BOARD OF STUDIES
NEW SOUTH WALES

1995 HSC

EXAMINATION REPORT

MATHEMATICS

Includes:
- **Marking criteria**
- **Worked answers**
- **Examiners' comments**

Published by
Board of Studies NSW
PO Box 460
North Sydney NSW 2059
Australia

ISBN 0 7310 6364 3

95227

MATHEMATICS

General Comments.

The Mathematics examinations in 1995 resulted in a good discrimination between candidates at all levels. Specific comments follow on each particular course. There were changes in 1995 in the examinable content of each examination except 4 Unit Additional, due to the division of the courses into Preliminary and HSC sections. There were more substantial changes in Mathematics in Practice and Mathematics in Society in that the time allocated to each examination was reduced to two and a half hours and the number of free response questions was reduced to 5 in both courses. Mathematics in Society candidates were asked to attempt only two options.

The Multiple Choice questions in MiS and MiP were machine marked with each question being initially marked either right (1 mark) or wrong (0 marks). These marks were then scaled to the required value in each course, the computer carrying more than enough decimals to ensure no loss of discrimination in the process.

All free response questions in all courses except 4 Unit Additional were marked in whole numbers out of 12. In 4 Unit, questions were marked out of 15 marks. No half marks are ever awarded. Neither are negative marks awarded.

Again marks are scaled to the required value in each course, with the computer carrying sufficient decimals to ensure no loss of discrimination at any level. Board scores are calculated for each candidate in accordance with well known and published procedures. The Tertiary Entrance Rank (TER) is calculated by the Committee of Chairs of Academic Boards of Universities in New South Wales and the ACT independently of the Board of Studies, which has no input into this process, beyond supplying results.

As a general rule, candidates should be encouraged to bear the following facts in mind.

1. At all times, answers should indicate in some way to examiners how they were derived. Hence, for example, candidates should not give single word or figure responses. If correct, such answers usually, though not always, receive full marks.

On the other hand, if incorrect, they will most often receive no part marks, since the examiner may have no idea how the answer was arrived at.

2. Graphs and diagrams should be clearly marked and reasonably executed to assist both the candidate and the examiner to know what they are doing. They should also be larger rather than smaller, for the same reason. When the paper asks the candidate to copy the diagram into their exam booklet, they well not receive marks for the exercise since it usually involves a tracing out or a simple reproduction of a given diagram into their booklet. Unfortunately many candidates neglect this instruction to their detriment, since to complete the question, they will invariably have to insert additional information into the diagram and examiners (and candidates!) are unable to follow arguments involving non-existent diagrams.

3. Candidates should use pages in the booklet in which they are answering the given question for any rough work and not set aside a booklet for all such work for all questions. This is because all such work is read and all may receive part marks if appropriate. While it is possible to assign part marks to rough work on the question in the booklet involved, it is virtually impossible to locate work on specific questions in an assorted collection of rough work on different questions in a separate booklet.

   Examiners do read everything written by every candidate, whether it is written on the backs of pages or booklets crossed out or even carrying an instruction ‘Do not mark this work’. Since negative marks are never awarded, any written work will receive marks if it deserves it.

4. Candidates in the higher levels should be reminded not to forget the existence and utility of tables of Standard Integrals and formulas.

5. Candidates should write answers to different questions in different booklets. On the other hand, if they should forget this detail under the stress of the moment, they should ensure that working for specific questions is identified as such and then continue with their remaining answers as required. They should be assured that all their work will be read and marked in any event and they will suffer no disadvantage or penalty for any such slip.

MATHEMATICS IN PRACTICE

SECTION II

Question 31 — The Consumer

(a) Diagram of a sign advertising a sale with '20% off all marked prices'.

(i) (1 mark)
Candidates were asked to calculate the sale price of a shirt marked at \$75. This was well answered.

(ii) (2 marks)
This involved calculation of the sale price, given the saving in dollars. This was found to be much more difficult. One mark was awarded for correct calculation of the marked price, or for an essentially correct calculation but with one error.

(b) Table of Monthly Repayments versus Amount Borrowed (over 15, 20 & 25 years)

(i) (1 mark)
Candidates were asked to indicate the maximum amount which can be borrowed if the maximum monthly repayment was \$700. This was generally answered well.

(ii) (2 marks)
In this question candidates were asked to calculate the amount saved if a \$50 000 loan was repaid over 15 years rather than 25 years. Quite a reasonable proportion managed to do this correctly, but many used incorrect figures from the table.

1 mark was awarded if 1 error appeared in an otherwise correct calculation.

(c) All costs associated with owning a mobile phone were given.

(i) (1 mark)
Given the total cost of 1 year's calls, candidates were asked to calculate the

total amount spent on the phone over the year. This was not very well done, as many omitted one or more of the associated costs.

(ii) (1 mark)
Find the cost of a 2 minute call. This was fairly strongly answered, with most answers accounting for the different price for the first 30 secs.

(iii) (1 mark)
Asked to calculate the length (in minutes) of a $1.24 call. Reasonably well answered, but with many neglecting to convert to minutes, or doing so incorrectly. Accounting for the different costs for the first 30 secs and the rest was found difficult by many.

(iv) (1 mark)
Given that 20 calls were made at an average cost of $1.24, candidates were asked to calculate a month's bill. Many omitted the monthly access fee.

(v) (2 marks)
The question was to calculate the difference between the monthly costs for 2 people using different cost structures. Once again many omitted the monthly access fees, but it was generally well answered.

Question 32 — Travel

(a) (3 marks)
The questions related to a train timetable.

(i) (1 mark)
Involved correct reading of the table to ascertain the latest possible train that could be caught to arrive at a certain destination by a specified time. The question was well answered.

(ii) (1 mark)
Candidates had to calculate a time difference to work out the length of the train trip. Again, this was well answered.

(iii) (1 mark)

Involved a correct reading from the table and a calculated time difference. This was not as well answered as the previous two parts however it is difficult to comment on their errors as they are loathe to show any working.

(b) (4 marks)

Candidates had to answer questions related to a table on cruise fares.

(i) (2 marks)

This was poorly answered. There was a general misunderstanding of the information contained in the table. The majority of students calculated the fare for 1 person instead of 2 (1 mark was awarded for this). Another common error was to ignore the transfer fee and give 3160 as an answer (awarded 1 mark).

(ii) (1 mark)

Reasonably well answered. Candidates were asked to calculate the extra cost of taking a child on the trip. The main error was again, in ignoring the transfer fee.

(iii) (1 mark)

The calculation of the refund for cancelling the trip was handled very poorly.

(c) (5 marks)

Students had to answer questions relating to a scale map of New Zealand.

(i) (2 marks)

This involved a calculation (using scale) of the distance between two cities. It was well answered. 1 mark was awarded for use of scale but there was much inaccuracy in estimation. 1 mark was also awarded for 'as the crow flies' estimation of distance.

(ii) (1 mark)

Candidates had to calculate average speed for the trip, where the formula was given. This was well answered.

(iii) (2 marks)

Candidates were given a rate of petrol consumption and asked to find the amount of petrol the car used for the trip. Not well answered.
1 mark was awarded for (3×8.2) and a ‘bit more added on’

Question 33 — Accommodation

(a) (5 marks)

The question gave a table showing the comparative prices of four houses in Sydney suburbs, at the beginning and end of 1993.

(i) (1 mark)

Candidates had to calculate the percentage increase of one house. This was not well answered. A large number did not even attempt the question.

(ii) (2 marks)

Candidates had to calculate which of the four houses had the greatest profit. 2 marks were awarded for \$38 870, 1 mark for a bald ‘Clovelly’. The question was well answered, the most common error being the correct calculation of the four profits *without* a conclusion (1 mark).

(iii) (2 marks)

Candidates were given the percentage increase during 1994 for one of the houses, and were asked to find the increase in the value of the house from 1993 until the end of 1994. This question was poorly answered. It involved several steps, however most only got as far the first step which involved finding $11/100 \times 280\,000 = 30\,800$. This earned them 1 mark.

(b) (4 marks)

The question gave a floor plan of a house, scale 1:100.

(i) (1 mark)

Candidates were asked to find an *actual* length. To gain the mark, units were necessary. This was very poorly done. Answers were very unreasonable, and ranged from 15 cm to 670 m.

(ii) (1 mark)

This involved a calculation of area and costing . It was poorly answered, with many simply multiplying their answer in (i) by $35. Again it was very evident that candidates did not consider how *appropriate* their answer was, as $3.5 million dollars and 35 cents were seen.

(iii) (2 marks)

The question supplied information about costs of two different Carpet Cleaning services.

1. (1 mark) Candidates had to calculate the cost for choosing 'A' Carpet Cleaning service. This was very well answered.

2. (1 mark) Candidates had to work out how much would be saved by using Service 'B' instead of 'A' – this was also well answered.

(c) (3 marks)

Candidates were given a diagram showing the plan of a block of land with a rectangular house on it.

(i) (1 mark)

Candidates had to find the area of the house – not well answered, with the majority finding the perimeter.

(ii) (1 mark)

Candidates had to find the area of the block of land which involved the addition of an triangle and rectangle. This was poorly answered. Missing side lengths had to be found, and this proved too difficult for most people. Again the most common error was in the calculation of *perimeter*.

(iii) (1 mark)

Candidates were told that the house could only occupy 60% or less of the area of the block of land. They were asked to state whether the house would satisfy this regulation, and give a reason. Not well answered – many candidates who correctly found the percentage then made an incorrect conclusion.

Question 34 — Design

The number of students who disadvantaged themselves by not using a rule or other straight edge for the construction questions was significant.

(a) (2 marks)
The question asked for a continuation of the pattern shown by carefully drawing in the next element. Most could generally recognise the pattern, although many created variations on it. Many were unable to draw in carefully the next shape, with incorrect answers ranging from just outside the accepted tolerances to roughly sketched highly inaccurate responses.

1 mark was awarded if an answer exhibited 3 of the 5 recognised features of an accurate answer, or if an otherwise accurate answer was drawn freehand.

(b) (2 marks)
The question was to name the 2 geometrical shapes which made up a tiling pattern. This was done well, with a mark each being awarded for square and octagon. Diamond was not accepted unless qualified by the word square, eg square diamond or diamond (square on its point). Incorrect spelling was not penalised, but the marker needed to be sure the correct answer was intended.

(c) (2 marks)
The diagram of an incomplete flag was provided, with a description of the flag. Candidates had to draw in 2 horizontal stripes of the correct width and spacing.

1 mark was awarded if the stripe width was within an accepted tolerance, and 1 mark was awarded if the spacing was within an accepted tolerance. Not well done.

(d) (2 marks)
The question was to construct a scale drawing of a rectangular poster. The lack of accuracy was notable in this question, with poor use of geometrical equipment to measure right angles and lengths.

1 mark was awarded if one dimension of the rectangle was correct, or the rectangle dimensions were in the correct proportion.

(e) Diagram of 2 boxes made to pack chocolates of given dimensions.

(i) (1 mark)
How many chocolate blocks would fit in Box 1? Well answered.

(ii) (2 marks)
What is the surface area of the outside of Box 1? Not well done. Students who included lid were not penalised (1392 cm^2). This was also the answer if the surface area of Box 2 was correctly calculated, and so was accepted.
1 mark was awarded if 1 error was exhibited in an otherwise correct calculation.

(iii) (1 mark)
What was the height of Box 2? This was well answered, with students showing a range of strategies.

Question 35 — Social Issues

(a) (2 marks)
The question asked for an interpretation of a table on wind speeds.

(i) (1 mark)
Candidates had to describe the type of wind that had a speed of 30 knots by correctly reading the table. It was well answered.

(ii) (1 mark)
Candidates were required to convert knots to km/h they were given the conversion rate).
Correct answer 118.4 (64×1.85)
Errors included $64 \div 1.85$, 65×1.85

(b) (4 marks)

The question gave a table showing a relationship between Residents in full time employment, Residents who were full time students, and where they lived in respect to proximity to the city centre.

(i) (1 mark)
Candidates were awarded the mark if they correctly added the two required percentages. It was poorly answered, most writing down the two percentages with no attempt at addition.

(ii) (2 marks)
Candidates had to construct a column graph referring to Full time employment *only*. It was well answered.

1 mark was awarded for an incorrectly drawn graph if the percentages were plotted correctly (according to the vertical axis and scale).

1 mark was awarded if the percentages were incorrectly plotted but the graph was a correctly drawn column graph.

(iii) (1 mark)
Candidates were asked to describe the trend shown in their graph. This was poorly answered – the majority having no idea of what ‘trend’ implied. For the 1 mark candidates had to at least infer ‘decreasing’

(c) (2 marks)
The question gave a table of primary votes in an election, and how preferences were to be distributed. Candidates had to state who would win, *with reasoning*.
About half correctly stated the winner but had great difficulty explaining how they arrived at their answer. Numerical calculations scored marks more easily than ‘essay’ answers.
1 mark was awarded for ascertaining that Clarke won; the additional mark was gained for sound reasoning.

(d) (4 marks)
The question related to a graph showing population changes and predicted population growth.

(i) (1 mark)

Candidates had to state which continent had the least change in population. This was well answered.

(ii) (1 mark)

Candidates had to predict the population of Asia in the year 2000. It was poorly answered. The majority did not understand the implication of '÷ 10' on the graph and consequently ignored it or divided their answer by 10.

(iii) (2 marks)

Candidates were asked to calculate a percentage increase in population (given the formula). Most received at least 1 mark by correctly calculating either the numerator or denominator, however very few were able to continue to obtain the final answer.

MATHEMATICS IN SOCIETY

SECTION II

General Comments

- Students' work was usually well set out, with working out shown for most parts.
- Students need to be careful when transcribing work from their own working or from a calculator.
- It was good to see that many students left their rounding off as the last step in their calculation.

Question 21

Question 21 consisted of four parts, one of which involved solving an equation and another was an application of volume with substitution into a formula. The last two parts of the question were on probability.

(a) (2 marks)

Candidates were asked to solve a simple equation which first involved expanding the left side of the equation.

The first mark was awarded for the correct expansion. The mark was only awarded if the right side of the equation was also written down correctly as some multiplied the right side by 3 to obtain $3x - 6 = 15 - 3x$. The second mark was awarded for the correct answer from the expansion.

This part of the question was done very well, with many scoring 2 marks. Many who did not complete the correct expansion were still able to obtain a mark for solving their equation (provided the equation was not over simplified). A common error was to subtract x (take away the x) from the left side of the equation.

The answer $x = 3$ required $4x = 11$ to be shown as working out.

(b) (i) (1 mark)

This question involved finding the volume of 15 gold ingots, when given the diagram (with dimensions) of one.

Most were able to obtain the correct answer, although a good number found the volume of one ingot only.

(ii) (1 mark)

This question involved finding the volume of gold in one medal when the volume of the 15 gold ingots is melted down to make 200 gold medals. Again, most were able to obtain the correct answer or obtain the correct answer from their answer in part (i).

Many who obtained the answer of one ingot in part (i) first found the total volume of the 15 ingots and then proceeded to find the correct answer of 11.25 cm^3.

(iii) (2 marks)

In this part of the question, candidates had to substitute for V and h in the formula $V = \pi r^2 h$ and then find the value of r.

One mark was awarded for substituting for V and h and then the second mark for the correct numerical expression (or the answer) for r.

More successful answers solved $11.25 = \pi r^2 \times 0.4$ step by step.

A common error in evaluating $\dfrac{11.25}{\pi \times 0.4}$ was to divide by π and then multiply the answer by 0.4. Some 'solved' the equation $11.25 = 1.26 \times r^2$, by subtracting 1.26 from both sides.

(c) (i) (1 mark)

This question asked for the probability of selecting a yellow golf ball from a bag which contained 16 white and 4 yellow golf balls.

Most were able to answer this question correctly although a number drew tree diagrams and became totally confused.

(ii) (2 marks)

This question asked for the probability of selecting two yellow balls.

Candidates were awarded one mark for $\frac{3}{19}$ (or for a correct reduction from their answer in part (i)) and one mark for the calculation of the product of the two probabilities.

Those who obtained the wrong answer in part (i) were often able to obtain the correct answer of $\frac{3}{95}$. A large number used a tree diagram, but many put incorrect probabilities on the branches of the tree diagram.

(d) (3 marks)
This question asked for the probability of winning at least two games if three games are to be played, with the probability of winning each game given as a percentage. Candidates were guided into using a tree diagram (or otherwise).

Most candidates could draw a tree diagram, but then were not able to proceed. A large number drew the correct tree diagram and listed the outcomes beside each of the 8 branches. They then proceeded to state that the P (winning at least 2 games) $= \frac{4}{8}$ without indicating the 4 correct outcomes and/or not considering the 80% chance of winning.

One mark was awarded for correctly identifying the 4 correct outcomes of WWW, WWL, WLW, LWW.

Many who identified the 4 outcomes were then unable to proceed. A large number who did work with the probabilities of 80% (or $\frac{4}{5}$) and 20% obtained the correct answer of 0.896.

Some candidates just listed the probabilities as 80,80,80 for the outcome WWW, etc. without proceeding.

Others added the probabilities (such as 80 + 80 + 80 + WWW) and then obtained an answer greater than 1.

Question 22

This question was on trigonometry and graphing. As a core question it covered these aspects of the syllabus well. Well prepared candidates were able to score full marks, with the average mark around seven. There were very few non-attempts of this question. Several students had calculators set in the wrong mode. Their calculations were followed through and marked accordingly.

(a) (2 marks)

In order to score full marks, candidates were expected to set up a correct trigonometrical statement from the given information and then to perform the calculation correctly.

Many calculated the hypotenuse. This yielded zero marks. A correct answer without working scored a maximum of one mark as it was possible to come up with a correct answer using an incorrect method.

Those who drew diagrams representing the given information made it easier to follow their calculations, particularly when an error had been made.

(b) (2 marks)

To score full marks, candidates needed to substitute correctly into the given formula and then perform the calculation correctly.

Some had difficulty assigning a value to the angle from the cosine of the angle.

Those who rounded to 35˚ without showing a full calculator display received a maximum of one mark.

Common errors included incorrect substitutions. With a correct calculation in this case it was still possible to score one mark. This may have been due to an inability to make a correlation between *a, b, c* and *X, Y, Z.*

A correct answer without working a scored a maximum of one mark as it was possible to calculate a correct answer using an incorrect method.

(c) (3 marks)

(i) Many had difficulty drawing a correct diagram. To score full marks candidates had to mark the angle correctly and then indicate either North or mark 23 km correctly on the diagram. The orientation of the diagram presented problems to some.

(ii) It was pleasing to see many understood this question and were able to set up an appropriate trigonometric equation using either 42˚ and 48˚ to calculate east. Candidates needed to clearly indicate which side of the triangle they found. If they did not calculate the 'east' side, no marks were awarded. Calculations with 138˚ received zero marks.

(d) (5 marks)

(i) The majority successfully substituted into the formula and completed the table, though those who made an error were still able to score one mark.

(ii) Many had difficulty with the scale on the graph and counted each subdivision as an increment.

(iii) The majority answered this question incorrectly by giving the maximum height of the ball as the answer.

(iv) When reading the time from the graph many candidates made errors with the scale, although generally this question was well answered. It was not possible to score a mark in this section if (i) gave a linear graph.

Question 23

This core question consisted of four unrelated parts, addressing Algebra (Equations), Measurement (Volume), Graphs and Probability, and Statistics. The 12 marks were spread in 1's and 2's across the nine parts. Candidates with a sound knowledge in the areas covered by the question scored well.

(a) The first step of this question involved squaring both sides of the given equation. The squaring of the equation was not well done, incurring a loss of 2 marks.

A bald answer of 40, substituted into the equation into the equation to establish its correctness, was awarded two marks.

Algebraic errors following successful squaring of the equation were penalised.

(b) This question was a composite volume question worth two marks involving a cylinder and a cone with key formula given.

- Generally well done.
- Main errors were: incorrect substitution (loss of first mark) and failure to complete calculation correctly (loss of second mark).
- Transcription errors were allowed where the difficulty of the question was maintained.

Example of error: $V = \frac{1}{3}\pi r^2 h$ written as $V = \frac{1}{2}\pi r^2 h$

However candidates should be encouraged to be more careful in their written working and transcription from the printed question.

(c) (i) This question required the reading of basic column graph. This part of the question was answered well. (1 mark awarded).

(ii) Unfortunately many allowed themselves to be misled by part (i) and hence in this part only focused on the faulty tubes rather than on the entire sample.

This resulted in a very common incorrect answer of $\frac{6}{23}$ (loss of 1 mark) rather than the correct answer of $\frac{6}{400} = \frac{3}{200}$

(iii) Well done. Most identified fault A as the correct answer for 1 mark.

(d) (i) (1 mark)
A majority interpreted the frequency distribution table correctly to state that there were 40 sentences in the letter.

Most common error of 9 resulted from a misinterpretation of the table.

(ii) Almost always a correct answer for 1 mark following a correct answer in part (i). If 5 was given following a 9 in part (i) one mark was awarded.

(iii) This part required the calculation of the mean and standard deviation worth 1 mark each. Some were able to do this successfully on the calculator. Common errors were to enter the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10 which resulted in the mean = 6 and s.d. = 2.58 as distinct from the correct answer: mean = 6.05 and s.d. = 2.20 – 2.23.

The mode and the median were also both 6 and consequently a mean of 6 and s.d. of 2 could not be accepted as the correct answer.

(iv) As many would suspect, this question illustrated the inability of many M.I.S. candidates to express their mathematical understanding succinctly.

Many answers repeated the wording of the question without further reasoning given. The simple answer ‘greater mean and smaller standard deviation’ was awarded 1 mark. Simply waffling on for a half a page or more with little reference to the mean and standard deviation indicated a lack of understanding of the question. A handful of students even wrote their own threatening letters.

Question 24 Space Mathematics

This question on Space Mathematics consisted of 3 separate parts. Part (a) provides half of the total marks. It only requires candidates to understand such terms as semi minor axis, semi major axis, eccentricity and focus. This gives those who understand ellipses the opportunity to score well in this section. Part (b) involves the understanding of escape velocity which provides a third of the total marks. Many candidates were obviously not used to writing down meaningful statements in mathematics and there were others who did not understand what *m* and *r* stood for. Part (c) poses difficulties for many candidates who had no idea how to convert 28 days into seconds.

(a) (6 marks)

Those who understood this part scored well. Confusion between semi major axis and semi minor axis caused the e^2 to be negative. Many worked out e^2 but forgot to take the square root. Once e was found, candidates had no trouble working out the distance from the focus to the centre. Part (iv) was poorly done. Many could not handle the algebraic manipulation to find the length of the semi minor axis.

(b) (4 marks)

This part was generally well done. Many candidates had difficulties explaining 'escape velocity'. Quite a few candidates anticipated V as 11.2. Very few converted km to m. There were others who made life very difficult for themselves by trying to change values for m and r.

(c) (2 marks)

Most could find the distance travelling by the moon in one orbit of the moon in one orbit of the earth but had difficulties in converting km per 28 days to km per second.

Question 25 Mathematics of Chance and Gambling

This question required students to calculate some selections and arrangements and to answer some questions related to gambling when given a set of odds.

(a) (i) (1 mark)

This part was not well done. Many had no knowledge of how to approach the question and hence wasted much of their time trying to write out the possible arrangements.

(ii) (1 mark)

This part was well done and was often the only correct part in the question.

(b) (i) (2 marks)

This part was poorly done. Many did not know the difference between odds on and odds against.

(ii) (1 mark)
This part was poorly done. Many obviously did not know the way of calculating a bookmaker's margin.
NB. Teachers should read the syllabus carefully as texts exist which calculate the bookmaker's margin in a different way to the syllabus.

(iii) (1 mark)
This part was reasonably well done. However many forgot to add on the amount invested when calculating the amount received from the bookmaker.

(c) Part (c) was very poorly done. Few candidates had any idea of what was required by the part and many tried to do it by using the throw of two dice.

(i) (1 mark)
This part was poorly done. Common incorrect answers were 30 and 36

(ii) (1 mark)
This part was poorly done. Few related it to part (i).

(iii) (1 mark)
This part was poorly done. Many did not attempt to 'mathematically' answer the question but instead tried to invent ways in which a person drawing the balls could draw none out.

(iv) (1 mark)
This part was poorly done. Few realised that the part could be done using the answer to parts (i), (ii) and (iii).

(v) (2 marks)
This part was poorly done. Few related parts (ii) and (iv) to this part. In general candidates who showed working gained 1 or 2 marks and those who did not received no marks.

Question 26 Land and Time Measurement

This question had three separate parts to it. It was a very straight-forward question where candidates with a fair knowledge of the topic were able to score well.
As a general rule for this question, round-off and truncation errors were ignored. Also incorrect calculations from correct substitution into formulae were not always penalised as the examiners thought that calculator work had already been tested in other parts of the examination paper.

(a) (i) (3 marks)

Candidates were asked to find a cross-sectional area of a creek using two applications of Simpson’s Rule. These were many mistakes in their substitutions, mainly to do with the zero substitutions for the sides of the creek. However an allowance of one mark for a correct substitution into one Simpson’s Rule enabled most candidates to score marks.

(ii) (1 mark)

This mark was given for an indication that the answer from part (i) was multiplied by eighty-five. An easy mark for most candidates.

(b) Part (b) was a triangulation question.

(i) (1 mark)

The correct angle was needed to get the mark. There was quite a bit of confusion amongst the candidates as to which angle they were finding.

(ii) (2 marks)

This part involved a substitution into a Sine Rule given. There were a lot of incorrect substitutions as an angle not given in the diagram had to be found before applying the Sine Rule. Again there was a lot of confusion as to which side of the triangle they were actually finding and the use of the angle found in part (i).

(iii) (1 mark)

A correct substitution involving their answers from part (i) and (ii) into the area of a triangle formula (given) earned the mark. Incorrect calculation from the substitution was not penalised. This was a fairly easy mark for the candidates although there was still the confusion as to which sides and which angle they were to use.

(c) Part (c) was longitude and latitude question involving time differences and distance along a circle of latitude.

(i) (2 marks)
The first mark was awarded for finding the time difference and the second mark was for the correct subtraction of the time from Grafton time. Adding the time on rather than subtracting it was the most common error.

(ii) (1 mark)
Most candidates knew the formula for finding the radius of a circle of latitude and the mark was awarded for the correct trig. statement with the correct numerical values.

(iii) (1 mark)
This was the hardest mark to gain in the question as the candidates needed to know how to find a proportional part of the circumference of a circle and then use their answers from parts (i) and (ii). For this reason a correct substitution into the appropriate formula was awarded the mark and calculation mistakes were not penalised.

Question 27 Personal Finance

(a) (2 marks)
Candidates were asked to use a table to determine an interest rate and a monthly interest payment. This part was not well done.

(i) Many candidates who selected 9.50% did not subtract 0.25%. Some reduced 9.50% by 25% or 0.25% of itself.

(ii) The correct answer of $38.54 was not often given with $39.58 [$39.55 to $39.60] from 9.50% being more common. Candidates often gave 9.25% as their answer to this part. Division by 12 was frequently not done, with many others dividing by 24 rather than 12.

Some candidates who found the correct answer, first found the interest for 2 years then divided by 24.

(b) (2 marks)
Candidates were required to find the amount of interest payable on 2 separate purchases on a credit card account. This was very poorly done.

Difficulty using 0.04% was evident with most who responded using 0.04, with many also ignoring the number of days. Most did not demonstrate understand the term ‘no interest-free period’ many finding the interest in the total cost for 30 days.

Responses assuming interest was charged on the day of purchase, giving periods of 21 and 11 days, were accepted as well as those assuming it wasn’t, giving periods of 20 and 10 days. It was not always clear whether candidates thought interest was charged on the day of purchase or whether they thought there are 31 days in June.

(c) (i) (2 marks)
Candidates were asked to calculate the price of an item reduced by 2 successive ‘mark downs’.

This part was well done with most giving the correct answer of $98.44 ($98.40 – $98.50 accepted).

The most common error was to add to 25% and 12.5% together to give $93.75 (1 mark was awarded for this calculation). Some misinterpreted the question to mean either $150 was the price after the first mark down or that only the Saturday mark down was applicable.

(ii) (1 mark)
Candidates were required to express the saving as a percentage of the original price.

This was generally well done although many simply quoted 37.5%, even having correctly done (i). Some expressed the sale price as a percentage of the original.

(d) (5 marks)

Candidates were required to interpret two graphs one showing a differential taxation rate, the other showing a flat taxation rate.

(i) Most correctly stated no tax payable on $4000 although a significant number also stated the flat tax amount.

(ii) Well done although a number of candidates gave the tax on $15 000 as $1900 rather than $1800, misinterpreting the scale in the vertical axis.

(iii) Well done with most finding $1200 correctly. Some subtracted from $5000 (extra income) rather than $3000 (extra tax).

(iv) Most did not demonstrate understanding and simply divided $5000 by answer (iii). Some divided answer (iii) by $5000 without correctly stating the number of 'cents in the dollar' giving 0.24 rather than 24 as their answer.

(v) Poorly done. Many candidates responded to the question of what range of incomes would pay more tax under a flat rate by writing a paragraph on the effects of the two systems (or one of the two systems) without specifying amounts. Others interpreted the graph incorrectly, reading the point of intersection as $35 400 rather than $37 000. Some also used $7000 as the lower limit rather than $0.

General Comments

- Candidates had difficulty interpreting the language and did not demonstrate understanding of terminology.
- Candidates not always aware of the need to express answers in dollars and cents (allowance was made for 'rounding').
- Some whole centres performed quite poorly indicating, perhaps, they had done more than 2 option questions. This seems a poor decision considering the time spent responding.

Question 28 Mathematics in Construction

The question consisted of nine parts. It required candidates to read a plan, use a ruler and protractor, convert from one unit of measurement to another and to calculate and apply a scale. In general, basic calculations only were required in the question. There was a large increase in the number of candidates attempting the question compared to previous years. Many lost marks as they were unable to convert correctly from one unit to the other, but on the whole the question was well done.

(a) (1 mark)

This part was well done. However, some confused length with width and others used a ruler and scale incorrectly in an attempt to calculate the width rather than read it off the plan.

(b) (1 mark)

This part was well done. Unfortunately some candidates gave only one measurement.

(c) (1 mark)

This part was not well done. Many attempted to use a variety of (often complex) methods to calculate the angle and usually got it wrong. Those who used a protractor inevitably got the mark.

(d) (1 mark)

This part was well done. However, a number incorrectly calculated 1:1000 and others stated that it was 1:50, presumably because plans they studied had the common 1:50 scale.

(e) (1 mark)

This part was poorly done. Rather than using a simple conversion from millimetres to metres before the calculation of the area many students attempted to convert their answers from square millimetres to square metres and got it wrong.

(f) (1 mark)

This part was not well done. A common error was incorrectly changing from millimetres to metres and another common error was to find the height of the extension rather than the house.

(g) (2 marks)

This question was done reasonably well. However too many did not realise that their answers bore little resemblance to real life stairs and hence did not query an answer such as 3 millimetres. Also many simply stated that the stairs were a certain size presumably because stairs that they studied at school had a standard size.

(h) (2 marks)

This part was not well done. Many did not use the plan and scale but rather assumed the eaves to be a certain standard width and hence got the wrong answer. Many candidates showed no working and hence lost both marks. Usually those who showed their working gained 1 or 2 marks.

(i) (2 marks)

Many elevations were poorly done. Candidates need to realise that this is a specialist mathematics extension topic and as such requires the use of acceptable industry standards.

2/3 UNIT (COMMON)

Question 1

This question consisted of seven parts covering a variety of topics. Each part had a maximum mark of 2 except parts (a) and (c) which were each worth 1 mark. At least sixty per cent of candidates scored a mark of 10 or better. Most marks were lost through lack of knowledge with few marks lost through errors.

(a) Although many were awarded the mark for finding the factors of $9x^2 - 16$, some of these candidates suffered a time penalty by creating an equation and then attempting to solve it.

(b) Nearly every candidate received 2 marks for this question. However some confused $19^{-0.5}$ with the display found on a calculator and wrote instead $19 \times 10^{-0.5}$. This error yielded a maximum of 1 mark if $19 \times 10^{-0.5}$ was correctly evaluated to two decimal places.

(c) Many calculated $\dfrac{3\pi}{5}$ and as a result this part had most common error of 1.88 radians instead of 108°.

(d) As in part (a) many attempted to solve an equation. Here candidates were penalised if initially they multiplied the given expression by 6 and as a result 'lost' the denominator. This part also attracted a very large number of transcription errors. If, as a result of this action, the degree of difficulty of the question remained the same, the candidate was not penalised.

(e) Here the candidates lost marks through lack of knowledge. This was the part of Question 1 which had the greatest number of non-attempts. Of those who knew the meaning of 'primitive', almost all were awarded the maximum 2 marks.

(f) Candidates often knew the correct fraction, $\frac{23}{99}$, but also attempted to show working which was incongruous with their answers. About 25% used a geometric progression to find their answer and most of these were successful.

(g) Quite a few candidates changed this question from a simple inequality, $5 - 3x < 7$ to an absolute value question. This was the part where candidates who lost marks were most heavily penalised. Although a common error was to give a final answer with the inequality reversed, in general the calculation of $-\frac{2}{3}$ was correct.

Most who received a mark less than 5 did so, not because they tried and got the parts wrong, but because they did not try at all.

Question 2

This question consisted of seven separate parts related to lines and points on a number plane. It was generally well attempted with many candidates awarded full marks. One pleasing aspect of the scripts was the improved and more extensive use of mathematical language (in response to the request to 'show ... ') in attempting to explain or justify answers. Approximately half of the candidates neglected to copy the diagram into their Writing Booklet.

(a) (2 marks)

The question asked for the equation of a line given its intercepts on the *x* and *y* axes. The most common approach was to find the gradient of the line and then substitute their gradient into one of the 'general forms' for the equation of a line. Very few candidates went directly to the intercept form $\left(\frac{x}{a} + \frac{y}{b} = 1\right)$.

In general, full marks were awarded for correct substitution into any correct formula. A single mark was allowed in either of the following two instances:

- correctly finding the gradient of the line.

- correct substitution of an incorrect gradient into a correct general form.

The most common error was to state the gradient as -3/4, most often deduced from the fact that the *x* intercept was -4

(b) (1 mark)

Here candidates were requested to show that a given point (16,15) lay on the line whose equation they (hopefully) found in part (a).

The mark was awarded for the following most common approaches:

- Substitution of (16,15) into their equation from part (a).
- Showing gradients of intervals joining any two of the three given points were equal.
- Finding the equation of a line with gradient 3/4 through (16,15) and showing that it corresponded to their answer in part (a).

Attempting to show on a diagram that the line ℓ when extended would pass through (16,15) was awarded no marks.

(c) (2 marks)

The question asked students to show that ΔPLM was isosceles. Almost all students recognised the need to show that two sides of this triangle were equal. Attempts to find the length of LM were equally divided between applying Pythagoras' Theorem to ΔLOM and using the distance formula. The length of PM was often stated simply as a distance along the *y* axis.

A mark was awarded for each correct length found by candidates. A significant number of students having found two sides to be equal went on to find the length of the third side, thereby penalising themselves on a ‘loss of time’ basis.

(d) (1 mark)

The requirement here was to find the gradient of the line PL. Errors here were consistent with incorrect attempts to find the gradient in part (a). The most common incorrect answer came from writing ‘ gradient $= \dfrac{8-0}{0-4} = -2$. Many of these students, realising from the diagram that they should get a positive slope, simply converted their answer to +2.

(e) (2 marks)

In this part, the question asked for the co-ordinates of the point N (marked on the diagram), given that M was the mid-point of the interval LN. Many were able to obtain the correct answer through a variety of methods:

- use of the mid-point formula
- dividing an interval in a given ratio
- similar triangles
- recognition that a gradient of 3/4 represented the ratio $\frac{\text{rise}}{\text{run}}$.

The marking scheme awarded 1 mark for each correct co-ordinate of N found. It also allowed 1 mark for identifying the point M as having co-ordinates $\left(\frac{-4+x}{2}, \frac{0+y}{2}\right)$. The most glaring error involved $2 \times 0 = 2$.

(f) (2 marks)

Here it was required to show that $\angle$NPL was a right angle. The candidates attempted to do this (in order of frequency) by:

- finding the gradients of PL and PN (first mark awarded at this stage) and showing that their product was –1 (second mark awarded).
- finding the lengths of PN, PL and NL (first mark awarded) and then applying Pythagoras' Theorem or the cosine rule to show that ΔNPL was right angled (second mark awarded).
- finding the equation of PL and then showing the perpendicular distance of the point N from this line was the same as the length of PN obtained by the distance formula.
- a geometric approach involving the base angles of the two isosceles triangles (ΔPML and ΔPNM) and the angle sum of ΔPLN.
- recognising that LN was the diameter of a circle passing through P and that the angle in a semi-circle was a right angle.

It should be pointed out that students who found incorrect co-ordinates for the point N in part (e) were not penalised by the marking scheme for correctly applying the methods of solution above and then making a supporting statement for their results.

(g) (2 marks)
Candidates were requested to find the equation of the circle passing through the points N, P and L. This was the part that received the poorest attempts. Many had difficulty recognising that M was the centre of the circle and merely gave answers of the form $x^2 + y^2 = r^2$. Others attempted to find the equation using locus techniques for a point equidistant from N and/or P and/or L.

The marking scheme allowed 1 mark for an answer of the form $(x-g)^2 + (y-h)^2 = r^2$ for any *g, h, k* (*k* >0) consistent with their stated centre. Subsequent algebraic errors in attempting to expand and simplify were ignored.

Question 3

Part (a) of the question involved differentiation of a negative index $\left(2x^{-3}\right)$, an exponential function $\left(4e^{2x}\right)$, and application of the product rule with the logarithmic function $\left(x\log_e x\right)$.

Part (b) required the use of trigonometry applied to two right angled triangles, or sine/cosine rules could have been used either in the right angled triangles or combining the triangles to create a non-right angled triangle. Additionally candidates needed to understand that horizontal and vertical lines are perpendicular to each other and the concept of the midpoint of an interval. A diagram was provided.

Part (c) involved finding the indefinite integral of e^{3x} and the definite integral of sin 2x, both of which are standard integrals.

Overall the question was very well done (average 9) with 30% of candidates scoring 12 marks. However there was also a notable group who scored very little, indicating little understanding of some of the basic concepts of the 2 Unit course.

(a) (i) Differentiate $\left(2x^{-3}\right)$. (1 mark)

By far the most common mistake was $-6x^{-2}$.
It was interesting that many candidates lost their only mark for the question here, perhaps rushing too much, whereas for some this was their only mark.

Another error was $6x^{-4}$.

(ii) Differentiate $\left(4e^{2x}\right)$. (1 mark)
Well done with common errors $8e^{x}$, $4\times\frac{1}{2}e^{2x}$, $8xe^{2x}$.
Some treated $4\times e^{2x}$ as a product, usually without success while some had not encountered the notation $\log_e x$ replacing it with $\log e^{x}$.

(iii) Differentiate $\left(x\log_e x\right)$. (2 marks)
Most recognised the need to use Product rule.
However, although not penalised here, their algebraic simplification was very poor, especially $x\times\frac{1}{x}$ which equalled x^2, $\frac{1}{x^2}$, $or\ \ 0$.

Candidates were awarded 1 mark for two out of the three obvious elements in the Product Rule.

(b) Mainly well done, particularly those who used trig ratios in each of the two triangles. Others wasted valuable time doing 2 or 3 step solutions involving Pythagoras, Sine and Cosine rules which increased both the level of difficulty and the chance of error. Many believed the question had to be more difficult than it was, and calculated sides and angles that were not subsequently used.

Some assumed (from the diagram?) that $\angle PSR = 90°$. Some candidates found the hypotenuses SR, SP to be less than 8 without concern!

(i) (2 marks)
1 mark was awarded for a correct trig expression involving SR, and 1 mark for the calculation giving SR between 10 and 10.2.

Most common mistakes were $\sin 52° = \dfrac{8}{SR} \Rightarrow SR = 8\sin 52°$ (the use of the Sine rule eliminated this problem for some candidates) or finding side QR, or $\tan 52° = \dfrac{8}{SR}$ or $\sin 52° = \dfrac{SR}{8}$

Successful alternative solutions included (i) using Sine rule in ΔSQR, (ii) finding side QR and then using Pythagoras Thm., (iii) firstly finding $\angle SPQ$ and side PS then Sine rule in ΔSPR.

(ii) (2 marks)

1 mark for a correct expression involving PS and 1 mark for the angle 36.02°. Degrees and minutes were also accepted.

The most successful method was using $\tan 36° = \frac{8}{11}$, followed by those who found side PS and then used Sine rule in ΔSPQ or ΔSPR. Less successful were those who used the Cosine Rule because they could not manipulate the subject of the formula or they mixed up the sides and found $\angle PQS$ or $\angle PSQ$.

Too many were not able to find $\angle P$ correct to the nearest degree because they rounded off intermediate steps eg. PS = 13 or 14.

Some could not find the length of PQ (P midpoint of AQ) as indicated by their ability to correctly find $\angle SAQ$ but go no further. Others divided 22 by 2 to give 12.

Common mistakes involving Sine/Cosine rule were

(a) $\dfrac{\sin P}{8} = \dfrac{\sin 52°}{SP}$ (b) $\cos P = \dfrac{SP^2 + PQ^2 - SR^2}{2.SP.PQ}$

(c) Overall well done, however many students obviously did not use the table of Standard Integrals.

(i) (1 mark)

Most common errors were $3e^{3x}$, $\dfrac{e^{4x}}{4}$, e^{3x+1}.

Use of notation was often poor eg. $= \frac{1}{3}\int e^{3x}$

(ii) (3 marks)

1 mark for correct integration, 1 mark for the correct substitution of their limits in their integral, 1 mark for the correct evaluation of their substitution.

When a final answer was incorrect, students (including capable ones) who missed steps sacrificed marks, because markers were unable to determine the nature of error(s). Conversely the weaker students often scored their only mark for their correct substitution.

Overall the use of parentheses and correct accepted notation was lacking. Students ran into difficulty as soon as they attempted to use their calculator. The understanding that trig. functions are defined in radians needs attention.

The concepts of finding a definite integral and finding the area under a curve are still not clearly understood by a significant number of students. Many answers included units or units^2, but others changed their (correct from their integral) negative answers to positive, while some added a constant c.

Most common problems were:

Step 1 Incorrect value for k in $(k)\cos 2x$, $\left(e.g.\ k = \pm 2, \pm 1, \tfrac{1}{2}\right)$.
Also $\sin x^2$ *or* $\cos x^2$ *or* $\sin 2x$

Step 2 Many students assumed that cos 0 = 0 and omitted –F(0). Many of the parentheses were invisible to the markers but obvious to the students as implied by their evaluation step.

Step 3 Many students believed they should be working in degrees. Some left their answer as $\tfrac{1}{2}(-\cos\pi + 1)$ not realising that $\cos\pi = -1$, while others found $\cos\pi°$.

Question 4

The three parts (a), (b) and (c) of this question tested, respectively, understanding of arithmetic series, drawing graphs of simple functions and interpreting information from them, and Simpson's rule. The level of difficulty was such that most well-prepared candidates scored full marks. Many centres had difficulty with at least one part of the question, usually part (a), slightly less frequently part (c)(ii). A significant number of those candidates who scored no marks in part (a) either did not seem to realise that it was a question involving series, or believed it involved the geometric series; many did not attempt this part.

Incorrect arithmetical calculations were evident far too often, especially in (a)(i) and (c)(ii).

(a) (i) (2 marks)

Successful candidates obtained their first mark by recognising that substitution of the three pieces of information into the sum formula $S_n = \frac{n}{2}(a+\ell)$ was required; indeed, a few candidates arrived at this result arguing in the same manner as the seven-year old Gauss that the number of strings is equal to the sum divided by the average of the first and last. Many of those candidates who used $S_n = \frac{n}{2}\left(2a+(n-1)d\right)$ either could not proceed further, or had difficulty solving this equation simultaneously with that for $T_n = a+(n-1)d$, though some demonstrated excellent algebraic skills.

The second mark was awarded for the calculation, from the candidate's equation(s), of a positive integer value for the number of strings. Many candidates with incorrect substitutions, or substitutions into incorrect formulae, overcame this difficulty by rounding or changing '–' to '+'; others did not realise that this was a problem. A significant number of candidates believed that the subscript in S_n is to be interpreted as $S \times n$. Many candidates did not know the formulae, or the significance of the symbols contained therein.

(ii) (2 marks)

The most poorly done part of Question 4. Candidates who were unsuccessful in (i) rarely proceeded with this part, though a few candidates found d first, and then n, when solving their simultaneous equations in (i). Many candidates who correctly found the number of strings were unable to proceed to find the common difference of the arithmetic series; many of these candidates did not understand what T_n stands for. Others quoted incorrect formulae, often automatically setting an expression to zero, then proceeding to solve the resultant equation. Another large section of the candidature used the more cumbersome second form quoted in (i) for S_n to find d; some used both S_n and T_n, as a check one against the other.

Many intuitively thought that the common difference was obtained by the quotient 20/31; very few who did not use a formula realised that the difference was 20/(31–1).

(b) (i) (2 marks)

About half the candidature were able to draw the two graphs successfully, though many candidates had to rely on plotting points, sometimes a large number of them. Many candidates did not recognise that $y = x + 4$ is a straight line, and only requires intercepts to be calculated in order to draw it, for one mark. Many believed that, despite plotting 5–7 points about $x = 0$, the graph of $y = |x|$ is curved, particularly at the origin, and 'parabolic' in shape; those with a reasonable '⁄' at the origin obtained one mark. A significant number drew poor freehand sketches, including the axes, with inconsistent and variable scale. Some exhibited little knowledge of where negative numbers fall on the real number line, which others believed covers only non-negative numbers. Many candidates believed that the graph of y = $|x|$ is coincident with that of either $y = x$, $y = –x$ or both, or even that of $x = 0$, or $y = 0$. A significant number of candidates proceeded to plot the graph of $x = |y|$ and/or that of $y = x – 4$ as the result of mis-labelling their box calculations, eg.,

y	–2	–1	0	1	2
x	2	1	0	1	2

(ii) (2 marks)

The majority of candidates realised that the question was in some way related to the answer of (i).

Candidates who drew careful sketches in (i) were able to easily read off the answer (–2, 2) to gain full marks. Many more, including otherwise very weak candidates, were able to pick (–2, 2) from their box calculations.

Even some candidates with freehand sketches, no scales and no labelling, were able to 'read' (–2, 2) from their graphs. Those candidates who realised they could read the answer from their graph received one mark for this recognition, and the second mark for a non-trivial, correctly-stated point of intersection. The examiners would have felt much more confident of candidate's bare answers if words such as 'from the graph' had been included.

But the majority of candidates felt compelled to find (or perhaps justify) their answer by solving algebraic equations. Those who realised that, for $y = |x|$, the case $y = -x$ was needed had little difficulty, as did those who followed $|x| = x + 4$ with $x^2 = (x+4)^2$, from $|x| = \sqrt{x^2}$. A significant number realised that the abscissa of the point of intersection was –2, and proceeded to incorrectly adjust their algebra to suit; they were penalised one mark. More interesting were the algebraic manipulations of those students whose incorrect graphs intersected at (2, 2), (2, –2), or had two points of intersection.

(c) (i) (1 mark)

Almost all candidates were able to gain the mark for correctly completing the table of powers of 2. The most common errors were $2^{-1} = -2, -\frac{1}{2},$ or 0.2; $2^0 = 0$ or 2; $2^1 = 1$, $2^3 = 9$; occasionally 2^x was interpreted as $2x$, e^x or $2e^x$.

(ii) (2 marks)

One mark was awarded for a correct version of Simpson's rule, one for the strip width (>0), and one for substitution of the five ordinate values into the candidate's formula. Most candidates scored at least one mark.

The main problems encountered by those students who used the syllabus formula, applied twice, were in interpreting 'a' and 'b' as x-values in the table in (i) (and not y-values), and in $f\left(\frac{a+b}{2}\right)$ which many interpreted as $\left(\frac{f(a)+f(b)}{2}\right)$, or even as $\left(\frac{a+b}{2}\right)$. Successful candidates used 4×'middle ordinate'. Too many students disregarded '... with these five function values, ...' and proceeded to use the formula with three, or nine

($h = \frac{1}{2}$), function values, or used two different band widths and created two new y-values.

The various forms of the composite formula, including TOFE and FOTE, with or without $\frac{b-a}{3n}$, created problems for the examiners, who had to determine what 'even' and 'odd' meant by investigation of the subsequent substitution into the 'formula'. For some candidates, 'even' meant even x-values, or (all, including $\frac{1}{2}$) even y-values, and similarly for odd, with no regard for the 'weightings'. Additions of groups of ordinates was often replaced by multiplication (even when summation notation was included in the formula). Similar confusion existed over the meaning of n in this formula; candidates who used $\frac{h}{3}$ generally did better. Those students who used functional notation eg. $A = \frac{h}{3}\left(y_1 + y_5 + 4\left(y_2 + y_4\right) + 2y_3\right)$, were often uncertain as to the meaning of the subscript, firstly on the final point (y_4 or y_5) depending on whether they started from y_0 or y_1, and secondly on its correspondence with values in the table in (i). The more successful ones used $A = \frac{h}{3}\left(1. + 4. + 2. + 4. + 1.\right)$. A few candidates confused these methods, producing hybrid rules, such as $\frac{b-a}{6}\left(f(a) + f(b) + \text{FOTE}\right)$, and some used the Trapezoidal method, or a hybrid such as $\frac{h}{3}\left(y_0 + 2\left(y_1 + y_2 + y_3\right) + y_4\right)$.

Question 5

The question consisted of two parts: to calculate the area of the major segment of a circle and then to sketch an inverted quartic function. The question allowed many 2 Unit candidates to score well and the mean was around 8 marks. Many did not attempt the first part of this question, the reason for this remains a mystery. In the second part most had difficulty sketching a stationary point of inflexion even though they had correctly answered all the previous parts. So a score of 11 marks was common.

(a) (i) 1 mark was allowed here for showing ΔAOB is equilateral and hence <AOB = 60° (other satisfactory explanations such as using the cosine rule were of course allowed).

Candidates needed to explain why <AOB = 60°. Often the simple explanation eluded them and they wrote half a page. Many used the cosine rule successfully. Just as many used reasonable arguments such as '*the chord AB equals the radius hence* <AOB = 60°'. However many tried to use $\ell = r\theta$ and failed to convince the marker.

(ii) 3 marks were allowed; 1 for correct recognition that the area could be found using an addition or subtraction method, then 1 for correct formula, and finally 1 for correct substitution

Candidates confused the words sector and segment. There were many different correct methods applied using both addition and subtraction methods. The common problems were those of wrong formulae and incorrect substitutions; using degree measure rather than radian measure. The formula for the area of a circle appeared not to be known by a large percentage of students!

(b) (i) 2 marks were allowed; 1 mark for $12x^2 - 12x^3 = 0$ and then 1 for $x = 1, 0$. Obviously (1,8) (0,7) gained 2 marks.

This was usually well done. Candidates often got into difficulties with the negative signs if they rearranged the polynomial. There were the usual problems with differentiating, and solving the resulting equation. Often the solution $x = 0$ was lost and many other students had 3 solutions (0,1,–1) for x.

(ii) 2 marks were allowed; 1 for y'' correct and then 1 mark for $x = \frac{2}{3}, 0$.

This part was well done by most candidates. Even those who could not solve $y' = 0$ in (i) could successfully do this part. Often the quadratic equation was incorrectly solved and the value $x = \frac{3}{2}$ was given.

(iii) 2 marks were allowed; 1 for horizontal inflexion at (0,7) and then 1 for the maximum at (1,8).

There were wide variations in tests used to investigate the stationary points, but such tests were often misapplied or misinterpreted. When the second derivative was used candidates would test *x* values too far away from 0 and neglected to notice how clustered the significant *x* values $(0, {}^{2}/_{3}, 1)$ were. The words minimum and maximum were often interchanged. Candidates assumed that $y'' = 0$ automatically gave points of inflexion. It did work in this question.

(iv) 2 marks were allowed for a clear sketch of the curve.

The question guided candidates through the features of the curve. Only good students managed to get the correct sketch. Many did not understand the restricted domain and did not seem to use their correct answers in earlier parts as a guide. The points of inflexion were often omitted or poorly drawn. Many had small diagrams and did not notice the limited domain which indicated a larger scale on the *x* axis.

It is worth mentioning that a mistake in say differentiating in (i) did not stop a candidates getting full marks in the other parts. Candidates are never penalised twice for the one mistake.

Question 6

This question consisted of three unrelated parts. Parts (a) and (b) required candidates to set up integrals for volume and area and use their integration and substitution skills. Part (c) involved exponential decay.

(a) (3 marks)
This part asked for the volume contained by a glass formed by rotating part of the parabola $x = \dfrac{y^2}{30}$ around the *y* axis. Candidates needed to find x^2 and then integrate between limits of $y = 0$ and 10.

The 3 marks were allocated according to each step in the solution. One mark was gained for the correct statement of integral with x^2 replaced by $\frac{y^4}{900}$, the second mark for the correct integration of their expression in y and the third for correct substitution and evaluation of the integral. A number of candidates used the expression in x (for around the x axis) and could gain 2 marks out of the total of 3 if they did so correctly. Because this required a change of limits, students were unlikely to gain the maximum allowed marks for this method.

While around 30% of candidates gained full marks on this question, the majority of mistakes came either by not simplifying $\left(\frac{y^2}{30}\right)^2$ and trying to integrate as is, or by incorrectly simplifying this expression. There were numerous mistakes when working with the square of a fraction, with many showing significant misunderstandings about fractions, indices and integration of functions involving fractions. Candidates often, for example, reversed the numerator and denominator, or included log 900 as part of their result from integration! Other students who had successfully gained $\frac{y^4}{900}$ either forgot the 5 during integration or did not know what to do with it, often multiplying by it. The degree of approximation was a problem for students, with students leaving their answer as 22.2π for $\frac{200\pi}{9}$.

Most students did recognise the need for an integral involving the square of the expression and included π at some stage through the question.

(b) (5 marks)

This part required students (i) to find the x values of the points of intersection of $y = \frac{4}{x}$ and $y = 5 - x$ and then (ii) to find the area between the line and the curve.

(i) Two marks were awarded for the correct answer from correct working, with, in this case, the bald answer gaining full marks. One mark was awarded for only one correct value, or the statement $5 - x = \frac{4}{x}$.

While the majority of students correctly managed to find the points of intersection (x = 1 and 4), there were a number of elaborate and quite incorrect methods (often resulting in the same values!). A common example of this is:

$\frac{4}{x} = 5 - x$, giving $4 = x(5 - x)$, so $x = 4$ and $5 - x = 4$, ie $x = 4$ and 1.

A number obviously did not refer to the diagram, gaining impossible x values and not considering their validity.

(ii) Candidates could consider two integrals separately and subtract the results or consider one integral with the subtraction included. Those who treated their areas separately and subtracted (the most common method) could obtain one mark for each correct integral, and the final mark for the subtraction in the correct order or with the use of absolute value of a negative result. Those who combined their functions initially, could receive one mark for the correct statement of integral, one mark for the correct integration of their function, and one mark for the correct substitution and one step of evaluation beyond first step (markers were looking to see recognition that $4 \log 1 = 0$). Candidates could get full marks in this part if they correctly used their incorrect limits from part (i).

The most successful candidates subtracted a log integral from the area of a trapezium or the area under the straight line. However, a large percentage of the candidature could not integrate $\frac{4}{x}$, with many getting $4x^0$, $4x^{-2}$, $4\log x$ or $\frac{1}{4}\log x$ as a result. Many 'simplified' the problem by multiplying $5 - x - \frac{4}{x}$ through by x and then integrating $5x - x^2 - 4$.

A large number of candidates did not worry about setting up their subtraction of areas in the correct order initially and relied on taking the absolute value of their answer if necessary. This method is risky since they often make mistakes in the evaluation of their integral and get an incorrect negative or positive answer. A number made statements such as the negative result equals the positive result without any attempt to explain what they are doing.

(c) (4 marks)

This part gave candidates the exponential equation of decay for coal being extracted from a mine. Rather than give initial amounts, the question gave information about when half of the coal remains and asked (i) to find *k*, and (ii) to find how many more years before 30% of the original amount remains.

(i) One mark was given for the correct interpretation of the question into the equation $0.5R_0 = R_0e^{-kt}$, with the second mark for the correct taking of logs of both sides of the equation after cancelling the R_0.

Those who were able to interpret the words of the question into the initial equation were usually able to go on and often gained full marks. In some centres this part of the question was the only part attempted, while in other centres this part was often not attempted at all. The common mistakes included the placement of 0.5 on the wrong side of the equation, failure to cancel the R_0, the use of 50 instead of 50%, changing signs incorrectly and incorrect use of natural logarithms, with logarithms to the base 10 used in some instances.

(ii) One mark was given for a correct next step from $0.3R_0 = R_0e^{-kt}$, and the second mark for the correct evaluation of *t* from their expression involving natural logs and their *k*.

It was surprising to see the number of candidates who wrote $\frac{1}{3}$ instead of 30%. A number of did not use exponentials at all, with some trying a linear proportion approach. Some must have used the memory function on their caclulator to store the value of *k*, and while they wrote an approximate value or even an incorrect value, their final result from their calculator was accurate. Another successful approach was to start with $\frac{3}{5} = e^{-kt}$. This gave the correct answer directly, without the need to subtract 20 years (most students forgot to do so when using the other method).

Overall, the question provided good discrimination between the candidates, with the marks obtained being spread evenly over the range from 0 to 12. It was particularly pleasing to see that candidates who were unable to attempt earlier sections of the question usually attempted later sections, often gaining a substantial number of marks.

Question 7

This question contained four parts taken from four areas of the syllabus, namely probability, geometry, interpretation of graphs and the rules of logarithms.

(a) (2 marks)

This probability question was well answered. Few candidates needed to draw a tree diagram and saw that they only had to multiply the complements of each event, ie. 0.94×0.96 to obtain 0.9024. The most common error was $1 - 0.04 \times 0.06$, but as this indicated an idea of complementary events it was awarded 1 mark, as was the recognition 0.94 and 0.96, even if used incorrectly.

(b) (i) No marks were awarded for this part.

(ii) (1 mark)

Most were able to explain why the angles were equal, ie. 'the opposite angles of a parallelogram are equal', and some even proved it through co-interior angles. Reasons such as 'property of a parallelogram', 'parallelogram', 'corresponding angles' and 'vertically opposite' were not accepted.

(iii) (1 mark)

Many saw the link between AD = BC (opposite sides of a parallelogram) and the given information, BC = AX.

Those that failed to see the link, tried to prove the result through isosceles triangles but made the assumption that AX was parallel to YC. These attempts were awarded zero.

(iv) (2 marks)

This part of the question was poorly answered. Most used the lead in parts (ii) and (iii) and their 'proof' was,

AD = BC (opposite sides of a parallelogram)
AX = YC (given)
<ADX = <YBC (opposite angles of a parallelogram)
OR (proved in (ii))
Therefore the triangles are congruent (SAS).

Many stated two sides and an angle rather than two sides and the *included* angle. It appeared that thought that writing the 'proof' in order side, angle, side justified the use of SAS. These attempts were awarded zero, since only facts previously given or proved were used.

One mark was awarded for the recognition of the need to use angles in isosceles triangles or for stating that angles DAX and YCB were equal.

Two marks were awarded for the correct use of SAS using angle DAX and YCB or for correctly using AAS.

(v) (1 mark)

In the main this part was poorly answered. Many candidates assumed that one pair of opposite sides and one pair of opposite angles equal was a test for a parallelogram, while others listed all the properties of a parallelogram without any justification or linkage to a specific test.

The 'simplest' proof was to prove AY and DX equal and parallel.

(c) (1 mark)

Most noted that the level of the pollution had increased, however many failed to comment correctly on the rate of change. Those candidates who wrote their comments in one sentence often used 'it' and in marking, it was difficult to determine whether the 'it' referred to the level or the rate of change.

Many just referred to the derivatives, ie. $\frac{dp}{dt} > 0$ and $\frac{d^2p}{dt^2} < 0$. Some students had a conflict when using a description in words and the derivatives in symbols.

(d) (i) (1 mark)

Generally well done except the answer was often given as $\log_a 2.5$ rather than the correct answer of 2.5

(ii) (2 marks)

Many found this part difficult, particularly the squared. There were many variations of the rule eg.

$$\log_a(bc)^2 = (\log_a b + \log_a c)^2 = 2\log_a b + \log_a c$$

Question 8

This question consisted of two parts. The first part asked candidates about a golf tournament in which two players, Greg and Jack play two rounds of golf against each other. While the two players are equally likely to win the first round, the probability of winning the second round depends on the result of the first round, reflecting the psychological influence of success and defeat.

(a) (i) (2 marks)

The candidates were required to draw a tree diagram, indicating the appropriate probabilities for each branch. One mark was given for a diagram of the correct shape showing the probability of 0.5 for each outcome of the first round. The second mark was given for labelling the branches corresponding to the second round with the correct probabilities.

Over 55% of the candidates gained both marks, and about 20% gained 1, usually for having the right probabilities for the second round.

(ii) (2 marks)

The candidates were required to compute the probability that Greg wins exactly one game. Full marks were awarded for either the correct answer or an answer which was consistent with their diagram in a (i). One mark was available for clearly showing that one needed to add the probability of Greg winning first, then losing to the probability of Greg losing first and then winning. Candidates who calculated both of those probabilities but then multiplied them also were awarded one mark.

Just over 50% of candidates scored full marks, while about 5% of the candidates gained 1 mark. About 10% of the candidates did not respond at all.

(b) Part (b) of the question involved the investment of $10 000 in a bank account followed by 10 further deposits of $1000 at one year intervals, with the funds in the account earning interest at a constant rate of 8% compounded annually. The candidates were required in the first two sub-parts to compute the balance in the account just after the last deposit of $1000, and in the third sub-part they were asked to find the interest rate which would have provided a given balance, very close to the correct answer to the previous sub-part, if only the initial $10 000 deposit had been made.

(i) (2 marks)

Here candidates were required to compute the balance in the account at 8% interest if only the initial $10 000 deposit was made. Errors in the number of cents were ignored, and one mark was available for those who wrote down the correct expression but failed to compute it correctly.

One mark was also awarded for a correct computation using the compound interest formula but with the number of years incorrectly calculated as 9 or 11. However, candidates who computed the balance each year, or who showed no working at all scored zero unless they had the correct answer.

This was well done, with over 70% of the candidates being awarded full marks. Another 5% received one mark, usually for the computation with the incorrect number of years.

(ii) (4 marks)

In this part of the question, the candidates needed to recognise that interest was to be paid on the initial $10 000 and that there were further deposits; understand that the effect of the additional deposits was to form a geometric series with last term $1000; to write down an expression for the sum of this geometric series and finally, to evaluate this expression. Each of these steps was awarded one mark.

Half the candidates did not score at all on this sub-part, partly because over 15% of the candidature wrote nothing at all, but also because a large proportion of the remainder simply wrote down an incorrect answer, with no discernible explanation of how it arose. The half of the candidature earning marks were fairly evenly divided between those scoring 1, 2, 3 or 4.

The most common problem occurring in candidates responses arose from their identifying this question as a superannuation question and applying a remembered formula for the result of the additional deposits. Candidates did not realise that the formula they were using did not take account of the last deposit, and such students usually scored 3. Students who laboriously computed the balance of the account at the end of each year scored 4 marks if it was done correctly, 3 marks if the only mistake was to neglect the last deposit, but otherwise could obtain at most 2 marks.

(iii) (2 marks)

As mentioned earlier, candidates were asked to compute the rate of compound interest which would result in \$10 000 growing to \$35 478 after 10 years. One mark was awarded for work which progressed as far as indicating the need to find $3.5478^{\frac{1}{10}}$, or, for those who proceeded by using logarithms, reaching the stage of computing (log3.5478)/10, with full marks being awarded for the correct answer of 13.5%. Candidates who had used the wrong number of years in b(i) were able to continue using that number in this part without any further penalty.

About 40% of the candidates obtained full marks for this section, while on the other hand, just under 20% of the candidates made no response at all. Many of the correct answers were as the result of trial and error.

A large number apparently knew that they needed to compute $3.5478^{\frac{1}{10}}$, but instead computed $10^{\frac{1}{3.5478}}$, by reversing the order required by their calculator, or alternatively, they knew that one could compute fourth roots by taking two successive square roots, and so they attempted to find the tenth root by taking 5 successive square roots. Many such candidates scored zero instead of 1 because the only evidence they provided was the incorrect answer obtained from these errors, emphasising once again the need to show working.

However, most of the zeros obtained by candidates attempting this sub-part were due to the incorrect expansion $(1+r)^{10} = 1+r10$.

Question 9

This question consisted of two parts each of which involved a physical problem which was to be solved or interpreted mathematically and this made the question difficult for many candidates. There were many non-attempts for (a) but most candidates did attempt part (b). The average mark was about 4 but there was fair spread of marks and a mark of 12 was not unusual.

(a) (7 marks)

In this part candidates were given the formula $S = kd^2w$ for the strength of a rectangular beam of wood of width *w* and depth *d* and where *k* is constant of proportionality. Since the beam has been cut from a log of circular cross-section the beam must have a diagonal length of 15 cm. There were the three parts to the question: in (i), candidates were asked to show that $S = k\left(225w - w^3\right)$, in (ii), to find the dimensions of the beam of maximum strength and in (iii), to show that the beam of maximum strength is stronger than a square beam by more than a given percentage.

The majority of candidates were unable to begin this problem or to get further than part (i) which suggests that they were unable to interpret the problem mathematically. Those who did recognise it as a routine maxima/minima problem were usually able to score quite well. However these were in a small minority and the average mark was less than 2.

(i) Only 1 mark was awarded for this part which involved a simple application of Pythagoras' Theorem. The those who realised this were generally able to derive the formula. Some who did observe that $d^2 + w^2 = 15^2$, were unable to go any further.

(ii) This was simply a problem of finding the maximum value of *S* in terms of w which meant solving $\frac{dS}{dw} = 0$ to find *w* and then using Pythagoras' Theorem to find *d*. Since the question asked for a justification of the answer the candidate was expected to show that these dimensions gave the maximum strength and this could be done, for example, by using the second derivative test. There were 4 marks for this section, 1 each for finding the derivative, solving to find *w*, finding *d* and showing that the strength was a maximum.

Many candidates were confused by the constant *k* which was often treated as a variable and some were unsure whether to treat *S* as a function of *w* or *k* or both. Many simply dropped *k* altogether during their calculations. It was also quite common for the justification step to be left out and although the question asked for the dimensions some candidates did not bother to calculate *d*.

(iii) This was badly done, even by many of those candidates who were able to do (ii) correctly. Again the main problem was that most were unable to translate what was being asked into a mathematical problem. Two marks were awarded, the first for finding the dimensions of the square beam and the second for calculating the strengths and finding their ratio (or a ratio of the difference to the maximum). Common mistakes were to use a square of length 15 or $\sqrt{75}$.

(b) (5 marks)

In this part candidates were given a graph of the velocity *v* of a particle moving in a straight line as a function of time *t* and, in parts (i) and (ii), were asked to mark on the time axis with a *Z* whenever the acceleration was zero and with a *G* when the acceleration was the greatest. In part (iii) candidates were asked to give the value of *t* when the particle was the furthest distance from its initial position, giving reasons for the answer.

In order to answer this question correctly, candidates needed to know the relationship between distance, velocity and acceleration but for about half of the candidates this was poorly understood (at least from a graph). However, the average mark was slightly more than 2 and most candidates were able to get at least one or two marks from this section.

(i) & (ii)

There were two correct values for *Z* (the turning points) and one for *G* (although the *exact* location for *G* was not clear) and a mark was awarded for each correct value. The most common mistake was to treat the graph as that of acceleration versus time and hence to mark as *Z* the points where the curve cut the axis and as *G*, the turning points.

Some candidates employed the strategy of marking every root and turning point as *Z* but marks were deducted for additional incorrect points and so this approach yielded no marks. However the origin almost looked like a turning point and candidates who marked this point did not have a mark deducted.

For (ii), the mark was given if *G* was marked as any point along the rising, ‘straight’ section of the curve to the left of the maximum. Again marks were deducted for wrong values and if the correct value was marked together with at least one incorrect value then no marks were awarded.

(iii) This section was probably the best done for the whole question. One mark was given for choosing the correct value of $t = 7$ from the three given possibilities and most candidates were at least able to guess correctly. The second mark was for the reason and here many candidates had difficulty in explaining why they had chosen the correct value

Either the reason was incorrect or was poorly expressed but if the candidate indicated that there was a change in direction at $t = 7$ then the mark was awarded. This could be explained in terms of the velocity becoming negative (as most did) or in terms of the area under the graph (as few did).

Question 10

Question 10 required candidates to sketch a trigonometric and linear graph on the same set of axes and then to use the graphs as an aid in the solution of a motion in a straight line problem. The majority began the problem and most were able to score marks, however very few were able to produce a quality, full mark scoring solution.

(a) (i) Locating the relative positions of $\frac{\pi}{2}$, 2, and π on the x axis was the major problem encountered and source of most errors in constructing the original graph. A significant number were unable to correctly sketch the line $y = 2 - x$ despite having drawn up a table of function values. Many candidates showed no indications of scales on either axes and others drew a cosine graph which was almost the shape of a W.

(ii) When attempting to explain why all the solutions to the equation $4\cos x = 2 - x$ must lie between $x = -2$ and $x = 6$, the majority were unable to express themselves despite giving the impression that they possibly did know what they were trying to say! Many stated the solutions were *at* –2 and 6. Most were unable to competently use mathematical terminology and confused terms like ‘domain’ and ‘range’ while others use expressions similar to ‘smaller than –1’, when they probably meant $x > -1$. Many indicated they thought –2 to 6 was the same as the domain -2π to 2π given in the question.

(b) (i) The majority of candidates were able to correctly differentiate to find expressions for the velocity of both particles. Small numbers attempted to find velocity by integration while others clearly indicated they believed the second derivative gave velocity (this approach tended to be centre-specific).

(ii) Most ignored the instruction ‘Use part (a)’ when they attempted to show there were two times when the particles had the same velocity. The curves shown in (a) intersected 3 times; twice above the x axis, twice to the right of the y axis. A significant number chose the two points above the x axis as ‘they were positive’. Few appreciated the necessity for time to be positive.

The majority attempted an algebraic solution to $4\cos t = 2 - t$. The most common approach consisted or attempts to solve $4\cos t = 0$ and $2 - t = 0$.

(iii) Few were correctly able to cope with positive and negative velocity when they struggled to find the total distance travelled by the particle. Those who attempted the problem usually found the expression for the final displacement rather than the distance travelled. Only a few used an area under the curve approach.

(iv) To show the two particles never met, candidates who used a graphical, rather than an algebraic approach were generally more successful. A very common response included the belief that because the candidate was unable to solve the equation it therefore had no solution! Few considered the possibility of the curves meeting at $x = -4$.

3 UNIT (ADDITIONAL) AND 3/4 UNIT (COMMON)

Question 1

This question consists of five unrelated parts.

(a) (3 marks)

Candidates were asked to indicate the region satisfied by two inequalities $y \le |x-1|$ and $y \le 1$. The three marks were awarded for

(i) showing the correct position of the line y = 1

(ii) showing the graph of $y = |x-1|$ with its 'V' correctly positioned, and

(iii) correctly shading or otherwise indicating the region below the graphs (i) and (ii).

In order to gain the second mark the candidate needed to show that the 'V' of the curve $y = |x-1|$ occurs at $x = 1$ on the x-axis. This could be done by labelling this point or as an inference from other labelled points. It was somewhat surprising that some failed to show any scale or to label any points. Whilst it is not always necessary to include detailed scales on the axes candidates should be encouraged to ensure that *key* points (in this case (0,1) and (1,0)) are clearly indicated.

Many could not shade in the correct region; a common error was to shade only between y=0 and y=1. The shading often was ambiguous, especially when using hatching in different directions for each of the component regions. Candidates should clearly indicate their intended region rather than trust the discretion of the marker. Some appear to have wasted time meticulously testing points when the correct region was intuitively obvious.

Those who believed that the aim of the question was to graphically solve an inequality in one variable x, thereby obtaining number line solutions were not penalised if their solution contained a two-dimensional graph which would otherwise deserve marks.

(b) (2 marks)

This question required the candidates to make y the subject of the formula $xy = 1$ and to integrate the resulting function of x. The first mark was awarded for the correct formation of the integral in terms of one variable; the second was for evaluating the integral. Whilst most answered this correctly, some had trouble recognising what had to be done.

Some of the common errors were

$$xy = 1 \quad \Rightarrow \quad y = 1 - x$$

$$\int_1^4 y\,dx = \left[\frac{y^2}{2}\right]_1^4$$

$$\int_1^4 y\,dx = \int_{\frac{1}{4}}^1 \frac{1}{x}\,dx$$

The last example shows a failure to realise that $\int_1^4 \ldots\,dx$ gives limits in terms of x not y.

(c) (1 mark)

This question tested knowledge that $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$. Clearly many students were not familiar with this result. Some common answers were 0, $\dfrac{0}{0}$, 5 or ∞. Some attempted to substitute small numbers (but usually not in radians).

(d) (2 marks)

The candidates were required to factorise $2^{n+1} + 2^n$ in order to simplify a special case where $n = 1000$. This question highlighted a lack of ability in manipulating powers of actual numerals as well as some poor index law practices. Responses involving

$$3 \times 2^n = 6^n$$

$$2^{n+1} + 2^n = 2^n\left(1^{n+1} + 1^n\right) \text{ or } 2\left(1^{n+1} + 1^n\right)$$

$$2^{n+1} + 2^n = 2^{n+1} \text{ or } 2^{n(n+1)}$$

were not uncommon. Some were satisfied with an unsimplified correct answer $\dfrac{2^{1000}(2+1)}{3}$. The marking scheme did not penalise this failure to employ elementary arithmetic.

(e) (4 marks)

Here candidates were required to use a substitution to evaluate a definite integral. The first mark was awarded for correctly differentiating in order to obtain a link between *dx* and *du*. The second was for correctly changing the limits of integration. The third was for correctly forming an integrand in terms of *u* and the fourth was for evaluating the integral. Those who returned to the original variable *x* thus avoiding the need to change to new limits were awarded the second mark (even if they incorrectly continued with old limits in the intermediate steps).

The most common error was to put the new limits in the wrong places (without adjusting the sign) ie. $\int_8^9$ instead of $\int_9^8$, in the mistaken belief that the larger number must always be on the top. Some had difficulty substituting $dx = \dfrac{du}{-2x}$ and cancelling the *x*'s. $-\dfrac{1}{3}\int \sqrt{u}\ du$ instead of $-3\int \sqrt{u}\ du$ was not infrequent. Attempts to integrate when the integrand contained two different variables were not awarded marks.

An answer in terms of fractional indices eg. $2\left(9^{\frac{3}{2}} - 8^{\frac{3}{2}}\right)$ was considered to be an adequate evaluation. Many had problems dealing with further simplification of surds or in obtaining a decimal approximation but this was not penalised. As a general principle candidates should be encouraged to round sensibly to one or two decimal places rather than to the nearest integer. Writing $2 \times 9^{\frac{3}{2}} - 2 \times 8^{\frac{3}{2}} = 54 - 45 = 9$ tends to disguise the correct numerical answer.

Question 2

(a) On the whole this question was poorly done. Many appeared to have difficulties understanding what the question meant and showed no real understanding of the concept of a root.

(i) (1 mark)

Most gained this mark for showing that *f*(0) and *f*(2) were opposite in sign. A statement to this effect was not necessary. Some students used Newton's Method to prove that a root existed between 0 and 2 and consequently scored 0.

(ii) Candidates were able to gain a mark easily by merely attempting to evaluate $f(1)$. A great number showed no knowledge of 'halving the interval'. Many used Newton's Method to reduce their interval. This gained no marks. Some halved the interval more than once. They were still able to score full marks if they correctly stated the sub-interval from their working.

(iii) Many did not know how to answer this question. Even though they appeared to know what they were doing in (ii) they did not understand what was expected here. Most assumed the root was closest to 0 since $|f(0)| < |f(1)|$. Some deduced that the root was between 0.5 and 1. This scored full marks.

(b) On the whole this part was well done. The *dx* or *dy* in their integral expression was ignored but they had to integrate with respect to *y*. 1 mark was awarded for the correct expression for the volume of each and a further mark for the correct evaluation of their expression. If *p* was omitted from each expression a maximum of 2 marks was possible. If the same expression was given for the top and bottom only one was marked.

For the top: incorrect responses included $\int_3^6 \frac{9}{x^2}$ or $\pi\int_3^6 \frac{3}{y}$

For the bottom: many were not able to quote the formula for the volume of a cylinder. Some used 3 as the radius instead of 1. Many believed the bottom volume to be $\pi\int_0^3 9y^{-2}\,dy$ and were not fazed by the 0 in the evaluation thus giving the answer as 3*p* .

(c) Most made reasonable attempts. Those who used the Factor Theorem arrived at the roots much more easily than those who used sum and product of roots.

Question 3

The question was designed to test a variety of skills from 3 main areas of the syllabus: combinations and permutations, binomial theorem, and coordinate geometry. Although not apparently a demanding question, the mean was only 7 and the question spread the candidature fairly evenly over the whole range 0 – 12.

(a) This part dealt with counting skills; selecting 3 objects from 8. The selections were:

(i) with replacement, order important;

(ii) without replacement, order important;

(iii) without replacement, order unimportant.

Although the question was quite clear, answers of the correct type; ie. 8^3, ${}^8P_3$, ${}^8C_3$ were often sprayed out in almost any haphazard order; some candidates choosing to cut their losses and giving, say, ${}^8P_3$, two or three times although they must have realised the situations were different. Very few seemed to use a common-sense approach; ie. writing $8 \times 7 \times 6$ rather than ${}^8P_3$, and there was clearly confusion about what ${}^8P_3$ and ${}^8C_3$ stood for. Skill on this part often seemed to be centre-based.

(b) Candidates were asked to find the term independent of x in the expansion of $\left(x^3 + \frac{3}{x}\right)^6$, and close to half the candidature found this an easy source of 3 marks. The marks were basically given for: being able to write down the general term for the expansion; being able to determine the term independent of x; and of course the final mark for doing this correctly.

Of the partially correct solutions many confused themselves with their own notation ($T_{k+1}={}^nC_k$ … became $T_5={}^6C_5$ … , etc.), and too large a minority gave expansions with *no* Binomial coefficients.

(c) The parabola (co-ordinate geometry) question was split into 5 parts, with the first 3 being of the 'Show that' type, so that there was no subsequent penalty for failing on any one part of the question.

Some got off to a bad start by confusing the a in $(2a, a^2)$ which was the given point on $y = \frac{x^2}{4}$, with the focal length of the parabola which was 1, and this trivialised the question for them.

The skills required for each part were:

(i) Differentiation and substitution of find gradient of tangent – Well done.

(ii) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, with simple algebra to obtain given result – Well done.

(iii) $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$, slightly more difficult algebra to obtain given result – 20% used incorrect formula; many manipulation errors such as errors of the form $a - (b - c) = a - b - c$, and a lot of fudging the given result rather than checking back to find what was usually an elementary algebraic slip.

(iv) & (v)
Although these should have been simple deductions from the results so far ((iv) started with 'Hence'), candidates did not always follow through.

An overall impression was that candidates were reacting to supposed familiar situations rather than THINKING and PLANNING responses.

Question 4

The question attempted to lead candidates through the processes of analysing and interpreting the properties of a particular function: $f(x) = \dfrac{3x}{3 + e}$. Many were able to cope with the instruction 'find' in parts (b) and (g) but unable to determine succinct appropriate responses to instructions such as 'show', 'describe', 'sketch' and 'explain why' found in other parts of the question. Candidates need more practice in interpreting the results they find in algebra and calculus. Overall the average was slightly above 6.

(a) (2 marks)
Candidates needed to differentiate, using a quotient rule or product rule, to get the first mark.

ie. $f'(x) = \dfrac{(3 + e^x)\, e^x - (e^x)\, e^x}{(3 + e^x)^2}$

The second mark was for simplification of $f'(x)$ leading to the conclusion that $f'(x) \neq 0$.

ie. $f'(x) = \dfrac{3e^x}{(3 + e^x)^2} \neq 0$ (because $3e^x \neq 0$).

Part (a) was generally well done.

Common errors:

- Unnecessary attempts to expand the denominator regularly resulted in errors such as $9 + e^{2x}$ or $9 + 2e^x + e^{2x}$.
- $(e^x)(e^x)$ was often simplified as e^{x^2}.
- Quotient rule often misquoted with '+' in the numerator.
- Showing $e^x \neq 0$ rather than $f'(x) \neq 0$.

(b) (1 mark)

Candidates needed to find the point of inflexion: $(ln\ 3, \frac{1}{2})$

This part was generally well done.

Approximations for *ln* 3 were accepted but definitely not preferred.

Common errors:

- failing to find the y value.
- failing to see that the question implied the existence of the point of inflexion. Valuable time was wasted unnecessarily showing change of concavity, and even deriving $f''(x)$ in some cases.
- $y = \dfrac{e^{ln 3}}{3 + e^{ln 3}} = \dfrac{3}{3+3} = \dfrac{1}{3}$ was a frequent simplification.

The mark was still awarded for the correct numerical expression, but students who failed to show the substitution, merely quoting $y = \frac{1}{3}$, lost the mark.

(c) (2 marks)

In showing $0 < f(x) < 1$ candidates were far more successful if they treated the two cases separately, ie. $f(x) > 0$ and $f(x) < 1$

Elaborate prose was not necessary. Successful responses could be as succinct as:

'Denominator > Numerator, $\therefore f(x) < 1$,

Numerator and denominator are both positive, $\therefore f(x) < 1$'

OR: ‘$e^x > 0$, $\therefore$ f $(x) > 0$, and $e^x < 3 + e^x$, $\therefore$ $\dfrac{3x}{3 + e^x} < 1$’

Part (c) was the part with the poorest responses.

<u>Common errors:</u>

- Simply stating $x \to \infty$, $f(x) \to 1$ and $x \to \infty$, $f(x) \to 0$ without understanding the need to show $f(x)$ is located *between* the two limits. (1 mark).

- Attempting proof by contradiction, ie. $f(x) < 0$ and $f(x) > 1$ (1 mark, if done correctly) without realising the need to negate the equality as well ie. $f(x) \leq 0$ and $f(x) \geq 0$. (2 marks, if done correctly). Very few completed this method correctly.

(d) (2 marks)

Verbosity in ‘describing’ the behaviour of $f(x)$ was prevalent but not necessary.

An adequate description could be simply: as $x \to \infty$, $f(x) \to 1$

as $x \to -\infty$, $f(x) \to 0$

Students generally scored the full 2 marks on this part.

<u>Common errors:</u>

- Reasoning using ∞ as a finite real number.
  eg. $\dfrac{e^{\infty}}{3 + e^{\infty}} = \dfrac{\infty}{\infty} = 1$. (not penalised when correct conclusion resulted).
- Stating $f(x) = 1$ and $f(x) = 0$, rather than ‘approaching’ the values of 0 and 1.
- Candidates often regurgitated the same limits in (c) and (d), failing to see the different implications of each question.

(e) (2 marks)

- 1 mark for the two horizontal asymptotes with a curve, in between, approaching both asymptotes.
- 1 mark for shape and position.

Plotting $(ln3, \frac{1}{2})$ was the best way to show position. Showing the change of concavity at $(ln3, \frac{1}{2})$ was the best way of presenting the shape. The y intercept of $\frac{1}{4}$ was not critical for the marks. (It was often found incorrectly as $\frac{1}{3}$)

<u>Common errors:</u>

- Showing a *horizontal* point of inflexion (penalised 1 mark)
- Simply plotting points did not usually gain two marks.
- In spite of the help from previous parts many graphs were hyperbolic outside the range $0 < f(x) < 1$. (0 marks).
- It was common to draw a graph similar to $y = e^x$. (0 marks)
- Stopping the graph at the y-axis, assuming the domain $x > 0$. (penalised 1 mark)

(f) (1 mark)

General confusion as to what constituted an inverse function's existence was evident. Correct responses could be as simple as: ' $f(x)$ is one-to-one'; or 'A horizontal line cuts the curve only once'; or 'For each y value there is only one x-value'

<u>Common errors:</u>

- Contradictions such as:
  'horizontal line test ie. for every x value there is only one y value'
  Or '$f(x)$ is one-to-one, ie. it passes the vertical line test.'
- Believing the process is proof of existence; eg. ' it has an inverse because it can be reflected in $y = x$' (0 marks)
- Reflecting in $y = x$ was sometimes called 'rotating', 'inverting', 'translating' or 'tipping upside down'.
- Stating 'The curve passed the horizontal *and* vertical line tests' (or equivalent algebraic double-attempts).
- 'Similar to $\tan^{-1}x$, $\therefore$ it has an inverse' indicates the need to spend more time on inverse functions before progressing to inverse trigonometric functions.

(g) (2 marks)

1 for interchanging the variables to get $x = \frac{e^y}{3 + e^y}$, and 1 for making *y* the subject to get $y = ln\,(\frac{3x}{1-x})$ or an equivalent form.

This part was generally well done, even by those who could not answer part (f).

Common errors:

- algebraic errors in making *y* the subject.
- not attempting to progress beyond $x = \frac{e^y}{3 + e^y}$
- confusing 'inverse' with 'reciprocal' to get $y = \frac{3 + e^x}{e^x}$ (0 marks)
- writing In instead of *ln*, a misunderstanding from reading the calculator (not penalised).

Question 5

This question had two distinct parts: the first on trigonometry and the second on probability.

(a) (6 marks)

This section consisted of three interrelated parts. A surprising number of candidates, however, failed to link the parts: many who correctly solved the given equation could not make use of the information to correctly address part (ii); and, even though the result was given in part (ii), many still failed to correctly set up the integral for finding the area in part (iii).

(i) (2 marks)

Two notable points:

- many divided by sin *x* (or simply ignored it) to solve the simpler equation $\sin x = \cos x$
- those who used $\cos 2x = 1 - 2\sin^2 x$ were faced with solving the more difficult equation $\sin 2x + \cos 2x = 1$. Some correctly changed it to the form $A\sin(x + a) = 1$ (or some similar form), others could not go any further. A number used this 'auxiliary

angle method' to solve $\sin x - \cos x = 0$. Others used the *'t'* method, often striking difficulty.

(ii) (2 marks)

It was obvious that candidates have difficulty with '*show* ...' questions, failing to give adequate explanations or reasons. An overwhelming number started with the result and tried to work backwards. A typical incorrect response was:

$$0 < x < \frac{\pi}{4} \qquad 2\sin x \cos x > 2\sin^2 x$$
$$\cos x > \sin x$$
$$\therefore \sin 2x > 2\sin^2 x$$

Many simply substituted a suitable value for x to show that $\sin 2x > 2\sin^2 x$. This was awarded 1 mark if the substitution was correct. The second mark was awarded for realising that, from part (i), $\sin 2x \neq 2\sin^2 x$ for $0 < x < \frac{\pi}{4}$ and that the substitution of any value of x chosen in this interval would determine which expression was greater.

A significant number reversed the *'if ... then ...'* statement, and did not deal with this successfully.

Candidates need to be reminded that graphical solutions are often not adequate in a question such as this.

(iii) (2 marks)

Two common errors:

- $\int_0^{\frac{\pi}{4}} 2\sin^2 x - \sin 2x \, dx$ despite the fact that part (ii) clearly told them that $\sin 2x > 2\sin^2 x$ for $0 < x < \frac{\pi}{4}$
- incorrect expressions for $2\sin^2 x$. Many who correctly wrote $\cos 2x = 1 - 2\sin^2 x$ then went on to write $2\sin^2 x = 1 + \cos 2x$. A number substituted (perhaps carelessly, or perhaps having learnt by rote) for $\sin^2 x$ instead of for $2\sin^2 x$

A surprising number could not give the primitive of $\sin 2x$ despite the fact that the table of standard integrals is given. It is also worth reminding candidates that the substitution of the limits should be shown, and not to just give the final numerical answer.

(b) (6 marks)

(i) (1 mark)

Generally well done. Those who scored 0 generally had left the binomial coefficient off; or had incorrectly evaluated the probability of the jackpot prize not being won; or gave the probability as $9(0.985)(0.015)$

(ii) (2 marks)

Two common errors:

- probability = 1 – (answer to part (i))
- probability = 1 – P(no wins) – P(at least one win)

Some wrote out the sum of the probabilities of winning once, twice, ten times. In this solution the coefficients were sometimes omitted, and the values of p and q were often switched around .

(iii) (3 marks)

Understanding of 'exceed' appeared to be confused. Having worked out that $\$200000 \div \$8000 = 25$, students did not know whether the jackpot would exceed \$200 000 after 24, 25, or 26 draws with no win.

Either the jackpot idea is not understood, or the binomial coefficient is meaningless to a large number of students. Many, after stating clearly that the jackpot would exceed \$200 000 if it was not won in the first 25 draws, gave the probability as ${}^{26}C_1(0.985)^{25}(0.015)$.

Others tried to link this part to the previous part and had the probability as $\left[(0.985)^{10}\right]^{25}$. A very small number recognised that the jackpot could be won on the 26th, or 27th, or 28th, or 29th .... draw, setting up a GP with a limiting sum.

Question 6

From the point of view of 3 Unit topics, the question consisted of two parts, one on circle geometry, and another which posed a rate of change problem involving an inverse trigonometric function. However the question had as a central theme a problem about a length of cable, part of which was in contact with the circumference of a large wheel. A candidate did not have to recognise the connection between the two parts to score full marks.

Although the vast majority scored some marks for this question, many found the question as a whole difficult, as perhaps would be expected for the second last one on the paper. Many were clearly pushed for time. The mean was about 6, and about 1% of candidates obtained full marks.

In four places, and for a total of 7 out of the 12 marks, the question required the candidate to show or to prove a given result. The onus is on the candidate in these cases to write down all the intermediate steps which make it clear why the result follows from the given information.

(a) A total of 3 marks

(i) (2 marks)

Candidates were asked to prove that two triangles, formed in this case from a tangent and a secant to a circle, were similar. One mark was awarded for attempting to show that correspondingly angles of the triangles are equal, and one for giving the reason.

The question was not well answered. Less than half scored full marks. Many gave barely recognisable, or just plain incorrect versions of the reason which refers to 'the angle in the alternate segment' theorem. Some confused a similarity test with a congruence test. Some wrote down more information than was needed, claiming for example that sides were equal or in the same ratio, and then did not give the reason why the triangles were similar, so that the examiner did not know why they thought the result followed.

Some gained full marks by correctly applying the result concerning the square of the distance of a tangent from an external point to a circle, and the product of the intercepts formed by the secant. However, this made it difficult for them to gain the mark in part (ii), since the argument was then likely to be circular.

(ii) (1 mark)

Here candidates were to prove, using part (i), and for this particular situation, the result referred to in the previous paragraph. The mark was awarded for writing down the required ratios and showing them as equal, or for stating that the corresponding sides of similar triangles are in the same ratio.

Again the part was not well answered, with about half the candidates getting it correct. It was good to note though that occasionally were able to obtain this mark, even though they scored 0 or 1 on part (i).

(b) A total of 9 marks

(i) (1 mark)

This was the first of two sections in which candidates were asked to 'explain' a result, in this case why the cosine of an angle in the diagram was equal to a certain ratio. There were clearly several desirable steps in this explanation, but only one mark was available. The mark was awarded if the candidate indicated, in a diagram or otherwise, that the angle between the cable and the radius of the wheel at the point where the cable left the wheel was a right angle.

It followed that the required angle was in a right angled triangle, so that the cosine ratio, defined as 'adjacent over hypotenuse', could be used, but the candidates did not have to state this.

The vast majority obtained this mark.

(ii) (2 marks)

The candidates had to show that a length in the diagram was given by a certain expression. One mark was awarded for indicating that the formula for arc length (in any one of its several forms) was needed, and one for deriving, by use of a diagram or otherwise, the expression for the required angle.

The part was well answered, with well over half scoring full marks. Some answered very well by noting that the θ in the diagram was different from the θ in their formula $l = r\theta$, and then renaming the latter; and others by noting that the required length was equal to half the circumference minus the arc opposite the θ in the diagram. However it was a concern of the examiners that many students did not include enough detail in their argument. See the general comment above. Occasionally students mixed degrees and radians in the one expression.

(iii) (2 + 1 marks)

Here the candidate was given the derivative of the formula obtained in part (ii), and asked to show how to obtain it. The two marks were awarded according to the stage reached by the candidate in the differentiation process. The candidate then had to explain the significance of the value of the derivative being always negative in the context of this problem. The candidate obtained the mark if they wrote that it meant that the function found in part (ii) was decreasing. Some went on to apply this to the problem and say that it meant the amount of cable in contact with the wheel was decreasing, but they did not have to go this far to be awarded the mark.

Most found the differentiation difficult, with about one quarter obtaining full marks. Those who used the chain rule, either explicitly or implicitly, had a high rate of success. Those who tried to apply a standard formula for the derivative of $\cos^{-1}\left(\frac{x}{a}\right)$ nearly always scored zero. Once again the examiners were concerned because many students did not show the separate stages of their argument clearly. For example, there was often carelessness with negative signs.

Very few students obtained the mark for the explanation. The great majority did not even attempt it. Since this often happened even with very strong candidates, they perhaps forgot to come back to it after doing the differentiation. Of those that attempted it, very many said that the rate of change was decreasing, or simply gave the ambiguous answer ‘it is decreasing’.

(iv) (1 mark)

At this stage the candidates were asked to find an expression for the total length, s, of the cable, in terms of x. The question suggested they could use the result proved in part (a). However most used an alternative approach, with Pythagoras' theorem being the most popular. The mark was awarded for the correct answer.

About one student in three obtained the mark. Many students omitted to remove the variable θ from their answer. Often there was a careless error, for example not taking the square root, or writing $\sqrt{9+x^2} = 3 + x$.

(v) (2 marks)

Finally the candidate had to find an expression for $\dfrac{ds}{dt}$ in terms of x, and evaluate it. The first mark was awarded for correct differentiation with respect to x of the expression found in part (iv). The second for the correct application of the chain rule followed by at least the explicit substitution of $x = 10$. The candidates did not have to find the correct numerical answer to gain this mark.

For those who had persevered to this point, the first mark was usually easily obtained. About one half of these went on to score the second mark. Frequent errors were to not use the chain rule at all, or else to quote some form of it which was not helpful here, for example by including variables such as l that had arisen earlier in the problem.

Question 7

This question, on projectiles, consisted of five linked parts involving a ball hit towards the fence of a softball field. As the final question on the paper it was a searching question, with the last two parts in particular designed to sort out the best candidates. Even so, it allowed any well-prepared candidate to score marks, as the first five marks were standard bookwork. It was a successful question as the average mark was around 4, while very few candidates scored full marks.

In order to enable candidates to attempt later parts of the question even if they were unsuccessful in earlier parts, parts (a) to (d) were of the 'Show that ...' type, where the required result is given. It was good to see that many were able to take advantage of this format, and score marks in later parts of the question. However it must be emphasised that this format places the onus on them to indeed *show* that they have derived the result, and thus must not leave out intermediate steps.

(a) (4 marks)

Candidates were asked to use calculus to derive the equations for the position of the ball in terms of t. The 4 marks were awarded essentially for each of the four required integrations. In order to score full marks it was necessary to show the constants of integration as well as to explicitly show their evaluation using the initial conditions.

This part of the question was generally well done, with an average of around 3 marks being awarded. The most common errors were in the evaluation of constants of integration, and sloppiness in showing the evaluation steps by candidates who may well have known exactly what they were doing but failed to clearly show their working.

Although this was a familiar exercise for most, many obviously were not used to dealing with projectiles starting from points other than the origin, and did not know how to deal with the non-zero constant in the expression for y.

It was pleasing to see that very few used motion formulae such as $v = u + at$. This approach yielded a maximum of 1 mark.

(b) (1 mark)

This question asked students to derive the Cartesian equation of the trajectory of the ball. The mark was awarded for correctly substituting $t = \dfrac{x}{V\cos\alpha}$ in the equation for y. This part of the question was very well done.

(c) (2 marks)

Here candidates needed to show $V^2 \geq \dfrac{gR}{2\sin\alpha\cos\alpha}$ if the ball cleared the fence. The best way to show this was to state that $y > h$ when $x = R$, and substitute these values in the equation from part (b). The first mark was awarded for an attempt at substitution of these values or equivalent, and the second mark was for successful completion of the algebra, including dealing correctly with the inequalities. A very common error was to reverse the inequality when substituting for *y*.

Many used $y = h$ in the working, resulting in an equation for V^2. If they went on to say that the ball would *clear* the fence for V^2 *greater* than this value, they were awarded full marks. Candidates could earn marks quoting a range formula provided they clearly showed that the 'range' given by this formula was the *x* value where the ball returned to the height of projection.

(d) (3 Marks)

This part asked candidates to show that $\tan\alpha \geq \dfrac{Rh}{(R+r)r}$ if the ball hit a cap at *C*.

The first two marks were for (i) substituting $x = (R+r),\ y = 0$ in the equation given in (b), and (ii) substituting for V^2 from (c). The third mark was for successfully dealing with the algebra.

This part of the question was not easy because it was not clear from the question where to begin to derive the inequality, and the algebra, particularly dealing with the inequality signs, was tricky.

(e) (2 marks)

Candidates were asked to find the closest point to the fence (outside the fence) that the ball can land, given certain numerical values. This was very difficult, with few successfully completing it.

The best solutions re-arranged the inequality from (d) as a quadratic inequality in *r*, and then recognised that the minimum value for *r* occurs when $\tan\alpha$ is the maximum value satisfying the inequality in (c).

More commonly, candidates substituted the given values in the inequalities from (c) and (d), and attempted to make some progress from there. Even having shown that $\sin 2\alpha \geq \dfrac{gR}{V^2} = \dfrac{80}{2500}$, it was rare for them to take into account that 2α could be obtuse, and so they missed the value of α which led to minimum *r*. This approach usually yielded a maximum of 1 mark, with the second mark being awarded only for the correct answer.

4 UNIT (ADDITIONAL)

Question 1

This question, on integration, was fully attempted by almost every candidate. It was generally well done, giving them confidence to tackle following questions. Many errors were of an arithmetical nature rather than in the method used. The question covered many techniques of integration such as 'substitution', 'integration by parts', 'partial fractions' and 'integration as anti-differentiation'. Repetition in method was minimal. If preparation has been adequate, candidates could benefit by using the reading time to identify each question part with the relevant method to be used. For example, if the denominator contains factors, then partial fractions appears to be the method to try. To expand these factors immediately seems to be undoing an obvious lead.

(a) (2 marks) $\displaystyle\int \frac{dx}{x(\ln x)^2}$

A mark was lost here for the omission of the constant, but not if stated in parts (b) or (d). One mark was gained if the candidate recognised that the differentiation of $\ln x$ gave $\dfrac{1}{x}$. This usually led to the correct substitution of $u = \ln x$. Many candidates gave their final answer in terms of 'u' which cost them a mark. Some used integration by parts and eventually reached the answer at the expense of valuable time. The correct answer was $-(\ln x)^{-1}$. Variations of logarithm laws and omission of brackets gave incorrect results such as ln x, and $-\ln x\left(\dfrac{1}{x}\right)$.

(b) (2 marks) $\displaystyle\int xe^x\,dx$

This part was extremely well done and straight-forward. Problems arose for those few students who chose the parts incorrectly and let $u = e^x$ and $\dfrac{dv}{dx} = x$. One mark was gained by choosing the correct parts and realising the formula similar to $(\ ) \pm \int (\ )dx$

(c) (4 marks) $\displaystyle\int_1^4 \frac{6t+23}{(2t-1)(t+6)}dt$

Basically this question was done by one of two main methods:

(i) partial fractions

(ii) expanding the denominator

A maximum of two marks could be gained for preparation and the remaining two marks were awarded for the integration process and evaluation.

On the whole, method (i) was done well. However errors occurred too often in either finding *A* and *B* by simultaneous equations or substituting a suitable value for *t* to eliminate either *A* or *B*.

Method (ii), if a set of factors was not recognised, was most tedious and usually ended in ‘completing the square’ on the denominator. It is worth noting how many students, at this stage, carried fractions such as $\frac{6}{4}$ through the complete operation.

A few candidates tried to ‘fudge’ the given answer, ln 70, while others obviously became frustrated and persevered at length, sometimes to no avail.

(d) (3 marks) Find $\frac{d}{dx}\left(x\sin^{-1}x\right)$ and hence find $\int \sin^{-1}x\,dx$.

One mark was awarded for the correct use of the product rule; one for applying the ‘hence’ instruction and one for the integration. Many used integration by parts and ignored the ‘hence’, giving themselves a maximum of two marks, eg., $\left[\int 1.\sin^{-1}x\,dx\right]$.

A common error was the final sign in $-\int \frac{x}{\sqrt{1-x^2}}dx$

(e) (4 marks) $\int_0^{\frac{\pi}{2}} \frac{dx}{5+3\sin x+4\cos x}$ using $t=\tan\frac{x}{2}$.

Basically one mark was awarded for the correct *dx* equivalent in terms of *t*. This process was extremely well done. One mark was awarded for the correct *t* substitution *and* simplification. For many, simplifying algebraic fractions was a weakness, whereas rote *t* substitution was fairly successful. The final two marks were for integration and evaluation including limit change. These marks, as with previous sections, were almost independent of each other. This means that integration of their incorrect substitution or simplification if not made easier, could gain the mark.

Quite often the relatively simple integration, $\int \frac{2dt}{(t+3)^2}$, was transformed into something most complicated and if the integration *process* was wrong, the evaluation mark could not be gained. Overall, the processes used were correct and efficient, testing many aspects of integration method. Main faults usually originated in algebraic or arithmetical errors.

Question 2

This question on complex numbers consisted of four main parts. There were only two non-attempts, very few scored 0, and about 2% scored the full 15 marks. The most pleasing aspect of the responses was the large, clear diagrams employed by many candidates. The most disappointed aspect was the complicated methods and convoluted explanations employed in answering relatively simple problems.

Although a few candidates understandably thought that part (c) required the intersection of two loci, most considered the discrete cases.

It should emphasised that a number of marks throughout the question were lost because of careless errors which could have been picked up by simple checks (for example, checking the sum of the roots of the quadratic in (b) (ii) against $-\frac{b}{a}$).

(a) (1 mark)

Most candidates gained the mark for this part, although a number changed the sign of the real part in determining the conjugate. Occasional careless addition errors meant the loss of any easy mark.

(b) (i) (1 mark)

A surprisingly large number of candidates chose to find the square roots of $-3-4i$ by solving two simultaneous equations. Most were successful, particularly since the answer was available, but it was a time-consuming exercise to gain a single mark. Of the ones who simply expanded the expression, only the occasional careless error deprived a few candidates of an easy mark.

(ii) (2 marks)

About two-thirds of the candidates made use of the result in part (i) to evaluate z by the quadratic formula. Many of the others, however, did not see this connection and proceeded to find the square root again! Carelessness in expressing the roots as $\frac{5 \pm 1 - 2i}{2}$ instead of as $\frac{5 \pm (1 - 2i)}{2}$ led to incorrect solutions which gained only 1 mark instead of the two on offer.

A large number tried expressing z as $x + iy$, substituting for z in the quadratic equation, equating real and imaginary parts, and then solving two equations in x and y simultaneously. Very few were successful with this method. A number of candidates attempted the use of the quadratic formula as a last resort, conveying the impression that it was appropriate only quadratic equations with real coefficients.

(c) (i) (2 marks)

Roughly half the candidates obtained both marks for drawing the locus correctly. Most had been well drilled and needed no working. Many candidates were penalised for failing to mark the size of the angle or either intercept with the co-ordinate axes. Extending the locus below the real axis also resulted in the loss of a mark. Some candidates also failed to realise that the locus had a negative gradient and were penalised for this. As mentioned previously, the standard of the large, clear diagrams was very pleasing.

Candidates should be encouraged to investigate critical points (in this case, (4, 0) was specifically excluded, although candidates were not penalised for overlooking this) and to indicate by an arrowhead if the locus continues indefinitely (again, no penalty was incurred unless the candidate explicitly indicated that the locus was an interval and not a ray).

(ii) (3 marks)

A mark was lost in a large number of cases for failing to terminate the ray at the origin. Again, no penalty resulted from failing to draw an arrowhead. One frequent problem, however, was the failure to mark the locus carefully. In some cases, particularly with those candidates who showed no working, it was of extreme importance to indicate clearly the non-negative section of the y -axis. Shading was ignored unless the candidate stressed that a region rather than a ray or line was the answer.

Many candidates mistakenly wrote x or iy for $Im\,(z)$ and/or $x^2 + y^2$ for $|z|$. Unfamiliarity with basics like these thus deprived them of the opportunity of gaining more than one mark for this part.

(d) (i) (2 marks)

Candidates were required simply to appeal to the triangle inequality (either by name or by expression in terms of the sides of the triangle) and then to draw the required conclusion. Many used the word 'hypotenuse' to refer to the longest side, perhaps because they had concluded from (iii) that the figure contained a right angle. Others tried to prove the inequality from statements (often based on the cosine rule) that indicated that they were unaware that the real and imaginary parts of w and z, being arbitrary, could be of either sign.

(ii) (2 marks)

The construction of R was often done inexactly. Unless it was clearly in the wrong place ('below' PQ, for instance) or stated to be the fourth vertex of a kite or of a cyclic quadrilateral, the mark was awarded. A large number of candidates gained the second mark for stating that the figure was a parallelogram, though the spelling was frequently questionable. Those who simply stated the properties of the figure without classifying it as a parallelogram did not gain the second mark.

(iii) (2 marks)

Many candidates translated this question into a geometrical context, but a disappointingly large number concluded that the quadrilateral $OPRQ$ had to be a square, a rhombus or a kite. Even those who correctly concluded that it was rectangle often wrote that the value of w/z was i, instead of ki. Stated restrictions on k (for example, positive or integral) were ignored. A number of candidates, having incorrectly stated that $w/z = i$ then went on to conclude (correctly) that w/z was purely imaginary. This failed to gain full marks since in effect it restricted z/w to the single point (0,1) rather than to the line $x = 0$.

In conclusion, it should be emphasised to every candidate that every part of a question should be attempted. Since the first mark is usually relatively easy to gain, no student should ignore a question because no clear path to the final solution can be seen immediately.

Question 3

This question was on the sketching of a parabola and then applying some transformations to it in the next 4 parts. The second part asked a question on volumes. This contained a circle to be rotated about a line where the cross-sectional area was to be annulus, taking a slice perpendicular to the axis of rotation. Result: the volume of a torus.

Question 3 was well attempted by the great majority of candidates, many gaining full marks as the statistics will indicate.

Unfortunately, on the negative side, many candidates often lost marks due to unwarranted carelessness in setting out, basic numerical errors, not reading and/or not following the instructions given in the question.

Many candidates could do the sketches in part (a) well but they then left, or had no idea or had trouble in attempting part (b) on volumes.

(a) Some candidates appeared inexperienced and/or unskilled in this area of sketching graphs. Also it must be stated that there were a few candidates who tried to use calculus. It appeared that they were not really conversant with these types of sketching approaches. Calculus was not required. A few used a table of values.

(i) (2 marks)
1 mark for correct concave downwards shape and passing through (2, 0) & (4, 0)

1 mark for y passing through (0, –8) or indicated.

The errors occurred when they did not take enough care in organising the $-x^2+6x-8$ properly (ignoring the negative coefficient of the x^2, factoring incorrectly and so forth) with no clear indication of the y-intercept.

(ii) (1 mark)
1 mark for correct part reflection, passing through (0, 8) with at least 1 cusp shown at (2, 0) and/or (4, 0).

(iii) (2 marks)
1 mark for basically circular in shape through only $x = 2$ & 4, no change in concavity above or below x-axis.

Many candidates did not realise it was actually a circle, but their shape was basically circular and symmetrical about the x-axis.

(iv) (3 marks)
1 mark for showing the vertical asymptotes at $x = 2$ & 4.
1 mark for showing the horizontal asymptote at $y = 0$ with branches from below.
1 mark for correct shape passing through $\left(0,-\frac{1}{8}\right)$.
Candidates could show the 2 vertical asymptotes at $x = 2$ and $x = 4$ and maybe the horizontal asymptote at $y = 0$, but after that they were inconsistent as to where the branches of the curve should be placed.

Lack of clear indication of the y-intercept of $-\frac{1}{8}$.

(v) (2 marks)
1 mark for correct shape indicating through $\left(0,e^{-8}\right)$ or either $(3,e)$ or either (2, 1) and (4, 1).
1 mark for indicating horizontal asymptote at $y = 0$ from above.

This part was frequently the only one of the 5 sketches not attempted. It appeared that many of the candidates were not really experienced with this type of sketching transformation.

e^{-8} is very close to zero so the candidates were not really penalised with their y-intercept with their sketch.

If the candidate's sketch in (i) was incorrect but then the correct interpretation was achieved in (ii) to (v), marks were awarded.

(b) As stated earlier some left this part out, but other centres and candidates were well versed in this topic.

(i) (2 marks)

1 mark for obtaining correct inner radius of $9 - x_1$ and outer radius of $9 + x_1$ or equivalent in y.

1 mark for showing clearly and correctly that the area was $36\pi\sqrt{16-y^2}$, using the difference of two squares.

It was noticeable that some candidates took a few attempts at this part before they could show that the area of the annulus was $36\pi\sqrt{16-y^2}$. Others became confused and could not get it. They did not visualise the annulus, nor draw it, nor organise their radii clearly or correctly.

There were numerical errors in eg., squaring 9, adding $18\pi\sqrt{16-y^2}$ to $18\pi\sqrt{16-y^2}$ to give $32\pi\sqrt{16-y^2}$! They did not check that the question was to be eventually $36\pi\sqrt{16-y^2}$ (maybe exam pressure?!).

The candidates who stayed in terms of x's had the least problems in showing the given result. There were a few who tried to treat the annulus as a rectangle. They were neither clear nor successful.

(ii) (3 marks)

Basically: 1 mark for the correct definite volume integral expression.
1 mark for evaluating the definite integral correctly.
1 mark for the correct volume.

Again many careless errors in this part. It was common enough for them to write $\int_{-4}^{4} 36\pi\sqrt{16-y^2}\ dx$ and go on to state $\int_{-4}^{4} 36\pi x\ dx$ etc.

They were not thinking about the question nor visualising the diagram and that the slice's height was to be *dy*. Many had the wrong end points eg., –4 to 9; 0 to 9. Many able/knowledgeable candidates misinterpreted their integral eg., $\int_0^4 \sqrt{16 - y^2}\ dy$ to be a semicircle of radius 4 and so wrote $\frac{1}{2} \times \pi \times 4^2 = 8\pi$.

Due to unclear (and invariably squashed up) setting out in applying the trig. substitution technique, many wrote eg. for dy they wrote $\cos\theta\ d\theta$, the usual errors in knowing and manipulating the double angle results for $\cos^2\theta$ and $\sin^2\theta$.

Wrong signs and on integration – incorrect coefficients and trig. expressions and then on evaluation more numerical errors. Some confused their change of variable values in going from x to θ, eg. $-4 \to \frac{3\pi}{2}$

The candidates who used a geometric approach to the integral had least errors. There were only a few candidates who used cylindrical shells and they were able to gain the correct response. Some centres tried to apply Pappus's Theorem. (Fine if they used it as a check).

Question 4

This question consisted of three parts. Part (a) examined complex numbers (and de Moivre's Theorem), part (b) was on Mathematical Induction, and part (c), which contained 5 linked parts, dealt with trigonometric identities and trigonometric integration. The question was attempted by most candidates, with possibly 15 – 20 non-attempts. The average mark for this question was around 8, while a good number of students (possibly 50) obtained full marks.

Part (c) caused problems with a large number of students – possibly 20% of the candidature: they misinterpreted $\sin(2m+1)x$ as $x.\sin(2m+1)$ and expanded this as $x(\sin 2m \cos 1 + \cos 2m \sin 1)$. These candidates usually left (c)(i) incomplete, and proceeded with the rest of the question, but some candidates panicked and did not attempt any more of the question.

(a) (i) (1 mark)

Candidates were asked to find the least positive integer *k* such that $\cos\left(\frac{4\pi}{7}\right)+i\sin\left(\frac{4\pi}{7}\right)$ is a solution of $z^k = 1$. Quite often the words 'least positive integer' were ignored, so that common final answers were either 0 or $3\frac{1}{2}$.

(ii) (3 marks)

Candidates were asked to 'show that if the complex number *w* is a solution of $z^n = 1$, then so is w^m, where *m* and *n* are arbitrary integers'. The word 'arbitrary' caused many problems – many candidates either ignored this word, or thought, mistakenly that it meant 'consecutive' or 'reciprocal'. Of those who took the 'de Moivre' approach, the majority assumed that *w* was the smallest complex solution. Many students also asserted that *m* must be smaller than *n*. A small number of candidates ignored the word 'complex', claiming that *w* must equal 1, so therefore so will w^m. Most candidates who attempted this part were awarded 2 marks.

(b) (i) (1 mark)

Candidates were asked to solve a quadratic inequality – note that the quadratic did not factorise. After incorrect algebraic manipulation, many claimed that $(x-1)^2 > 0$ while others had as their final solution either $\left(1-\sqrt{2}\right) < x < \left(1+\sqrt{2}\right)$ or $\left(1-\sqrt{2}\right) > x > \left(1+\sqrt{2}\right)$. Successful candidates usually had arrived at the solution via clearly labelled graphs.

(ii) (3 marks)

One mark was awarded for showing that the statement was true for $n = 5$. A number of candidates, out of habit, showed that the statement was true for $n = 1$ instead. Two marks were awarded for correct algebraic manipulation – for using both the assumption statement for $n = k$, and for using the solution of the inequality in (b)(i). Some students, not realising that (b)(i) was required, successfully showed that $(k-1)^2 - 2 > 0$ for $k > 5$.

Too many worked with both LHS and RHS at the same time, and then became confused. The successful candidates were those who started with LHS, ie. with 2^{k+1}, and then worked through until they obtained the required expression on the RHS.

(c) (7 marks)

As already mentioned above, the expression $\sin(2m+1)x$ did confuse many of the candidates. It should be noted, though, that as the candidates worked through the rest of this part, many did realise the mistake they had made in (i), and then went back and attempted to fix up their blunders.

(i) (2 marks)

Two methods were correctly employed:

(1) 'differences to products' formula;

(2) correct expansions of $\sin(2mx+x)$ and $\sin(2mx-x)$ followed by collecting of like terms.

Both of these methods were used by approximately the same number of candidates. Only one candidate successfully started with RHS.

(ii) (1 mark)

A surprising number of candidates did not refer to the standard integrals sheet provided, and thus incorrectly calculated $\int_0^{\frac{\pi}{2}} \cos(2mx)\, dx$.

A number of candidates felt uncomfortable about the presence of the 'm', rather than a number, and could not handle the numerical evaluation.

A number of candidates incorrectly established that $\cos(2mx)$ is an odd function, and thus the answer is 0!

Many students (mostly unsuccessful due to lack of relevant information) took a graphical approach to this part, and talked about areas cancelling out.

(iii) (1 mark)

This part was very well done, with the candidates, on the whole, referring to the results of (i) and (ii) successfully.

(iv) (1 mark)

Was also very well done, and was attempted by all the candidates who went ahead and did (c).

(v) (2 marks)

The candidates, to achieve their 2 marks, were required to:

(1) recognise, from (iv), that $\int_0^{\frac{\pi}{2}} \frac{\sin(2m+1)x}{\sin x}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin 3x}{\sin x}\,dx = \frac{\pi}{2}$, when $m = 1$;

(2) use the result from (iii) when $m = 2$ to then relate $\frac{\sin 5x}{\sin x}$ to $\frac{\sin 3x}{\sin x}$.

Unfortunately, even though the question said 'hence', this part was not well done, with many candidates wasting much time trying to expand, and then simplify $\frac{\sin 5x}{\sin x}$. Other candidates, without any supporting evidence, claimed that, when $m = 1$, then $\int_0^{\frac{\pi}{2}} \frac{\sin(2m+1)x}{\sin x}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin 5x}{\sin x}\,dx = \frac{\pi}{2}$.

It should be noted that a number of candidates used the method of (2) outlined above, then successfully expanded sin $3x$ and finally correctly evaluated $\int_0^{\frac{\pi}{2}} \frac{\sin 3x}{\sin x}\,dx$, showing all necessary working.

Question 5

This question contained five parts which were relatively straight-forward, and two parts which were a little tricky. Few candidates scored full marks. Many were able to score the nine easy marks, but parts (b)(iii) and (c)(ii) were generally handled very badly. It is clear that candidates, even at this level, are not adept at algebraic manipulation, and are easily put off if the algebra becomes messy.

(a) Candidates were required to prove a trig identity in part (i), and to use the identity to solve an equation in part(ii).

(i) (1 mark)
The expectation was that students would write

$$\sin x + \sin 3x = \sin(2x - x) + \sin(2x + x) \ .$$

Only a minority did so. A large number wrote

$$\sin x + \sin(2x + x) = \sin x.(1 + \cos 2x) + \sin 2x \cos x,$$

and then used

$$\sin x.(1 + \cos 2x) = \sin x.\cos^2 x = \sin 2x \cos x$$

to successfully get the result.

Many used the formula $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$, and were awarded the mark so long as the formula was clearly apparent. Similarly, those who used a formula for sin 3x derived from de Moivre's theorem were awarded the mark. Most were able to score this mark one way or another.

(ii) (3 marks)

Candidates were expected to make a substitution using the identity in (i) and factorise the resulting expression (for 1 mark), and then to obtain the two equations sin 2x = 0, or $\cos x = -\frac{1}{2}$ (for 1 mark).

The final mark was awarded for the correct solutions to these two equations for $0 \le x \le 2\pi$. (The inclusion of $x = 2\pi$, and/or the exclusion of $x = 0$ as solutions were ignored.) Candidate who gave the solution as $x = \frac{n\pi}{2}$, $x = 2n\pi \pm \frac{2\pi}{3}$ were not awarded the final mark. A common error was to miss the solution $x = \frac{3\pi}{2}$, by failing to realise that $0 \le 2x \le 4\pi$. Happily, only a few failed to realise that they should use part (i), and many students were able to score 3 marks.

(b) Parts (i) and (ii) required candidates to know something about sums and products of the roots, t_1, t_2, t_3 of the cubic $f(t) = t^3 + ct + d = 0$. Part (iii) required differentiation to find values at which the cubic had turning points, and then substitution of these values into a (given) inequality, followed by some algebraic manipulation in order to find a (given) relationship between the coefficients c and d of the cubic. Part (iii) had absolutely nothing to do with parts (i) and (ii), a fact which caused those who assumed it did, to come to grief.

(i) (1 mark)

Almost all gained 1 mark for stating that the sum of the roots is 0.

(ii) (2 marks)

Candidates were asked to show that $t_1^2 + t_2^2 + t_3^2 = -2c$. The first mark was awarded for the identity

$$t_1^2 + t_2^2 + t_3^2 = (t_1 + t_2 + t_3)^2 - 2(t_1t_2 + t_2t_3 + t_3t_1)$$

and the second mark for substituting 0 for $(t_1 + t_2 + t_3)$ and c for $(t_1t_2 + t_2t_3 + t_3t_1)$. Most used this method, and successfully gained 2 marks.

Another successful approach was to find an equation with t_1^2, t_2^2, t_3^2 as roots, and then write down the sum of the roots of this equation. Fewer candidates would have gained the marks had the question asked them to *find* $t_1^2 + t_2^2 + t_3^2$, since quite a few thought $(t_1t_2 + t_2t_3 + t_3t_1)$ was $-c$, until they realised this gave the wrong result.

(iii) (3 marks)

This part was handled quite badly, and many scored 0. The first mark was awarded for differentiating, and clearly identifying two *distinct* values at which the cubic has turning points. The two equations $u + v = 0, \quad uv = \frac{c}{3}$ were sufficient to gain this mark. (The required result is relatively easy to obtain by using these two relationships in an expanded form of $f(u).f(v)$) An alarming number found $u = v = \sqrt{\frac{-c}{3}}$, and then wrote $f(u)^2 < 0$.

The second mark was awarded for a correct, relevant substitution for u and/or v, in terms of c, in $f(u).f(v) < 0$. The third mark was for the algebraic manipulation leading to the required result. Since the result was given, this third mark was awarded only if the algebra was error-free. Candidates were most successful when they found some neat way of simplifying the expression $f(u).f(v)$ in terms of c and d, before multiplying out. Those who stated, correctly, that $f\left(\sqrt{\frac{-c}{3}}\right) < 0$, unfortunately scored 0, since there is no way of getting the result from this. (Although many did!)

(c) Part (i) asked for the equation of a normal to a parabola in a specified form. In part (ii), candidates were required to recognise that the conditions stated allowed them to form a cubic equation which satisfied the inequality in (b)(iii), and then to use that inequality to find another (given) result.

(i) (2 marks)

Most scored the 2 marks (one for finding the gradient and writing down the equation, and one for rearranging). Unsuccessful candidates included those who found the equation of a tangent, or used $\frac{dy}{dx} = \frac{-1}{2x}$ as the gradient.

(ii) (3 marks)

Very badly done, with very many non-attempts. The first mark was for explaining that the conditions stated meant that there were three distinct real roots to the equation $t^3 + \left(\frac{1-2y_0}{2}\right)t + \left(\frac{-x_0}{2}\right) = 0$. Very few scored this mark. (There appears to be little understanding of the fact that mathematics involves more than just being able to do the mechanical stuff.)

The second mark was for substituting $c = \frac{1-2y_0}{2}$ and $d = \frac{-x_0}{2}$ in the inequality in part (b)(iii). Of those who attempted this part, most scored this mark. (The subscripts did not have to be included to get the mark.) The final mark was for the rearrangement which, in most cases, was not handled well. Once again, since the result was given, the mark was awarded only if there were no errors. Candidates had trouble getting the signs correct, and with the indices.

Question 6

This question consisted of two sections, the first involving some 3-dimensional trigonometry and the second some geometry. Although it was near the end of the paper, almost all candidates attempted some part of it. The policy of attempting the easy bits from all questions certainly paid off for them. The average mark was in the range 5 to 6 out of 15. No candidate, however, scored full marks and only a handful scored 13 or 14 marks.

(a) (6 marks)

In this section the candidates were required to find the direction of flight of an aeroplane given its bearing and elevation at two different times. This is a harder inverse variant of the fairly standard problem on the height of a flagpole given its elevation from two different directions. Most candidates made hard work of it.

Part (i) required the candidates to draw a single diagram to represent the information. The 1 mark for this part was given generously for any diagram that contained the information and showed the candidate had some idea of the 3-dimensionality of the situation. About 65% scored the mark, although many of them had very poor diagrams. Some candidates got the directions wrong, others drew multiple diagrams and the rest had no idea.

Part (ii) required the candidates to find a horizontal distance from an elevation and height, which they were able to do even if they had not drawn a good diagram in part (i). 75% got this correct and scored the very easy 1 mark. About half of the others got their trig formulae wrong (usually having their tangent upside down) and the rest made no serious attempt at the part.

Part (iii) caused problems with not many more than 10% getting the full 4 marks. About half the candidates either did not attempt this part or had no idea how to do it. Even among those who scored full marks, most did not express the answer as a correct bearing (the mark scheme allowed the marks for getting the angle of the direction in any form).

There were a large variety of correct methods used, the most common being an application of the cosine rule followed by the sine rule. Others used were two applications of the cosine rule, the sine rule with angles of $(45+\theta)$ and $(90-\theta)$ which were then expanded and collected with an inverse tangent used to get the answer, or similar with angles of α and $(135 - \alpha)$ (where θ is the angle W of N and α the angle N of W), dropping perpendiculars and using projection-type techniques, and so on. Unfortunately very few candidates were able to carry their methods to completion, having problems with their algebra or calculator work. The algebraic ability of most of the candidates leaves much to be desired – it would seem that correctly manipulating surd and other expressions is beyond most of them.

Some candidates penalised themselves by not drawing separate diagrams to show the triangles involved in their calculations and consequently confusing which angle was which.

1 mark was given for being able to sort out what were the relevant angles and distances, 2 marks were given for correct method and the last mark for calculation of the angle of the direction (in any form, not necessarily as a bearing).

(b) (9 marks)

In this section the candidates were led step by step through a proof of a difficult geometric theorem. It was good to see that many of them realised that they could use the earlier parts of the question to answer the later parts even if they had not proved them. Only one candidate completed the whole proof and scored full marks for this section.

In part (i) the candidates were asked to 'prove' that, in the diagram given, two lines were perpendicular. This was poorly done, with only about 10% scoring the 3 marks allocated, despite the fact that the result was the one that 'the line of centres of two intersecting circles is perpendicular to the common chord', a result which they should have seen. About 10% of the candidates just quoted this theorem as if that were sufficient and only obtained 1 mark. Many more than this used non-fundamental results without proof, such as 'the bisector of the vertex angle of an isosceles triangle is perpendicular to the base', while others invented or partly remembered results coming up with absurdities like 'a radius is perpendicular to a chord' etc. Such students scored 2 or less marks depending on how much they had correct.

There needs to be much more emphasis on what is required in a 'proof' of a geometric result. In this particular case the only acceptable proof was one that used the properties of congruent triangles, sums of angles in triangles and sums of angles on a line. Results that could be proved from these were not allowed except that it was acceptable to use 'equal angles opposite equal sides' in a triangle. Basically, 1 mark was given for each of two correct congruence arguments and 1 for an argument about equal angles on a line being 90 degrees. Other correct methods were marked similarly.

Many candidates used an SSA argument (although many called it SAS), ie. side, side and non-included angle. This is an invalid method of congruence but still is a favourite of many. About 10% of the candidates proved OVPW was a 'kite' and then claimed the diagonals of this are perpendicular. Again this is not a complete proof of the result – it needs to be proved that the diagonals are perpendicular. This sort of argument scored 2 marks. The word 'proof' was used rather than 'show' to doubly emphasise what was required.
Another 10% of the candidates assumed things about the diagram that were not given, such as OV is a tangent to the circle centred P, or OV=VP, or TU is parallel

to VM. Sometimes this was done in an attempt to find a cyclic quadrilateral. There were no cyclic quadrilaterals, but many candidates seemed to need one, possibly because so many previous exam questions have needed them.

On the other hand part (ii) was well done. This was simply the application of Pythagoras to two triangles with a shared side. Since exactly the same argument was needed for the two parts, the 2 marks were given if the candidate had at least one of them correct. 1 mark was for a correct use of Pythagoras and the other for the algebra. About 60% of the candidates received these 2 marks, while most of the rest made no attempt.

In part (iii) (worth 2 marks) there was almost universal confusion in the candidates' logic, mainly as to what was meant by 'T lies on VM exactly when ...'. Many read it as 'T lies exactly on VM when ...'. Only a couple of candidates realised it meant proving the forward result and its converse (ie. it was an 'if and only if' proof) and only one was able to carry out the full proof. The difference between the forward result and its converse was very often confused.

Of the less than 50% of candidates who seriously attempted this part most claimed to be proving 'T lies on VM when (or if) ...' when in fact they actually proved 'if T lies on VM then ...' which is the converse of this. Only 10% actually proved the result in the direction 'if ... then T lies on VM'. The mark scheme was designed as easily as possible with no requirement that they properly show that if $OU^2 - PU^2 = OM^2 - PM^2$ then U and M are the same point. Most simply assumed it was true.

The last part (iv) was worth 2 marks and only a handful of candidates received both these marks. Hardly any candidates could see the relation to the earlier parts and, of those that did, almost all assumed that there was a point lying on all three lines AB, CD and EF. From this they deduced something that was 'true' and then claimed that the result was proved. Again the candidates' logic was not sufficient for the task. The correct method is to consider the intersection say of AB with CD and then show it lies on EF using both the forward and converse directions of the part (iii). Amazingly a few candidates did part (iv) correctly using both directions of (iii) then did not realise that they had to prove both directions in (iii).

Question 7

This question is divided into two related sections. Part (a) consists of four linked parts to test candidates' ability to derive and use a recurrence relation. Part (b) consisted of three linked parts and tested ability to manipulate binomial probabilities, and in the final part the inequality obtained in part (a) is used to obtain a bound for a binomial probability.

This question was late in the paper and was meant to sort out the best candidates. While several parts were standard questions that had occurred in recent papers, in order to do well a candidate needed to demonstrate an understanding of the topics and very good algebraic manipulation skills. The average mark was around 4 and fewer than 10 scored 14 or 15.

The parts of the question were designed so that they could be answered independently of the other parts. Candidates should be encouraged to read the complete question and attempt any parts that they can. For example, many did not attempt (b)(ii) which was a very easy 1 mark part.

(a) (i) (3 marks)

A standard trigonometric integration by parts. This was generally well done. The most common error that occurred was starting with

$$\mathrm{I}_n = \int_0^{\frac{\pi}{2}} \sin^n x\, dx = \left[x.\sin^n x\right]_0^{\frac{\pi}{2}} - n\int_0^{\frac{\pi}{2}} x \sin^{n-1} x.\cos x\, dx$$

rather than using only trigonometric functions when integrating by parts.

(ii) (3 marks)

Many had difficulty using the recurrence relationship to deduce expressions for I_{2n} and I_{2n+1}. One mark was given for translating the expression in (i) to get an expression for I_{2n} *or* I_{2n+1}. The remaining two marks were allocated for identifying *and* evaluating the final terms in the representations, that is, I_0 (or I_2) and I_1. Generally students either gave a clear reasoned argument and scored full marks or did not proceed successfully beyond $\mathrm{I}_{2n} = \left(\dfrac{2n-1}{2n}\right)\mathrm{I}_{2n-2}$. A common error was to write $\mathrm{I}_{2n-2} = \left(\dfrac{2(2n-2)-1}{2(2n-2)}\right)\mathrm{I}_{4n-6}$.

(iii) (1 mark)

This part was not done well with people wasting time trying to use (ii) rather than noting $(\sin x)^k > (\sin x)^{k+1}$ when $0 < x < \frac{\pi}{2}$. A number of candidates sketched $y = (\sin x)^k$ and $y = (\sin x)^{k+1}$ (often incorrectly) and argued from their diagram.

(iv) (1 mark)

This mark was awarded for correctly manipulating *one* of the two given inequalities to obtain one of the required bounds. This part was often done successfully by candidates who had had difficulty with the previous parts. An alternative slick method for obtaining the result was noting $\mathrm{I}_{2n+1}.\mathrm{I}_{2n} < \mathrm{I}_{2n+1}.\mathrm{I}_{2n-1} < \mathrm{I}_{2n}.\mathrm{I}_{2n-1}$

(b) (i) (3 marks)

Although this part dealing with binomial $\left(2n, \frac{1}{2}\right)$ probabilities was standard bookwork, it was not answered well. Of those attempting this part many stated general expressions for $\frac{\mathrm{U}_{r+1}}{\mathrm{U}_r}$ or $\frac{\mathrm{T}_{r+1}}{\mathrm{T}_r}$ without defining U_r or T_r, often confusing n and $2n$. There was also the problem of confusing the term involving *k* and the *k*th term in the binomial expansion. Of those who successfully obtained an inequality by determining when the ratio of successive terms was greater than 1, very few went on to explain why this inequality meant that the most likely outcome occurred when $k = n$ and so did not gain the final mark for this part.

The marks for this part were allocated for noting $\mathrm{P}_k = \binom{2n}{k}\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{2n-k}$ only depends on *k* through $\binom{2n}{k}$ (or for manipulating the indices within a ratio calculation); obtaining an expression for the ratio of $\binom{2n}{k}$ and $\binom{2n}{k+1}$; and for reasoning that $\binom{2n}{n}$ is the maximum of the $\binom{2n}{k}$ coefficients.

Alternatively, to earn the final 2 marks, candidates could argue that $\binom{2n}{n}$ was the middle term in the $(2n)^{\text{th}}$ row of Pascal's triangle and so was the maximum of the $\binom{2n}{k}$ terms. To gain these 2 marks, candidates needed to give a clear reason.

(ii) (1 mark)
Well done by those who attempted it.

(iii) (3 marks)
The incorrect reference to (a)(iii) instead of the related (a)(iv) did not appear to cause any confusion. Most people attempting this part went on to use the inequality in (a)(iv). Candidates could and did successfully complete this part without attempting all of the preceding parts. Most successful attempts started with the inequality in (a)(iv) rearranging this to obtain the bounds $\frac{1}{\sqrt{\pi\left(n+\frac{1}{2}\right)}}$ and $\frac{1}{\sqrt{\pi n}}$. The products in the bounded expression were then completed to obtain $\frac{(2n)!}{2^2.4^2\ldots(2n)^2}$ then this term manipulated to obtain the expression for P_n given in (b)(ii). One mark was awarded for each of these three steps.

Question 8

This question had three parts. The first, worth one mark, involved a form of the geometric mean/arithmetic mean inequality, and was intended as hint to the second part, concerned with properties of ellipses, and particularly the chord of contact. The third and final part of the question, worth 8 marks, dealt with motion on a circular track, in the presence of friction.

There were easy marks and hard marks on this question, and many candidates made effective use of their time by tackling only some or all of parts (a), (c) (i) and (c) (ii). Many others tackled these parts effectively, but then wasted a lot of time on the rest of the question. There were quite a few non-attempts, not surprising for a final question on a demanding examination, and a fair number of students who made a good fist of most or all of the question. Careful explanations and logic were required, particularly in (b) (ii), (c) (ii), and (c) (iii), and many attempts at these parts failed to score full marks because of a lack of clarity. The most common marks on the paper were 0, 1, 4, and 5, and I estimate the mean to be about 3.

Now to details:

(a) (1 mark)

Most candidates who attempted this part successfully derived the inequality from the inequality $(p-q)^2 \geq 0$ but a disappointingly large number tried to use $(p+q)^2 \geq 0$ (to no avail). Those who appealed to the arithmetic mean-geometric mean inequality were not awarded a mark, as this was essentially what they were required to show.

(b) (i) (4 marks)

Two approaches were successfully used to tackle this part. By solving simultaneously the equations for ℓ and E, and showing that there are no real roots, some students were able to show that ℓ and E do not intersect, and hence ℓ lies outside E. However, the algebra for doing this is quite technical, and most of those who attempted this approach made slips which left them floundering. The second method starts with the equality

$$\frac{x_0 x_1}{a^2} + \frac{y_0 y_1}{b^2} = 1$$

(since P lies on ℓ), and then uses the inequalities $x_0 x_1 \leq \dfrac{x_0^2 + x_1^2}{2}$ and $y_0 y_1 \leq \dfrac{y_0^2 + y_1^2}{2}$ to deduce that

$$\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} + \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} \geq 2 \ .$$

The required result then follows from the inequality $\dfrac{x_0^2}{a^2} + \dfrac{y_0^2}{b^2} < 1$. A good number found this method, but regrettably many of these lost marks along the way through careless manipulation of inequalities (eg., using > where ≥ is appropriate).

(ii) (2 marks)

This part has a very short solution: since Q lies on ℓ, $\frac{x_0 x_2}{a^2} + \frac{y_0 y_2}{b^2} = 1$ (A), so (x_0, y_0) satisfies the equation $\frac{x x_2}{a^2} + \frac{y y_2}{b^2} = 1$ (B). Consequently a high standard of logic was needed to obtain full marks, and unhappily many candidates failed to express themselves clearly or concisely. There was much confusion between variables (x and y) and fixed values (x_0, y_0, x_2, y_2) and between relations involving fixed values (eg., (A) above) and equations (eg., (B)).

(c) (i) (3 marks)

This part is essentially bookwork, and many had no difficulty with it. The best solutions were accompanied by diagrams which indicated clearly the candidates' understanding of resolution of forces, and while these were not required by the question, were very helpful in getting signs right and sines and cosines in the right place (many candidates did not). In addition, those who resolved forces correctly in a diagram but did not proceed still gained one mark for the question; in short, those who drew diagrams were generally better off, and the practice of drawing them should be encouraged. Unfortunately, quite a few candidates did not know what horizontally and vertically mean, and a number left m out of the equations.

(ii) (2 marks)

This part, like the previous one, was in the ‘show that’ format, which allows candidates to tackle later parts of a question without necessarily having done (correctly) the earlier ones. However, this format requires that steps taken be justified, and in particular in this case the key substitution $F = -\mu N$ *had* to be justified, either physically (frictional forces towards the centre need to be as large as possible to provide maximum centripetal force) or mathematically (to maximise the numerator and minimise the denominator), to obtain full marks, and *no* marks were given unless some attempt was made. A number obtained at least one mark on this part without having done the previous part.

(iii) (3 marks)

This part of the question was hard. Several methods of solution are available, but all require sophisticated justifications. One approach, not worth full marks, considers the case where $v = 0$, shows that the particle does not slide down, and then argues that the particle is less likely to slide down when it is moving. Another argues that, as in (ii) (but with some sign changes), there is a minimum velocity $v_{\min}$ given by

$$\frac{v_{\min}}{Rg} = \frac{\tan\theta - \mu}{1 + \mu\tan\theta}$$

and that when $\mu \geq \tan\theta, \quad v_{\min}^2 \leq 0$, so the particle will not slide down. When phrased in the form ‘the particle will only slide down if $v < v_{\min}$, where $v_{\min}$ is as above, and this cannot happen because v is non-negative and $v_{\min}$ is either 0 or imaginary’, this solution is worth full marks. Yet another approach starts from the equation and inequality

$$0 \leq \frac{mv^2}{r} = N\sin\theta - F\cos\theta$$

and deduces that $\dfrac{F}{N} \leq \tan\theta$. The argument proceeds that the particle only slides down when $\dfrac{F}{N} > \mu$, because in this case the frictional force needed to stop the particle sliding down is more than the system can provide, but this does not happen, because $\mu < \tan\theta$. All these arguments are very subtle, and very few candidates obtained full marks. A number were awarded two out of three. Perhaps the most disappointing feature of this part was the large number who started with the speed $v_{\max}$ of the previous part and attempted, obviously unsuccessfully, to derive the required result from this, usually by fudging.