |
| To be able to: | | |
| - simplify expressions of the type  
  $a \times a \times a \times a = a^4$  
  $a^2 \times a^3 = a^5$  
  $a^4 + a = a^5$  
  $(a^2)^3 = a^6$ | | |
| **A9 Factorisation** | - Simplifying formulae derived from patterns in earlier topics,  
  eg Perimeter of a rectangle $= 2l + 2b$  
  $= 2(l + b)$ | - A clear understanding of factorisation of numbers is essential. The highest common factor is to be sought. |
| To be able to: | | |
| - factorise a single term,  
  eg $6ab = 3 \times 2 \times a \times b$ | | |
| - find a common factor,  
  eg $6a + 12 = 6(a + 2)$  
  $3a + 3b = 3(a + b)$  
  $ab + ac = a(b + c)$ | | |
Solving equations provides a good opportunity to incorporate ideas from the Problem Solving section, especially 'teaching through problem solving' where problem solving is the methodology adopted. Students could be introduced to solving equations by some concrete representation of two equivalent algebraic expressions. This allows students to discover for themselves the general principles needed for solving equations.

**Model 1:** uses a two pan balance and objects such as coins or centicubes. A light paper wrapping can hide a 'mystery number' of objects without distorting the balance's message of equality.

The sketch shows a balance set up to model the equation $3x + 1 = 2x + 3$.

Keeping the balance level while removing certain items, students soon solve the problem of the value of $x$.

**Model 2:** uses small objects (all the same) with some hidden in containers to produce the 'unknowns' or 'mystery numbers', eg place the same (unknown) number of small objects in a number of paper cups and cover them with another cup.

Arrange an equation (say, $3x + 2 = 2x + 7$) using the cups and then remove the objects in equal amounts (those with crosses) to arrive at:

A similar exercise could, if necessary, be carried out for the equation $4x - 3 = x + 12$.

Note: Students remove the minus zone by adding three objects to each side (outside the minus zone), thus arriving at:

With each of these models, each step could be recorded diagrammatically, summarised orally and then algebraically, assisting in the development of formal solutions to equations. The models are inappropriate for equations with fractional or negative solutions.

Guidelines for A10 are continued on page 118.
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</thead>
<tbody>
<tr>
<td>A10 Algebraic sentences</td>
<td>Problem solving</td>
<td>The difference between algebraic expressions and equations will need constant emphasis.</td>
</tr>
<tr>
<td>To be able to:</td>
<td>Solving equations resulting from substitution in a formula</td>
<td>The equality meaning of the = sign needs to be dwelt on as some students see it only as 'makes' or 'gives'.</td>
</tr>
<tr>
<td>• recognise a simple equation</td>
<td></td>
<td>Solution of simple equations may have been treated informally in A2.</td>
</tr>
<tr>
<td>• solve a simple equation by algebraic methods.</td>
<td></td>
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</tr>
</tbody>
</table>
Guidelines A10 (Cont.)

Practical note: When using these models, it is most important that the correct number of objects be hidden before presenting an equation problem to a student. One way to do this in groups is to have an instruction card for one student to set up a problem with the model, and then the other students in the group use the model to solve the problem.

Model 3: uses one to one matching of terms on each side of the equation,

\[ 3x + 1 = 2x + 3 \]

can be rewritten as

\[ x + x + x + 1 = x + x + 2 + 1 \]

giving

\[ x = 2 \]

through one to one matching.

Model 4: uses a substitution approach. By trial and error a value is found for the unknown which produces equality for the values of the two expressions on either side of the equation (this highlights the variable concept),

\[ 4x - 3 = x + 12 \]

<table>
<thead>
<tr>
<th>Guess</th>
<th>4x - 3</th>
<th>x + 12</th>
<th>Verdict</th>
</tr>
</thead>
<tbody>
<tr>
<td>x = 3</td>
<td>9</td>
<td>15</td>
<td>x</td>
</tr>
<tr>
<td>x = 6</td>
<td>21</td>
<td>18</td>
<td>x</td>
</tr>
<tr>
<td>x = 5</td>
<td>17</td>
<td>17</td>
<td>✓</td>
</tr>
</tbody>
</table>

Conclusion: \( x = 5 \).
### Content and Skills Objectives

**A11 Solving word problems using algebra**

To be able to:
- translate a word problem into an equation
- solve the equation
- translate this solution into an answer to the problem.

### Applications

- Problem solving

### Implications and Considerations

- Problem solving is of importance at every stage of this algebra sequence. At the end of this course students should have a wide experience in solving problems by use of algebra in the forms of terms, expressions and equations.