

## BOARD OF STUDIES <br> NEWSOUTH WALES

## HIGHER SCHOOL CERTIFICATE EXAMINATION

1995

## MATHEMATICS IN SOCIETY

## 2 UNIT

Time allowed-Two hours and a half (Plus 5 minutes' reading time)

## Directions to Candidates

- Board-approved calculators may be used.
- Show all necessary working for Section II and Section III.
- Marks may be deducted for careless or badly arranged work.
- You may ask for extra Writing Booklets if you need them.


## Section I (25 marks)

- This Section contains 20 multiple-choice questions.
- Attempt ALL questions.
- All questions are of equal value.
- Mark your answers in pencil on the Answer Sheet provided.
- Allow about 45 minutes for this Section.

Section II (45 marks)

- Attempt ALL questions.
- Each question is worth 15 marks.
- Answer each question in a separate Writing Booklet.
- Allow about one hour for this Section.

Section III (30 marks)

- Attempt TWO questions.
- Each question is worth 15 marks.
- Answer each question in a separate Writing Booklet.
- Allow about 45 minutes for this Section.


## SECTION I

Attempt ALL questions.
All questions are of equal value.
Select the alternative A, B, C, or D that best answers the question.
Mark your answers in pencil on the separate Answer Sheet provided.

1. A property was advertised for $\$ 1 \cdot 1$ million.

This is the same as
(A) $\$ 110000$
(B) $\$ 1010000$
(C) $\$ 1100000$
(D) $\$ 11000000$
2. In ten different soccer games, Mario scored the following number of goals:

$$
0,1,0,2,1,2,0,1,1,1 .
$$

What is his median score?
(A) 0.9
(B) 1
(C) 1.5
(D) 2
3. $2^{7} \times 2^{-3}=$
(A) $\quad 2^{4}$
(B) $4^{4}$
(C) $\quad 2^{10}$
(D) $4^{10}$
4. The circumference of a bicycle wheel is 210 cm .

How many revolutions will the wheel make if it travels a distance of 6.3 km ?
(A) 30
(B) 33.3
(C) 300
(D) 3000
5.


In a game show there are nine boxes, three of which contain money. All the other boxes are empty. Two boxes have already been chosen as shown ( $\$=$ money).

What is the probability that the next box chosen holds a money prize?
(A) $\frac{2}{9}$
(B) $\frac{1}{4}$
(C) $\frac{2}{7}$
(D) $\frac{1}{3}$
6. Three partners, Joe, Zelsa, and Jadon, invest in a business in the ratio $3: 4: 1$. The total amount invested is \$125000.

How much did Joe invest?
(A) $\$ 15625$
(B) $\$ 41667$
(C) $\$ 46875$
(D) $\$ 75000$
7.


What is the surface area of this solid triangular prism?
(A) $120 \mathrm{~cm}^{2}$
(B) $124 \mathrm{~cm}^{2}$
(C) $172 \mathrm{~cm}^{2}$
(D) $184 \mathrm{~cm}^{2}$
8. A microsecond is one millionth of a second.

In scientific notation, 8 microseconds is written as
(A) $10^{-6}$ seconds
(B) $8 \times 10^{-6}$ seconds
(C) $8 \times 10^{-3}$ seconds
(D) $8 \times 10^{6}$ seconds
9.


The graph gives information about the weights of boys aged between 2 and 18 .
The middle $50 \%$ of 16 -year-old boys weigh
(A) 30 kg
(B) 60 kg
(C) between 55 kg and 60 kg
(D) between 55 kg and 65 kg
10. At the end of 1994 the price of a house in Cronulla was $\$ 267500$. This price had fallen by $11 \%$ since the beginning of 1994 .

What would have been the price of the house at the beginning of 1994 ?
(A) $\$ 238075$
(B) $\$ 240991$
(C) $\$ 296925$
(D) $\$ 300562$
11. $v=\sqrt{u^{2}+2 a s}$

Find the value of $v$, given that $u=5, a=2 \cdot 2$, and $s=10$.
(A) 6.63
(B) 7
(C) 8.31
(D) 31.6
12. If $\frac{1}{2} \mathrm{~L}$ of paint covers $3 \mathrm{~m}^{2}$, how much paint is needed to cover $17 \mathrm{~m}^{2}$ ?
(A) $\quad \frac{17}{6} \mathrm{~L}$
(B) $\frac{6}{17} \mathrm{~L}$
(C) $\frac{34}{3} \mathrm{~L}$
(D) $\frac{3}{34} \mathrm{~L}$
13. Solve the equation $\frac{3 x}{x+1}=6$.
(A) $x=-2$
(B) $x=-\frac{1}{3}$
(C) $x=\frac{2}{3}$
(D) $x=2$
14. $A \leadsto B$


## NOT TO SCALE

(Sine rule : $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ )

Using the sine rule, find an expression for the length of $A B$.
(A) $\frac{15 \sin 80^{\circ}}{\sin 40^{\circ}}$
(B) $\frac{15 \sin 40^{\circ}}{\sin 60^{\circ}}$
(C) $\frac{15 \sin 80^{\circ}}{\sin 60^{\circ}}$
(D) $\frac{\sin 40^{\circ}}{15 \sin 80^{\circ}}$
15. Three students have an average mass of 46 kg . A fourth student with a mass of 66 kg joins the group.

What is the average mass of the four students?
(A) 48 kg
(B) 51 kg
(C) 56 kg
(D) 62 kg
16. The graph shows the population growth in a certain town. $P$ is the population of the town in thousands and $t$ is the number of years after 1970.


By how much did the population increase between 1975 and 1985 ?
(A) 7000
(B) 14000
(C) 17000
(D) 21000
17. Simplify $2(3 x-1)-2(x-5)$.
(A) $4 x-6$
(B) $4 x+4$
(C) $4 x-12$
(D) $4 x+8$
18. The time taken for a journey varies inversely with a car's average speed. The journey takes 4 hours when the car travels at an average speed of $88 \mathrm{~km} / \mathrm{h}$.

How long would the same journey take at an average speed of $66 \mathrm{~km} / \mathrm{h}$ ?
(A) 3 h
(B) 5 h 20 min
(C) 5 h 33 min
(D) 22 h
19. The diagram shows a spinner. When you spin, you can win either a $\$ 10$ or a $\$ 5$ prize. The arrow points to the amount won.


In two spins, what is the probability of winning a total of $\$ 15$ ?
(A) $\frac{1}{9}$
(B) $\frac{2}{9}$
(C) $\frac{4}{9}$
(D) 1
20. Anderville $(A)$ is 30 km due east of Daytown $(D)$. Haston $(H)$ is on a bearing of $040^{\circ}$ from Daytown and $325^{\circ}$ from Anderville.

Which of the following diagrams best represents this information?
(A)

(B)

(C)

(D)


## SECTION II

Attempt ALL questions.
Each question is worth 15 marks.

QUESTION 21. Use a separate Writing Booklet.
(a) Solve the equation $3(x-2)=5-x$.
(b) A goldsmith is making gold medals. She has 15 gold ingots, each in the shape of a rectangular prism. Each ingot measures 10 cm by 5 cm by 3 cm .

(i) What is the total volume of all 15 gold ingots in cubic centimetres $\left(\mathrm{cm}^{3}\right)$ ?
(ii) After melting down the ingots, the goldsmith uses all the gold to make 200 gold medals. What is the volume of gold in each medal?
(iii) Each gold medal is cylindrical. If a medal is 0.4 cm thick, find the radius of the medal.

(c) A golfer carries 16 white golf balls and 4 yellow golf balls. At the driving range, she selects one ball at random and hits it. She then selects another ball at random and hits it. What is the probability that:
(i) the first ball hit is yellow?
(ii) both balls are yellow?
(d) Peter has an $80 \%$ chance of winning each game of chess he plays in a particular competition. He is to play three games. He must win at least two of these games to make a final play-off.

By using a tree diagram, or otherwise, find the probability that he makes the final playoff.

QUESTION 22. Use a separate Writing Booklet.
(a) The observation deck of Sydney Tower is about 250 m above the ground level of the Sydney Football Stadium. The angle of depression of the Sydney Football Stadium from the deck of Sydney Tower is $6^{\circ}$.


Calculate, to the nearest 100 m , the horizontal distance from the Tower to the Stadium.
(b)


The triangle $X Y Z$ drawn above has sides $X Y=19 \mathrm{~cm}, Y Z=12 \mathrm{~cm}$, and $X Z=21 \mathrm{~cm}$.
Use the cosine rule to find the size of $\angle Y X Z$ to the nearest degree.

QUESTION 22. (Continued)
(c) A boat sails in a straight line from a point $O$ in the direction $138^{\circ}$. After a while it is at a point $G$, which is 23 km from $O$.
(i) In your Writing Booklet, draw a diagram showing all the above information.
(ii) Calculate how far east the boat is of its starting point.
(d) Annie throws a ball from the top of a building. The height, $h$ metres, of the ball above the ground is given by the formula

$$
h=20+21 t-5 t^{2}
$$

where $t$ is the number of seconds after the ball is thrown.
(i) Copy and complete the table below in your Writing Booklet.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | ---: | ---: | ---: | :---: | :---: | :---: |
| $h$ |  | 36 |  | 38 |  |  |

(ii) On the graph paper provided on page 25, plot these points and draw a smooth curve to join the points.
(iii) What is the height of the building?
(iv) Use your graph to find the time in seconds when the ball is at height 10 metres above the ground.

QUESTION 23. Use a separate Writing Booklet.
(a) Solve the equation $\sqrt{2 x+1}=9$.
(b) A grain silo is in the shape of a cylinder with a cone at the bottom. It has dimensions as shown in the diagram.


What is the volume of the silo? (Cylinder: $V=\pi r^{2} h$. Cone: $V=\frac{1}{3} \pi r^{2} h$.)
(c) The quality controller for a toothpaste manufacturer inspected a sample of 400 tubes of toothpaste. The number of faults of different types is recorded in the column graph below.

FAULTS IN TOOTHPASTE TUBES

(i) What was the total number of faults found in the 400 tubes of toothpaste?
(ii) A tube of toothpaste is chosen at random. What is the probability that it will have fault $B$ ?
(iii) The company wants to improve its product by reducing the number of faulty tubes it makes. Which one of the fault types should be fixed to give the greatest improvement?

QUESTION 23. (Continued)
(d) Jillian received a threatening letter. It is being analysed by the police in an attempt to identify its writer.

The number of words per sentence has been counted. These data are summarized in the following table.

| Number of words <br> per sentence | Frequency |
| :---: | :---: |
| 2 | 2 |
| 3 | 3 |
| 4 | 6 |
| 5 | 6 |
| 6 | 7 |
| 7 | 5 |
| 8 | 4 |
| 9 | 4 |
| 10 | 3 |
| TOTAL | 40 |

(i) How many sentences were in this letter?
(ii) How many sentences contained more than five words?
(iii) Calculate the mean number of words per sentence, and the standard deviation.
(iv) Jillian received a second letter. The police suspect that this second letter was written by a different person, because the sentences were longer on average and more consistent in length.

Write a statement comparing the mean number of words per sentence and the standard deviation for the second letter to your results in part (iii).

## SECTION III

Attempt TWO questions.
Each question is worth 15 marks.

QUESTION 24. Space Mathematics. Use a separate Writing Booklet.
(a) The ellipse below has been drawn in an auxiliary circle of radius 5 cm .

(i) Give the measurement, in centimetres, of the length of the semi-minor axis.
(ii) Calculate the value of the eccentricity of the ellipse. $\left(e^{2}=1-\frac{b^{2}}{a^{2}}\right)$
(iii) Calculate the distance from the focus $(S)$ to the centre $(C)$ of the ellipse. ( $C S=e C A$ )
(iv) An ellipse with eccentricity 0.3 is to be drawn on the diagram above. What would be the length of its semi-minor axis, to the nearest millimetre?

You do NOT need to draw this ellipse.

QUESTION 24. (Continued)
(b) (i) Explain what is meant by 'escape velocity' from a planet.
(ii) The escape velocity from the Earth, $V_{E}$ in $\mathrm{m} / \mathrm{s}$, is given by

$$
V_{E}=\sqrt{2 g r},
$$

where $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$ and $r$ is the radius of the Earth in metres.
Calculate $V_{E}$ if the radius of the Earth is 6400 km .
(iii) The escape velocity from any planet can be calculated using

$$
V_{M}=V_{E} \sqrt{\frac{m}{r}}
$$

where:
$V_{M}$ is the escape velocity from the planet;
$V_{E}$ is the escape velocity from Earth;
$m \quad$ is the ratio of the mass of the planet to the mass of Earth;
$r$ is the ratio of the radius of the planet to the radius of Earth.
The mass of Mercury is 0.05 times that of Earth and its radius is 0.38 times that of Earth.

Calculate the escape velocity $V_{M}$ from Mercury.
(c) The distance from the Moon to the centre of the Earth is 386000 km . It takes the Moon approximately 28 days to orbit the Earth in a circular path.

Calculate:
(i) the distance (in kilometres) travelled by the Moon in one orbit of the Earth ( $C=2 \pi r$ );
(ii) the average speed (in kilometres per second) at which the Moon orbits the Earth.

## QUESTION 25. Mathematics of Chance and Gambling

Use a separate Writing Booklet.
(a) A teacher is arranging 7 children for a group photo. The children will sit in a row.
(i) In how many different ways can the teacher arrange the seven children in the row?
(ii) Bart is one of the children. What is the probability that Bart will be sitting in the middle of the row?
(b) In the final of the Australian Tennis Open in January 1995, Andre Agassi played Pete Sampras. Just before the match a bookmaker was offering the following odds:

| Agassi | $3 / 2$ on |
| :--- | :---: |
| Sampras | $10 / 9$ |

(i) For each player, what was the estimated probability of winning?
(ii) Calculate the bookmaker's percentage margin.
(iii) Agassi won the match. If you had bet $\$ 120$ on Agassi how much would you have collected from the bookmaker?
(c) Connie has devised a game of chance called Blotto. The rules of her game are:

1. It costs $\$ 1$ to buy one card.
2. A player is given a card as shown.

| 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |

The player circles two different numbers on the card.
3. A box contains 6 balls, each marked with a different number from 1 to 6 . Two balls are drawn at random from the box.
4. If the numbers on the 2 balls chosen are the same as those circled on the card, you get $\$ 5$.
5. If only one of the numbers is the same, you get $\$ 1$.
6. If no numbers are the same, you get no money.
(i) How many different pairs of two balls could be drawn from the box?
(ii) What is the probability that you get $\$ 5$ from one card?
(iii) Two balls can be drawn from the box so that no numbers on the balls match those circled on the card. In how many ways can this be done?
(iv) What is the probability that you get only $\$ 1$ from one card?
(v) For one game, Tilin buys enough cards to cover every possible pair of two numbers. The two balls are drawn. How much money will he get back?

QUESTION 26. Land and Time Measurement. Use a separate Writing Booklet.
(a) The diagram shows a vertical cross-section of a creek.

(i) By using Simpson's rule twice, find an approximation for the area of this crosssection of the creek.

$$
\left[A \approx \frac{h}{3}\left(d_{F}+d_{L}+4 d_{M}\right)\right]
$$

Give the answer correct to one decimal place.
(ii) Assume that an 85 metre length of this creek has approximately the same crosssection as above. Estimate the volume of water in this section of the creek, to the nearest 100 cubic metres.

QUESTION 26. (Continued)
(b) A student using the method of triangulation has written down the information he needs on the sketch below. The sketch shows a base line $P Q$, and the compass bearings of a post $K$ from each of $P$ and $Q$. The baseline $P Q$ lies in the east-west direction.

(i) The student has noted that $\angle K P Q=125^{\circ}$. Find $\angle K Q P$.
(ii) Find the distance from $P$ to the post $K$, to the nearest metre.

$$
\left(\text { Sine rule }: \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}\right)
$$

(iii) Find the area of the triangle $P K Q .\left(\right.$ Area $\left.=\frac{1}{2} a b \sin C\right)$
(c) Durban in South Africa has latitude and longitude coordinates ( $30^{\circ} \mathrm{S}, 31^{\circ} \mathrm{E}$ ), and Grafton in New South Wales has coordinates $\left(30^{\circ} \mathrm{S}, 153^{\circ} \mathrm{E}\right)$.
(i) If it is $3 \mathrm{p} . \mathrm{m}$. at Grafton, what time should it be in Durban (ignoring time zones)?
(ii) Show that the radius of the $30^{\circ} \mathrm{S}$ circle of latitude is 5543 km to the nearest kilometre. (Radius of Earth $=6400 \mathrm{~km}$ ).
(iii) Find the distance between Durban and Grafton along the $30^{\circ} \mathrm{S}$ circle of latitude.

QUESTION 27. Personal Finance. Use a separate Writing Booklet.
(a) The table below gives details for fixed-term deposits.

| Time period | Interest rate per annum |
| :--- | :---: |
| 3 months to less than 4 months | $6.50 \%$ |
| 4 months to less than 6 months | $7.00 \%$ |
| 6 months to less than 13 months | $7.75 \%$ |
| 13 months to less than 24 months | $8.75 \%$ |
| 24 months to less than 36 months | $9.50 \%$ |
| 36 months to less than 48 months | $9.75 \%$ |
| 48 months to less than 60 months | $10.00 \%$ |
| 60 months exactly | $10 \cdot 25 \%$ |
| An option is available for interest to be paid monthly. |  |
| In this case the interest rate is reduced by $0.25 \%$. |  |
|  |  |

Stephanie would like to invest $\$ 5000$ for 2 years with the interest paid monthly.
(i) What interest rate will Stephanie receive?
(ii) How much interest will she receive each month?
(b) Diane received a new credit card in early June. There is no interest-free period on purchases and the daily interest rate is $0.04 \%$.

The table lists her purchases for June.

| Date | Purchase | Cost |
| :--- | :--- | ---: |
| 10 June | Clothing | $\$ 350$ |
| 20 June | Petrol | $\$ 25$ |

She receives an account for the month of June.
What is the total interest she will be charged?
(c) A store was having a summer sale where all items were marked down by $25 \%$. On Saturday, a special one-day sale was held where all sale items were reduced by a further $12.5 \%$ off the marked-down price.
(i) On this Saturday, Julianne bought a dress which was marked at $\$ 150$ before the summer sale. How much did she pay for this dress?
(ii) Write her savings as an overall percentage of the original price of the dress.

QUESTION 27. (Continued)
(d) The solid line on the graph shows the tax payable on taxable incomes up to $\$ 50000$ in Australia in 1993.

The broken line shows a possible $25 \%$ flat tax rate.


KEY

-     -         -             - 25\% flat rate
- 1993 tax

QUESTION 27. (Continued)
(i) What was the tax payable on an income of $\$ 4000$ in 1993?
(ii) Bernie's taxable income in 1993 was $\$ 15000$. What was the tax payable on his income?
(iii) Kerry's taxable income in 1993 was $\$ 5000$ more than Bernie's. How much more tax did Kerry pay than Bernie?
(iv) How many cents in the dollar did Kerry pay in tax on the $\$ 5000$ ?
(v) The broken line on the graph represents a flat tax rate of $25 \%$. Suppose that the taxation system changed to a flat tax rate of $25 \%$. In what range of incomes would more tax be paid under this new system?

QUESTION 28. Mathematics in Construction. Use a separate Writing Booklet.

The drawings on page 21 show a first-floor extension to a house.
(a) What is the width of the extension?
(b) Use the plan to write down the dimensions of bedroom 1.
(c) Measure the angle of the pitch of the roof $X Y$.
(d) Using the measurement on Section A, find the scale used.
(e) Calculate the area, in square metres, of timber decking on the balcony.
(f) Use the scale to find the actual height of the house at point $X$.
(g) There are 17 stairs joining the ground floor to the first floor. What is the actual height of each step?
(h) The guttering runs all the way round the roof of the extension. What is the total length of guttering?
(i) Sketch the west elevation of the first-floor extension.


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| 1995 |
| HIGHER SCHOOL CERTIFICATE EXAMINATION |
| MATHEMATICS IN SOCIETY |
| 2 UNIT |

Student Number
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Centre Number


This page is to be completed, torn off, and attached to your Writing Booklet for Question 22.
QUESTION 22
(d)
(ii)


