2013 Notes from the Marking Centre – Mathematics

Introduction

This document has been produced for the teachers and candidates of the Stage 6 Mathematics course. It contains comments on candidate responses to the 2013 Higher School Certificate examination, highlighting their strengths in particular parts of the examination and indicating where candidates need to improve.

This document should be read along with:

- the *Mathematics Stage 6 Syllabus*
- the 2013 Higher School Certificate Mathematics examination
- the marking guidelines
- *Advice for students attempting HSC mathematics examinations*
- *Advice for HSC students about examinations*
- other support documents developed by the Board of Studies, Teaching and Educational Standards NSW to assist in the teaching and learning of Mathematics in Stage 6.

Question 11

(a) Most candidates completed this question correctly, rounding to 3 significant figures.

(b) Many candidates had difficulty with this question.

Common problems were:

- substituting ‘2’ into the function and giving an answer of ‘0’ or ‘undefined’
- not factorising $x^3 - 8$ or $x^2 - 4$ correctly; some factorised the difference of 2 cubes, then incorrectly factorised $x^2 + 2x + 4$ so they could cancel
- thinking that finding the limit as $x \to \infty$ was required
- confusing the limit with differentiation by first principles or the quotient rule.

(c) Most candidates completed this question correctly.

Common problems were:

- not differentiating $(\sin\theta - 1)$ correctly
- forgetting to write the power in their solution.

(d) Most candidates completed this question correctly.

Common problems were:

- confusing the product rule with the quotient rule
- not recognising that the function was a product.

(e) Most candidates completed this question correctly.

Common problems were:

- confusing differentiation with integration of exponential functions
- not knowing how to differentiate an exponential function
- including a logarithm as part of the answer.

(f) Many candidates had difficulty with this question.

Common problems were:

- not recognising that the integral involved a log function
- treating the integral like a polynomial, trying to simplify by using index laws poorly
- not being able to calculate the constant multiplier
- multiplying by a fraction involving *x*
- not substituting the limits into the primitive or not doing it correctly, often resulting in $[f(a)] + [f(b)]$.
- a weak understanding of logarithms and brackets, with common errors after the substitution, as in:

$$\frac{1}{3}[\ln 1^3 + 1] - \frac{1}{3}[\ln 0^3 + 1]$$
$$= \frac{1}{3}(0+1) - \frac{1}{3}(0)$$
$$= \frac{1}{3}$$

(g) In successful responses, candidates linked the radius of 2 with the centre of the circle at (2,3), thus drawing a circle that is tangential to the *y*-axis. They also considered the coordinates of the points on the circumference of the circle in the north, south, east and west positions.

Common problems were:

- not finding the centre or radius of the circle correctly and hence when they tested a point to find where to shade, their conclusions were incorrect
- drawing graphs poorly, so that the important features were not clearly distinguishable
- shading so lightly on the graphs that it was difficult to see if the shading surrounded the whole circle
- algebraic attempts to solve for *x* and *y* with no attempt at a sketch
- confusing the circle equation with that of the parabola and attempting to find vertex, focus and directrix and then sketching a parabola.

Question 12

(a) This question was generally well done, with most candidates recognising that they were required to solve $f'(p) = 0$ for the point of inflexion.

Common problems were:

- finding $f(p)$ before differentiating
- differentiating y or y' incorrectly.

(b) (i) Most candidates found the gradient and then, using an appropriate point, found the required equation. Some correctly substituted two appropriate points into the given equation to establish the required result.

Common problems were:

- using the incorrect formula to find the gradient
- using an incorrect point, which meant they could not establish the equation of *AD*.

(b) (ii) Most candidates found the required distance using the correct formula. Some attempted to find the equation of the perpendicular line from *B* to *AD*, then the point of intersection of this line with *AD*, and then applied the distance formula to give the result of 20 units.

Common problems were:

- not including absolute value notation in their response
- using the incorrect formula for perpendicular distance.

(b) (iii) This question was well done by most candidates.

Common problems were:

- arithmetic errors, such as

$$\sqrt{(18-22)^2+(39-42)^2} = 4+3$$
$$= 7$$

- using the incorrect formula

$EC = \sqrt{(18-22)^2 - (39-42)^2}$ for the distance of *EC*.

(b) (iv) Most candidates correctly used either the formula for the area of a trapezium or the formula for the area of a triangle.

Common problems were:

- not using their result from (b) (ii) in their answer
- using the incorrect value of $h = 25$ for the height of the trapezium.

(c) (i) Candidates substituted into the correct T_n formula: for Alex the arithmetic progression formula $T_n = a + (n-1)d$ and for Kim the geometric progression formula $T_n = ar^{n-1}$.

Some candidates listed the sequence for each salary and were generally successful with this method.

Common problems were:

- using $n - 1 = 10$, rather than the correct $n - 1 = 9$
- using an incorrect formula, but could state that $d = 1500$ for Alex and $r = 1.05$ for Kim
- using T_2 as a for both Kim and Alex
- mixing up the starting salary for either Kim or Alex
- using the incorrect value for r.

(c) (ii) Most candidates used the correct formula for the sum of a geometric progression. Those candidates who listed Kim's salary for the first 10 years were generally successful in calculating the correct total.

Common problems were:

- using the incorrect starting salary
- using the incorrect value for n or r
- using the formula for the sum of an arithmetic progression.

(c) (iii) Many candidates recognised the sum as an arithmetic progression. Some candidates stated the correct S_n formula and showed the substitution before any calculation was made. This allowed markers to allocate marks to those candidates who made numerical errors.

Candidates who arrived at the correct quadratic equation were successful in either factorising or using the quadratic formula to find the correct solution. Some candidates listed the series and were generally successful using this method.

Common problems were:

- using the incorrect combination of $S_n = 262\ 500$ with $a = 33\ 000$ and $d = 1500$
- using Kim's salary of \$30 000, arriving at a quadratic equation that did not factorise
- using the geometric progression formula, resulting in an equation involving a log function
- using the method applying to loan repayments to answer the question.

Question 13

(a) (i) Candidates were able to access at least one mark by realising that $P(t) = 375$ and simplifying to get $\cos\left(\frac{\pi}{6}t\right) = -\frac{1}{2}$.

Common problems were:

- substituting $t = 12$ into the equation
- converting to degrees
- separating the $\frac{\pi}{6}$ and the t, hence not being able to proceed to any meaningful conclusion
- not realising that two quadrants should be considered.

(a) (ii) Common problems were:

- sketching the graph incorrectly or inaccurately
- not realising that they were sketching a periodic curve
- not labelling or poorly labelling axes or critical points
- sketching an incorrect shape
- sketching such a small diagram that it was difficult to ascertain if necessary features were correct.

(b) (i) Most candidates could equate the two functions $f(x)$ and $g(x)$ and set up a quadratic equation, leading to a correct answer $x = \frac{1}{2}$.

Common problems were:

- not factorising and solving correctly
- differentiating the given function.

(b) (ii) Most candidates integrated and used their x value from part (b) (i) to achieve full marks for this part of the question. A small number subtracted $f(x)$ from $g(x)$, hence requiring the use of absolute values. Most candidates realised that the area had to be positive.

Common problems were:

- using the y-value in part (i) as their upper limit for integration (ie $x = 1$)
- differentiating instead of integrating.

(c) Candidates who began by finding the area of sector ABC $(A = 450\theta)$ were able to access one mark. Generally, those who equated the areas solved the equation to get the correct answer in the exact form $\left(CD = 15\sqrt{2} \right)$.

Common problems were:

- not knowing how to use the fact that the area of the small sector was half the area of the large sector to find the radius of the small sector
- misquoting the area formula
- using incorrect formulae (eg of a segment).

(d) (i) Most candidates scored full marks for this part. Candidates who substituted the correct values of n and r into the correct geometric series formula and equated to zero were able to access the first mark. Candidates who went on to evaluate the correct value of M were awarded full marks.

Common problems were:

- using $n = 30$ instead of 360 months (not converting 30 years into months)
- not writing the correct decimal answer $M = \$2\,997.75$ before rounding to the required accuracy of \$2998, as was given in the question.

(d) (ii) Most candidates substituted, summed up the resulting geometric progression series and evaluated correctly to get \$269 903.63.

A common problem was using $n = 20$ instead of 240 months.

(d)(iii) Many candidates had difficulty with this part. Candidates were able to access one mark by successfully substituting and equating to zero. Candidates who went on to introduce logarithms eventually found the value of n =192.46.

Common problems were:

- not solving accurately for n
- solving for $370000 = 500000(1.005)^n - 599600(1.005^n - 1)$
- beginning the question using incorrect amounts owed
- writing the initial equation as an expression but going no further.

Question 14

(a) (i) Most candidates completed this section successfully.

Common problems were:

- mismanaging $+/-$
- substituting $t = 0$.

(a) (ii) Candidates needed to solve $10 - 2t = 0$ to find $t = 5$. In most responses, the stated condition ($\frac{dx}{dt} = 0$) was clearly evident.

A common problem was:

- not associating 'at rest' with $\dot{x} = 0$ or $\frac{dx}{dt} = 0$.

(a) (iii) In correct responses, candidates calculated $\int_0^7 10 - 2t\, dt$ and then added the initial position. Most candidates integrated the velocity equation $\dot{x} = 10 - 2t$, and then used the initial conditions to produce $x(t) = 10t - t2 + 5$, and subsequently $x(7) = 26$.

(a) (iv) Candidates who drew a diagram had much greater success with this part. Only a small number of candidates solved the question by calculating absolute value of sections and adding.

A common problems was:

- not understanding the difference between distance travelled and position.

(b) (i) A significant number of candidates could not establish that $RB = 100 - 50t$ and $RA = 80t$. Most candidates correctly identified the cosine rule.

Common problems were:

- not substituting correct values into the cosine rule
- making algebraic and simplification errors, as more than one algebraic step was required to prove the given result.

(b) (ii) Finding the derivative of r^2 (rather than making r the subject) made the algebra much easier.

Common problems were:

- when making r the subject, finding the second derivative posed a problem
- not testing the value found to justify a minimum.

(c) Candidates used a range of successful strategies to find the exact value of $\sin x$. Some recognised that they were required to use the sine rule and Pythagoras' theorem and they used the appropriate notation correctly.

Common problems were:

- phrasing their 'exact' answer in terms of inverse trig functions
- using Pythagoras' theorem on non-right triangles.

(d) Candidates cancelled out the triangular areas, and then calculated the areas above and below using rectangles. In many cases, they included a diagram.

Common problems were:

- taking the areas above the x-axis as negative areas
- including inequalities such as $-2 < a < 3$ when the question asked 'what is the value of a'.

Question 15

(a) (i) Most candidates used the trapezoidal rule correctly. These candidates quoted the formula and used it correctly, showing their substitution into the formula. Candidates who used the weighted table method were generally successful.

Common problems were:

- poor use of brackets
- having the correct numerical expression but an incorrect answer
- not finding the correct value of h
- confusing trapezoidal and Simpson's rules and using them in reverse.

(a) (ii) Most candidates used Simpson's rule correctly. These candidates quoted the formula and used it correctly, showing their substitution into the formula. Candidates who used the weighted table method were generally successful.

Common problems were:

- showing confusion with the use of the extended formula, eg odds and evens
- multiplying the middle function value by 2 instead of 4
- not finding the correct value of h
- poor use of brackets
- having the correct numerical expression but an incorrect answer
- confusing trapezoidal and Simpson's rules and using them in reverse.

(a) (iii) Candidates who used a diagram were more successful in relating the concave down parabola used in Simpson's rule and the straight lines used in the trapezoidal rule to the actual roofline to illustrate the difference in areas.

Common problems were:

- not knowing that Simpson's rule uses parabolas to estimate area
- not mentioning concavity
- commenting on the number of function values, applications used and subintervals, all of which had no relevance to the question.

(b) Common problems were:

- finding the volume of the solid of revolution around the x-axis, not the y-axis
- not correctly making x the subject
- not breaking the parabola up into 2 branches
- after making x the subject, incorrectly stating that $x^2 = 4 + y$ or $x^2 = 4 \pm 2\sqrt{y} + y$
- using the right-hand branch $x = \sqrt{y} + 2$ instead of the-left hand branch $x = 2 - \sqrt{y}$
- expanding a perfect square incorrectly
- not being able to identify and use limits
- mixing the two variables of x and y together in the integrand
- omitting π
- not squaring the function
- giving decimal approximations instead of exact answers.

(c) (i) Common problems were:

- not recognising the shape represented by an absolute value function
- drawing a curve that resembled a 'U' shape
- knowing it was a 'V' shape graph but not always correctly finding the *x* and *y* intercepts
- finding the *x*-intercept as $\frac{2}{3}$ instead of $\frac{3}{2}$
- drawing diagrams that were too small
- not using rulers, not marking intercepts and/or using an inconsistent scale.

(c) (ii) Most candidates made the assumption that one answer for *m* was sufficient and did not look for all possible values of *m*.

Candidates who earned one mark often simply:

- stated that $m = 2$ (quoting the gradient of the right-hand branch of the absolute value curve)
- attempted to solve 2 cases of the absolute value equation
- attempted an algebraic solution, leading to a quadratic equation, and then used $\Delta = 0$ or substituted $\left(\frac{3}{2}, 0\right)$ into the given equation.

Common problems were:

- missing the value for *m* found by substituting the point $\left(\frac{3}{2}, 0\right)$
- making errors with the inequality signs.

(d) (i) Most candidates scored one mark for this part.

(d) (ii) Common problems were:

- not adding in the probability from the first throw
- not realising that taking 'turns' was a necessary component of the problem
- not realising that the game stopped if Pat won and that the branches in a tree diagram only continued from the person who had lost
- not realising that for Pat to have a second turn, Pat needed to lose, then Chandra needed to lose, and then Pat would have a turn again; they needed P(LLW) for the calculation of Pat winning on the second throw $\frac{35}{36} \times \frac{35}{36} \times \frac{1}{36}$
- one of the most common answers was $\frac{1}{36} + \left(\frac{1}{36} \times \frac{35}{36}\right)$, in which it had been forgotten that Chandra needed to lose the second throw so Pat could then throw to win.

(d) (iii) A significant number of candidates did not attempt this part, with those attempting it finding it a very difficult question.

Common problems were:

- a common but incorrect answer was a bald $\frac{1}{2}$ or 1 – *P* (Chandra losing), making the assumption that there was a 50–50 chance with two people playing
- not writing down a series before attempting a calculation.

Question 16

(a) Most candidates got at least one mark for finding the correct primitive or finding the point of contact of the tangent by solving $4x - 3 = 5$. Many candidates then used this to calculate the correct constant and receive full marks. Not as common, but still quite successful, was the method of using the discriminant of the equation $2x^2 - 3x + c = 5x - 7$ to get the correct value of the constant.

(b) (i) Most candidates used the substitution of $t = 0$.

Common problems were:

- not realising that $e^0 = 1$
- bald answer of 375.

(b) (ii) Most candidates earned full marks for this part, successfully using logarithms to solve the equation.

(b) (iii) This sketch proved to be difficult for many candidates, although some used a table of values.

Common problems were:

- the sketch of the graph not touching both horizontal and vertical axes, as was indicated in parts (i) and (ii)
- incorrect concavity of the curve.

(b) (iv) This part was answered poorly. Some candidates were able to quote the result of $P = P_0 e^{kt}$ from $\frac{dP}{dt} = kP$, where $k = 0.02$. Most candidates commenced to solve $\frac{dP}{dt} = \frac{dN}{dt}$. Some candidates understood the need to ignore the negative sign, as the questions involved the comparison of rates.

(b) (v) This part was answered poorly, or not attempted at all, by most candidates.

(c) (i) Most candidates gained full marks.

A common problem was not giving a final reason (ie the test) for the similarity.

(c) (ii) Many candidates identified the correct ratios of sides for similar triangles.

A common problem was not seeing connection with the ratio of sides involving *XY* in both triangles *ABD* and *ABC*.