BOARD OF STUDIES
NEW SOUTH WALES

HIGHER SCHOOL CERTIFICATE EXAMINATION

2000

MATHEMATICS IN SOCIETY

2 UNIT

Time allowed—Two hours and a half
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Only Board-approved calculators are to be used.
- Show all necessary working for Section II and Section III.
- Marks may be deducted for careless or badly arranged work.
- You may ask for extra Writing Booklets if you need them.
- The mark out of 80 will be converted to a mark out of 100.

Section I (20 marks)

- This Section contains 20 multiple-choice questions.
- Attempt ALL questions.
- All questions are of equal value.
- Complete your answers in either blue or black pen on the Answer Sheet provided.
- Allow about 45 minutes for this Section.

Section II (36 marks)

- Attempt ALL questions.
- Each question is worth 12 marks.
- Answer each question in a SEPARATE Writing Booklet.
- Allow about one hour for this Section.

Section III (24 marks)

- Attempt TWO questions.
- Each question is worth 12 marks.
- Answer each question in a SEPARATE Writing Booklet.
- Allow about 45 minutes for this Section.

SECTION I

Attempt ALL questions.

All questions are of equal value.

Instructions for answering multiple-choice questions

- Complete your answers in either blue or black pen.

- Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 =   (A) 2   (B) 6   (C) 8   (D) 9

A ◯   B ●   C ◯   D ◯

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ●   B ✖   C ◯   D ◯

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ✖   B ✖   C ◯   D ◯

1 The graph shows the number of goals scored by the Southern Ghosts soccer team in its matches over the season.
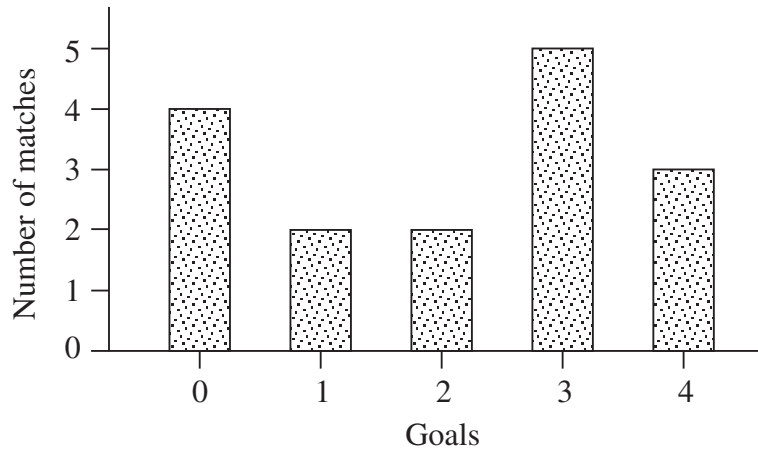
How many goals did the team score throughout the season?

(A) 5   (B) 16   (C) 33   (D) 37

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2 Christine receives a telephone bill for her mobile phone. This bill is 20% lower than her previous bill. If the previous bill was $62·20, what is Christine's phone bill now?

(A) $12·44   (B) $42·20   (C) $49·76   (D) $62·00

---

3 Solve the equation $4x - 2(x - 3) = 14$.

(A) $x = 4$ (B) $x = 5\dfrac{1}{2}$ (C) $x = 8\dfrac{1}{2}$ (D) $x = 10$

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4 A verandah is in the shape of a rectangle and a quarter circle, as shown in the diagram.
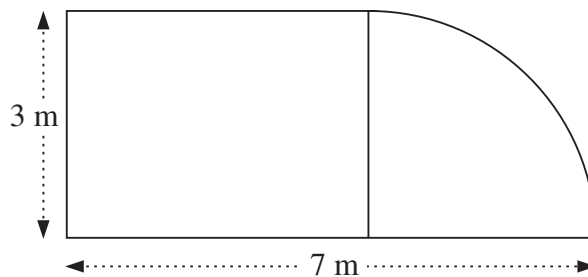
The area of the verandah is closest to

(A) 16 m^2 (B) 19 m^2 (C) 21 m^2 (D) 22 m^2

5 Given that $T = \dfrac{d}{\sqrt{s}}$, find the value of d when $T = 3 \times 10^{-2}$ and $s = 4 \times 10^4$.

(A) $7 \cdot 5 \times 10^{-7}$ (B) $1 \cdot 5 \times 10^{-4}$ (C) 6   (D) 1200
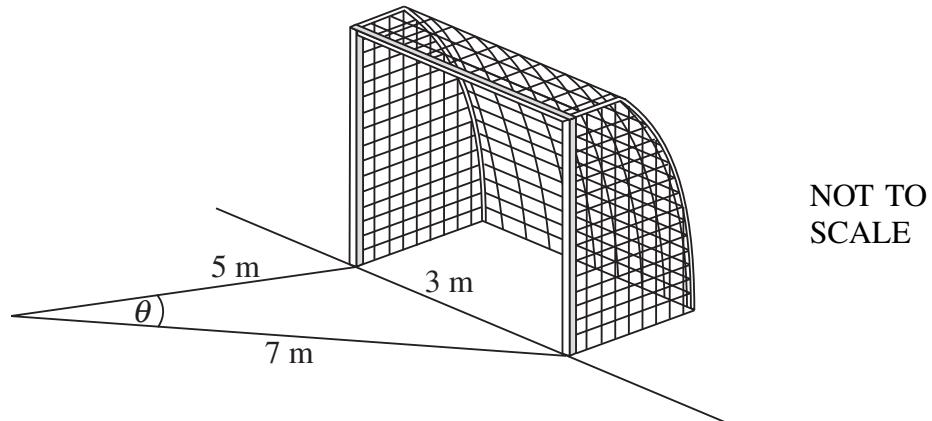
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6 A box of chocolates contains 12 with hard centres, and 12 with soft centres. Two chocolates are chosen at random.

What is the probability that either both have a hard centre, or both have a soft centre?

(A) $\dfrac{1}{3}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{11}{46}$ (D) $\dfrac{11}{23}$

7

NOT TO SCALE

5 m   3 m θ 7 m

In a game of Goal-ball the goal is 3 m wide. Sook-Yee has the ball in a position 5 m from the closest post and 7 m from the other post.

Calculate the size of the angle θ (to the nearest degree).
(Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$)

(A) 13°   (B) 22°   (C) 38°   (D) 64°

8

NOT TO SCALE   10 m
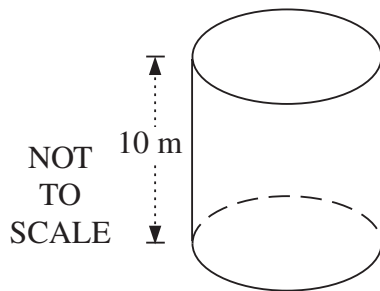
A cylindrical tank, used to store liquid chemicals, has a diameter of 6 m.

What is the storage capacity of this tank to the nearest litre? ($V = \pi r^2 h$)

(A) 283   (B) 1131   (C) 282 743   (D) 1 130 973

9   The ages of the children who live in Olympic Street are:

2   5   6   7   7   8   9   12

A new family, with one 7-year-old child, moves into the street.

Which one of the following changes?

(A)   The mean age

(B)   The median age

(C)   The range of the ages

(D)   The standard deviation of the ages

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10   Yousef was asked to solve an equation for homework. His solution, shown below, has two errors.

$$\sqrt{\frac{6x-2}{5}} = 36$$

$$\frac{6x-2}{5} = 6 \quad \text{.............. Line 1}$$

$$6x - 2 = 30 \quad \text{.............. Line 2}$$

$$6x = 28 \quad \text{.............. Line 3}$$

$$x = \frac{28}{6} \quad \text{.............. Line 4}$$

Which lines do NOT follow correctly from the previous line?

(A)   Line 1 and line 3   (B)   Line 1 and line 4

(C)   Line 2 and line 3   (D)   Line 2 and line 4

11 In the diagram, *B* represents the position of a boat on the ocean, and *L* represents the top of a lighthouse.
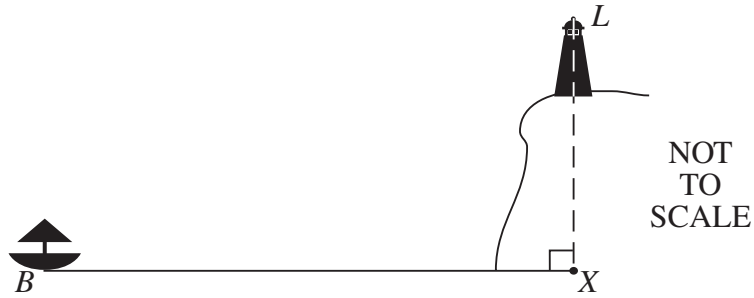
The distance *BX* is 3 km and *LX* is 50 m. The angle of elevation from *B* to *L* is θ.

Which of the following expressions is correct?

(A) $\sin\theta = \dfrac{3}{50}$

(B) $\sin\theta = \dfrac{5}{300}$

(C) $\tan\theta = \dfrac{3}{50}$

(D) $\tan\theta = \dfrac{5}{300}$

---

12 In Australia, a birth occurs every 2 minutes and 35 seconds.

Approximately how many births occur in one hour?

(A) 23   (B) 25   (C) 141   (D) 155

---

13 At a sale, Jason obtained a 40% discount and saved $240 on the normal purchase price of a TV.

How much did Jason pay for the TV?

(A) $96   (B) $144   (C) $360   (D) $600

---

14 At Sunshine Resort, the probability that it will rain on any particular day in January is 0·1. Gloria will spend 3 days at Sunshine Resort in January 2001.

What is the probability that it will rain on at least one of those 3 days?

(A) 0·001   (B) 0·271   (C) 0·3   (D) 0·729

---

15 In order to pass his university course, Dimitri must average at least 50% over 5 assessment tasks. After the first 4 assessment tasks, Dimitri has a mean mark of 45%. All tasks have equal weight.

What is the minimum mark, out of 100, that Dimitri must score in the fifth assessment task to pass the course?

(A) 50   (B) 55   (C) 65   (D) 70

---

16 Blue and yellow paints are mixed in the ratio 5 : 3 to make green paint. In making a batch of green paint, the number of litres of blue paint was 12 more than the number of litres of yellow paint.

How much green paint was produced in this batch?

(A) 19·2 L   (B) 32 L   (C) 48 L   (D) 96 L

---

17 Vanessa used five 2-litre bottles of lemonade and three 1-litre bottles of grape juice to make a special drink for a party. The lemonade cost $2·20 per bottle and the grape juice cost $1·80 per bottle.

Find the cost per litre of the special drink.

(A) $1·26   (B) $2·05   (C) $2·11   (D) $3·43

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18 The cost per student of hiring a bus for a school excursion is inversely proportional to the number of students on the bus. If there are 20 students, the cost per student is $36.

What is the cost per student when there are 30 students on the bus?

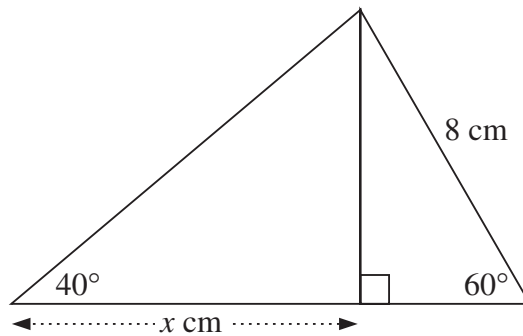(A) $18   (B) $24   (C) $46   (D) $54

19   The heights of 20 boys were recorded. The following diagram is a cumulative frequency polygon illustrating the measurements.
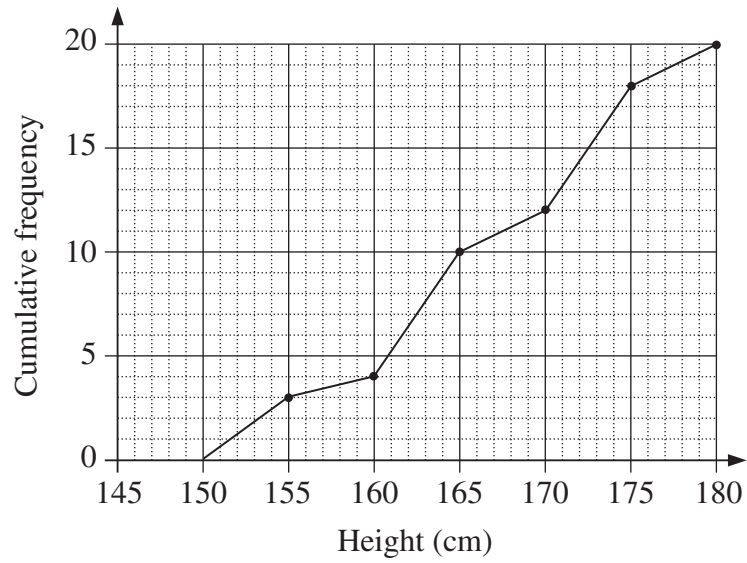
How many of these boys were taller than 160 cm, but no taller than 170 cm?

(A)   4   (B)   8   (C)   10   (D)   12

20

The value of x in the diagram can be calculated using the expression

(A) $\dfrac{8\sin 60°}{\tan 40°}$.

(B) $8\sin 60° \tan 40°$.

(C) $\dfrac{8\tan 40°}{\sin 60°}$.

(D) $\dfrac{8}{\sin 60° \tan 40°}$.

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SECTION II

Attempt ALL questions.

Each question is worth 12 marks.

Show all necessary working.

QUESTION 21 Use a SEPARATE Writing Booklet.

(a) Calculate $(3{\cdot}21 \times 10^{23}) \times (7{\cdot}34 \times 10^{-21})$. Give your answer in scientific notation.

(b) Solve the equation $3 + \dfrac{2}{x} = \dfrac{5}{2x}$.

(c) A stone is thrown at a flock of birds flying at a height of 15 metres.

   The height h (in metres) of the stone, t seconds after it is thrown, is given by $h = 2 + 20t - 5t^2$.

   (i) Draw a graph of height against time on the graph paper provided on page 33.

   (ii) At what time does the stone first reach the same height as the birds?

   (iii) What is the maximum height reached by the stone?

(d) Two triangular pieces of shade-cloth, equal in area, are sewn together.
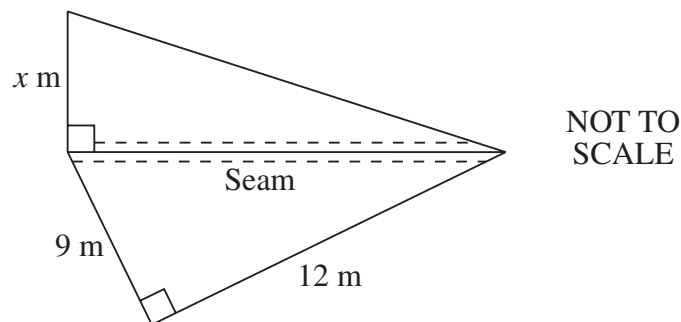
   (i) Find the length of the seam.

   (ii) Find the value of x.

QUESTION 21 (Continued)

(e)   At a Maths Fun Day, one of the games involves randomly selecting one of the following cards from a box.

| M | A | T | H | E | M | A | T | I | C | S |

Rules of the game:

- Choose | M | and win! (Game over)

- If you do not choose | M | replace the card and choose again.

  Choose | M | this time and win, otherwise you lose! (Game over)

   (i)   If a letter is chosen from the box, what is the probability that it is an | M | ?

   (ii) What is the probability of winning this game?

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QUESTION 22 Use a SEPARATE Writing Booklet.

(a)   A racing car completed a 6·3 kilometre lap in 2 minutes 15 seconds. What was the car's average speed in km/h?

(b)   In the following diagram, *A* and *B* represent the positions of two communication towers. Tower *B* is due east of Tower *A*, at a distance of 5 km.

$$A \times \xmapsto{ 5 \text{ km} } \times B$$
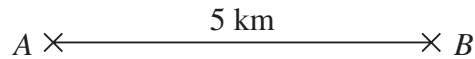
   (i)   Copy the diagram into your Writing Booklet. A third tower, *C*, bears 210° from *B*, and 170° from *A*. Mark the position of Tower *C* on your diagram.

   (ii)   Find the distance, in kilometres, from Tower *A* to Tower *C*. (Give your answer correct to one decimal place.)

$$\left(\text{Sine rule} : \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \right)$$

(c)   In a large population, the percentages of people with blood groups *O*, *AB*, *A* and *B* are as shown in the table.

O	*AB*	*A*	*B*
46%	3%	11%	40%

   (i)   A person is chosen at random from this population. What is the probability that this person has blood group *O* or *A*?

   (ii)   A random sample of ten people is chosen from the population. Find the probability that all ten people have blood group *O*. (Give your answer to two significant figures.)

   (iii)   Two people are selected at random from this population. What is the probability that at least one of these people has blood group *B*?
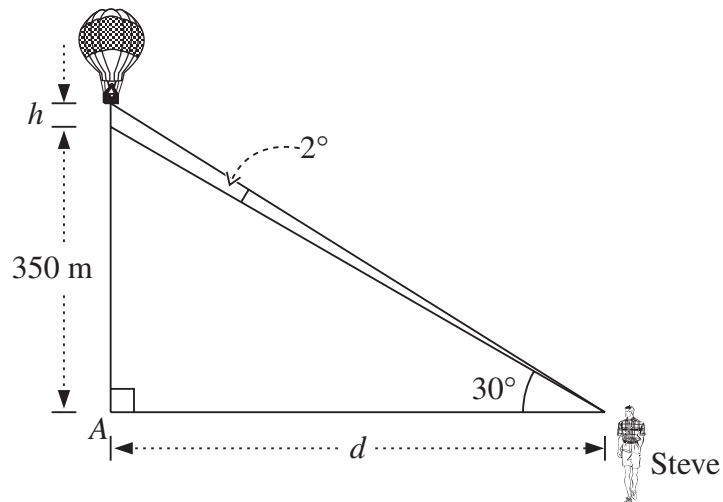
QUESTION 22 (Continued)

(d)   Steve notices a hot air balloon at an angle of elevation of 30°.

(i)   If the balloon is 350 m directly above point A, calculate the distance (d) between Steve and point A (to the nearest metre).
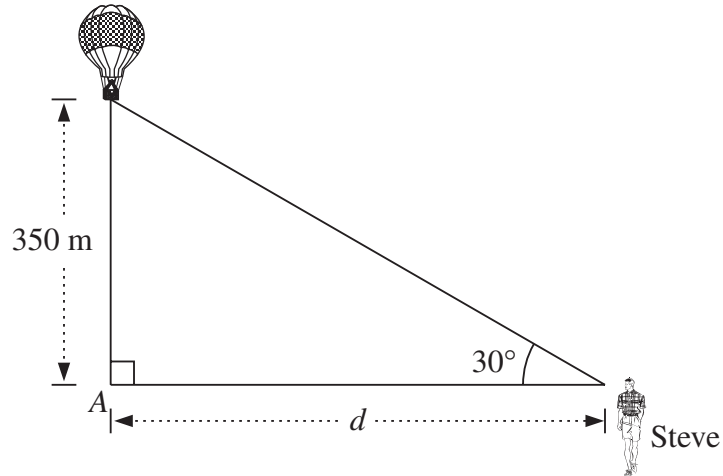
(ii)   If the balloon rises so that the angle of elevation is now 32°, calculate the extra height (h) of the balloon above point A.

QUESTION 23 Use a SEPARATE Writing Booklet.

(a) When Bill sets the thermostat on his central heating to d degrees Celsius, the actual temperature in his house has a mean of d degrees and a standard deviation of 0·5 degrees. Bill believes that if he sets the thermostat at 24°C he can be almost certain that the temperature will always be at least 22°C.

Is Bill correct in this belief? Explain your answer.

(b) Vicki and Marcelle are two members of a tenpin bowling team.

The scores for Vicki's last ten games are given in the table.

Game	1	2	3	4	5	6	7	8	9	10
Score	120	130	98	143	132	88	125	136	156	142

(i) What is the mean of Vicki's scores?

(ii) Calculate the standard deviation of Vicki's scores.

(iii) Vicki's bowling handicap is 80% of the difference between her mean score and 200.

Calculate Vicki's bowling handicap. (Give your answer to the nearest whole number.)

(iv) The mean of Marcelle's last ten scores is the same as Vicki's. The standard deviation for Marcelle's scores is 10·3.

Is Vicki or Marcelle the more consistent player? Explain your answer.

QUESTION 23 (Continued)

(c) The following table contains data about the heights of 40 Year 12 students.

Height range (cm)	Class centre (cm)	Frequency
155–159	157	3
160–164	162	7
165–169	167	6
170–174	172	6
175–179	177	9
180–184	182	A
185–189	187	3
190–194	192	2
195–199	197	1

(i) The frequency for the 180–184 height range is written as A in the table. What is the value of A ?

(ii) What is the modal class?

(iii) What is the median class?

(iv) Use the class centres to calculate the approximate mean height.

(d) PURA-WATER comes in a large cylindrical container with diameter 30 cm and height 40 cm.

PURA disposable cups are cylindrical. They have a diameter of 5 cm and can be filled to a depth of 6 cm.
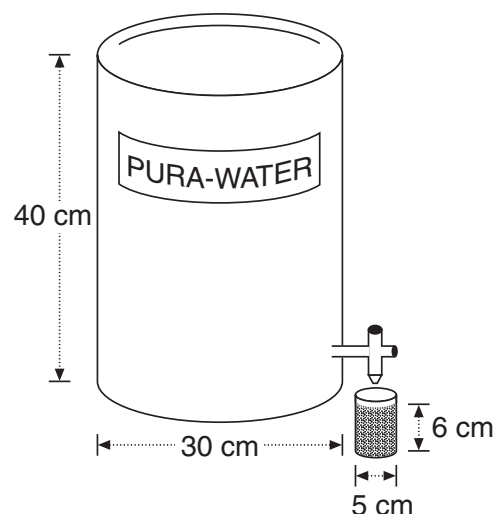
(i) Find the volume of water in a full PURA-WATER container. ($V = \pi r^2 h$)

(ii) If cups are filled to a depth of 6 cm, how many cups can be filled from the full container?

(iii) A company uses two containers of PURA-WATER per week. Cups can only be bought in boxes of 1000, at a cost of $18 per box. Find the cost of cups for one year. (Assume that only PURA disposable cups are used and that they are filled and used only once.)

SECTION III

Attempt TWO questions.
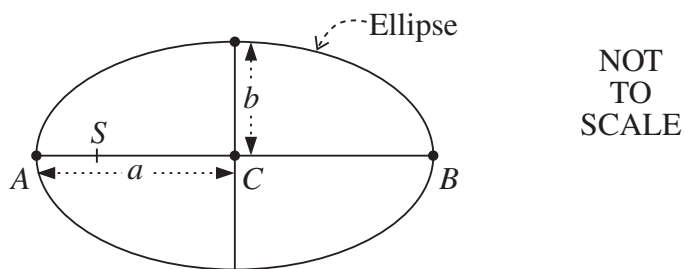
Each question is worth 12 marks.

Show all necessary working.

QUESTION 24 Space Mathematics Use a SEPARATE Writing Booklet.

(a) A galaxy has a diameter of $2 \cdot 365 \times 10^{20}$ kilometres.

Convert this measurement to light years. (1 light year $= 9 \cdot 46 \times 10^{12}$ km)

(b)

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For the ellipse shown, the ratio $\dfrac{b}{a} = 0 \cdot 75$.

(i) Calculate the eccentricity of this ellipse. $\left(e^2 = 1 - \dfrac{b^2}{a^2} \right)$

(ii) If the distance AB is 40 cm, how far from A is the focus S? ($CS = e \times CA$)

(iii) Describe the position of the other focus of the ellipse.

(iv) In the context of planetary motion, what is the significance of an ellipse and its foci?

QUESTION 24 (Continued)

(c)   The table shows details for the three planets that are nearest to the Sun.

Planet	Orbital eccentricity	Average distance from Sun (km)
Mercury	0·206	$5{\cdot}79 \times 10^7$
Venus	0·007	$1{\cdot}08 \times 10^8$
Earth	0·017	$1{\cdot}49 \times 10^8$

(i)   Which planet has an orbit that is closest to a circle?

(ii)   What is the average distance of Mercury from the Sun in astronomical units? (1 AU = $1{\cdot}49 \times 10^8$ km)

(iii) Use the formula $R^3 = T^2$ (where R is in AU and T is in years) to find the period of Mercury.

(iv)   Calculate the length of Mercury's orbit. ($C = 2\pi r$)

(v)   Find the average speed (in km/h) at which Mercury orbits the Sun.

Please turn over

QUESTION 25 Mathematics of Chance and Gambling
                Use a SEPARATE Writing Booklet.

(a)   What are the odds of throwing a 6 when a single unbiased die is rolled?

(b)   The racehorse Miss Mipp will run in its final race at odds of $4 : 1$ on.

   Rodney bets $100 on Miss Mipp in this race. How much will Rodney collect if Miss Mipp wins?

(c)
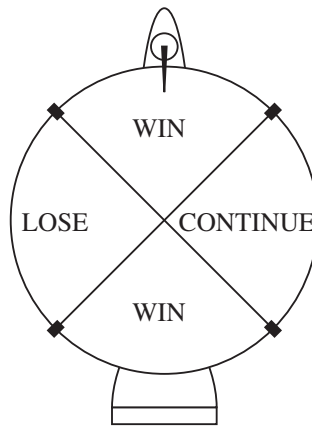
   On a spinning wheel, two sections are labelled WIN, one section LOSE, and the other section CONTINUE.

   If the wheel stops on CONTINUE, you have another spin.

   Find the probability of:

   (i)   winning on the first spin;

   (ii)   losing on the first spin;

   (iii) taking exactly two spins to win.

QUESTION 25 (Continued)

(d) A poker machine consists of four reels that spin when the PLAY button is pressed.
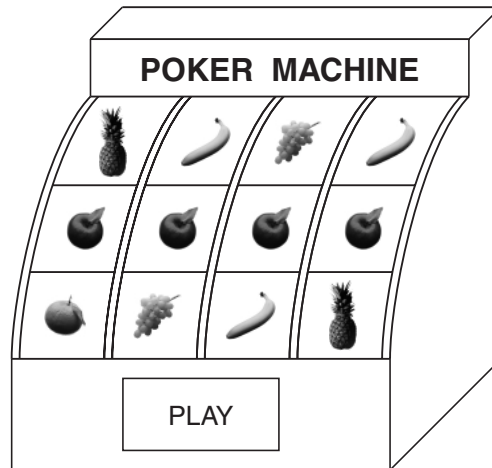
Each reel has 20 symbols. The table describes how many symbols of each type are on each reel.

Symbol / Reel	Apple	Orange	Grape	Banana	Pineapple
1	1	2	7	4	6
2	1	3	6	5	5
3	1	4	5	6	4
4	1	5	4	7	3

(i) What is the probability that there are four apples on the centre line (as shown in the diagram) when the reels stop spinning?

(ii) What is the probability that there are either four apples or four oranges on the centre line when the reels stop spinning?

QUESTION 25 (Continued)

(e) In the game of 'Double Barrel' there are two barrels.

- Barrel 1 contains 4 balls, numbered ①, ②, ③, ④

- Barrel 2 contains 6 balls, numbered ⑤, ⑥, ⑦, ⑧, ⑨, ⑩

	DOUBLE BARREL	
	Level 1 Game	*Level 2 Game*
Step 1	Nominate one numbered ball from each barrel	Nominate two numbered balls from each barrel
Step 2	Select one ball from each barrel	Select two balls from each barrel
Step 3	WIN a prize if all the nominated balls are selected	

(i) What is the probability of winning a Level 1 Game?

(ii) If the prize for a Level 1 Game is $12, what is a fair price to pay for one Level 1 Game?

(iii) What is the probability of losing a Level 2 Game?

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QUESTION 26 Land and Time Measurement Use a SEPARATE Writing Booklet.

(a)   The notebook entries shown below are the results of a traverse survey of a field.

$$B$$

		90		
D	30	50		(Measurements are in metres.)
		30	15	C
		0		

$$A$$

(i)   Use these notebook entries to draw a neat sketch of the field. Mark the interval and offset measurements.

(ii)   Find the area of the field.

(b)   The diagram shows the results of a radial survey.

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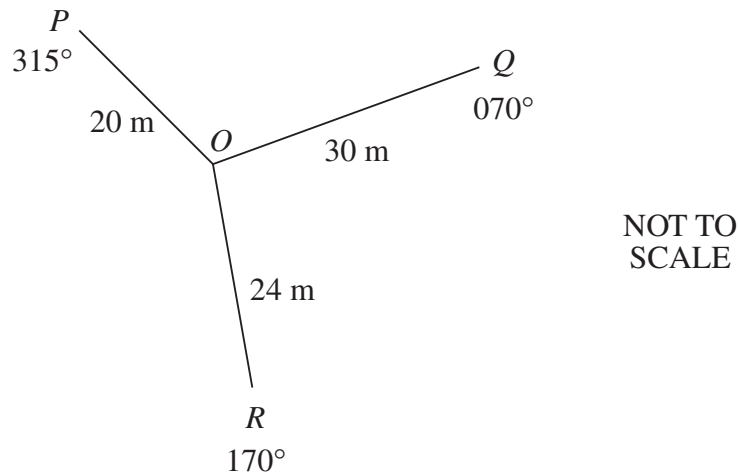
(i)   Calculate the size of $\angle QOR$.

(ii)   Calculate the distance QR correct to the nearest metre.
(Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$)

(iii) Calculate the area of triangle POQ correct to the nearest square metre.
$$\left(\text{Area} = \frac{1}{2} ab \sin C \right)$$

QUESTION 26 (Continued)

(c)   Alexandria is located at 31°N 30°E, and Shanghai is located at 31°N 121°E.

(i)   Find the radius of the parallel of latitude that passes through Shanghai. (Take the radius of Earth to be 6400 km; $r = R \cos \theta$.)

(ii)   Find the shortest distance between Alexandria and Shanghai along the parallel of latitude joining them.

(iii)   What will the time be in Shanghai when it is 7:30 am in Alexandria? (Ignore time zones.)

(d)   The diagram below shows an area of land bounded by a river and fences.
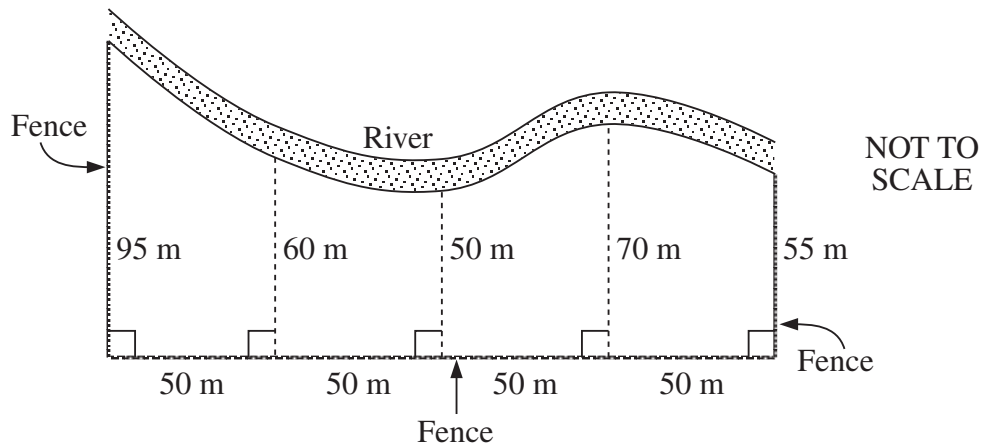
Use two applications of Simpson's rule to find the approximate area of this land.

$$\left(Area \approx \frac{h}{3}\left(d_f + d_l + 4d_m \right) \right)$$

QUESTION 27 Personal Finance Use a SEPARATE Writing Booklet.

(a) Simona is a salesperson for Nova cosmetics. She is paid $350 per week and receives, in addition, a commission of 5·5% on her sales in excess of $600.

    (i) What does Simona earn in a week when she makes sales of $1050?

    (ii) What are Simona's sales in a week if she is paid $397·30?

(b) Lisa buys 2000 shares in a toy company, for $1·27 each. She sells them 12 months later, through a stockbroker, for $2·75 each.

The stockbroker charges 3% of the selling price.

    (i) Find the gross profit.

    (ii) Find the net profit as a percentage of the cost of the shares.

(c) The following information was taken from a credit card statement that Sebastian received in May 2000.

Payment due date	4 June 2000
Minimum amount due	$25·00

Overdue	Opening Balance	New Charges	Payments/refunds
$0·00	$44·95	$925·90	− $15·00

		Annual rate	15·100%	Daily rate	0·04137%

    (i) Sebastian made a payment of $450 on the due date. What was the outstanding amount on his credit card after he made this payment?

    (ii) How much interest would be charged on this outstanding amount from 4 June 2000 to 4 July 2000?

QUESTION 27 (Continued)

(d) Michael obtains a personal loan from a bank to buy a computer.

The loan is for $4500. Michael also pays a loan protection fee of $1·24 per $100 borrowed. This fee is added to the $4500.

  (i) Find the total amount borrowed.

 (ii) The loan is to be repaid over 3 years, and interest is charged on the total amount borrowed at 8% p.a. flat.

Calculate the interest charged.

(iii) Michael repays the loan in equal monthly instalments.
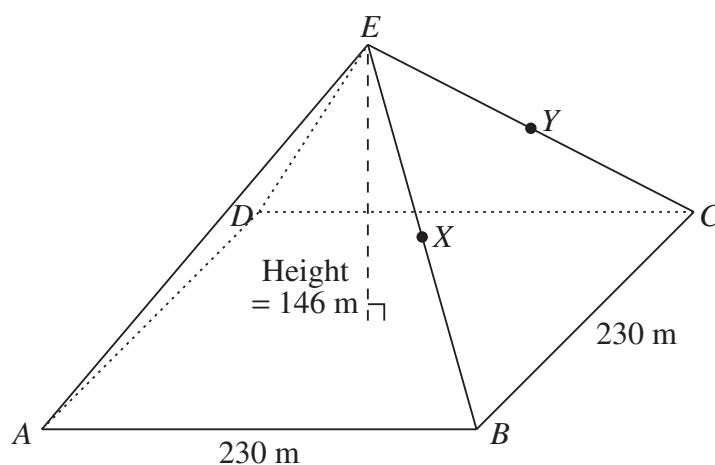
What is the amount of each instalment?

(iv) Find the effective annual rate of interest, a, using the formula $a = \dfrac{2Rn}{n+1}$,

where R is the flat rate of interest and n is the number of instalments.

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QUESTION 28 Mathematics in Construction Use a SEPARATE Writing Booklet.
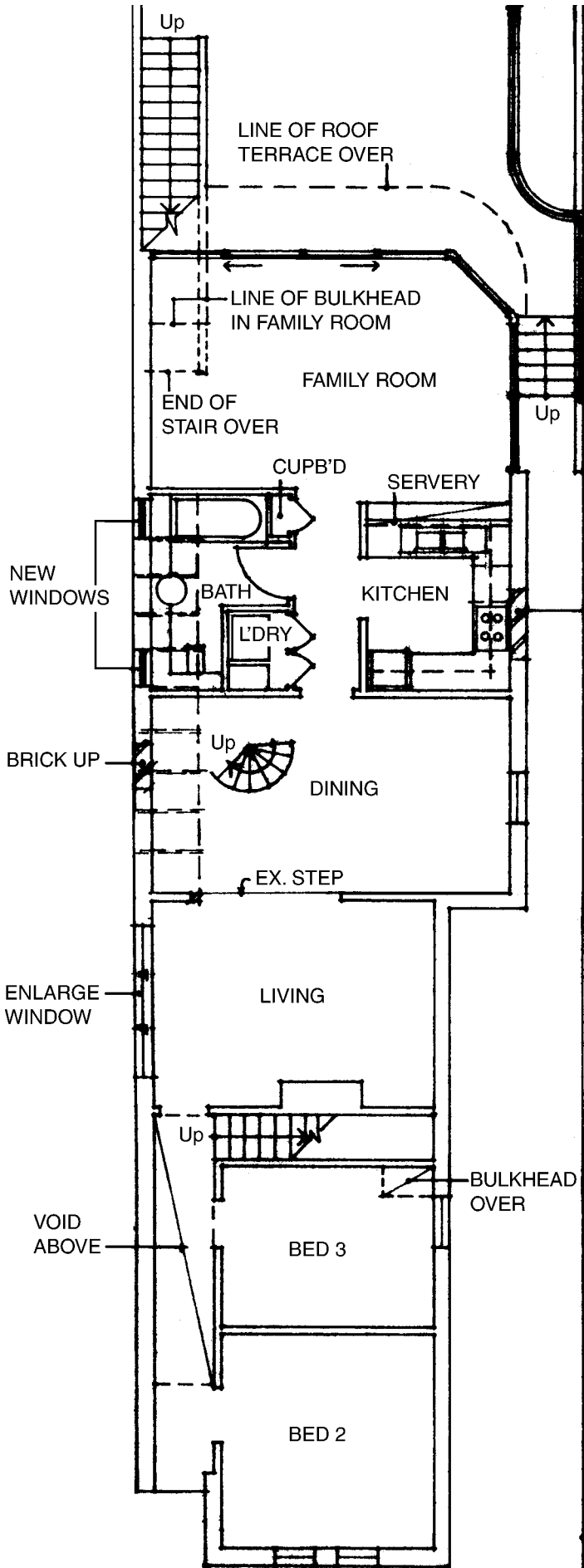
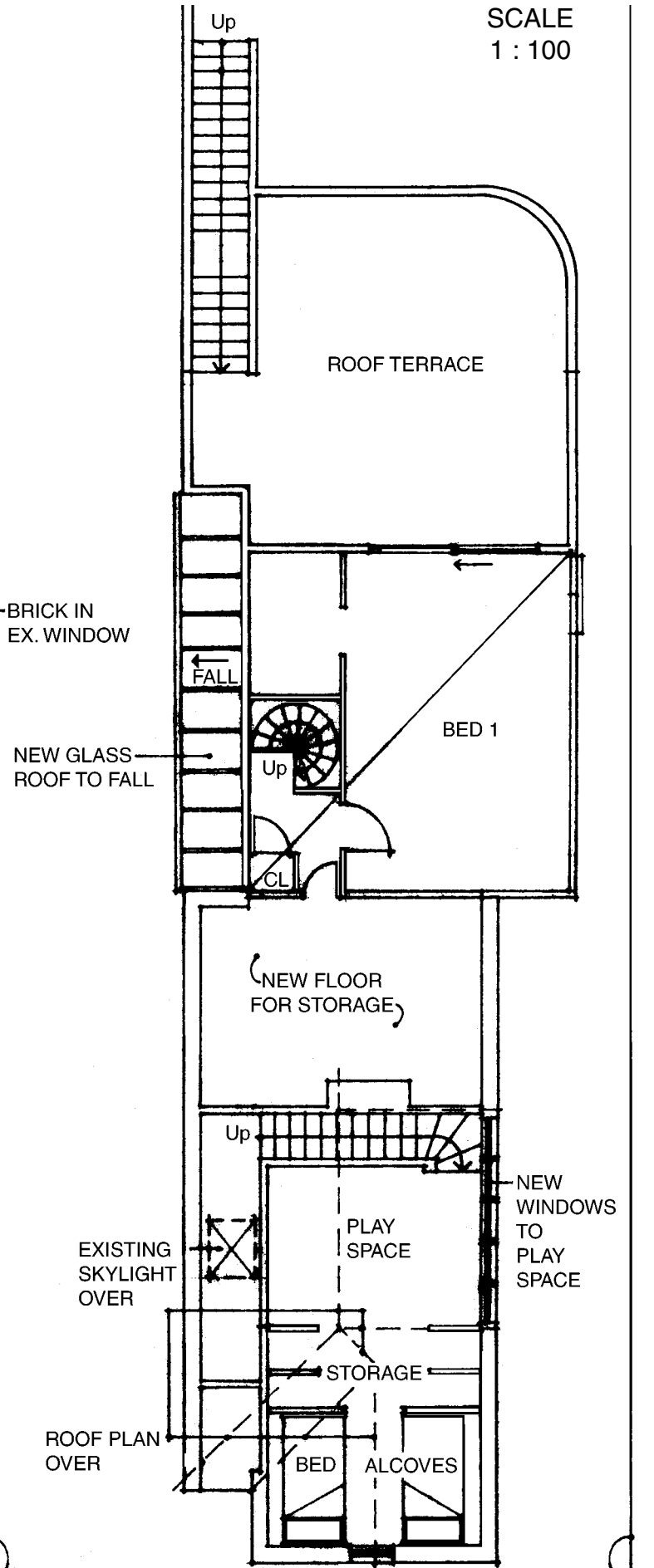(a)   The Great Pyramid of Egypt is a right square pyramid with dimensions as shown.

(i)   *X* is the midpoint of *EB*, and *Y* is the midpoint of *EC*. Sketch a vertical cross-section of the pyramid, through the line *XY*.

(ii)   Draw a sketch showing the projection of the pyramid onto its base *ABCD*.

(iii)   The three points *E*, *B* and *Y* determine a plane. Name another point in this plane.

(iv)   What is the line of intersection of the plane *AEB* and the base *ABCD*?

(v)   Find the size of the angle between the plane *AEB* and the plane *ABCD*.

(b)   The floor plans and the south elevation (on pages 28 and 29) for proposed alterations to a house are shown.

(i)   How many flights of stairs are there?

(ii)   What is the area, in square metres, of Bedroom 2?

(iii)   Sketch the east elevation of Bedroom 1.

(iv)   What is the maximum height of the building from ground level?

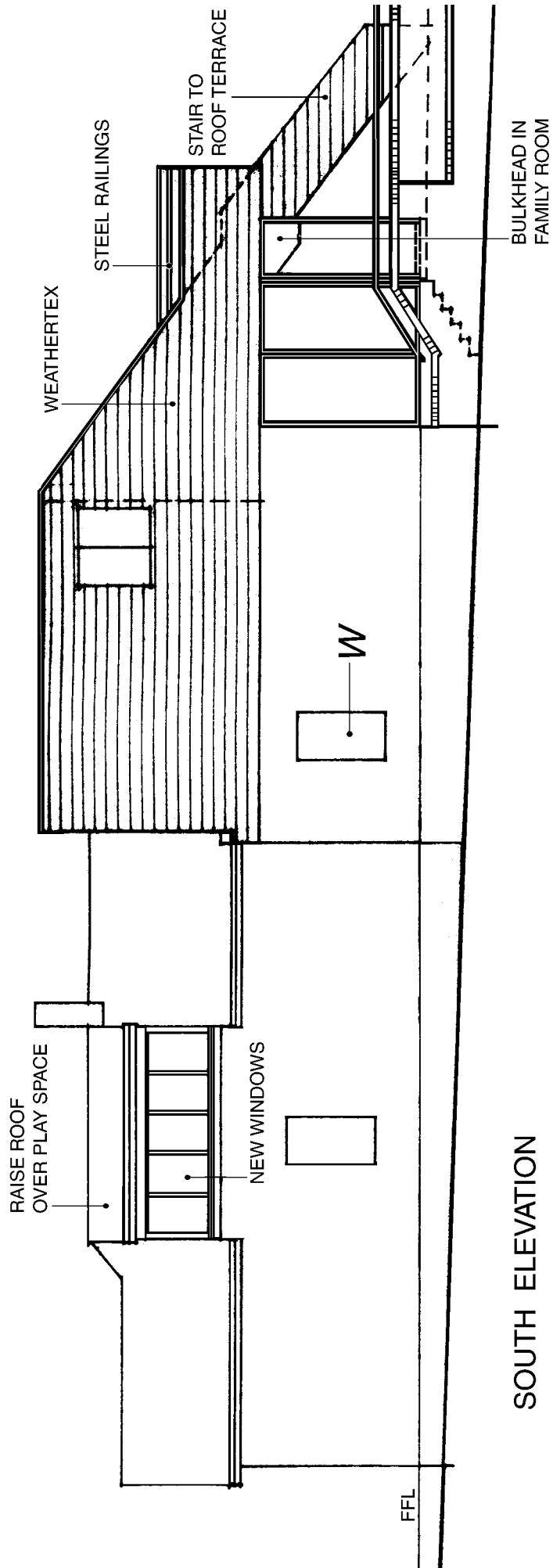(v)   On the south elevation a window is labelled *W*. To which room does this window belong?

Up

LINE OF ROOF
TERRACE OVER

LINE OF BULKHEAD
IN FAMILY ROOM

FAMILY ROOM

END OF
STAIR OVER

Up

CUPB'D

SERVERY

NEW
WINDOWS

BATH

KITCHEN

L'DRY

BRICK IN
EX. WINDOW

BRICK UP

Up

DINING

EX. STEP

ENLARGE
WINDOW

LIVING

Up

BULKHEAD
OVER

VOID
ABOVE

BED 3

BED 2

LINE OF
PROPERTY

GROUND FLOOR PLAN

Up

SCALE
1 : 100

ROOF TERRACE

FALL

NEW GLASS
ROOF TO FALL

Up

CL

BED 1

NEW FLOOR
FOR STORAGE

Up

EXISTING
SKYLIGHT
OVER

PLAY
SPACE

NEW
WINDOWS
TO
PLAY
SPACE

STORAGE

ROOF PLAN
OVER

BED

ALCOVES

LINE OF
PROPERTY

FIRST FLOOR PLAN

STAIR TO
ROOF TERRACE

STEEL RAILINGS

WEATHERTEX

BULKHEAD IN
FAMILY ROOM

RAISE ROOF
OVER PLAY SPACE

NEW WINDOWS

W

FFL

SOUTH ELEVATION

SCALE
1 : 100

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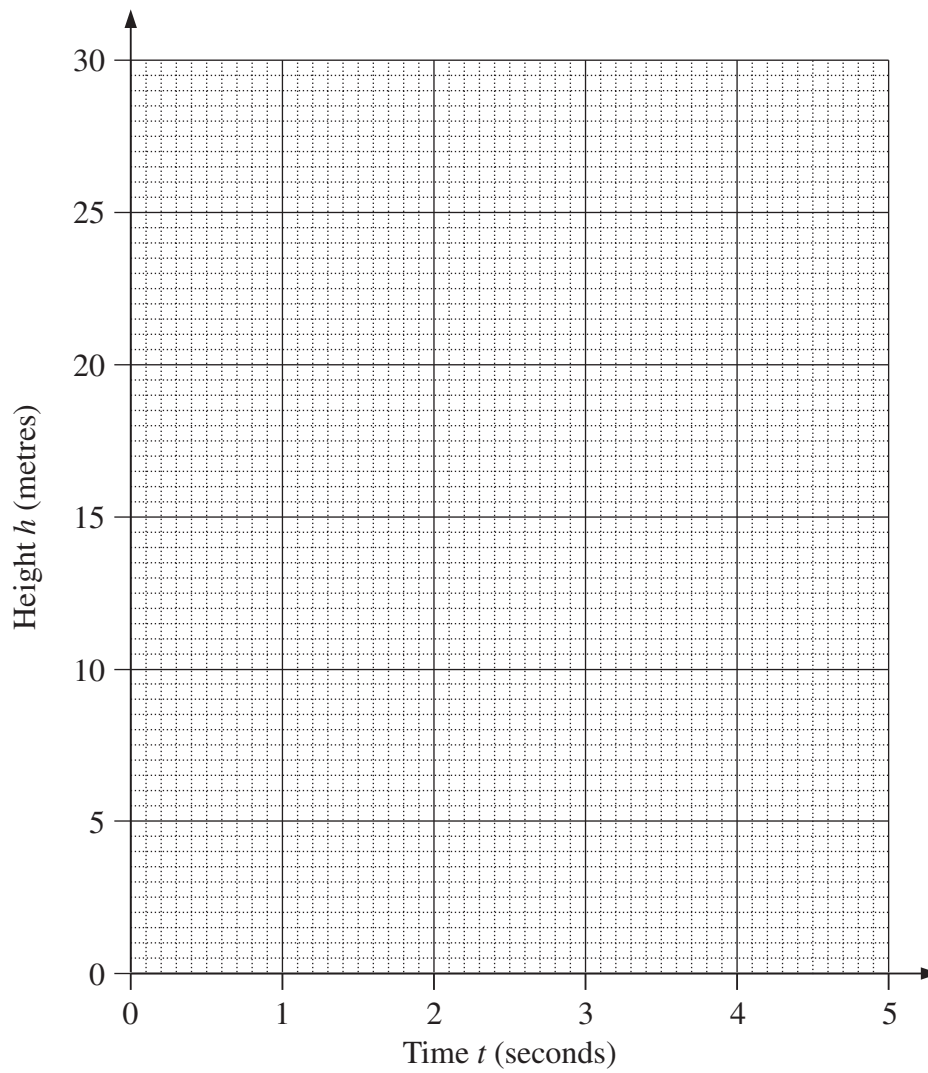
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STUDENT NUMBER

2000
HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS IN SOCIETY
2 UNIT

CENTRE NUMBER

This page is to be detached, completed and attached to the inside front cover of your Writing Booklet for Question 21.

QUESTION 21

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