2014 Notes from the Marking Centre – Mathematics Extension 1

Introduction

This document has been produced for the teachers and candidates of the Stage 6 Mathematics Extension 1 course. It contains comments on candidate responses to particular parts of the 2014 Higher School Certificate examination, highlighting candidates' strengths and indicating where they need to improve.

This document should be read along with:

- the Mathematics Extension 1 Stage 6 Syllabus
- the 2014 Higher School Certificate Mathematics Extension 1 examination
- the marking guidelines
- Advice for students attempting HSC Mathematics examinations
- Advice for HSC students about examinations
- other support documents developed by the Board of Studies, Teaching and Educational Standards NSW to assist in the teaching and learning of Mathematics in Stage 6.

Question 11

(a) Many candidates recognised and used an appropriate substitution, performed appropriate basic algebraic processes to establish the final result of $x = 1$ and $x = 2$.

A common problem was:

- expanding the expression but not demonstrating an appropriate method to solve for x.

(b) Candidates who recognised and correctly used the binomial expansion gained full marks.

Common problems were:

- not understanding what was meant by 'fewer than 3 days'
- not including the probability of rain on 'no days'.

(c) Many candidates correctly sketched the graph.

A common problem was:

- not determining the correct range.

(d) Most candidates found the derivative for the given substitution of $u = x^2 + 1$ and/or changed the limits correctly.

A common problem was:

- making errors when trying to re-arrange the integrand, sometimes leading to an expression which contained both x and u, making it difficult to go further.

(e) This question was answered quite well by most candidates. The most common method used to establish the critical points was to multiply LHS and RHS by the square of the denominator, in this case x^2. Candidates performed some basic algebraic processes to arrive at critical values of 0, 1 and 5.

Successful responses of '$0 < x < 1$ or $x > 5$' occurred where the values were tested, by trial and error, or where diagrams were drawn on a number line (or number plane) to verify the solution set.

(f) Some candidates used the quotient rule and others used the product rule correctly.

A common problem was:
- failing to differentiate the numerator correctly before actually applying the quotient rule. Nevertheless, a mark was awarded for the correct use of quotient or product rule, with at least one derivative correct and a common denominator of x^2.

Question 12

(a)(i) This part of the question was done poorly by a large number of candidates.
Many candidates drew a diagram and based their argument for the distance to be 4 metres on their diagram. A large number of candidates used basic trigonometry to determine the amplitude (2 metres) and then realised that the particle has to travel 4 metres to return to the starting point.

There were a number of candidates who used the velocity–time graph to determine the distance travelled. This method required a substantial amount of work to achieve the correct solution.

Common problems were:

- thinking that the area under the displacement–time graph represented the distance travelled
- considering the amplitude of 2 as the total distance travelled
- only determining the time taken to return to the origin or the period of motion
- confusing the distance travelled with displacement from the origin.

(ii) Most candidates obtained correct expressions for velocity and acceleration in terms of t.

In better responses, candidates realised that the particle was first at rest at positive amplitude and found acceleration in terms of x, using the formula: $\ddot{x} = -9x$.

Candidates who used a graphical approach for (i) generally continued with this approach into this part. These candidates were then able to determine the correct acceleration.

A large number of candidates who did not attempt, or were not successful in (i) were able to achieve highly in this part.

Common problems were:
- failing to realise that velocity equals zero at rest and put $t = 0$ (or $x = 0$) instead
- not solving $v = 0$ correctly for time
- not understanding the direction of the acceleration.

NB: Candidates should be careful using terms such as *deceleration* to describe negative acceleration. Also, acceleration towards the origin does not necessarily distinguish between positive and negative acceleration. All SHM has acceleration towards the centre of motion.

(b) Candidates were required to use the formula for volume, use double angle formula involving $\cos^2 4x = \frac{1}{2}\left(1+\cos 2.4x\right) = \frac{1}{2}\left(1+\cos 8x\right)$ and complete two substitutions.

Common problems were:

- using the double angle formula incorrectly; for example, knowing the identity for $\cos^2 x$ but not making the correct substitution to find the identity for $\cos^2 4x$
- knowing the primitive of $\cos^2 x$ and attempting to jump straight to the primitive of $\cos^2 4x$ but making errors in determining their expression
- making errors with their substitutions of $\sin\pi$, with $\sin\pi = 1$ being the most common mistake
- omitting π from the expression for the volume
- assuming that $\left(\cos 4x\right)^2 = \cos 16x$ or $\cos^2 16x$.

(c) Candidates generally knew that $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ or $v\frac{dv}{dx}$ and applied this correctly to attain the desired result. Some candidates approached the question by separating variables and were generally successful.

Common problems were:

- not finding the correct primitive of $e^{-\frac{x}{2}}$
- not determining the value of the constant of integration correctly
- finding $\dot{x} = v = 2x + 2e^{-\frac{x}{2}} + c$ and then squaring to get v^2
- finding $\frac{1}{2}v^2 = 2x + 2e^{-\frac{x}{2}} + 6$ but then writing $v^2 = x + e^{-\frac{x}{2}} + 3$.

(d) Candidates who had fluency with the binomial expansion of either $\left(1+x\right)^n$ or $\left(1-x\right)^n$ were most successful.

One method that was often used was:

$$\left(1-1\right)^n = 0 = \binom{n}{0}.1^n.\left(-1\right)^0 + \binom{n}{1}.1^{n-1}.\left(-1\right)^1 + \binom{n}{2}.1^{n-2}.\left(-1\right)^2 + ... + \binom{n}{n}.1^0.\left(-1\right)^n$$
$$= \binom{n}{0} - \binom{n}{1} + \binom{n}{2} + ... + \binom{n}{n}\left(-1\right)^n$$

Common problems were:

- misusing the sigma notation for the expansion of $\left(1+x\right)^n$ or $\left(1-x\right)^n$

- failing to realise that the expansion of $(1+x)^n$ is not the same as $(x+1)^n$ as a direct binomial expansion
- approaching this part by considering specific cases (say proving true for $n=3$).

(e) This part was answered well by most candidates. Many candidates answered the question by drawing the diagram, showing the tangent at x_1 and an appropriate position for its x – intercept.

A substantial number of candidates approached the question by referring to the formula for Newton's method and stating that because $f'(x_1)<0$ and $f(x_1)>0$ then

$$\frac{f(x_1)}{f'(x_1)}<0\left(-\frac{f(x_1)}{f'(x_1)}>0\right) \text{ hence } x_2=x_1-\frac{f(x_1)}{f'(x_1)}>x_1 .$$

Candidates who argued from the formula needed to be clear with their understanding of the sign of the derivative and of the function at the first approximation for the root.

Common problems were:

- not drawing a tangent to the curve at x_1
- simply drawing x_2 on their diagram without any explanation
- basing an argument on the function values at x_1 and x_2
- failing to explain why x_1 is a better approximation – simply restating the question.

Candidates should pay attention to a question when a question states 'explain' and write clearly, re-read their statements and complete their arguments.

(f) It was evident that many candidates had a sound understanding of Newton's Law of Cooling. This allowed them to find the correct values for A and B and the correct solution.

Common problems were:

- not identifying the correct values of A and B
- failing to substitute $t=0$
- having incorrect values for A or B (or both A and B), leading to a subsequent exponential equation that involved the logarithm of a negative number (which was ignored)
- not realising that as $t\to\infty$, $T=23$ $\therefore A=23$.

Question 13

(a) A large number of candidates executed a correct induction proof. Those who were adept with index laws produced an efficient proof within a few lines.

Common problems were:

- knowing to use the assumption but not manipulating the resulting expression to arrive at the final step

- treating it as an equation and working on both sides.

(b)(i)
Common problems were:
- being confused by the use of an angle in the diagram, many differentiated with respect to θ, leading to convoluted solutions using $\frac{dL}{d\theta}$ and $\frac{d\theta}{dx}$
- not making the connection that $L=\sqrt{x^2+40^2} \Rightarrow \frac{dL}{dx}=\frac{x}{L}=\cos\theta$, leading to the incorrect substitution of $x=\frac{40}{\tan\theta}$ into $\frac{dL}{dx}=\frac{x}{\sqrt{x^2+40^2}}$ in order to eliminate x
- misunderstanding about variables as was evident in the process:
  $\cos\theta=\frac{x}{L} \Rightarrow L=\frac{x}{\cos\theta} \Rightarrow \frac{dL}{dx}=\frac{1}{\cos\theta}$
- substituting $x=3t \Rightarrow L=\sqrt{40^2+(3t)^2}$ and proceeding using this.

(b)(ii) Most candidates stated that $\frac{dx}{dt}=3$ and arrived at the correct result.

(c)(i) Those candidates who quoted the correct formula or process, substituted correctly and used basic skills such as adding like terms were able to gain full marks. Candidates who justified their answer were rewarded.

A common problem was:
- stating incorrect expressions without any basis.

(c)(ii)This part required using the gradient formula between two points and simplifying

$$\frac{\frac{2at^2}{1+t^2}-0}{\frac{2at}{1+t^2}-0}=t.$$

A common problem was:
- dealing with the compound fraction.

(c)(iii) There were many different approaches to this part where candidates were asked to show that Q lies on a fixed circle of radius a.

As the diagram was almost to scale, a large number of candidates made an intuitive guess that the centre of the circle was S since $SQ=SO$ and thus just stated the answer. Others justified it by finding the distance $SQ=a$.

Although the question suggested candidates use their answer to part (c) (ii), many candidates could not see how the gradient related to a circle. Those who found and used the relation

$t = \dfrac{y}{x}$ to eliminate the parameter t were often successful in reaching the result, whereas those who attempted to isolate t using $y = \dfrac{2at^2}{1+t^2}$ or $x = \dfrac{2at^2}{1+t^2}$ often floundered.

Another approach was to create another point, say $R(0,2a)$ and show that $m_{QR} \times m_{QO} = -1$ and then conclude that $\angle RQO$ was the angle in a semicircle. Some candidates used the gradient $t = \tan\theta$ and substituted into $y = \dfrac{2at^2}{1+t^2}$ or $x = \dfrac{2at^2}{1+t^2}$.

(d) Candidates who copied the diagram in order to execute a proof had a greater chance of earning marks in this part as the examiner could confirm their assertions and could observe markings on the diagram. A significant number of candidates did not copy the diagram yet referred to points and angles that they had constructed, leaving the examiners to guess their meaning and validity.

(d)(i) Candidates who quoted the appropriate circle geometry theorem, namely 'the exterior angle of a cyclic quadrilateral is equal to the opposite interior angle' (or equivalent) easily earned this mark. Candidates who used unclear language ran the risk of the examiners being unable to interpret their meaning.

(d)(ii) A common and efficient proof was to produce OP and use vertically opposite angles with the isosceles triangle ΔPAO to conclude that OP is a tangent using the converse of the alternate segment theorem.

Another popular response was to join OP, use the angle sum of ΔPQC and the sum of the angles at P on line CPA to prove the result.

Question 14

(a)(i) The most common approach was to make t the subject in the x equation, by substitution, and eliminate it from the y equation.

(a)(ii) This part provided the candidates with the biggest challenge in this question. There were a variety of approaches, with the most successful responses recognising that $y = -x$ and using this along with the fact that $D = 2\sqrt{x}$ to eliminate x and y from the equation in part (a)(i) and introduce D into this equation, producing something that could be rearranged into the desired result.

Another common approach was to try and use Pythagoras. This method required much more algebraic manipulation and was often unsuccessful.

Common problems were:

- not dealing with the minus signs (simply changing the sign and hoping that it won't be noticed rarely works)
- attempting to apply some of the common ideas (which unfortunately played no role in the solution of this question) such as: maximum height occurs when $\dot{y} = 0$; particle hits the ground when $y = 0$, maximum range occurs when $\theta = 45°$.

(a)(iii) This was answered well by the majority of candidates.

The most successful approach was to expand the parentheses in part (a)(ii) and then convert it to a double angle before differentiating.

Common problems were:

- not making a clear distinction between D and the derivative, and when one stops and the other begins
- putting a random collection of double angle formulae and expansion of parentheses in different positions on the page, with no clear indication of what was being used and when. (NB: A solution should follow a logical pattern which the markers can follow.)

(a)(iv) This part was done well by many candidates. Candidates who realised that the coefficient of the derivative took no part in the process of finding the stationary point were left with the simple trig equation $\cos 2\theta - \sin 2\theta = 0$ to solve.

The two main approaches to this equation were transforming the expression into a single trig function, possibly by noticing the similarity to multiple-choice Question 2, and dividing by $\cos 2\theta$ to create $\tan 2\theta = 1$.

Both methods were generally successful but there were a significant number of candidates who ended up with statements such as $\tan 2\theta = 0$ which then produces some impractical solutions such as $\theta = 0$ or $\theta = 90°$ or even $\theta = 180°$.

A common problem was:

- ignoring the instruction to 'show that D has a maximum value'. (Candidates are reminded that showing a substitution and the value(s) when testing for maximum or minimum is essential.)

(b)(i) Many candidates were confused about the rules of the game and hence had trouble gaining any marks for either part of this question. A significant number of candidates simply did not attempt this question.

Those who were able to interpret the rules were generally successful in showing the required result.

(b)(ii) Only a few candidates came up with a series that would lead to a correct solution. Candidates are reminded that when using the limiting sum formula it is important to state the condition on 'r' for the limiting sum to exist and not just assume that the series converges.