BOSTES
BOARD OF STUDIES TEACHING & EDUCATIONAL STANDARDS NSW

2014 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11–14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I Pages 2–5

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II Pages 6–13

60 marks

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

2620

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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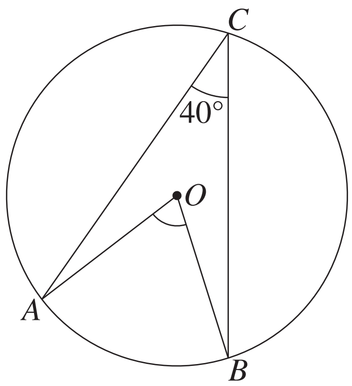
1 The points A, B and C lie on a circle with centre O, as shown in the diagram. The size of $\angle ACB$ is 40°.



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What is the size of $\angle AOB$?

(A) 20°

(B) 40°

(C) 70°

(D) 80°

2 Which expression is equal to $\cos x - \sin x$?

(A) $\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)$

(B) $\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$

(C) $2\cos\left(x + \frac{\pi}{4}\right)$

(D) $2\cos\left(x - \frac{\pi}{4}\right)$

3 What is the constant term in the binomial expansion of $\left(2x - \dfrac{5}{x^3}\right)^{12}$?

(A) $\binom{12}{3} 2^9 5^3$

(B) $\binom{12}{9} 2^3 5^9$

(C) $-\binom{12}{3} 2^9 5^3$

(D) $-\binom{12}{9} 2^3 5^9$

4 The acute angle between the lines $2x + 2y = 5$ and $y = 3x + 1$ is θ.

What is the value of $\tan\theta$?

(A) $\dfrac{1}{7}$

(B) $\dfrac{1}{2}$

(C) 1

(D) 2

5 Which group of three numbers could be the roots of the polynomial equation $x^3 + ax^2 - 41x + 42 = 0$?

(A) 2, 3, 7

(B) 1, −6, 7

(C) −1, −2, 21

(D) −1, −3, −14

6 What is the derivative of $3\sin^{-1}\dfrac{x}{2}$?

(A) $\dfrac{6}{\sqrt{4-x^2}}$

(B) $\dfrac{3}{\sqrt{4-x^2}}$

(C) $\dfrac{3}{2\sqrt{4-x^2}}$

(D) $\dfrac{3}{4\sqrt{4-x^2}}$

7 A particle is moving in simple harmonic motion with period 6 and amplitude 5.

Which is a possible expression for the velocity, v, of the particle?

(A) $v = \dfrac{5\pi}{3}\cos\left(\dfrac{\pi}{3}t\right)$

(B) $v = 5\cos\left(\dfrac{\pi}{3}t\right)$

(C) $v = \dfrac{5\pi}{6}\cos\left(\dfrac{\pi}{6}t\right)$

(D) $v = 5\cos\left(\dfrac{\pi}{6}t\right)$

8 In how many ways can 6 people from a group of 15 people be chosen and then arranged in a circle?

(A) $\dfrac{14!}{8!}$

(B) $\dfrac{14!}{8!6}$

(C) $\dfrac{15!}{9!}$

(D) $\dfrac{15!}{9!6}$

9 The remainder when the polynomial $P(x) = x^4 - 8x^3 - 7x^2 + 3$ is divided by $x^2 + x$ is $ax + 3$.

What is the value of a?

(A) –14

(B) –11

(C) –2

(D) 5

10 Which equation describes the locus of points (x, y) which are equidistant from the distinct points $(a + b,\ b - a)$ and $(a - b,\ b + a)$?

(A) $bx + ay = 0$

(B) $bx + ay = 2ab$

(C) $bx - ay = 0$

(D) $bx - ay = 2ab$

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Solve $\left(x+\dfrac{2}{x}\right)^2-6\left(x+\dfrac{2}{x}\right)+9=0$. **3**

(b) The probability that it rains on any particular day during the 30 days of November is 0.1. **2**

Write an expression for the probability that it rains on fewer than 3 days in November.

(c) Sketch the graph $y=6\tan^{-1}x$, clearly indicating the range. **2**

(d) Evaluate $\displaystyle\int_2^5 \frac{x}{\sqrt{x-1}}\,dx$ using the substitution $x=u^2+1$. **3**

(e) Solve $\dfrac{x^2+5}{x}>6$. **3**

(f) Differentiate $\dfrac{e^x \ln x}{x}$. **2**

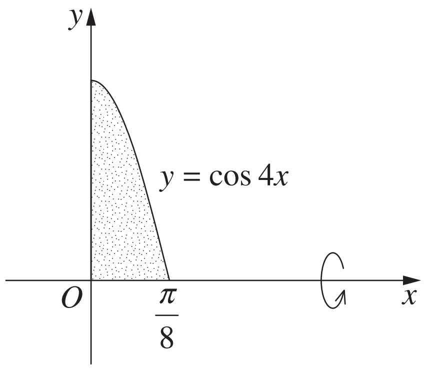
Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) A particle is moving in simple harmonic motion about the origin, with displacement x metres. The displacement is given by $x = 2 \sin 3t$, where t is time in seconds. The motion starts when $t = 0$.

(i) What is the total distance travelled by the particle when it first returns to the origin? **1**

(ii) What is the acceleration of the particle when it is first at rest? **2**

(b) The region bounded by $y = \cos 4x$ and the x-axis, between $x = 0$ and $x = \frac{\pi}{8}$, is rotated about the x-axis to form a solid. **3**



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Find the volume of the solid.

(c) A particle moves along a straight line with displacement x m and velocity v m s^{-1}. The acceleration of the particle is given by **3**

$$\ddot{x} = 2 - e^{-\frac{x}{2}}.$$

Given that $v = 4$ when $x = 0$, express v^2 in terms of x.

Question 12 continues on page 8

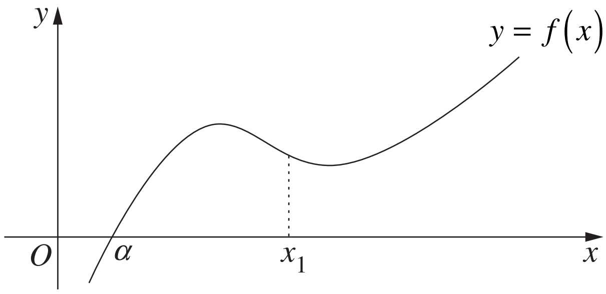
Question 12 (continued)

(d) Use the binomial theorem to show that **2**

$$0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n \binom{n}{n}.$$

(e) The diagram shows the graph of a function $f(x)$. **1**

The equation $f(x) = 0$ has a root at $x = \alpha$. The value x_1, as shown in the diagram, is chosen as a first approximation of α.



A second approximation, x_2, of α is obtained by applying Newton's method once, using x_1 as the first approximation.

Using a diagram, or otherwise, explain why x_1 is a closer approximation of α than x_2.

(f) Milk taken out of a refrigerator has a temperature of 2°C. It is placed in a room of constant temperature 23°C. After t minutes the temperature, T°C, of the milk is given by **3**

$$T = A - Be^{-0.03t},$$

where A and B are positive constants.

How long does it take for the milk to reach a temperature of 10°C?

End of Question 12

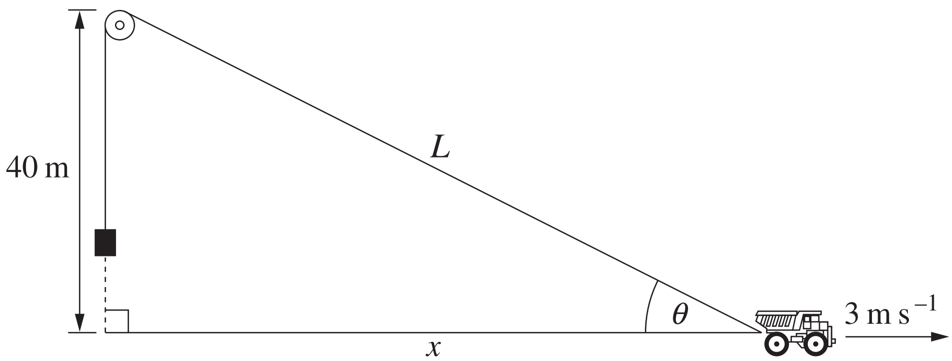
Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Use mathematical induction to prove that $2^n + (-1)^{n+1}$ is divisible by 3 for all integers $n \geq 1$. **3**

(b) One end of a rope is attached to a truck and the other end to a weight. The rope passes over a small wheel located at a vertical distance of 40 m above the point where the rope is attached to the truck.

The distance from the truck to the small wheel is L m, and the horizontal distance between them is x m. The rope makes an angle θ with the horizontal at the point where it is attached to the truck.

The truck moves to the right at a constant speed of 3 m s^{-1}, as shown in the diagram.



(i) Using Pythagoras' Theorem, or otherwise, show that $\dfrac{dL}{dx} = \cos\theta$. **2**

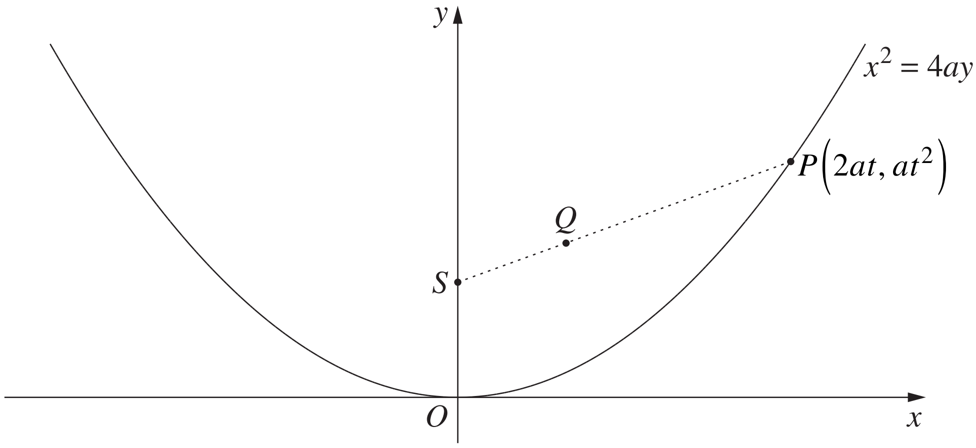
(ii) Show that $\dfrac{dL}{dt} = 3\cos\theta$. **1**

Question 13 continues on page 10

Question 13 (continued)

(c) The point $P\left(2at, at^2\right)$ lies on the parabola $x^2 = 4ay$ with focus S.

The point Q divides the interval PS internally in the ratio $t^2 : 1$.



(i) Show that the coordinates of Q are $x = \dfrac{2at}{1+t^2}$ and $y = \dfrac{2at^2}{1+t^2}$. **2**

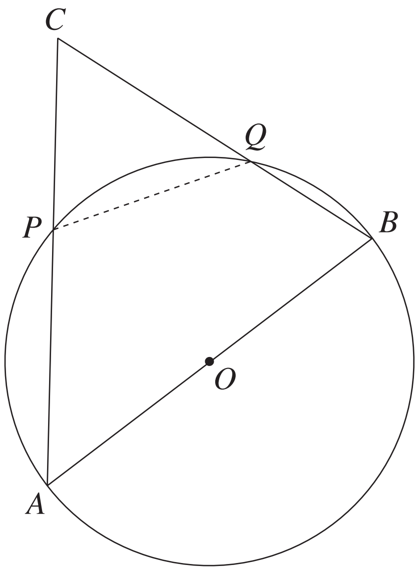
(ii) Express the slope of OQ in terms of t. **1**

(iii) Using the result from part (ii), or otherwise, show that Q lies on a fixed circle of radius a. **3**

Question 13 continues on page 11

Question 13 (continued)

(d) In the diagram, AB is a diameter of a circle with centre O. The point C is chosen such that $\triangle ABC$ is acute-angled. The circle intersects AC and BC at P and Q respectively.



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Copy or trace the diagram into your writing booklet.

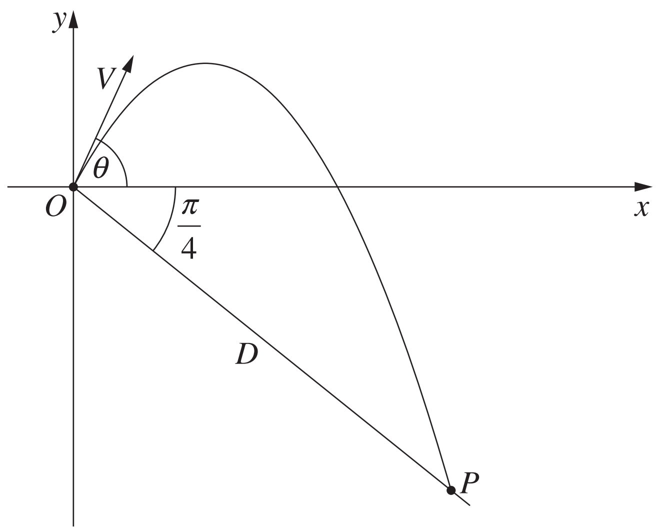
(i) Why is $\angle BAC = \angle CQP$? **1**

(ii) Show that the line OP is a tangent to the circle through P, Q and C. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) The take-off point O on a ski jump is located at the top of a downslope. The angle between the downslope and the horizontal is $\frac{\pi}{4}$. A skier takes off from O with velocity $V\text{ m s}^{-1}$ at an angle θ to the horizontal, where $0 \le \theta < \frac{\pi}{2}$. The skier lands on the downslope at some point P, a distance D metres from O.



The flight path of the skier is given by

$$x = Vt\cos\theta\,, \quad y = -\frac{1}{2}gt^2 + Vt\sin\theta\,, \qquad \text{(Do NOT prove this.)}$$

where t is the time in seconds after take-off.

(i) Show that the cartesian equation of the flight path of the skier is given by **2**

$$y = x\tan\theta - \frac{gx^2}{2V^2}\sec^2\theta\,.$$

(ii) Show that $D = 2\sqrt{2}\,\frac{V^2}{g}\cos\theta\left(\cos\theta + \sin\theta\right)$. **3**

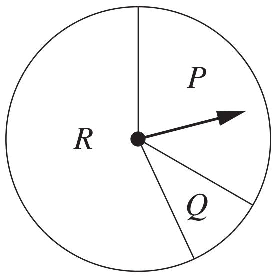
(iii) Show that $\frac{dD}{d\theta} = 2\sqrt{2}\,\frac{V^2}{g}\left(\cos 2\theta - \sin 2\theta\right)$. **2**

(iv) Show that D has a maximum value and find the value of θ for which this occurs. **3**

Question 14 continues on page 13

Question 14 (continued)

(b) Two players A and B play a game that consists of taking turns until a winner is determined. Each turn consists of spinning the arrow on a spinner once. The spinner has three sectors P, Q and R. The probabilities that the arrow stops in sectors P, Q and R are p, q and r respectively.



The rules of the game are as follows:

- If the arrow stops in sector P, then the player having the turn wins.
- If the arrow stops in sector Q, then the player having the turn loses and the other player wins.
- If the arrow stops in sector R, then the other player takes a turn.

Player A takes the first turn.

(i) Show that the probability of player A winning on the first or the second turn of the game is $(1-r)(p+r)$. **2**

(ii) Show that the probability that player A eventually wins the game is **3**

$$\frac{p+r}{1+r}.$$

End of paper

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$