2013 Notes from the Marking Centre – Mathematics Extension 1

Introduction

This document has been produced for the teachers and candidates of the Stage 6 Mathematics Extension 1 course. It contains comments on candidate responses to the 2013 Higher School Certificate examination, highlighting their strengths in particular parts of the examination and indicating where candidates need to improve.

This document should be read along with:

- the *Mathematics Extension 1 Stage 6 Syllabus*
- the 2013 Higher School Certificate Mathematics Extension 1 examination
- the marking guidelines
- *Advice for students attempting HSC mathematics examinations*
- *Advice for HSC students about examinations*
- other support documents developed by the Board of Studies, Teaching and Educational Standards NSW to assist in the teaching and learning of Mathematics in Stage 6.

Question 11

(a) Most candidates answered this part correctly.

Common problems were:

- not knowing the correct formula
- omitting the '–' sign.

(b) Many candidates recognised that the integral was an inverse trigonometric function.

Common problems were:

- dealing with the constants 2 and 7 incorrectly
- lack of accuracy.

(c) Most candidates found the correct expression for the binomial term required.

A common problem was:

- confusing 'success' and 'failure', assigning the wrong index to each probability.

(d) (i) This part required candidates to articulate a mathematical argument. A number of candidates were able to gain full marks in this part by finding the correct derivative in simplified form, $f'(x) = \dfrac{4+x^2}{\left(4-x^2\right)^2}$, and then correctly reasoning why it was positive, making reference to the numerator or denominator.

Common problems were:

- transcription or algebraic errors in differentiation, leading to incorrect expressions such as $f'(x)=\frac{4-3x^2}{(4-x)^2}$ or $f'(x)=\frac{4+2x^2}{(4-x^2)}$ which were not possible to justify as being positive
- using the derivative in expanded form, such as $f'(x)=\frac{4-x^2+2x^2}{16-8x^2+x^4}$, to justify as positive, which led to many spurious arguments.

(d) (ii) Many candidates did not link this part with part (i). Most candidates were able to sketch the two vertical asymptotes correctly labelled, or with an indication of scale.

(e) While candidates could observe that $\lim_{x\to 0}\frac{\sin\frac{x}{2}}{3x}$ involved two constants, 2 and 3, like part (b), various permutations of 2 and 3 were seen in the numerator and denominator, only some of them arriving at the correct answer $\frac{1}{6}$.

(f) Candidates gained some marks in this part by correctly finding $\frac{du}{dx}=3e^{3x}$ or changing the limits. Having done this, some candidates then used substitution successfully to obtain $\frac{1}{3}\int_1^e \frac{1}{u^2+1}\,du$, which then led to the inverse tan function. Of these candidates, many were able to execute the final step $\frac{1}{3}\left[\tan^{-1}e-\frac{\pi}{4}\right]$ or 0.14 to gain full marks.

Common problems were:

- reaching the step $\frac{1}{3}\left[\tan^{-1}e-\tan^{-1}1\right]$ then writing = 8.2, indicating that they were calculating the answer in degrees rather than radians
- not being able to obtain $\frac{1}{3}\int_1^e \frac{1}{u^2+1}\,du$ and so integrated using logs or single terms in e^{nx}.

(g) The constant 5 in the function $x^2\sin^{-1}5x$ proved troublesome, with only about half the candidature providing a correct solution.

Common problems were:

- understanding the notation $\sin^{-1}5x$ but not being able to differentiate it correctly; common incorrect derivatives were $\frac{1}{\sqrt{1-25x^2}}$, $\frac{1}{\sqrt{25-x^2}}$, $\frac{5}{\sqrt{\left(\frac{1}{5}\right)^2-x^2}}$
- calculating or substituting the incorrect derivative of $\sin^{-1}5x$ into an attempt to use the product rule

- writing $\sin^{-1} 5x = \dfrac{1}{\sin 5x}$.

Question 12

(a) (i) Most candidates gained 1 mark by establishing the correct trigonometric relationship, enabling them to calculate $\alpha = \dfrac{\pi}{6}$.

Common problems were:
- providing answers which were negative
- providing answers in degrees.

(a) (ii) Candidates who got the correct result in (a) (i) usually went on to gain the 2 marks by establishing the correct trig equation and solving it. Candidates who used an incorrect answer to part (i) and who demonstrated the relevant skills were not further penalised if they arrived at a correct solution for their incorrect α-value.

A common problem was:
- ignoring the given domain.

(b) Candidates went to the identity $\sin^2 x = \frac{1}{2}(1 - \cos^2 x)$, commonly used in integration, and proceeded to alter it to suit the question.

Common problems were:
- not correctly writing $\sin^2 \dfrac{x}{2}$ in terms of $\cos x$
- omitting π
- not squaring the function
- squaring $\dfrac{x}{2}$
- using $\sin^2\left(\dfrac{x}{4}\right)$
- writing the incorrect primitive
- careless substitutions and evaluations.

(c) Most candidates gained full marks for this part.

Common problems were:
- not identifying correct A and B values
- as a result of having incorrect A and B values, the subsequent log equation involved the log of a negative number, which was ignored
- truncating the k-value, leading to a less accurate answer.

(d) (i) Candidates were required to use the perpendicular distance formula appropriately, showing correct substitutions. It was also necessary to clearly justify/explain the removal of the absolute value sign.

(d) (ii) Candidates needed to recognise that this was a minimisation problem, and to correctly solve $D'(t) = 0$.

(d) (iii) Candidates needed to use $y = x^2 + 3$ to establish the gradient y' , use the substitution of $t = 1$ from (ii), and then link it to the gradient of $y = 2x - 1$. (Note: Some candidates were able to get full marks for (ii) and (iii) without necessarily getting (i) correct.)

(e) For 2 marks, candidates needed to show that the particle was moving in simple harmonic motion by proving the differential equation $\ddot{x} = -n^2 x$, or correctly using $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$, in the context of this question.

Common problems were:

- differentiating poorly, which limited the accuracy of some responses
- simply using a common identity such as $v^2 = n^2\left(a^2 - x^2\right)$ and stating that $n = 3$.

Question 13

(a) (i) Most candidates successfully applied the chain rule and showed that the correct result was $\frac{dr}{dt} = -10^{-4}$.

Common problems were:

- not simplifying and hence the result was not a constant
- incorrect derivatives.

(a) (ii) A common method used to find t was to integrate the rate found in (a) (i).

Most candidates tried to determine the initial radius.

Common problems were:

- errors in making r the subject of $\frac{4}{3}\pi r^3 = 10^{-6}$
- assuming that one could find the volume in terms of t from $\frac{dV}{dt} = -10^{-4}A$ without realising that A is a function of t.

(b) (i) Most candidates used the ratio formula correctly. Some recognised that T is the midpoint of NG and then used the midpoint formula. A few candidates used similarity or congruence to find the coordinates of G from the ratios of corresponding sides.

A common problem was:

- using $2ap + ap^2$ instead of $2a + ap^2$ for the y-coordinate of N.

(b) (ii) This part was challenging for most candidates.

Common problems were:

- substituting the coordinates of G into the initial parabola
- eliminating x or y and leaving the equation in terms of p
- making p the subject and then making careless errors with their algebraic manipulation

- difficulty in finding the correct equation to determine the focal length and the directrix
- stating the directrix to be $x = -a$ or $x = a$ and the focal length $4a$ units.

(c) (i) Most candidates correctly attempted this question by solving $\dot{y} = 0$ or by finding half the time of flight.

Common problems were:

- differentiating in terms of θ instead of t
- leaving the answer in terms of v and θ instead of u and α
- ignoring the instruction 'Do NOT prove this' and deriving the given expressions for x and y.

(c) (ii) Candidates used a variety of methods to show the result. By far the simplest approach was to equate the times taken by particles A and B to reach their maximum heights using the result found in (a) (i). Other longer solutions involved equating the values for y or $\dot{y}$ for the two particles. Some candidates used the initial vertical components of velocity to show the result, but these statements were not always accompanied by the appropriate reasoning.

(c) (iii) Candidates found this part challenging. Most realised that the distance required was the sum of the horizontal distances travelled to reach the maximum height.

Common problems were:

- not recognising the need to substitute the results for time found in (c) (i) and (c) (ii)
- not realising the need to use the result from (c) (ii) to make the switch in the expression that leads to the final result
- finding d to average the total horizontal distance travelled by both particles and then having difficulty with the algebra involved
- those who chose to expand the given result often could not show a complete solution in reverse.

(d) Only a small number of candidates used the fact that the 'circles touched', resulting in the use of a common tangent at T or the line of centres passing through the point of contact.

Common problems were:

- using the fact that QTP was a straight line in their proof, when this is what was required
- not giving supporting reasons for each step in the proof
- not annotating the diagram that they had to copy into their answer booklet.

Question 14

(a) (i) Some candidates were able to combine the terms into a single term, such as $-\dfrac{1}{k(k+1)^2}$.

Some candidates simply substituted a positive number, eg $k = 1$, into the inequality and obtained a negative value. It should be noted that this method only shows that the inequality is true for that particular value, and not all positive values.

Common problems were:

- poor algebraic manipulation of the expression
- poor handling of the signs of the terms in the numerator, after creating a common denominator
- not justifying why their final expression is always negative and just assuming that 'it is obvious'.

(a) (ii) Candidates were expected to prove the inequality using the technique of mathematical induction. Simply proving the result for the base case of $n = 2$ presented many candidates with problems.

Common errors included:

- using $n = 1$, leading to the incorrect conclusion that $1 < 1$
- using $n = 3$, possibly because 3 is the first integer greater than 2
- treating the LHS as a single term of $\frac{1}{2^2}$, instead of $\frac{1}{1^2} + \frac{1}{2^2}$ in trying to prove for $n = 2$; as it was an inequality, candidates who did this did not realise their error as $\frac{1}{2^2}$ is indeed $< 2 - \frac{1}{2}$
- not realising that in induction problems involving a series of terms, it is the sum of terms in the LHS that is being compared to the RHS, and not the general term
- not understanding the difference between proving an inequality and solving an inequation.

After making the correct assumption of $\frac{1}{1^2} + \frac{1}{2^2} + \ldots + \frac{1}{k^2} < 2 - \frac{1}{k}$, many candidates started their proof with: $\frac{1}{1^2} + \frac{1}{2^2} + \ldots + \frac{1}{k^2} + \frac{1}{(k+1)^2} = 2 - \frac{1}{k} + \frac{1}{(k+1)^2}$
or implied this by simply starting their proof with $2 - \frac{1}{k} + \frac{1}{(k+1)^2}$ instead of the correct substitution of $\frac{1}{1^2} + \frac{1}{2^2} + \ldots + \frac{1}{k^2} + \frac{1}{(k+1)^2} < 2 - \frac{1}{k} + \frac{1}{(k+1)^2}$.

Candidates who noticed the connection with part (i) were then able to use it to quickly complete the proof.

(b) (i) Most candidates successfully used the binomial theorem to find the correct coefficient.

(b) (ii) Most candidates could see that they needed to rearrange $\left(1 + x^2 + 2x\right)^{2n}$ to $[1+x(x+2)]^{2n}$ in order to get the desired result.

The most common mistake was to use the binomial theorem to obtain

$$\sum_{k=0}^{2n}\binom{2n}{k}(1)^{2n-k}[x(x+2)]^k \text{ (or similar)}$$

and then to simply state that this was equal to $\displaystyle\sum_{k=0}^{2n}\binom{2n}{k}x^{2n-k}(x+2)^{2n-k}$,

with no explanation as to how the index changed from k to $2n-k$.

(b) (iii) Very few candidates obtained full marks for this part.

Many candidates gained some marks by recognising the connection between parts (i), (ii) and (iii) and realising that $\left(1 + x^2 + 2x\right)^{2n} \equiv (1+x)^{4n}$.

Substituting the given result into the right-hand side of the expression created problems, with incorrect statements such as $\displaystyle\sum_{k=0}^{2n}\binom{2n}{k}\sum_{r=0}^{2n-k}\binom{2n-k}{r}2^{2n-k-r}x^{2n-k+r}$ or other expressions involving double sums being used.

(c) (i) Although most candidates knew Newton's method, many could not create a valid function.

Common problems were:

- not differentiating properly
- letting the function be $f(t) = \frac{1}{t}$ or e^t, which in both cases simplified the problem
- using 0.56 instead of 0.5.

(c) (ii) Most candidates realised that $e^{rx} = \log_e x$ at the point of intersection. Not many realised that the gradients also would be equal due to the common tangent.

Hence, they did not have a second equation, $re^x = \frac{1}{x}$, which was also required to solve the question.

A common problem was:

- finding the value of x and not going on to find the value or r, as was required.