BOSTES
BOARD OF STUDIES TEACHING & EDUCATIONAL STANDARDS NSW

2015 HSC Mathematics Extension 1 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	A
2	C
3	B
4	C
5	A
6	D
7	B
8	B
9	D
10	C

Section II

Question 11 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the correct relationship between $\cos 2x$ and $\sin^2 x$, or equivalent merit	1

Sample answer:

$$\int \sin^2 x\,dx$$

$$= \frac{1}{2}\int 1 - \cos 2x\,dx$$

$$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + C$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	2
• Attempts to use the correct slopes in an appropriate formula, or equivalent merit	1

Sample answer:

$m_1 = 2 \qquad m_2 = -3$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{2 - (-3)}{1 + 2 \times (-3)}\right|$$

$$= \left|\frac{5}{-5}\right|$$

$$= 1$$

$\therefore$ acute angle $= 45°$ or $\dfrac{\pi}{4}$

Question 11 (c)

Criteria	Marks
• Provides correct solution	3
• Identifies both important values, or equivalent merit	2
• Identifies –3 as an important value, or equivalent merit	1

Sample answer:

$$\frac{4}{x+3} \geq 1$$

$$(x+3)^2 \times \frac{4}{(x+3)} \geq (x+3)^2 \times 1, \ (x \neq -3)$$

$$4(x+3) \geq (x+3)^2$$

$$0 \geq (x+3)^2 - 4(x+3)$$

$$0 \geq (x+3)(x+3-4)$$

$$0 \geq (x+3)(x-1)$$

$$\Rightarrow -3 < x \leq 1$$

Question 11 (d)

Criteria	Marks
• Provides correct solution	2
• Finds A, or equivalent merit	1

Sample answer:

$$5\cos x - 12\sin x$$

$$A\cos(x+\alpha) = A\cos x\cos\alpha - A\sin x\sin\alpha$$

$$\therefore A\cos\alpha = 5$$

$$A\sin\alpha = 12$$

$$A = \sqrt{5^2+12^2} \qquad \tan\alpha = \frac{12}{5}$$

$$= 13 \qquad \alpha = 1.176005207\ldots$$

$$5\cos x - 12\sin x = 13\cos(x + 1.176005207\ldots)$$

or

$$5\cos x - 12\sin x = 13\cos\left(x + \tan^{-1}\left(\frac{12}{5}\right)\right)$$

Question 11 (e)

Criteria	Marks
• Provides correct solution	3
• Finds a correct expression for the integral, or equivalent merit	2
• Correctly attempts substitution, or equivalent merit	1

Sample answer:

Let $u = 2x - 1$

$$\frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2}du$$

when $x = 1,\ u = 1$

$x = 2,\ u = 3$

$$\therefore \int_1^2 \frac{x}{(2x-1)^2}\,dx$$

$$= \int_1^3 \frac{1}{2}\left(\frac{u+1}{u^2}\right)\cdot\frac{1}{2}du$$

$$= \frac{1}{4}\int_1^3 \left(\frac{u+1}{u^2}\right)du$$

$$= \frac{1}{4}\int_1^3 \left(\frac{1}{u} + \frac{1}{u^2}\right)du$$

$$= \frac{1}{4}\left[\ln u - \frac{1}{u}\right]_1^3$$

$$= \frac{1}{4}\left[\left(\ln 3 - \frac{1}{3}\right) - \left(\ln 1 - \frac{1}{1}\right)\right]$$

$$= \frac{3\ln 3 + 2}{12}$$

Question 11 (f) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$P(x) = x^3 - kx^2 + 5x + 12$

Since $x - 3$ is a factor, then $P(3) = 0$

$$\therefore (3)^3 - k(3)^2 + 5(3) + 12 = 0$$
$$27 - 9k + 15 + 12 = 0$$
$$9k = 54$$
$$k = 6$$

Question 11 (f) (ii)

Criteria	Marks
• Provides correct solution	2
• Identifies quadratic factor, or equivalent merit	1

Sample answer:

Using $k = 6$ $P(x) = x^3 - 6x^2 + 5x + 12.$

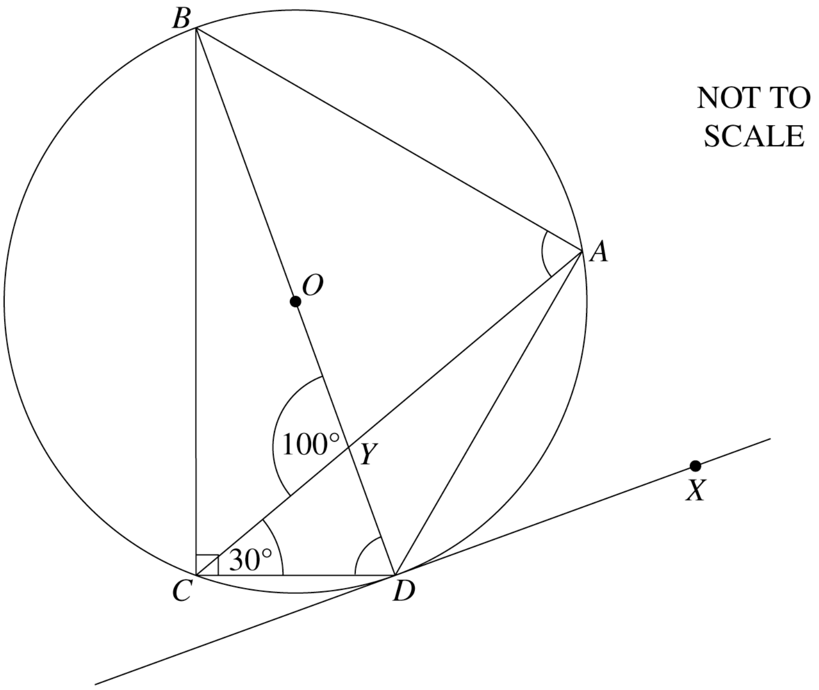
By long division

$$\begin{array}{r|l} & x^2 - 3x - 4 \\ \hline x - 3 & x^3 - 6x^2 + 5x + 12 \\ & \underline{x^3 - 3x^2} \\ & \quad -3x^2 + 5x \\ & \quad \underline{-3x^2 + 9x} \\ & \qquad -4x + 12 \\ & \qquad \underline{-4x + 12} \end{array}$$

$$\therefore P(x) = (x - 3)\left(x^2 - 3x - 4\right)$$
$$= (x - 3)(x - 4)(x + 1)$$

$\therefore$ zeros are 3, 4, –1

Question 12 (a)



Question 12 (a) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$\angle ACB + \angle ACD = 90°$ (Angle in a semi-circle is a right angle)

$\angle ACB + 30° = 90°$

$\therefore \angle ACB = 60°$

Question 12 (a) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$\angle ADX = \angle ACD$ (angle between tangent and chord = angle in the alternate segment)

$= 30°$

Question 12 (a) (iii)

Criteria	Marks
• Provides correct solution	2
• Makes some progress towards correct solution	1

Sample answer:

$\angle CDY + \angle YCD = 100°$ ($\angle BYC$, exterior $\angle$ of $\triangle CDY$)

$\therefore \angle CDY = 70°$

$\angle CAB = \angle CDB$ (Angles standing on same chord at the circumference are equal)

$\therefore \angle CAB = 70°$

Question 12 (b) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$PQ : (p+q)x - 2y - 2apq = 0$

At the focus $x = 0,\ y = a$

$\Rightarrow\ (p+q)0 - 2(a) - 2apq = 0$

$\therefore\ -2a = 2apq$

$\Rightarrow pq = -1$

Question 12 (b) (ii)

Criteria	Marks
• Provides correct coordinates	2
• Identifies the value of p, or equivalent merit	1

Sample answer:

$$P(8a,\ 16a),\ \therefore 8a = 2ap$$

$$p = 4$$

$$q = -\frac{1}{p} \quad \text{(from (i))}$$

$$\therefore q = -\frac{1}{4}$$

$$\therefore Q\left(2aq,\ aq^2\right) = \left(2a\left(-\frac{1}{4}\right),\ a\left(-\frac{1}{4}\right)^2\right)$$

$$= \left(-\frac{a}{2},\ \frac{a}{16}\right)$$

Question 12 (c) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

In $\triangle MOA$,

$$\tan 15° = \frac{h}{OA}$$

$$\cot 15° = \frac{OA}{h}$$

$$OA = h \cot 15°$$

Question 12 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Uses Pythagoras' theorem, or equivalent merit	1

Sample answer:

In $\triangle MOB$,

$$\tan 13° = \frac{h}{OB}$$

$$\cot 13° = \frac{OB}{h}$$

$$OB = h \cot 13°$$

In $\triangle OAB$,

$$OB^2 = OA^2 + AB^2$$

$$h^2 \cot^2 13° = h^2 \cot^2 15° + 2000^2$$

$$h^2 \left(\cot^2 13 - \cot^2 15\right) = 2000^2$$

$$h^2 = \frac{2000^2}{\cot^2 13° - \cot^2 15°}$$

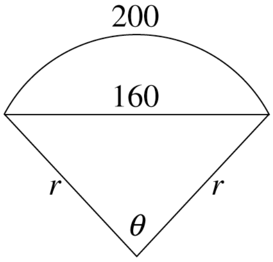
$$= 827561.0914\ldots$$

$$h = 910$$

Question 12 (d) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:



Using cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$160^2 = r^2 + r^2 - 2r.r.\cos\theta$

$160^2 = 2r^2 - 2r^2\cos\theta$

$\therefore 160^2 = 2r^2(1 - \cos\theta)$

Question 12 (d) (ii)

Criteria	Marks
• Provides correct solution	2
• Observes $200 = r\theta$, or equivalent merit	1

Sample answer:

Using $\ell = r\theta$

$200 = r\theta$

$r = \dfrac{200}{\theta}$

$$160^2 = 2\left(\frac{200}{\theta}\right)^2(1 - \cos\theta)$$

$$160^2\theta^2 = 2(200)^2(1 - \cos\theta)$$

$$8\theta^2 = 25 - 25\cos\theta$$

$\therefore 8\theta^2 + 25\cos\theta - 25 = 0$

Question 12 (d) (iii)

Criteria	Marks
• Provides correct solution	2
• Finds derivative of $8\theta^2 + 25\cos\theta - 25$, or equivalent merit	1

Sample answer:

$\theta_1 = \pi$ Let $f(\theta) = 8\theta^2 + 25\cos\theta - 25$

$f'(\theta) = 16\theta - 25\sin\theta$

$$\theta_2 = \theta_1 - \frac{f(\theta_1)}{f'(\theta_1)}$$

$$= \pi - \frac{8\pi^2 + 25\cos\pi - 25}{16(\pi) - 25\sin\pi}$$

$$= \pi - \frac{8\pi^2 - 50}{16\pi}$$

$= 2.565514\ldots$

$= 2.57$ (two decimal places)

Question 13 (a) (i)

Criteria	Marks
• Provides correct values	1

Sample answer:

$x = 3$ or $x = 7$

Question 13 (a) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Max speed when $v^2 = 11$

$\therefore$ Max speed $= \sqrt{11}$ m s^{-1}

Question 13 (a) (iii)

Criteria	Marks
• Provides correct solution	3
• Deduces the values of two of a, c, n, or equivalent merit	2
• Writes down the value of c, or equivalent merit	1

Sample answer:

$c = 5$ (particle oscillates about $x = 5$)

$a = 2$ (amplitude is $5 - 3$ or $7 - 5$)

$v^2 = n^2\left(2^2 - (x-5)^2\right)$

$v^2 = 11$ when $x = 5$

$\therefore \quad 11 = 4n^2$

$\Rightarrow n^2 = \dfrac{11}{4}$

$\Rightarrow \quad n = \dfrac{\sqrt{11}}{2}$

$(n > 0)$

Question 13 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Identifies $\binom{18}{2}$ or equivalent merit	1

Sample answer:

$$\left(2x+\frac{1}{3x}\right)^{18} = a_0x^{18}+a_1x^{16}+a_2x^{14}+\ldots$$

$$= \binom{18}{0}(2x)^{18}\left(\frac{1}{3x}\right)^0 + \binom{18}{1}(2x)^{17}\left(\frac{1}{3x}\right)^1 + \binom{18}{2}(2x)^{16}\left(\frac{1}{3x}\right)^2 + \ldots$$

$$\therefore a_2 = \binom{18}{2}2^{16}\frac{1}{3^2}$$

$$= 1\ 114\ 112$$

Question 13 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Identifies correct term, or equivalent merit	1

Sample answer:

$$T_{k+1} = \binom{18}{k}(2x)^{18-k}\left(\frac{1}{3x}\right)^k$$

For constant term, $x^{18-k}.\,x^{-k} = x^0$

$$\Rightarrow 18-2k = 0$$

$$\therefore k = 9$$

$\therefore$ constant term/term independent of x is $\binom{18}{9}.2^9.\left(\frac{1}{3}\right)^9$

Question 13 (c)

Criteria	Marks
• Provides correct proof	3
• Verifies initial case and attempts to use inductive assumption, or equivalent merit	2
• Verifies the identity for the initial case, or equivalent merit	1

Sample answer:

When $n = 1$, $RHS = 1 - \dfrac{1}{(1+1)!}$

$$= 1 - \frac{1}{2!} = \frac{1}{2} = LHS$$

Statement true for $n = 1$.

Assume the result is true for $n = k$

ie assume $\dfrac{1}{2!} + \dfrac{2}{3!} + \dfrac{3}{4!} + \ldots + \dfrac{k}{(k+1)!} = 1 - \dfrac{1}{(k+1)!}$ to be true.

Prove true for $n = k + 1$

ie $\dfrac{1}{2!} + \dfrac{2}{3!} + \dfrac{3}{4!} + \ldots + \dfrac{k}{(k+1)!} + \dfrac{k+1}{(k+1+1)!} = 1 - \dfrac{1}{(k+1+1)!}$

$\text{LHS} = 1 - \dfrac{1}{(k+1)!} + \dfrac{k+1}{(k+2)!}$ (from assumption)

$$= 1 - \frac{(k+2)-(k+1)}{(k+2)!}$$

$$= 1 - \frac{1}{(k+2)!} = \text{RHS}$$

Hence the result is proven by mathematical induction.

Question 13 (d) (i)

Criteria	Marks
• Provides correct proof	2
• Correctly differentiates $\cos^{-1} x$, or equivalent merit	1

Sample answer:

$$f(x) = \cos^{-1}(x) + \cos^{-1}(-x)$$

$$f'(x) = \frac{-1}{\sqrt{1-x^2}} + -1 . \frac{-1}{\sqrt{1-(-x)^2}} = \frac{-1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}$$

$$= 0$$

$\therefore f'(x) = 0$

$\therefore f(x)$ is constant

Question 13 (d) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Substitute any suitable value, eg $x = 0$

$$f(0) = \cos^{-1}(0) + \cos^{-1}(0)$$

$$= \frac{\pi}{2} + \frac{\pi}{2}$$

$\therefore f(x) = \pi$

$\therefore \cos^{-1}(x) + \cos^{-1}(-x) = \pi$

ie $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

Question 14 (a) (i)

Criteria	Marks
• Provides correct solution	2
• Finds correct time when $y = 0$, or equivalent merit	1

Sample answer:

When $y = 0$

$$0 = Vt\sin\theta - \frac{1}{2}gt^2$$

$$0 = t\left(V\sin\theta - \frac{1}{2}gt\right)$$

$$t = 0 \quad \text{or} \quad t = \frac{2V\sin\theta}{g}$$

Range: $x = Vt\cos\theta$ when $t = \dfrac{2V\sin\theta}{g}$

$$= V\left(\frac{2V\sin\theta}{g}\right)\cos\theta$$

$$= \frac{2V^2\sin\theta\cos\theta}{g}$$

$$= \frac{V^2\sin 2\theta}{g} \quad (\sin 2\theta = 2\sin\theta\cos\theta)$$

Question 14 (a) (ii)

Criteria	Marks
• Provides a correct angle to the horizontal	2
• Provides $\dot{x}$ and $\dot{y}$, or equivalent merit	1

Sample answer:

$$\frac{dx}{dt} = V\cos\theta$$

$$\frac{dy}{dt} = V\sin\theta - gt$$

$$\therefore \frac{dy}{dx} = \frac{V\sin\theta - gt}{V\cos\theta}$$

When $\theta = \frac{\pi}{3}$ and $t = \frac{2V}{\sqrt{3}g}$

$$\frac{dy}{dx} = \frac{V\sin\frac{\pi}{3} - g\left(\frac{2V}{\sqrt{3}g}\right)}{V\cos\frac{\pi}{3}}$$

$$= \frac{V\cdot\frac{\sqrt{3}}{2} - \frac{2V}{\sqrt{3}}}{V\cdot\frac{1}{2}}$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{3}}$$

Let α be the angle between the curve and the positive direction of the x-axis at that time.

$\therefore \tan\alpha = -\frac{1}{\sqrt{3}}$

$\Rightarrow \alpha = \frac{5\pi}{6}$ from positive x-axis

Question 14 (a) (iii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Since α is obtuse, the projectile is travelling downwards (or is descending).

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	3
• Writes v^2 as a perfect square, or equivalent merit	2
• Attempts to use $\dfrac{d\left(\frac{1}{2}v^2\right)}{dx}$, or equivalent merit	1

Sample answer:

$\ddot{x} = x - 1 \qquad t = 0,\ x = 0,\ v = 1$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = x - 1$$

$$\frac{1}{2}v^2 = \frac{1}{2}x^2 - x + C_1$$

$$v^2 = x^2 - 2x + 2C_1$$

At $x = 0,\ v = 1,\ \ 1 = 0 - 0 + 2C_1$

$$2C_1 = 1$$

$$\therefore v^2 = x^2 - 2x + 1$$

$$v^2 = (x-1)^2$$

$$\therefore v = \pm(x-1)$$

Now $v = 1$ when $x = 0$ so

$$1 = -(0-1)$$

ie take negative case

$$\therefore v = -(x-1)$$

$$\therefore v = 1 - x$$

Question 14 (b) (ii)

Criteria	Marks
• Provides correct expression	2
• Attempts to integrate a correct expression for $\frac{dt}{dx}$, or equivalent merit	1

Sample answer:

$$v = 1 - x$$

$$\therefore \frac{dx}{dt} = 1 - x$$

$$\therefore \frac{dt}{dx} = \frac{1}{1-x}$$

$$t = -\ln(1-x) + C_2$$

When $t = 0$ $x = 0$

$$\therefore C_2 = 0$$

$$\therefore t = -\ln(1-x)$$

$$-t = \ln(1-x)$$

$$\therefore e^{-t} = 1 - x$$

$$\therefore x = 1 - e^{-t}$$

Question 14 (b) (iii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$x = 1 - e^{-t}$$

As $t \to \infty$, $e^{-t} \to 0$

$$\Rightarrow x \to 1 - 0$$

$$= 1$$

$\therefore$ limiting position is $x = 1$

Question 14 (c) (i)

Criteria	Marks
• Provides correct explanation	1

Sample answer:

To get a prize in 7 games, *A* must win 4 of the first 6 games and then win the 7th.

$$\therefore \text{Prob} = \binom{6}{4}\left(\frac{1}{2}\right)^6 \times \frac{1}{2}$$

$$= \binom{6}{4}\left(\frac{1}{2}\right)^7$$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct expression	1

Sample answer:

A can get a prize by winning in 5 games or 6 games or 7 games.

$$\text{Prob} = \binom{4}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) + \binom{5}{4}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right) + \binom{6}{4}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)$$

$$= \binom{4}{4}\left(\frac{1}{2}\right)^5 + \binom{5}{4}\left(\frac{1}{2}\right)^6 + \binom{6}{4}\left(\frac{1}{2}\right)^7$$

Question 14 (c) (iii)

Criteria	Marks
• Provides correct proof	2
• Recognises that the probability that A wins the prize is $\frac{1}{2}$, or equivalent merit	1

Sample answer:

A can win in $n + 1$ games or $(n + 2)$ games or $(n + 3)$ games … or $2n$ games.

$$\text{Prob} = \binom{n}{n}\left(\frac{1}{2}\right)^{n+1} + \binom{n+1}{n}\left(\frac{1}{2}\right)^{n+2} + \binom{n+2}{n}\left(\frac{1}{2}\right)^{n+3} + \ldots + \binom{2n}{n}\left(\frac{1}{2}\right)^{2n+1}$$

But probability that A eventually wins is $\frac{1}{2}$

Equating these expressions and multiplying by 2^{2n+1} gives

$$\binom{n}{n}2^{n} + \binom{n+1}{n}2^{n-1} + \binom{n+2}{n}2^{n-2} + \ldots + \binom{2n}{n}$$

$$= \frac{1}{2} \times 2^{2n+1}$$

$$= 2^{2n}$$

2015 HSC Mathematics Extension 1 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	16.2E	PE3
2	1	14.2E	HE3
3	1	2.9E	PE3
4	1	18.1E	PE3
5	1	10.5E	H5, HE7
6	1	15.2E	HE4
7	1	15.5E	HE4
8	1	8.1, 8.2, 13.4, 13.5E	HE7
9	1	14.4E	HE3
10	1	13.7	H5

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	2	13.6E	HE6
11 (b)	2	6.6E	P4
11 (c)	3	1.4E	PE2, PE3
11 (d)	2	5.9E	P4
11 (e)	3	11.5E	HE6
11 (f) (i)	1	16.2E	PE3
11 (f) (ii)	2	16.2E	PE2, PE3
12 (a) (i)	1	2.8, 2.10	PE3
12 (a) (ii)	1	2.8, 2.10	PE3
12 (a) (iii)	2	2.8, 2.10	PE2, PE3
12 (b) (i)	1	9.6E	PE3, PE4
12 (b) (ii)	2	9.6E	PE3, PE4
12 (c) (i)	1	5.6E	PE6
12 (c) (ii)	2	5.6E	H5
12 (d) (i)	1	5.5	P4
12 (d) (ii)	2	13.1	H5
12 (d) (iii)	2	16.4E	HE7
13 (a) (i)	1	14.4E	HE3
13 (a) (ii)	1	14.4E	HE3
13 (a) (iii)	3	14.4E	HE3
13 (b) (i)	2	17.3E	HE7
13 (b) (ii)	2	17.3E	HE7
13 (c)	3	7.4E	HE2
13 (d) (i)	2	10.1, 10.2, 15.5E	HE4

Question	Marks	Content	Syllabus outcomes
13 (d) (ii)	1	15.4E	HE4
14 (a) (i)	2	14.3E	HE3
14 (a) (ii)	2	14.3E	HE3
14 (a) (iii)	1	14.3E	HE3
14 (b) (i)	3	14.3E	HE5
14 (b) (ii)	2	14.3E	HE5
14 (b) (iii)	1	14.2E	H3, H4, HE7
14 (c) (i)	1	17.1E, 17.3E, 18.1E, 18.2E	HE3
14 (c) (ii)	1	17.1E, 17.3E, 18.1E, 18.2E	HE3
14 (c) (iii)	2	17.1E, 17.3E, 18.1E, 18.2E	HE3