BOARD OF STUDIES
NEW SOUTH WALES

1996 HSC

EXAMINATION REPORT

Mathematics

Including:
- **Marking criteria**
- **Sample responses**
- **Examiners' comments**

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1996 HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS

ENHANCED EXAMINATION REPORT

Table of Contents

2/3 Unit (Common)

3 Unit (Additional) and 3/4 Unit (Common)

4 Unit (Additional)

General Comments

Each of the Mathematics examinations in 1996 provided good discrimination amongst the candidates, with a desirable spread of raw marks across almost the full range of available marks. Each paper contained questions which provided sufficient challenge for the ablest students. The average student in each candidature found that they were able to earn just over half of the raw marks on their more difficult paper, in keeping with the standard of difficulty of the Mathematics examinations over many years.

These examinations were the second set of examinations to be affected by the changes in examinable material introduced by the division of the courses into preliminary and HSC sections, and the changes to the length and number of free response questions in Mathematics in Practice and Mathematics in Society. No significant new changes were introduced in 1996.

Once again, the multiple choice questions in Mathematics in Practice and Mathematics in Society were marked by machine, with correct responses earning 1 mark while incorrect responses, multiple responses and non-attempts scored 0 marks. The total mark from the multiple choice questions was then scaled to the appropriate value for the course, with several decimal places being used to ensure that there was no loss of discrimination.

The free response questions in each paper were marked out of 12, with the exception of the 4 Unit (Additional) paper, where the questions were each marked out of 15. Only whole number marks are awarded, and, in all but the Mathematics in Practice and Mathematics in Society papers, the distribution of these marks between the major sections of the question was indicated on the paper as a guide to candidates. The marking process allocates each single mark to an ingredient which is required for the correct answer to the question asked, and candidates are awarded those marks if that ingredient can be identified in their answer. These marks, once earned, are not negated by subsequent errors. Thus, the marking process is one of awarding marks for knowledge displayed by the candidates, and is not a process of deducting marks for mistakes.

The marks from the free response questions are also totalled and scaled to the required value for the particular course, retaining an ample number of decimal places to ensure that there is no loss of discrimination. (In Mathematics and Society, the raw marks for the option questions are first scaled so that their distribution reflects the distribution of the candidates attempting that option on the multiple choice section of the paper, so as to avoid distortions which might arise if the option questions were not of equal difficulty.) The Board scores are then computed for each candidate in accordance with the published procedures.

The rest of this report consists of detailed comments on each of the free response questions for each paper in the 1996 examination. These comments provide valuable insight into the way in which each question was marked, the difficulties encountered by students and common deficiencies in the preparation of candidates for the examination, and are written with a view to assisting teachers and candidates alike in preparing for future Higher School Certificate examinations.

Candidates should also be encouraged to bear in mind the following facts, which are essentially repeated from the 1995 examination report.

1. At all times, answers should indicate in some way to examiners how they were derived. Hence, for example, candidates should not give single word or figure responses. If correct, such answers might receive full marks, but usually only in cases where the examiners are convinced that the correct answer is unlikely to be obtained from incorrect working. This is especially the case in the higher levels for questions which the paper indicates are worth several marks. For instance, in Question 9 e) of the 2/3 Unit (Common) paper, a bare ‘correct’ answer without explanation only received 1 of the 4 marks allotted to the question.

   Even in instances where the bare ‘correct’ answer would receive full marks, candidates should be aware that a bare incorrect answer will almost certainly receive no part marks, since the examiner will have no idea how the answer was arrived at. If the working is shown, trivial transcription errors and computational errors which can be clearly identified are often ignored in the marking process, and students who show their working may even be awarded full marks despite minor slips.

2. Graphs and diagrams should be clearly marked and reasonably executed to assist both the candidate and the examiner to know what they are doing. They should also be relatively large, for the same reason. When the paper instructs the candidate to copy the diagram into their exam booklet, the reproduction should be at least as large as the original. Candidates will not receive marks for such copying

since it usually involves either tracing or a simple reproduction of the printed diagram.

However, the instruction to copy the diagram is invariably an indication that the candidates will have to insert additional information, or make reference to the diagram in the course of completing the answer to the question. Those who neglect to copy the diagram as instructed place themselves at a severe disadvantage, as examiners (and especially candidates!) are unable to follow arguments involving non-existent diagrams.

3. Candidates should use pages in the booklet in which they are answering the given question for any rough work. Many candidates find the unruled left-hand pages useful for this purpose. All such work is read, and it is often the rough work of candidates which provides the evidence of the ingredients for the solution which results in the candidate being awarded part marks.

   It must be emphasised that examiners do read everything written by every candidate, whether it is written on the backs of pages or booklets, whether it is crossed out, or even if it carries the explicit request ‘Please do not mark this.’ This is always to the candidate’s advantage, as marks are never subtracted for errors, but are gained for appropriate work if it deserves it, no matter where it is displayed, unless, of course, the candidate’s clear answer explicitly contradicts correct crossed out or rough work.

   For these reasons, candidates should also be discouraged from using ‘white out’ to obliterate their unwanted work, and advised to cross out in a way which leaves the underlying work legible.

   Under no circumstances should candidates set aside a separate booklet for rough work for all questions. Even though examiners do try, this practice makes it virtually impossible to locate the work on a specific question and relate it to the working shown in the writing booklet for that question. Candidates may needlessly miss out on marks which would otherwise have been awarded.

4. Candidates in the higher levels should be reminded not to forget the existence and usefulness of the tables of standard integrals printed on the back page of the

examination booklet. For instance, question 3 c) in the 2/3 Unit (Common) paper was most easily answered by direct reference to the tables.

5. Candidates should write answers to different questions in different booklets. On the other hand, if they forget to do this under the stress of the examination, they should ensure that their working is clearly labelled with the question number as well as the part number so that it can be readily identified. A note on the cover of the writing booklets affected, such as ‘also contains part of Q. 6’ and ‘part of Q. 6 is in Q. 5 booklet’ will assist examiners in assembling the student’s complete answer to the question. Candidates should be assured that all of their work will be read, and that they will not be penalised in any way for such slips.

6. Candidates should be discouraged from writing their answers in several columns on the page, as this can make it difficult for the examiner to check that everything has been read, and that all the appropriate marks have been awarded. Candidates who cram their work out of concern for the environment can be assured that all the paper in examination booklets will be recycled when the booklets are no longer required.

Mathematics in Practice

Question 31. The Consumer

(a) (2 marks)

Simple interest had to be calculated over 18 months. It was poorly answered.

$6000 \times \frac{13.75}{100} \times \frac{18}{12}$ earned 2 marks, as did the correct answer.

Common errors included multiplying by 18 (not $\frac{18}{12}$), forgetting to divide by 100, and adding their answer to (or subtracting it from) 6000.

1 mark was awarded if 1 error was made.

(b) Two tables were given showing comparative costs for installing and renting Pay TV.

(i) (1 mark)

$19.95 + 39.95 \times 6 = \$259.65$

The majority of students calculated this correctly. However 1 mark was also awarded for the correct numerical expression.

(ii) (2 marks)

In calculating the yearly subscription students had to find a 10% reduction of the monthly fee, multiply by 12 and then add the installation costs. It was very poorly done. Two of the above three steps earned 1 mark. Common errors included taking off $10 or 10c instead of 10% and this they would do at **any** point in their calculation (usually at the wrong place).

(c) Cash price and terms were given for the purchase of a computer.

(i) (1 mark)

To calculate the cash price a 10% discount on $3995 needed to be found.

$3995 – $399.50 = $3595.50

Generally well answered.

(ii) (1 mark)

$36 \times \$135 = \$865$ was the cost of buying by 'terms'. Well answered.

(iii) (1 mark)

Interest had to be calculated

$4860 – $3995 = $865

Well answered.

(iv) (2 marks)

This question asked for the yearly interest **rate**. Poorly answered. Most students had no idea what 'rate' meant and wrote $288.33.

1 mark was awarded for $\frac{865}{3995} \times 100$ (= 21.7%).

(d) Questions relating to tables showing amounts of life insurance cover under different monthly plans.

(i) (1 mark)

Very well answered – straight reading from table $131570.

(ii) (1 mark)

Well answered

$131579 – $72816 = $ 58763

A common error was to write $72816.

Question 32. Travel

(a) Given a map of a street plan, students had to locate a position and draw in a route, given directions.

(i) (1 mark)

Well answered. Most students could locate Carnegie Hall.

(ii) (2 marks)

Given four instructions students had to draw in a route. 1 mark was awarded if three instructions were followed correctly.

A common error was to mark the starting point and finishing point only. This did not receive any marks.

(b) A table was given showing how frequent flyer points are earned. Points were given per kilometre.

(i) (1 mark)
For a return trip Cairns to Perth
$5568 \times 2 \times 1.25$ (= 13920)
Common error was ignoring the word 'return' and only calculating one way. Reasonably well answered.

(ii) (1 mark)
Given the points, a division of $1500 \div 0.7 = 21428.57$ needed to be calculated to find the kilometres travelled. Poorly answered. Many students made incorrect trial and error guesses.

(c) (2 marks)
Students were given a Sydney departure time, a San Francisco arrival time and the time difference between the two cities. They were then asked to calculate the time of flight.

This was very poorly answered, and the majority of the candidates left it unanswered.
1 mark was awarded if 4.30 pm or 0005 appeared in their answer.

(d) Two options were given for a 23 day holiday in Germany including air fare and car hire.

(i) (1 mark)
Daily cost of option A car hire
$2899 - 2010 = 889$
$899 \div 23 = \$38.65$ – well answered.

(ii) (2 marks)
Total cost of option B air fare and car hire
$1570 + (3 \times 315) + (2 \times 54)$ (= \$2623)

1 mark was awarded for two of the above three numerical expressions. Common errors were when students wrote 23 days as 1 week 16 days, or 2 weeks 9 days, instead of 3 weeks 2 days.

Reasonably well answered.

(e) (2 marks)
Students had to read a table for hotel rates and find the cost of their holiday taking into account a special 'clause'.
'$(235 \times 4) + (2 \times 30) = \$1000$' earned 2 marks.
1 mark was awarded for (235×4) or (2×30). The question was poorly answered indicating a language problem for many students.

Question 33. Accommodation

(a) (2 marks)
Students were given the fees that a real-estate agent charges for a property. This required three steps: $750 + \left(\frac{3}{100} \times 100\ 000\right) + \left(\frac{1.5}{100} \times 260\ 000\right)$

The correct numerical expression was awarded full marks.
1 mark was given for two of the above three steps.

A common incorrect answer was \$5355. This was obtained by calculating 3% of \$100 000, subtracting this value from \$360 000, then calculating 1.5% of this amount.

Reasonably well done.

(b) A diagram was given showing a rectangular building site with a house and garage on it. Questions were asked regarding area, percentage area, interpreting direction, and costing.

(i) (1 mark)

$(8 \times 18) + (8 \times 3.5) = 172 \text{ m}^2$ was the total area of the house and garage. Correct numerical expression earned full marks.

Many students calculated perimeter instead of area. Of those students who did calculate area, many then went on to subtract this from the total area of the site to get 296.

Not very well answered.

(ii) (2 marks)

$\frac{172}{468} \times 100 = 36.75\%$ was the percentage area taken up by the house and garage.

1 mark was awarded for students with one error in the above calculation. A common errors was forgetting to multiply by 100, giving an answer of : $\frac{172}{468}$.

Poorly answered.

(iii) (1 mark)

900mm had to be converted to metres. This was poorly done, hence most answers were incorrect. Also, many students did not know which boundary was the northern.

(iv) (1 mark)

Quite well done by the majority of students who attempted to transfer their answer from (iii).

A common error was to multiply by 500 and not by 50.

(v) (1 mark)

Many students, instead of finding the perimeter of the house, found the perimeter of the building site or alternatively used the *area*.

This was poorly answered.

(c) A floor plan of a house was given and questions asked relating to number of door, scales, and area.

(i) (1 mark)
Well done. Most students recognised there were 10 doors.
Common error was to not include the robe doors.

(ii) (2 marks)
Very poorly done. On the whole the question was unanswered. Many could not draw to scale with any reasonable accuracy. Many could not interpret ‘under the window’, drawing the bed in all sorts of positions.
1 mark was awarded for correct scale drawing in the wrong position.

(iii) (1 mark)
Reasonably well done.
A common error \$36540 $\times$ 42 instead of \$36540 $\div$ 42.

Question 34. Design

This question contained a range of constructions and some calculations from various sections of the Design topic. Three features of student performance were noted: marks tended to be low in this question; many students showed little or no working (and may have missed out on part marks); and many students disadvantaged themselves by not using mathematical equipment where appropriate. A student doing this question without a ruler and a pair of compasses was effectively working towards a maximum mark of 6 instead of 12. Freehand responses were penalised.

(a) (i) (1 mark)
This question required the recognition of the shapes determining a tessellation. While many students answered this question correctly, many were too imprecise (eg *triangles* instead of *equilateral triangles*), and many named truncated elements of the design as separate shapes.

(ii) (2 marks)
Students were required to complete the tessellation. Most showed some recognition of the pattern, but many responses were roughly drawn and inaccurate.

(b) (2 marks)
The task involved constructing a diagram from a written description. This was generally answered well, with lack of precision in the drawing the most frequent deficiency in the student's answer.

(c) The first 2 parts of (c) involved the calculation of surface area. This was not usually done well, with common errors being the omission of one or more surfaces, or a totally inaccurate response (eg multiplying all numbers together). Zero was a common result.

(i) (2 marks)
The calculation of the surface area of a triangular prism. Rarely answered correctly. One mark was awarded if only one mistake was made (eg omitting one triangular end).

(ii) (2 marks)
Surface area of a solid composed of six of the previous shapes. This was also rarely answered correctly, and one mark was awarded if only one mistake was made.

(iii) (1 mark)
This calculation required the students to show some level of initiative to determine the number of hexagonal prism containers that would fit within a rectangular prism. Very few students were able to provide the correct answer. Many ignored the stated fact that the diagram was not to scale.

(d) (2 marks)
This question involved the reconstruction of a provided diagram to a certain magnification. The common errors were: lack of precision in using mathematical equipment, an incorrect magnification, and freehand drawings.

Question 35. Social Issues

(a) (i) (1 mark)

This question required the reading of a percentage from a table and the calculation of a weight. It was reasonably well answered.

(ii) (2 marks)

This involved extracting two percentages from the table, combining them and calculating a weight, and subtracting from the total weight. Because of the complexity, students had less success with this, commonly not completing the final step.

(b) (i) (2 marks)

The task involved completing a graph from a table. This was generally answered well, with lack of precision in drawing the most frequent deficiency.

(ii) (2 marks)

Students were asked to describe the trends evident in the graph. Responses were very varied, ranging from highly lucid interpretations of a factual nature, to inaccurate assumptions. Describing more than the graph displayed or extrapolating were common mistakes. Language difficulties were frequently manifest in these responses. The assumption that low employment is equivalent to high unemployment was common. The opportunity to drift from factual features of the graph to comments regarding sexual politics was taken by many.

(c) (i) (1 mark)

This required the calculation of a figure in a table from the appropriately related figures. It was reasonably well done.

(ii) (1 mark)

This required a more difficult calculation of a figure from the appropriately related figures in the table. Very few people indeed were able to give the correct answer.

(d) (i) (1 mark)

This question involved the completion of a table of possible outcomes of a game. Mostly this was done well. Doing inappropriate things with each pair of numbers was a common error (eg multiplying).

(ii) (2 marks)

In this question, students were asked to explain whether or not the game was fair, and why. The fact that the numbers of outcomes favouring the players were not equal was regularly noticed. Most students assumed that the players would choose their numbers randomly with equal probabilities, although this was not stated in the question. Students who explained that the game was not fair for reasons along these lines received full marks. If the players consider what the opposition is likely to do and adopt appropriate strategies for choosing their numbers, it changes the nature of the game, and renders it fair. This was accounted for in the marking process, and was occasionally encountered.

Mathematics in Society

Question 21

Question 21 consisted of three parts taken from the areas of numeration and measurement and from probability. The last two parts of the question were mainly on probability.

(a) (i) (2 marks)

Candidates were asked to calculate the area of a shape which could easily be divided into a rectangle and a triangle or into a rectangle and a trapezium.

A mark was awarded if there was a correct area with the appropriate addition or subtraction. If three areas were used a mark was awarded for two correct areas.

Most students realised they had to divide the wall into sections, one area being a rectangle. They were then able to find the area of the rectangle but failed to find the other areas (a triangle or a trapezium) correctly. The majority of students who divided the wall into three areas failed to obtain the correct answer of 14.7.

Another common error was to find the length of the hypotenuse using Pythagoras, and then use this value in the calculation for area of a triangle.

(ii) (1 mark)

A large number of students did not see the connection with (a)(i). A number of students who did use the result $V = Ah$ proceeded to multiply 14.7 by 2.5.

Many recalculated the area or found composite volumes. Some of these students then used $6 \times 1.7 \times 5$, forgetting to divide by 2 even though they had 14.7 in part (i).

(b) (i) (1 mark)

Most students gave the correct answer of $\frac{6}{7}$. However a significant number of students gave the answer $\frac{1}{7}$, failing to see or just ignoring the 'not'. Perhaps the word 'not' should have been given more emphasis in the question.

Some students didn't pay attention to the word 'week' and gave the answer for the year.

(ii) (2 marks)

This part of the question was poorly answered. Students need to be encouraged to draw a tree diagram with the associated probabilities on the branches. The few students who did this scored well as a mark was awarded for a correct tree diagram with the probabilities or for a tree diagram with the sample space listed.

Attempts to list all 49 branches often led to errors.

Common answers were $\frac{1}{49}$, $\frac{6}{49}$ and $\frac{13}{49}$. If students had an answer with a denominator of 49 they were awarded a mark. A number of students obtained the answer $\frac{1}{7}+\frac{1}{7}=\frac{2}{7}$.

(c) Confusion reigned in this question, especially parts (iii), (iv) and (v). The three dice question with a sample space of 216 also seemed to be beyond the ability of most Mathematics in Society students.

(i) (1 mark)

There was some confusion over the interpretation of this question with some students taking 'uppermost' to mean the highest numbers on two out of the three dice and so a mark was awarded for the answer 12%. This question was well done by most students. The mark was awarded if they had 6 + 6 + 6 or 3×6. However a significant number of students then gave the answer 36%.

(ii) (1 mark)

This question was quite well done. A common error was for students to write 100 – 3% and then put down the answer 99.97. The answer \$98 was also awarded 1 mark due to the confusion over the meaning of 'uppermost'.

(iii) (1 mark)

There was a very low scoring rate in this part. Most students seemed to understand the question and know what combinations led to the desired outcome of 5% discount. Their difficulty arose from being unable to see that the result 2, 1, 2 was different from 2, 2, 1 and 1, 2, 1, counting all such outcomes as one rather than three. Also a great deal of time was wasted by those students who attempted to draw a probability tree diagram showing all 216 outcomes. Students who drew tree diagrams or attempted to write down all the permutations generally made an error and scored 0 (since the question was 1 mark).

(iv) (1 mark)

Most students knew how to use their result from part (iii) in this question but were unable to answer the question as they could not determine the number of elements in the sample space. Common errors were the result of using 36 or 108 as the size of the sample space.

(v) (2 marks)

This part of the question was very poorly done. A large number of students failed to grasp the meaning of the question and merely repeated their answer to the previous part. Of those who did appear to understand the question, many knew that discounts of 3%, 4% and 5% were to be counted, but were unable to successfully calculate the appropriate probabilities. Awarding 1 mark for showing evidence of including 3%, 4% and 5% helped some students to score a mark.

Another common error was to give the answer $\frac{5}{100}$.

Question 22

(a) Labelling of the diagram was often inaccurate and/or not completed. The positions of X and Y in the correct quadrants relative to Z was poorly done. Students showed a general difficulty in drawing the diagram from the information given.

(b) (i) Lack of understanding of bearings as illustrated in (a) made this question difficult. Many could find the angle but were unable to write the bearing in one of the correct forms. Often candidates contradicted a correct angle of 149° with a directional bearing, eg 149 SE.

(ii) Very poorly done. Many students did not understand what distance was required. Many students did not understand the concept of South. Many answered in degrees eg 31° S. The most common incorrect answer was 65 which came directly from the diagram. Clearly there was widespread ignorance of what the phrase ‘south of’ meant.

(iii) Most students could apply the cosine rule. Where errors were made it was in substitution of angles and/or calculation. Many students were unable to use the calculator correctly to achieve the answer. Many did not take the square root.

(c) (i) Explanations were difficult for students. Contradictory information was often given. Evidence of calculations used was not always given. Words like adjacent, corresponding and vertically opposite were confused for the correct alternate angle explanation.

(ii) The value 116.29 was often used to show that $QT = 116.29$

Many students did not calculate their expressions to check if it would give the correct answer.

(iii) Many students were able to identify the three correct angles of 73°, 5° and 102° but still:

- the lack of matching an angle with a vertex caused some candidates a problem in (c)(iv);
- quite a number had silly arithmetic errors;
- a number of students appeared to ignore completely the well known 180° in a triangle.

(iv) Many correct answers achieved with errors most commonly being caused by:

- lack of information from (c)(iii);
- poor application of the sine rule;
- inadequate calculating skills.

Often no relationship was seen between parts (iii) and (iv).

Question 23

This question involved algebra, statistics and graph reading as well as measurement.

(a) (i) Most students were able to find the slant height of the cone, although some students found the area of the triangle.

Approximately 70% of students scored full marks.

(ii) Some students ignored the formula given to find the curved surface area. Generally these students did not score any marks. Most students were able to use the answer given in (i) to successfully substitute into the given formula. Approximately 85% of students scored full marks.

(b) (i) 165 was a common incorrect answer. This was obtained from finding the middle height reading from the height axis. Approximately 43% of students scored full marks.

(ii) Many students found the upper and lower quartile limits but failed to do the required subtraction to score full marks. Overall, students had difficulty locating the 25% and 75% marks on the cumulative frequency axis. Approximately 13% of students scored full marks.

(c) Many students chose to rewrite this equation as $3x - 2 = 4x$, thus obtaining an answer of –2. This scored zero marks. Many other students attempted to take reciprocals. Again zero marks were awarded. Approximately 19% of students scored full marks.

(d) (i) A common error was to simply list the monthly temperatures. Zero marks were awarded. Many students gave 26 – 4 = 21 as an answer. Approximately 75% of students scored full marks.

(ii) Students who made calculator errors but who did not show working were not awarded any marks. Some students were able to find the mean by adding the temperatures and dividing by 12 but could not find the standard deviation. Some students found the median which happened to have the same value as the mean when rounded. Approximately 52% of students scored full marks.

(iii) Many students simply recalculated the mean and standard deviation without commenting on the effect (mean increases and standard deviation decreases). This resulted in zero marks. Some students understood that the standard deviation is a measure of spread but were unable to recognise that it has a numerical value. Approximately 42% of students scored full marks.

Question 24. Space Mathematics

(a) (i) (1 mark)

This was very well done. Many candidates actually measured the diagram 1·5 and 3·1 instead of reading the values of a and b from the diagram. There was a need to include 'not to scale' on the diagram.

(ii) (1 mark)

This was generally well done, although a number of candidates had problems with $\sqrt{0.75}$.

(iii) (1 mark)

Most candidates wrote down the distance CS rather than the coordinates of the focus S. Many tried:

$$CS = e \times C \times A$$
$$= 0.866 \times 0 \times 2.$$

(iv) (1 mark)

About 50% of the candidates obtained the correct answer. Errors arose from incorrect transposition $\frac{b^2}{a^2} = 1 + 0{\cdot}36$, incorrect use of the formula $0{\cdot}36 = 1 - \frac{b}{a}$ or not knowing how to find $\frac{b}{a}$ after reaching $\frac{b^2}{a^2} = 0.64$.

(v) (2 marks)

There were many good responses. Many candidates wrote 'ellipse gets larger' instead of 'ellipse becomes rounder' and/or 'foci gets smaller' instead of 'the positions of the foci are getting closer to the centre of the ellipse'.

(b) (i) (1 mark)

This was poorly done. The majority of the responses used $r = \frac{142984}{2}$ instead of $r = 7.86 \times 10^8$. Others used $r = 6400$. A few candidates used $r = \left(7 \cdot 86 \times 10^8 + \frac{142984}{2}\right)$. Others had problems with scientific notation (786 million = 7.86×10^6). Quite a number of candidates used a factor of 100 to convert kilometres to metres.

(ii) (2 marks)
This was poorly done. Most candidates had no idea how to convert from Jupiter year to seconds. Once again, many candidates had problems writing numerals in scientific notation form.

(iii) (2 marks)
This was fairly well done. A number of candidates used the length of Jupiter's orbit about the Sun as the distance of Jupiter from the Sun. Other errors included using $C = 3\times10^5$ instead of 3×10^8 and incorrect conversion from seconds to minutes.

(iv) (1 mark)
This was quite well done by those who knew how to use scientific notation and scales.

Question 25. Mathematics of Chance and Gambling

This question contained three parts taken from four sections of the option topic, namely language of chance, counting techniques, mathematical expectation and fairness.

(a) (i) (1 mark)
The majority of the candidates got the mark. A few did not read the question carefully and thus ended up with negative scores and scores greater than 5.

(ii) (1 mark)
Candidates who answered part (i) correctly usually also scored the mark here. A small number of candidates did not understand probability at all.

(iii) (1 mark)
This was well done. A few candidates showed lack of understanding by multiplying 5×90 instead of $\left(\frac{5}{18}\right)\times 90$.

(iv) (1 mark)

Once again, those candidates who completed the table correctly had no trouble in explaining that Mark had a two out of three chance of success compared with Tim's one out of three. Incorrect completion of the table usually led to the wrong explanation.

(v) (1 mark)

Candidates showed they understood about fairness with a wide range of good answers. Answers ranged from giving Mark and Tim different combinations of scores, to giving Tim 2 points to every point Mark gained.

(b) (i) (1 mark)

Most candidates showed some understanding of this question. However, most gave the answer as \$160 with only the better candidates giving the correct answer.

(ii) (1 mark)

This was poorly done with the most common answer being \$80, while answers of \$5 and \$25 were divided equally. It was extremely rare for students who had not earned the mark in (b)(i) to gain the mark here.

(c) These questions had the poorest responses with only the better candidates gaining marks.

(i) (1 mark)

Common responses were $\frac{1}{8}$ or $\frac{1}{8} \times \frac{1}{8} \times \frac{1}{8}$.

(ii) (1 mark)

Candidates showed a lack of understanding of a box trifecta ($\times 3!$) and the usual responses were $\frac{3}{8}$ or $3\times$(the answer of part (i)).

(d) (i) (1 mark)

Candidates who appeared to have seen Pascal's triangle made good attempts at completing the 7^{th} row. Unfortunately, many candidates were careless and left out one or more of the numbers.

(ii) (1 mark)

This was reasonably well done. However, candidates who tried to list the combinations were usually unsuccessful.

(iii) (1 mark)

Once again, those candidates who had the correct answer did not obtain it by listing the different arrangements.

Question 26. Land and Time Measurement

The question contained three parts covering a scale drawing from a traverse survey, length and area calculations from a radial survey and a great circle question with distance and time involved. Parts (a) and (b) were reasonably attempted by most students with part (c) causing most confusion. A reasonable number of students gained almost full marks for parts (a) and (b).

Students made part (c) more difficult than necessary by not being able to identify the appropriate methods and formulae to use.

The examiners speculated that students may have been unfamiliar with the locations used in part (c) and were then unable to visualise the situation and 'make sense' of their answers.

(a) (2 marks)

- fairly well done overall
- common mistakes were use of incorrect scale ie 1 mm : 1 m, 1 mm : 2 m, 1 cm : 2 m
- the accuracy of measurements was fairly good.

(b) (i) (1 mark)

- well done, most obtained 99 or 261
- most students understood the subtraction of true North bearings.

(ii) (1 mark)

- well done since formula was given
- A significant number of students used calculators which were set in RAD or GRAD mode, which caused them to obtain the wrong answer.

(iii) (2 marks)

- most students gained 1 mark through a correct substitution into the formula
- the usual errors of taking the square root and doing the subtraction before multiplying led to mistakes
- transcription errors were common in this question
- the use of sin in the formula with an otherwise correct calculation appeared.

(c) Part (c) was poorly done with many non attempts and much confusion.

(i) (2 marks)

Since formula for circumference was not given many students had difficulties, including finding the radius of the small circle, miscalculation of the fraction of the circumference and errors in calculating the circumference without using π.

The use of 60 nm $\times$ 15 was an easier method but was used by only a few students.

Common Mistakes

- finding the radius of the small circle using $r = R \cos \theta$
- some then went on to find the distance using the small circle radius
- $\frac{15}{360} \times 6400$
- radius angles used eg 17, 16, 167

(ii) (2 marks)

Many students recognised the need to divide by $8 \times 1.852 = 14.816$. However a common error after a correct division was a failure to understand the units of their answer, eg did a further division by 60 to give an answer in 'hours'.

Other most common error was to divide by only 8 or 1.852.

(iii) (2 marks)

It was very difficult to award full marks in this question as students had over-simplified their time through incorrect calculations in (ii) or converted their time in hours to days incorrectly, eg 113 hrs = 4.7 days = 4 days 7 hours.

Another mistake was the confusion of the 15 difference in latitudes as difference in longitude that lead to a one hour time difference, ie left 10 am and arrived Tuesday at 11.00 am (15 = 1 hr).

Question 27. Personal Finance

The question was generally well done. Most students gained some marks for this question although very few gained full marks. Parts (a)(ii) and (c)(ii) were the parts which appeared to cause students the most difficulty.

Students who showed working generally gained at least 1 mark for the 2 mark parts. Students need to read questions carefully and respond to all words used such as 'rate' in (a)(ii) and 'extra' (d)(iv).

(a) Students were given the cost of a refrigerator and the deposit and weekly instalments if bought on terms over a year.

(i) (1 mark)

The total cost on terms was required. This part was very well done. There were errors made in the number of weeks 12×4 or 12 being used instead of 52.

(ii) (2 marks)

The rate of interest was asked for. This part was poorly done. Most students were able to calculate the amount of interest and were awarded 1 mark if this was shown. Many students stopped here. Those who attempted to find a percentage often based their calculation on the cash price (1699) rather than the balance after deposit (1199). Very few gained the 2 marks available.

(b) (2 marks)

Students were asked to calculate the amount earned by a casual worker employed for one weekday, one Saturday at time-and-a-quarter and one Sunday at time-and-a-half. The number of hours worked each day also needed to be calculated as part of the working. The majority of students found the correct answer.

There were a significant number of errors in finding the number of hours worked each day particularly 'from 8:30 am till 1 pm', being calculated as $5^1/_2$ hours. Students were familiar with penalty rates but the time-and-a-quarter caused problems. A number of students used time-and-a-half and double-time instead, while others added $^1/_4$ and $^1/_2$ to the hourly rate, ie used $9^1/_4$ and $9^1/_2$.

(c) Three types of savings accounts were described in a table giving account service fees, number of fee-free transactions and fee per transaction over this limit.

(i) (1 mark)

Students were asked to find the monthly fee for account 'A' with a balance above the minimum and three transactions over the fee-free limit.

This was well done although some errors were made by adding on the service fee. Some students did not correctly express their answer in dollars and cents and were not awarded the mark.

(ii) (1 mark)

Students were asked to explain why account ‘C’ would minimise fees for a person with a low balance making few transactions. This part was not satisfactorily answered by the vast majority of students. The usual responses were to reword the question as the answer, or to calculate the cost of account ‘C’ only.

Students should be prepared to express their reasons in mathematical terms and, in this case, show a comparison of the three costs.

(d) Students were given a taxation table.

(i) (1 mark)

They were asked to find the tax on $49 540. Most students calculated this correctly although it was clear some students had no experience in interpreting this information. Errors made included subtracting 38 001 rather than 38 000, dividing by 43 or 0.43 and subtracting 8942 as well as 3800.

(ii) (1 mark)

Students were asked to find the total amount of tax including a 1.5% Medicare levy. This part was not well done with many students calculating 1.5% of Answer (i) rather than the taxable income. The most common error was to fail to add on answer (i) to get the total tax. Most students were able to calculate 1.5% correctly.

(iii) (1 mark)

Students were given fortnightly tax instalments and asked to calculate the refund. This part was not well done with many students giving the total tax paid as their answer or subtracting this amount from the taxable income.

(iv) (2 marks)

Students were asked to find the extra tax paid when the taxable income increased by $3142. Most students were able to calculate the new amount of tax payable including the Medicare levy either correctly or consistent with their working in (i) and (ii). (A number who failed to get (i) and (ii) correct did this correctly but did not go back to correct (i) and (ii)). Many gained 1 mark for this step but did not go on to subtract the original tax to gain the second mark available.

A number of students were unable to correctly (or consistently) calculate the new tax payable but responded appropriately to find the extra tax due and were awarded 1 mark.

Question 28. Mathematics in Construction

This question required candidates to read and interpret house plans and elevations. It also tested their knowledge of three-dimensional geometry.

(a) (i) (1 mark)

This part required candidates to measure part of the floor plan and use the measurement to determine the scale that was used. Many candidates did it well. However, a number of candidates stated that there was no scale as they could not find it written on the examination paper.

(ii) (1 mark)

This part required candidates to sketch the verandah, and to place its external measurements on their sketch. Many candidates confused the terms 'external' and 'exterior' and hence were led into error.

Note: candidates need to realise the importance of neat accurate sketches and correct measurements as only a small tolerance is allowed.

(iii) (2 marks)

This part required candidates to calculate the area of the verandah. Many candidates did the part well, but a significant number divided the figure into shapes whose areas could not be calculated. Quite a few candidates did not know, or could not apply, simple area formulae.

(iv) (1 mark)

This part required candidates to determine which elevation was shown in the examination paper. Only about 50% of them could do so. A significant number referred to the 'back', 'front', or 'side' of the house.

(v) (1 mark)

This part required candidates to calculate the difference in height between two rooms. A large number of candidates incorrectly determined the number of steps from one room to the other, and hence gave an incorrect response, while other candidates could not interpret the plan of the house.

(b) (i) (1 mark)

This part required candidates to name a geometric solid given in a diagram. Only a small percentage of them could. The most common error was 'triangular prism or pyramid'.

(ii) (1 mark)

This part required candidates to name a skew line in the given diagram. This was poorly done with many candidates confusing parallel and skew lines.

(iii) (1 mark)

This part required candidates to name a projection line. It was obvious from the responses that many students had little if any idea what a line of projection was.

(iv) (2 marks)

This part required candidates to calculate an angle between a line and a plane. It was calculated correctly by very few candidates, including some of those who could not answer part (iii). Unfortunately, a great number of candidates who did find a correct trigonometric equation were unable to use their calculator to correctly find the angle.

2/3 Unit (Common)

Question 1

There were six parts in this question, dealing with arithmetic (calculator skills and surds), algebra (expansion of binomial product, collection of fractional terms and graphing an inequality) and calculus (primitive function). Most candidates handled the question well (the average was over 9 marks) although it was alarming to find some candidates who were unable to score even one mark.

(a) (2 marks)
Many candidates demonstrated that they could use the $x^{\pm y}$ function key(s) but were unable to correctly give a two-significant figure value for their answer. Most errors were in giving two decimal-places (= 0.05) while many candidates only gave a rounded-off answer (which was incorrect) for which it was impossible to award any mark. It is important (if not vital) that students be taught to write down a more accurate calculator answer before making any attempt to round-off (eg 0.046054.... ≈ 0.046).

(b) (2 marks)
A variety of methods were used. The factorising was generally well done although many candidates seemed not to know the difference between 'solve' and 'factorise' as they went on to give a 'solution' for an equation that they made from the original expression.

(c) (2 marks)
Well done by most candidates. The most common error was in carrying out the product of the conjugate surds in the denominator (eg getting 5 + 4 or 25 – 4 or 5 – 2).

(d) (2 marks)

A surprising number of mechanical errors including expansion of $3(x - 1)$ to get $3x - 1$, failure to handle the double-negative and inability to add 8 and 3 (doing 8 + 3 = 12). Also, many candidates either increased by a factor of 12 to 'eliminate' the fraction or turned the problem into an equation which they proceeded to solve.

(e) (2 marks)

Generally well answered. Almost all candidates set out to find a primitive (that is, not a derivative). Some were able to obtain the $\frac{-x^{-2}}{-2}$ term but not the $6x$.

(f) (2 marks)

A variety of methods were used. Many candidates had difficulty using the ≥ and ≤ signs in their solution steps even though they arrived at the correct graph in the end. Many candidates could not handle $-x \le 5$ (getting $-x \ge 5$) or just ignored the minus to get $x \le 5$, while others only found one solution ($x \le 1$). Many incorrect graphs resulted from errors in the algebra steps that produced conflicting solutions which the candidates were totally unequipped to interpret.

Overall, the graphs were appalling. There were open circles/dots, shading as though it was a region, arrowed rays that didn't indicate a final solution, curves and arcs joining the ends, and even number planes with regions shaded.

Question 2

This question, involving coordinate geometry, was reasonably well answered although many students made silly or careless errors. Many students could not cope with the 'show that' style of questions. Perhaps they need to be reminded that they can substitute the instruction 'convince me that you know' for 'show that'. A surprising number indicated they thought the question, rather than their calculations, was wrong.

Part (a) required students to find the coordinates of the *x* intercept of a line. Many students recorded the answer as (–8,–4) which was a 'combination' of the two intercepts (–8,0) and (0,–4).

In part (b) many students were unable to 'explain' why two lines were parallel. Most students indicated that the gradients of the lines were relevant although a not uncommon response was 'their gradients add to –1'. Many students reported that 'the gradients are equal' despite their working resulting in two different gradients.

When the candidates were drawing the graph of line *k*, which they has just shown was parallel to the other line on the graph, many drew a line which was obviously not parallel – some even drew perpendicular lines! A significant number ignored the instructions to indicate where it crossed the axes, others used arrows pointing to the positions where the line crossed the axes and wrote statements like 'this is where it crosses the *x* axis' without indicating the coordinates of the point. Common algebraic errors included $\frac{1}{2}x = 6 \Rightarrow x = 3$.

Many students demonstrated a lack of understanding when they tried to shade the region satisfying the inequality. Many deliberately excluded parts of the correct area, especially the area above the *x* axis in the 2nd quadrant.

Most students were unable to express the two inequalities that defined the region between two parallel lines. Most made errors in the direction of the inequality signs while others wrote expressions similar to $-8 \le x \le 12$ or $y = -\frac{1}{2}x + 6 \le 0$. Many left the '*y*' out completely when they tried to combine the two inequalities.

The majority of students were able to algebraically demonstrate that a point lies on a line. However those who didn't score the mark in this part usually just indicated the point on the line on their graph.

Most students knew the perpendicular distance formula although many were unable to use it correctly.

Part (h) required the students to show a particular point on one line was the closest point to another point on a parallel line. This part was poorly done. Many were unable to see the link with the previous part, that is, the perpendicular distance of a point to a line being the shortest distance possible, and most were unable to express themselves to explain their reasoning. Common unsuccessful approaches included testing the distance to points either side of the given point and using the perpendicular distance formula twice.

Question 3

This question consisted of four parts. The first part and the last two parts were 'bookwork' or routine differentiation and integration problems, whilst the second part was a problem which could be approached a number of ways, all of which required the use of a calculator.

(a) (i) (1 mark)

This part required students to differentiate $\sqrt{x}$, and was well done. About 84% of the candidates scored one mark. The most common errors were to incorrectly write $\sqrt{x}$ in index form, or to find a primitive of $x^{1/2}$. Many students put their answer back into 'radical' form, correctly or incorrectly, but this was not necessary for the mark.

(ii) (2 marks)

Those students who recognised xe^{2x} as a product and proceeded to use the product rule generally performed well, and about two thirds of the candidates gained full marks. Very few (< 5%) of the candidates scored one mark, and these were usually candidates who attempted to use the product rule and made some error. Those who failed to recognise xe^{2x} as a product scored zero, and $2xe^{2x}$ was a common (bald) incorrect answer.

(iii) (2 marks)

This part involved differentiating $\cos^2 x$, and was done poorly by the majority of the candidature, with over 50% scoring 0. These students used neither the 'function of a function' rule nor the 'product rule', or did not show sufficient working to convince the markers that any of the above techniques had been used. Many knew that $\frac{d}{dx}\cos x = -\sin x$, but did not know how to deal with the 'square'. Common incorrect answers (often with no working) were $-\sin^2 x$ and $-2\sin x$. Some candidates used the identity $\cos 2x = 2\cos^2 x - 1$. This approach was often rewarded with a correct answer, but was fraught with possibilities of error, and was clearly 'the long way' to solve the problem.

(b) This part of the question asked students to consider the effect of layers of plastic, each of which 'cuts out' 15% of the light striking it.

(i) (1mark)
The question asked the students to 'show' that two layers of the panels allowed 72.25% of the light through, and whilst most candidates could do this (0.85×0.85 would suffice) many did not really know how to 'show' or explain, and often tried to use the 72.25% which they were required to 'show'. Percentages were often confused with decimals and whole numbers.

(ii) (2 marks)
The students were asked to find the number of layers required to cut out at least 90% of the light. The interesting feature of this part was the variety of approaches. Many calculated laboriously layer by layer, writing down their 'subtotals' to umpteen decimal places, where perhaps the calculator memory may have been useful, whilst others experimented with $(0.85)^n$ and quickly narrowed '*n*' down to 14 or 15. The main message here is that confidence and proficiency in all aspects of calculator use is a very worthwhile skill, as well as being able to interpret the calculator answer! Substantial numbers of students solved $0.85^n < 0.1$ using logarithms, and did so well until they came to interpreting their answer, when many left $n = 14.17$ or rounded down to 14. A small number of students used an exponential decay model, and after calculating their decay constant often performed well. Many students tried a geometric progression formula, often the S_n formula rather than the T_n formula or equating T_n to 90% rather than 10%. A common error was to divide 90% by 15% and conclude that 6 layers were necessary! Only about 25% of the candidates gained full marks, whilst part marks often resulted from candidates not making the correct conclusion from their working. Two thirds of the candidates received zero for this part, and 21% made no attempt at all.

(c) (1 mark)
This part was well done, with over $^3/_4$ of the students correctly integrating $\sec^2 6x$ using the table of standard integrals or otherwise.

(d) (3 marks)

This part required the integration of $\frac{5}{x}$, the substitution e^3 and 1 into their logarithmic primitive, and the evaluation of the resultant expression, usually involving $\ln e^3$ and ln 1. Each of these steps was worth one mark. This part was reasonably well done, and almost half the candidates scored full marks. The '5' caused many problems, and answers like $\ln 5x$ or ${}^1/_5 \ln x$ were common. For students who recognised that the primitive involved logarithms, showing the substitution step clearly was necessary to ensure part marks. Finally, many candidates were unsure how to simplify their logarithmic expression correctly or fully, and left their answer unsimplified, as $5 \ln e^3$ rather than 15. Many of those who did simplify correctly obviously used their calculator, rather than their knowledge of log properties, and this again emphasises how useful calculator skills are either directly or as a 'fallback' option if theory fails.

Question 4

This question contained two parts, both of which were concerned with the topic of integration. The first was a fairly straight forward volume of a solid of revolution and the second involved an area using the trapezoidal method and an understanding of what an integral represents. The question was fairly well done by most students. Many students would have lost valuable time during this question attempting to do the integration $\int_0^5 \sqrt{25 - x^2}\, dx$ even though it was not asked for.

(a) (3 marks)

This part of the question was reasonably well done, with many students scoring full marks.

Many students who began with the correct expression $\pi \int_0^3 (x+1)^2 \, dx$, were unable to correctly expand $(x + 1)^2$.

Those who used the fact that $\int (x+1)^2 \, dx = \frac{(x+1)^3}{3}$ avoided many of the common mistakes.

Some students carelessly dropped the π throughout their working.

Although the question could be done by calculating the difference in volume between two cones, students who used this technique often lost possible marks by not explaining what they were attempting to calculate.

In general, setting out of this question was quite good.

(b) (i) (2 marks)

Extremely well done. Most students were able to gain two easy marks for being able to substitute into the expression $f(x) = \sqrt{25 - x^2}$.

(ii) (3 marks)

Well done. Those students who knew a correct form of the trapezoidal formula usually scored the full three marks. Students who used a tabular method for the trapezoidal rule were particularly successful.

Some students failed to use all six function values as the question directed.

A number of students had trouble with removing grouping symbols from their formula.

(iii) (2 marks)

Well done. Most students were aware that $x^2 + y^2 = 25$ was a circle, centre the origin and radius 5 units. A small number of students failed to mark intercepts on the axes to indicate the radius.

Shading the region caused problems for a number of students. Many recognised $y = \sqrt{25 - x^2}$ as a semicircle but failed to see that the limits 0 and 5 implied only the first quadrant.

(iv) (1 mark)

Many students had difficulty with this part, usually because they did not appear to understand the relationship between the integral and the area of a circle. Some explanations were little more than a restatement of the question or an attempt to fudge the answer. A clear understanding that the integral was equal to a quarter of the area of a circle with radius 5 was needed.

Some students who had (iii) incorrect recognised the contradiction of their answers and were able to correct themselves, while other students failed to detect the contradiction in their answers and consequently did not gain the mark.

(v) (1 mark)
Weaker students missed this out, but those that attempted it usually gained the mark. Some students used $\frac{25\pi}{4}$ = 19.63. … … to find an approximation for π rather than their answer in (ii) as requested.

Question 5

The four parts (a), (b), (c) and (d) of this question test respectively, understanding of when the primitive function is uniquely determined, solving a quadratic equation with irrational roots, various properties of exponential functions, and the graphing of a function given information on its first and second derivatives. Too many candidates were unable to solve the quadratic equation; the pronumeral *u* confused many of these, with a few being able to proceed on substituting ‘*x*’ for ‘*u*’. A greater variety of pronumerals should be used in classroom and textbook examples and exercises.

The linkage of (c)(ii) with (c)(iii), and the latter with part (b), caused many candidates considerable difficulty. Candidates from centres emphasising graphical work scored easy marks on part (d) whereas candidates from centres where little interpretive graphical work is done rarely attempted the part, and even more rarely scored a mark.

(a) (2 marks)
Many candidates did not realise that any two primitive functions of a given $f'(x)$ differ by a constant function, failing to add ‘+ *c*’ or ‘+ *k*’ to $x^3 - 2x$ to gain the first mark. A considerable proportion of those with ‘+ *c*’ could not use the additional information to correctly evaluate the constant, either failing to recognise the

significance of the '3' in (1, 3) or not being able to solve $3 = 1 - 2 + c$; many candidates replaced $f(x)$ with y before proceeding with the evaluation.

Some candidates thought that the question required them to find the tangent at (1, 3), with gradient $3.1^2 - 2$, or even $3x^2 - 2$. These answers attracted no marks.

(b) (2 marks)

The main solution technique was by explicit, or implicit, substitution into the quadratics formula of the candidate's a, b and c which themselves often were only given implicitly. Over fifty versions of the quadratic formula were recorded by the markers, with two of the erroneous ones leading to the correct surd expressions $\frac{1 \pm \sqrt{5}}{2}$ for the roots in this question; these still failed to gain the first mark. Some candidates could not handle $b = -1$, some used $a = u$, or $a = u^2$, $b = -u$; others could not simplify $(-1)^2 - 4 \times 1 \times (-1)$ without error. In evaluating their surd expression many candidates made calculator errors, for example $\frac{1+\sqrt{5}}{2}$ being evaluated as $1 + \frac{\sqrt{5}}{2}$, or even as $1 + \sqrt{5/2}$. The use of the grouping properties of the vinculum is worthy of attention by teachers. Most candidates with a negative discriminant ignored that sign in their calculation; few commented that there would be no real solutions.

Disappointingly few candidates used the completion of the square technique. The main errors were the appearance of $(-{}^{u}/_{2})^2$ where $(-{}^{1}/_{2})^2$ would be expected, and $+\sqrt{5/4}$ where it should have been $\pm\sqrt{5/4}$.

Too many candidates attempted to factorise the quadratic expression looking for integer roots!

(c) (i) (3 marks)

One mark was awarded for a (correct) mathematical expression for the area between the curves, one for the primitive of the total integrand, and one for the substitution of the upper and lower limits. Most candidates managed to score at least one mark; all too often the only marks awarded a candidate for question 5 came on this part.

Those candidates who expressed the area as $\int_1^2 (e^x - 1 - e^{-x})dx$ were the most likely to secure full marks. Those who approached the question by computing $\int_1^2 (e^x - 1)dx - \int_1^2 (e^{-x})dx$ frequently made errors with negative signs, often due to incorrect use of grouping symbols, especially in simplifying the expression achieved at the substitution of limits stage. Many candidates believed the required area was only one of these latter two integrals. Those candidates who tried to proceed directly to the primitive step rarely achieved full marks. Candidates from some centres seem to believe that a lack of understanding of the geometrical notion of area under the curve for the definite integral can be negated by absolute value signs; the incorrect use of such signs often resulted in a lower mark than would have been obtained if they had been omitted. Some candidates achieved their expression for the area by the curious method of integrating the left hand side of the equation $e^x - 1 - e^{-x} = 0$.

Many candidates could not write down correct primitives for all of e^{-x}, e^x and 1; the most common errors were $-e^x$ for e^{-x}, and either $\frac{e^{mx+1}}{m+1}$ or $\frac{1}{2mx}e^{mx^2}$ for e^{mx}. Other candidates differentiated the integrand, or believed that the primitive was identical to it.

Some candidates used limits other than 1 and 2, either not reading the question or not understanding '... from $x = 1$ to $x = 2$', and a few failed to gain part marks as a result of their changing the limit values between the 'expression for area' and the 'substitution of limits' steps. Some candidates used calculators to evaluate their primitive at the limit values, before reading 'Leave your answer in terms of *e*.' (meaning that $e^2 + e^{-2} - e - e^{-1} - 1$ is an acceptable answer). All too often assertions such as $e^2 - e^1 = e$, and $e^{-2} - e^{-1} = e^{-1}$ appeared in the simplification to answer step.

(ii) (1 mark)

Most candidates realised that the variable *y* had to be eliminated from the equations of the two functions, but few could proceed from $e^x - 1 = e^{-x}$ to the given equation. Those who realised e^{-x} is also $1/e^x$ had little trouble; some succeeded by multiplying through by e^x.

(iii) (2 marks)

This was the least attempted section in question 5, and the most poorly done. Only a handful of the candidates who could not do (b) made progress here. All too often candidates re-solved $u^2 - u - 1 = 0$ before proceeding on to x.

To gain both marks candidates had to draw conclusions from both their results (hopefully 1.618 and –0.618) in part (b), that ln(1.618) = 0.481 and, either that ln(–0.618) cannot be calculated or that the exponential function is always positive. Those candidates who wrote '$e^x = 1.618$, $\therefore x = 0.481$' penalised themselves by not making the connection with 'ln' explicit. Many candidates who dealt with 1.618 first either did not bother to examine the second root or incorrectly believed that as $x > 0$ for the point of intersection in the diagram then $u > 0$.

Most of those candidates who calculated 0.618 and –1.618 in part (b) forced errors in their working to ensure they ended with $x = 0.481$ rather than $x = -0.481$; only a small percentage backtracked to find their error in part (b). Some of those candidates with both roots wrong in (b) achieved full marks here, by correctly dealing with their roots.

It was also clear that a small percentage of the candidature either did not know the relationship between 'exp' and 'ln' or could not use the 'ln' button on the calculator.

(d) (2 marks)

One mark each was awarded to a graph which was decreasing over the required domain, and concave down. Many candidates wrote down that $g'(x) < 0$ means negative gradient or slope, and that $g''(x) < 0$ means concave down but more than half of these produced a graph which satisfied only one of these conditions (concave down appears to be a synonym for maximum); other candidates drew a graph which contradicted both their stated conditions. Too many candidates sacrificed a mark by their failure to be consistent over the required domain, which was often implied. A few drew no graph even though both conditions had been interpreted correctly and scored no marks. It is extremely important in such questions that the students DRAW SOMETHING.

Some candidates successfully sought a particular function satisfying both conditions. A small percentage of these candidates did not obtain full marks because of a poorly presented graph.

Some candidates redrew the diagram in part (c) and added their $y = g(x)$ to it. Others had multiple graphs on the one set of axes, sometimes labelled as $y = g(x)$, $g'(x)$ and $g''(x)$, but more often leaving it to the marker to guess which one represented $g(x)$.

Question 6

This question contained two parts: the first dealt with curve sketching and the second with an application of sequences and series. Overall, students found the question difficult. Good attempts in (a) were often followed by poor attempts in (b) and vice versa. Sketches generally were poor. Many students had problems interpreting (b) and relating it to an arithmetic progression.

(a) This question involved a cubic function which related the amount of medicine in the blood over a period of 3 hours.

(i) (4 marks)
The students were asked to sketch the function $M = 4t^2 – t^3$. It was easy to find M' and M'' and solve them equal to 0. The maximum turning point occurred at $t = 8/3$, which was close to the boundary condition of $t = 3$, and this caused problems. Many students had difficulty showing the nature of the graph near $t = 0$ as it was the minimum turning point and the extremity of the domain. Many students only plotted points. As a result they could not accurately locate the maximum point in the domain. Many used calculus and did not plot points! So they often had weird looking graphs.

At least 70% of students could not accurately sketch a cubic. Most would draw a straight line from the max turning point to the min turning point at (0,0). The point of inflection was often over-emphasised, to look like a horizontal inflection, or it was ignored. A great many students disregarded the boundary conditions which were twice stated in the question. This often resulted in their graphs showed negative amounts of medicine in the blood stream or amounts present before the medicine was administered.

(ii) (1 mark)

Generally this part was well done. A small percentage gave the value of M as their answer. It is interesting that many students who only plotted points in (i) managed to use calculus correctly here and find $t = 8/3$. However they could not see the connection with (i). Many gave the answer for part (iii) here.

(iii) (2 marks)

This was a good question. Not many students appreciated that the greatest value of M' occurred at the point of inflection. Some students who solved M'' equal to 0 and found $t = 4/3$ gave answers such as $4/3 < t < 8/3$, or $t < 4/3$. Many gave descriptive answers such as 'when the gradient is steepest'. Very few students realised that the answer lay half way between the stationary points.

(b) This question tested students' knowledge of series. It used a venetian blind with 25 slats with a separation of 27 mm.

(i) (1 mark)

The students had to show that the last slat rose 675 mm. Most students were able (some after several trials) to show that the bottom slat rose by this amount. There was a lot of fudging.

(ii) (1 mark)

The students had to find how far the next slat rose. Most could show that this was 648 mm. However many wrongly added or subtracted 3 mm or 30 mm indicating that they were confused about what number constituted the common difference. Adding rather than subtracting was a common error.

(iii) (1 mark)

Most students gave their version of a common difference in showing that the distances that the slats rose formed an arithmetic sequence. Many considered the closed blind and gave the common difference as the thickness of the slats. Often answers were long and unclear and marginal in their correctness.

(iv) (2 marks)

The students had to sum all the distances that the slats rose. As expected, many students did not know the correct formula. Many of those whose formula was correct, wrongly substituted. There were problems matching *a* and *d*. In particular many used 27 when they should have used negative 27, eg $S_{25} = \frac{25}{2}\left(2 \times 675 + (25 - 1) \times 27\right)$, which was wrong. An interesting and correct response was $27 \times (1+2+ \ldots +25) = 27 \times 325 = 8775$.

Question 7

This question consisted of two parts taken from two separate areas of the syllabus. Part (a) required the sketching of two trigonometric graphs on the same set of axes, namely $y = 2\cos x$ and $y = 2\cos x - 1$, both in the domain $0 \leq \ \leq 2\pi$. It also required candidates to find the exact values of the x coordinates of the points where the latter graph crossed the *x* axis in the given domain. Part (b) was a geometry question related to the concepts of congruency, isosceles triangles and parallel lines.

The question was generally well attempted with the majority of candidates scoring half marks or more. The stronger candidates found the question quite easy and were able to gain full marks or near full marks. One pleasing aspect of the responses was the improved attempt by students in the geometry part to present their proof in a logical and sequential form, giving reasons for each stage (step) of their argument.

(a) (5 marks)

(i) (2 marks)
Sketch $y = 2\cos x$.

Generally well done with most candidates having some idea of the general shape of the required graph, although in many cases the accuracy of their sketch left a lot to be desired. There was obviously confusion with regard to the difference between curves representing $y = \cos \frac{x}{2}$, $y = 2\cos x$ and $y = \cos 2x$.

The most common faults included:

- no scale given on either axis
- failure to 'label' either of their graphs
- graphs not symmetrical about the x axis
- a full wave-length occurring between $x = 0$ and $x = \pi$
- curves looking distinctly parabolic between $x = 0$ and $x = \pi$.

The marking scheme awarded 1 mark for drawing either one full wave-length of a sine or cosine graph about the x axis or for drawing a graph with the correct amplitude between $y = 2$ and $y = -2$. No marks were deducted for an otherwise correct graph which extended beyond $x = 2\pi$, provided the 2π was indicated on the x axis.

(ii) (1 mark)
Sketch $y = 2\cos x - 1$.

Most candidates realised that this graph was similar in shape to their graph in part (i) and had to be lowered one unit. A frequently occurring mistake was translating one unit to the left rather than one unit down. A minority drew graphs which intersected their previous curve, usually at $\left(\frac{\pi}{2}, 0\right)$ and $\left(\frac{3\pi}{2}, 0\right)$.

(iii) (2 marks)

Find the x coordinates of the points where $y = 2\cos x - 1$ crosses the x axis ($0 \le x \le 2\pi$).

Attempts at this part of the question were divided almost equally between candidates attempting to 'read off' the points where $y = 2\cos x - 1$ is at the x axis, and those that attempted to solve the equation $2\cos x - 1 = 0$. In the former, many assumed the point of intersection with the x axis to be halfway between 0 and $\frac{\pi}{2}$ with common incorrect solutions of $x = \frac{\pi}{4}$ and $x = \frac{7\pi}{4}$ being given. The latter method saw a significant number of candidates ignore the '4th quadrant' solution, giving $x = \frac{\pi}{3}$ as their sole answer.

A mark was awarded for each correct answer, with extraneous solutions being ignored. For students unable to obtain either correct answer, one mark was allowed for recognition of the fact that $y = 0$ was significant in their attempted solution.

(b) (i) (2 marks)

Here students were required to prove $\Delta ABC = \Delta ABD$. This part of the question elicited a full range of responses from excellent, well-documented answers, to a meaningless array of 'facts'. Many candidates were able to recognise that '*AB* is common' was an integral part of their proof (one mark awarded at this stage) and then went on to conclude their working with '*SSS*' as justification for their answer (second mark awarded).
Some incorrectly assumed angles in each triangle to be equal, making statements such as $\angle ABC = \angle BAD$ (base angles of a quadrilateral/trapezium/parallelogram, etc). In general, students did not 'use' their diagram, even drawing it at the bottom of one page and then starting their question on the next page.

(ii) & (iii)

(3 marks)

Prove Δ's *ABP* and *CDP* are isosceles.

These two parts were combined in the marking scheme with a total of 3 marks for a correct solution. The most common approach was to recognise that $\angle PAB = \angle PBA$ (the 'base' angles in ΔABP) because they represented corresponding angles in the congruent triangles in part (i). Students then went on to show that $PC = PD$ by a subtraction statement deduced from the fact that $AC = BD$ (given) and $AP = PB$ (proved above). For this method, one mark was awarded for a statement recognising that isosceles triangles have two equal sides and/or angles; the second mark for correctly proving ΔAPB was isosceles and the third mark for correctly proving ΔPDC to be isosceles.

A less frequently seen technique was to prove $\Delta APD = \Delta BPC$ *(AAS)* which immediately gave the required result for both triangles to be isosceles ($AP = PB$ and $PD = PC$ each being corresponding sides of those congruent triangles).

By far the most common problem experienced was candidates assuming information not in the data. Instances of this were illustrated by statements such as:

- the diagonals of a quadrilateral bisect each other ($\therefore$ *P* is the midpoint)
- the diagonals of a quadrilateral bisect the vertices of the quadrilateral
- the opposite angles of a quadrilateral are equal
- *AB* and *DC* are parallel
- *ABCD* was a parallelogram, rhombus, etc.

(iv) (2 marks)

Prove $AB \parallel CD$.

This part of the question proved to be the most difficult. It was, however, good to see students who correctly answered this question, presenting a logical and sequential line of reasoning. By far the most frequently attempted approach was to let $\angle PAB = \alpha$ (say), deduce $\angle APB = 180 - 2\alpha$ (isosceles Δ, angle sum Δ), $\angle DPC = 180 - 2\alpha$ (vertically opposite) leading to $\angle DCA = \alpha$ (Δ*PDC* isosceles, angle sum Δ) and a statement of equal alternate angles.

Other (infrequent) approaches were to:

- use ∠*CPB* as the exterior angle of both the isosceles triangles *PAB* and *PCD*
- realise ∠*DAB* = ∠*CBA* (part i) and ∠*ADC* = ∠*DCB* [∠*ADC* = Δ*BCD*] which yielded supplementary cointerior angles.

In the most common approach, one mark was awarded for a 'part proof' involving the statement that ∠*APB* and ∠*DPC* were a pair of equal vertically opposite angles. A necessary requisite for gaining the second mark was that candidates at the end of their working specifically *name* a correct pair of equal alternate angles.

One major difficulty experienced by markers was distinguishing between the students' writing of the letters *P*, *D*, *B*, *A*. It is in the students' best interests to write letters clearly in the geometry section of the paper, and elsewhere, to avoid any possible loss of marks.

Question 8

This question contained two main parts, the first on probability, and the second a minimisation question that initially required a geometrical proof for similar triangles. In general the question was done reasonably well for a question 8, with most students making some attempt at one or both parts of the question.

(a) (4 marks)

This question gave students information about the number of students who studied French and the number who studied Japanese, asking them for various probabilities of the random choice of students. It was clear that, while many students were familiar with the use of a Venn diagram, many others had no concept that there could be 18 students of French and 22 students of Japanese but only 28 students at the meeting. Most students did not obtain full marks for this part. Some students even suggested that the question was impossible!

(i) (1 mark)
This part required students to find the probability of choosing at random a student who studied French. Some students obviously confused the idea of the study of French with the study of French only. There was a range of denominators used with the most common incorrect one being 40.

(ii) (2 marks)
This part asked for the probability that two students studied French. By far, the most commonly made mistake in this part was to allow replacement and for students to square their answer to part (i). Some students based their answer on the simplified answer to part (i), ie $\frac{9}{14}\times\frac{8}{13}$.

(iii) (1 mark)
Students were asked here to find the probability that a student, chosen at random, studied both French and Japanese. Students who used the Venn diagram were usually more successful here than others. A number of students gave an answer that was greater than 1, while other students who had not correctly answered parts (i) and (ii) were able to give the correct answer here.

(b) (8 marks)
This question required students to establish, through similarity, a relationship between lengths of sides of right angled triangles and develop an expression for the area of a triangle that included a rectangle of fixed dimensions. Then students needed to find the dimensions that gave the minimum area of the triangle. It was a concern that a number of students, maybe about 10%, were not able to work with variables and used a particular value for x, often 6, but sometimes 8 or 3, throughout the question.

(i) (2 marks)
Many students did not obtain 2 marks for this part. While most were able to find and describe a pair of right angles ($\angle TPQ$ and $\angle QRU$), they were

not able to explain why other pairs of angles are equal. Many quoted reasons such as ‘alternate’, ‘collinear’, ‘co-interior’, or ‘same gradient’, while others gave long and obviously time-consuming answers that described every possible angle in the diagram. In some cases students were lazy about the naming of the angles, using $\angle P$ and $\angle Q$, when these angles are not uniquely determined. Students also tried to assume that the triangles were similar in the act of proving them, ie that there were pairs of proportional sides, some students even tried to use the fact that two sides are in proportion. There is obvious confusion about the meaning of the word ‘corresponding’ and some students mixed up congruence with similarity, or assumed that the triangles were isosceles. Some students correctly produced a length and used an ‘alternate angles’ argument, while a surprising number used the angle sum in both triangles to prove angles equal.

(ii) (1 mark)

Most students were able to earn a mark for this part from writing down a correct statement of proportionality relating to x and y from the similar triangles. What errors there were generally resulted from students citing the ratio incorrectly or by insisting that the triangles were isosceles.

(iii) (1 mark)

This part produced obvious attempts at ‘fudging’, where students were able to establish the $24 + 3x$ part of the area expression, but were unsure where the $\frac{48}{x}$ came from, not seeing the relationship shown in the previous part that $xy = 24$. About half of the students who were successful here used the whole triangle method, ie that $A = \frac{1}{2}(x + 4)(6 + y)$, although often without parentheses. Many students worked very hard, progressing in small steps to achieve the ultimate result.

(iv) (4 marks)

On the whole, this part was reasonably well done, with the majority of students gaining some marks. While a number of students obviously found this part quite easy and usually got 3 or 4 marks, there were some whole groups of students who did not attempt this part at all, particularly those who were unsuccessful in finding the expression for A in part (iii).

The most common mistake in differentiating resulted in the derivative $3 + 48 \ln x$. This made the rest of the question significantly more difficult, although some students were able to cope with $x = e^{-\frac{1}{16}}$. Some students did not simplify their answer, leaving it as $\sqrt{\frac{48}{3}}$, again making the rest of the question more difficult. More often mistakes were made in finding the second derivative, with students using the quotient rule surprisingly often. Some students changed the expression into a quadratic by multiplying through by x, and then either differentiated or simply solved the quadratic equation for the expression equalling zero.

A number of students needlessly failed to earn the final mark, by either not justifying their result, or by not answering the question which asked them to find the base and height of the triangle, not the value of x. More students used the second derivative test than the first derivative test. Students who use the first derivative test are more often in danger of losing marks because of their failure to be clear about what they are doing.

While it was pleasing to see the number of students who recognised that the value of x represented a length and so could not be negative, the willingness of a number of students to simply change a negative answer into a positive, without looking back to find their mistake also causes concern. Other students seemed to be quite happy to give the solution to $x^2 = -16$ as $x = 4$.

Question 9

The question contained five parts, basically from the syllabus topic of application of calculus to the physical world, linked to linear, quadratic and logarithmic functions and coordinate geometry. There was some linking between parts (a) to (d). However many students incorrectly linked (e) with (d). Overall many students were largely successful in parts (a) to (d) but most students had difficulty gaining marks in part (e).

(a) (2 marks)

Well done. Common mistakes were to ignore the constant term in the general primitive $x = t^2 + 4t$ or to be unable to evaluate the constant from the initial conditions. A number of students substituted $t = 4$, when $x = 0$.

(b) (2 marks)

Students did not gain as many marks as in part (a). The main difficulties were recognising the function as a parabola and its concavity and graphing the correct domain. The fact that the particles never meet, forced many students who answered (a) correctly, to graph $y = 4 + 3\log_e(t + 1)$. Others had the correct intercept but drew straight lines (using a ruler) or made their curve concave down, often because they used a variable vertical scale. A third group concentrated their efforts for $t < 0$. Overall, the larger the graph the better the chance of correct shape.

(c) (1 mark)

Students who attempted this part but who did not gain the mark usually just stated the result without showing the averaging of the x-coordinates. Many students knew the midpoint formula but could not apply it to the conditions. Students often incorrectly used length concepts, although they were not penalised.

Students who obtained an incorrect function for P in (a) often (understandably) fudged their answer to this part. The better of these students sometimes went back to part (a) to include a constant value of 4.

(d) (3 marks)

This part was quite well done with many students gaining 2 or 3 marks. Most students knew to derive $\frac{d^2x}{dt^2}$ and equate to zero.

Common mistakes were:

(i) not dealing with the $\frac{1}{2}$ correctly;

(ii) $\frac{d}{dx}(3\log_e(t + 1) = \frac{1}{t+1}, \frac{3}{t}, \frac{3t}{t+1}$;

(iii) $\frac{d}{dt}\left(\frac{3}{t+1}\right) = 3\log_e(t + 1), \frac{3}{(t+1)^2}, \frac{-3}{(t-1)^2}, \frac{(t+1).3-3.1}{(t+1)^2}$;

(iv) solution of the quadratic equation, poor manipulation and substitution skills. Students who didn't expand were usually more successful.

(e) (4 marks)

Poorly done, with few students gaining full marks. Students need to realise that the amount of explanation required is linked to the marks for each part of a question. Hence more was expected than a bald answer. Most students believed (e) was directly linked to (d) and attempted to find a minimum for x_M, and spent pages going round in circles.

Students who used the correct function for distance PQ often believed they had made numerical mistakes when their stationary points were both found to be for $t < 0$. It was only the best students who then examined the required domain and determined the minimum distance with additional comments.

Most successful answers involved a mixture of calculus and graphical skills.

Question 10

This question led students by an algebraic route to the final part. However, the students who did not master the necessary algebra also had an opportunity to show their ability using a logical argument. The question linked a variety of topics including circular measure. Many students made a good attempt at the question.

(a) (i) (2 marks)

Most students who attempted this part drew a correct graph of $y = \sin x$. Some students were not aware that graph $y = \frac{2}{3}x$ was a straight line through the origin. The most serious error was the attempt by some students to draw the graphs on the same set of axes but using a different scale for each graph.

(ii) (1 mark)

This part was well done by many students. Some students sacrificed time by finding the equation of the tangent to the curve at the origin.

(iii) (2 marks)

Few students attempted this part. Of those who did, some presumed that the solution would be a set of discrete values for m.

(b) (i) (2 marks)

Students correctly substituted into the formula $l = r\theta$ to receive their first mark. Few succeeded in gaining the second mark because they were not able to successfully use a trigonometric formula to establish the required relationship. It was sad to see that a significant number of students tried to fudge the answer to this part.

(ii) (1 mark)

Poor graphs in (a)(i) meant that some students were not able to read a value. This led to difficulties for these students in part (iii).

(iii) (2 marks)

Students who did the previous part nearly always scored the marks for this part. Very few students attempted to convert the value of angle *POQ* to degrees and consequently most calculations for the radius of the circle were correct.

(c) (2 marks)

Overall very, very few students attempted this part. The students who did use an algebraic approach had difficulty only with the inequality when they had to invert it so that l was no longer in the denominator. The students who used a logical approach frequently used sophisticated reasoning in presenting their solution. Of this group some looked only at the minimum value of the arc and subsequently forfeited the second mark to this part of the question.

3/4 Unit (Common)

Question 1

This question consisted of six unrelated parts.

(a) (1 mark)

Candidates were asked to find the value of a when $(x - 2)$ is a factor of $P(x) = 2x^3 + x + a$.

Most students successfully let $P(2) = 0$, to gain the value of $a = -18$. If a student reached $a + 18 = 0$ the mark was awarded, with no penalty for any subsequent error.

Clumsily, many students divided $(x - 2)$ into $P(x)$. The division process produces a remainder of $a + 18$, but errors were prominent when this approach was taken.

(b) (2 marks)

The question requested an acute angle, to the nearest degree, between the lines $2x + y = 4$ and $x - y = 2$.

The most successful method was to place the equations in the form $y = mx + b$ to get the gradients $m_1 = -2$ and $m_2 = 1$.

$$\begin{aligned}
\therefore \tan\theta &= \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \\
&= \left|\frac{-2 - 1}{1 - 2}\right| \\
&= 3 \\
\therefore \quad \theta &= 72
\end{aligned}$$

1 mark was awarded for method (correct use of correct formula, or subtraction of angles) and 1 mark for evaluation (correct answer from stated formula, but formulae required binomial numerators and denominators).

Common errors using this method included:

- the use of the incorrect formula: tan $\theta = |\frac{m_1 - m_2}{1 + m_1 m_2}|$, which gained one mark if the result was $\theta = 18$
- using $m_1 = 2$ (instead of –2) which also produced $\theta = 18$
- answering in radians (which didn't attract the evaluation mark)
- not rounding off, or rounding off incorrectly (neither was penalised)
- errors, in either gradient or formula, resulting in tan $\theta = 1$, $\therefore\ \theta = 45$ (this gained 1 mark, but $\frac{\pi}{4}$ was awarded 0).

Students who used $m_1 = -\frac{1}{2}$ also got the answer of 72, but were not awarded full marks.

Students should be encouraged to *quote* the formula, before substituting, so that a mark may still be awarded for evaluation from their incorrect formula or value(s).

Another successful method was to find each of the angles of inclination, and subtract them.

ie $\quad 116° 34' - 45°$
$\quad = 71° 34'$
$\quad = 72°$

Common errors, using this method, were:

- to misinterpret $\tan^{-1}(-2)$ from the calculator to produce $\pm\, 63° 26'$
- to add $63° 26' + 45° = 108°$ (1 mark was awarded for this answer, since the correct answer can be gained by finding the supplementary angle, using correct logic. A diagram was often used when this technique adopted.)

(c) (2 marks)

The question required an external division of the interval *AB*, where *A*(–1, 2) and *B*(3, 5), in the ratio 3 : 1.

The variety of attempts and errors on this relatively basic question was alarming. The approach which produced the correct solution most often was to let the smaller part of the ratio be negative (ie $m : n = 3 : -1$) and substitute into the formula:

$$\left(\frac{mx_2 + nx_1}{m+n},\ \frac{my_2 + ny_1}{m+n}\right)$$

$$= \left(\frac{3(3) - 1(-1)}{3-1},\ \frac{3(5) - 1(2)}{3-1}\right)$$

$$= \left(\frac{5}{6.5}\right)$$

ADVICE:

- Use the same formula for internal and external division, simply adjust the ratio, as explained above.
- Avoid the use of formulae with minus signs substituted for the plus signs. This was by far, the most common source of error, with students mixing + and – signs, and also not knowing whether to use a positive or negative ratio in their confused formula.
- Attempts at diagrammatic reasoning on the number plane seldom produced the correct result.
- Students should quote their formula, and show every step, especially the substitution of values, and make no attempt to evaluate until after this line.

Marks were awarded:

1 for method and 1 for evaluation.

A correct x or y value gained a mark, even if multiple errors occurred in evaluating the other coordinate.

Common errors, which were awarded 1 mark, included:

Internal division, $3 : 1 \Rightarrow (2,\ \frac{17}{4})$

Internal division, $1 : 3 \Rightarrow (0,\ \frac{11}{4})$

Internal division, $3 : 1 \Rightarrow (-3,\ \frac{1}{2})$

[These students at least demonstrated a basic knowledge of the use of the correct formula.]

NB. No marks were deducted for incorrect simplification of correct single, simple fractions, eg $(\frac{10}{2},\ \frac{-13}{-2}) = (4, -6.5)$.

Students who mixed *x* and *y* values into each ordinate, or who substituted a positive ratio into a formula with a mixture of + and – signs, were awarded zero.

(d) (1 mark)

Forming a committee of three men and four women from a group of eight men and six women required the simple expression (or its equivalent) of ${}^{8}C_{3} \cdot {}^{6}C_{4}$, and was generally well done. (Any subsequent evaluation or error was ignored.)

The most common error was to use permutations instead of combinations and was awarded zero.

Some students divided by ${}^{14}C_{7}$, confusing the question with one concerning probability. These students also scored zero.

Another common error was to form a sum instead of a product.

(e) (3 marks)

'Solve the inequality $\frac{2}{x-1} \le 1$.' produced a variety of approaches. Predominantly, four of these were correct. By far the most successful method was the 'critical points' method, which requires establishing critical points and testing the regions on a number line.

ie $\frac{2}{x-1} \le 1$

Critical points are $x = 1$, and when $\frac{2}{x-1} = 1$.

ie $2 = x - 1$

$3 = x$

$\therefore x , 1$ or $x \ge 3$

The marks were awarded as: 1 for the two critical points; 1 for the two inequalities; and 1 for excluding $x = 1$ from the inequality.

Other successful methods included:

- Multiplying by $(x - 1)$ and taking two cases, $x - 1 > 0$ or $x - 1 < 0$. This solution often resulted in students clumsily trying to resolve two cases ie $x > 1 \therefore x \ge 3$ and $x < 1 \therefore x \le 3$ and often having no idea how to resolve the final solution by combining the two partial solutions.

- Multiplying by $(x - 1)^2$, a positive number, and solving the resulting inequality $x^2 - 4x + 3 \geq 0$. Students often made algebraic errors in the simplification of the quadratic expression, and regularly failed to exclude $x = 1$ from $x \leq 1$.
- The use of a graphical solution from $y = \frac{2}{x-1}$ and $y = 1$ on a number plane was seldom used, and usually only analysed correctly by stronger candidates.

ADVICE:
Use the 'critical points' method for a greater success rate.

Avoid 'squaring both sides' to get $\frac{4}{(x-1)^2} \leq 1$, which is incorrect since the two expressions are opposite in sign for some values of x. This was a common, unsuccessful, attempt by students.

Using a number line clearly helps students to write two inequalities for two regions (and one inequality if the question requires the intersection of 2 regions).

Students, with great regularity tried to combine the two inequalities to produce:

$1 \leq x \leq 3$; $1 \geq x \geq 3$; $1 < x < 3$	(1 mark)
$1 < x \leq 3$; $1 > x > 3$	(2 marks)

1 mark was awarded for the simple attempt at a solution which produced $x \geq 3$.

It was common for good students to get the answer $x \leq 1$, $x \geq 3$ for 2 marks and lose their only mark, out of 12, for question 1, by failing to exclude $x = 1$ from the domain.

(f) (3 marks)

'Using the substitution $u = e^x$, find $\int \frac{e^x}{1 + e^{2x}} dx$.'

Students who understood the required solution often lost their final mark for answering $\tan^{-1} u$, failing to 're-substitute' e^x for u.

There was no penalty for omitting + *c* in the answer.

There was no penalty if *du* was omitted from the integral.

The marks were, basically, awarded: 1 for finding a correct derivative; 1 for substituting $u = e^x$ and simplifying the integral; and 1 for correctly integrating and re-substituting.

It was common for students to make an error in substituting, and produce a logarithmic function from their integral. One mark was awarded if they correctly integrated their integral to a logarithm *and* re-substituted.

The most common of these was:

$$\int \frac{u}{1+u^2} = \frac{1}{2}\ln\left(1+u^2\right) = \frac{1}{2}\ln\left(1+e^{2x}\right)$$

[The last step was necessary to gain the mark.]

Candidates would often attempt to transport *variables*, from inside their integrals, to the front of the integral sign, with disastrous results.

The greatest difficulty students found was how to use their derivative in the substitution process. Writing $dx = \dfrac{du}{u}$ to produce:

$$\int \frac{u}{1+u^2}.\frac{du}{u} = \int \frac{du}{1+u^2}$$

was a common good way for students to simplify the substitution step, and produce the correct integral.

1 mark was the mode for part (f), although the students who achieved 3 marks often only took a few lines to do it. The more ‘verbose’ attempts regularly led to confusion.

Question 2

This question contained three parts: (a) testing finding an approximation to the root by the halving-the-interval method, (b) reading an integral from the standard list and evaluating it using log theory, and (c) finding the area between a pair of intersecting circles.

The average was quite high, as it should have been since (b) and (c) are 2 unit work. Good to average students scored a relatively easy 6 marks on (b) and (c)(iii).

(a) (i) (2 marks)

Quite well done, but the 0.5 and 1 proved a distraction for many. Those who began by testing the sign of $f(0.5)$ and $f(1)$, then had to halve-the-interval several times before they reached the required interval.

One suspected that many of the comments 'since the function is continuous' were made in hope rather than in confidence.

Those attempting to use Newton's method rarely scored any marks since they failed to address the roots being *between* certain values.

'$f(0.8) < 0$ and $f(0.9) > 0 \therefore$ concavity changes' was frequently encountered but not penalised.

(ii) (2 marks)

Candidates who scored 10 or 11 marks most frequently lost the mark(s) here. *Very* many candidates failed to realise that *any* number between 0.85 and 0.9 rounds up to 0.9, and continued the process for many repetitions, often concluding for example 'root is 0.875 correct to 1 dec. pl.' or 'root is between 0.85 and 0.9 $\therefore$ 0.8 to 1 dec. pl.'

This response was popular: $\frac{0.8+0.9}{2} = 0.85 \cong 0.9$.

Those who began $f\left(\frac{0.5+1}{2}\right)$ sometimes continued for the 9 repetitions necessary to get the correct answer!

There was widespread confusion between the *root* and the *value of the function*, giving the answer 'f(0.85)= –0.001... so root = –0.0 (1 dec. pl.)'

Many said: $f(0.85) \cong 0 \therefore$ root $= 0.85 \cong 0.9$. Had the function been $f(x) = \ln(x+1) - x^3$, this approach would clearly have led to the incorrect answer.

(b) (3 marks)
This was an easy question for most. Of those who chose the correct standard integral (by no means all!) the ones who lost marks most frequently omitted parentheses, leading to statements such as $\ln(32-16) = \ln 32 - \ln 16$, or $\dfrac{\ln 32}{\ln 16} = \ln 2$.

(c) This was very difficult to mark. Some apparently assumed that this was the circle geometry question and launched into page-plus proofs for both (i) and (ii). Most such attempts involved assumptions that the triangle was equilateral.

(i) (1 mark)
Most attempts scored 1, provided they linked the three sides by their each being an equal radius.

(ii) (1 mark)
Well done, but omitted surprisingly often.

Too many candidates wrote at length to prove that ΔPOT was also equilateral, instead of writing: 'Similarly.....'.

(iii) (3 marks)
Many did not attempt this section.

We stopped counting at 15 different correct approaches, and seemingly more incorrect ones. **DIAGRAMS WERE OF GREAT BENEFIT,** particularly because of the difficulty in naming sections unambiguously and the inability to determine at times whether $\frac{\pi}{3}$ or $\frac{2\pi}{3}$ was the angle being used. The terms *segment, sector, arc* seemed to be entirely interchangeable.

Attempts at simplification were often very poor with, for example, answers which were clearly negative not causing obvious concern.

Some confusion was caused by *d* being the radius not the diameter, and many used $ST = d$.

We were astonished by the huge number of candidates who left answers in terms of $\sin \frac{2\pi}{3}$. In this part of this question, the evaluation was not required to earn full marks, but students need to be aware that this is not generally the case. (See the comments on question 3, for example.)

Some students thought 'in terms of *d*' meant 'give with *d* as the subject.'

Question 3

Overall this question was quite well done. The concentration of trigonometry in this question may have been the cause of some confusion among the students and may have led to careless and repetitive errors.

(a) (i) Most students understood the meaning of an even function but in showing that $f(x) = f(-x)$ they were not able to generalise and instead used specific values of *x*. There were a surprising number of students who believe that if the function is not odd then it must be even, thus only showing that $f(x) \neq -f(-x)$. Some students only commented on the symmetrical aspect of the even function. This did not earn any marks.

(ii) This question was generally very well done. Most students were able to use the standard integral sheet (and even stated this) to give the correct integral. It was difficult to award marks to some students who had the wrong answer as they omitted the intermediary steps and it was therefore hard to know how they arrived at their response. The most common errors included:

- giving the final answer as 60;
- $\sin^{-1} \frac{1}{2} = \frac{\pi}{3}$;

- $2\sin^{-1}\frac{1}{2}$. Although this was correct, it did not receive full marks as one mark was allocated to the evaluation.

(b) This question was very well done. Many students appear to learn the integral of $\cos^2 x$ off by heart. Some students knew that $\cos^2 x = \frac{1}{2}(1+\cos 2x)$, but made errors in integrating this. The most common error was having a minus sign instead of a plus sign but this was then integrated correctly. Fortunately only few students had $\dfrac{\cos^3 x}{3}$ as their integral. This was awarded no marks.

(c) This question seemed to lend itself to making many algebraic errors, the most common being:

$$(2+\cos x)^2 = 4+\cos^2 x$$
$$= 4+2\cos x+\cos^2 x$$

Most students knew how to find a volume but some omitted the π. Many students failed to recognise the relevance of the result obtained in (b) or simply chose not to use it. This meant they had to integrate all over again which led to errors which could otherwise have been avoided.

Some students used the result from (b) but failed to multiply it by π. Some included $\frac{\pi}{8}-\frac{1}{4}$ when evaluating the integral, thus cancelling itself out, eg $\left[4x+4\sin x+\frac{\pi}{8}-\frac{1}{4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$.

Some students used their incorrect answers from (b) instead of using the given result. This could not score full marks.

(d) (i) There were two main approaches used here. Most students derived the function from basics while others knew it was equal to $\frac{\pi}{2}$ and then realised the derivative was zero. Some had no idea or made careless mistakes in deriving from basics.

(ii) Disappointingly some students who had the correct derivative in (i) were unable to interpret that therefore the graph must be a straight line. The converse of this also occurred in that many were able to draw the horizontal line even though they could not do the previous part and having drawn it could not make the connection that the derivative must have been zero.

Many students drew the graph by adding ordinates but in a lot of cases this was most unsuccessful.

Some students ignored the required domain and sketched the function for all x. This did not attract full marks, while restricting the domain to between 1 and –1 did. (The correct domain is between 0 and 1.)

The drawing of $y = 90$ was not regarded as acceptable.

Question 4

The question was in three parts; a trigonometric identity, a rate of a change, and a three dimensional solid.

Within this question: (a), (b)(i) and (ii), (c)(i) and (ii) were all ‘prove that ...’ or ‘show that ...’. Although the candidature earned good marks (awarded if the candidate demonstrated that they knew why the result was true), the skill of setting out a mathematical proof, and ability to make clear statements with logical conclusions, was minimal. There was also an inordinate amount of fudging to obtain a given answer, when simple checking on their previous line would have unearthed the error made.

It was also disturbing to the examiners that skills were often ‘centre-based’, with whole centres not attempting (a), while other centres left out (b) or (c). This was especially upsetting when a student showed great sophistication and ability in earning full marks to (a) and (b) and then left out (c) completely.

(a) (3 marks)

Prove that $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$.

Many students obtained full marks, though solutions were often rambling, and the neat two or three line solutions were rare.

For those who did not have completely correct solutions, credit was given for skills used relevant to a plan which (without error) would have led to a solution. The plan for most students was to reduce the given expression to one just involving sines and cosines, by addition formulae followed by double-angle formulae; correctly done this earned 2 marks out of 3, but frequently led to complicated expressions which the student could not handle.

Some 10% of the candidature 'invented' formulae such as $\sin 3\theta = \sin 2\theta + \sin\theta$ or $\sin 2\theta \sin\theta$, and following a sign error $\cos^2\theta - \sin^2\theta = 1$, because it gave the 'right' answer.

(b) (i) (1 mark)

This easy mark just required the student to show one correct step, either tan 60 = $\sqrt{3}$ or comparing ratios with the similar 1:$\sqrt{3}$:2 triangle. Too many students thought the semi-vertical angle was 60° (still managing to fiddle the given answer!) and others made such statements as:

$$\left.\begin{array}{r} r = 1 \\ h = \sqrt{3} \end{array}\right\} \rightarrow r = \frac{h}{\sqrt{3}}.$$

(ii) (1 mark)

Well done, though still a large minority not knowing $V_{\text{cone}} = \frac{1}{3}\pi r^2 h$

(iii) (2 marks)

Any correct chain rule involving $\frac{dV}{dt}$ and $\frac{dh}{dt}$ earned the first mark.

Too many students did not seem to realise that the rate of change was time-based, and one mark was allowed for a correct value of $\frac{dh}{dV}$, but only if this was given as the candidate's honest response to the question. The most common error with a correct chain rule was to substitute $\frac{dV}{dh}$ instead of $\frac{dh}{dV}$.

(c) (i) (1 mark)

A surprising number of non-attempts suggested that these students did not have the spatial perception to see how the net folded up into the pyramid, and in fact this was what the first mark was awarded for, and could be implied at any stage of their response. To show that the 15–20–25 triangle was right-angled, most students simply verified that the sides satisfied Pythagoras, rather than the preferable (but equivalent) use of the cosine rule. Although 90% of the candidature were awarded this mark, perhaps only 20% *earned* the mark by setting out a clear argument with a *conclusion*. Frequently a bald $15^2 + 20^2 = 25^2$ (without so much as a QED) was deemed by students to be sufficient. Of the failures, many ignored the distances and managed to 'prove' that all triangles are right-angled! Also popular was the circular 'if a triangle is right-angled then it is a right angled triangle' by showing that tan–1 $\frac{3}{4}$ + tan–1 $\frac{4}{3}$ = $\frac{\pi}{2}$.

(ii) (1 mark)

The success rate on the question fell off dramatically at this point. Although the simple area argument: $\Delta TXY = \frac{1}{2} \times 15 \times 20 = \frac{1}{2} \times 25 \times TZ$ or similar triangles approach gave the height immediately, many students used Pythagoras in the two small triangles, and found the subsequent algebra too tough, or persevered for two pages to earn one mark.

(iii) (2 marks)

For identifying and calculating the angle between two planes. Overall the response was very disappointing – most students could not identify the required angle.

Question 5

This question contained three unrelated parts. The first two involved 3 unit applications of calculus; the third involved probability.

(a) (i) (3 marks)

The student was required to write down an explicit formula for the temperature T at time t of a cup of coffee which cools according to the differential equation $\frac{dT}{dt} = k(T - T_0)$ and to use given conditions to find the value of the constant k. The three marks were awarded for writing an explicit formula of the form $T = T_0 + Ae^{kt}$, for correctly evaluating A, and for correct logarithmic manipulations to find k. Many students derived the equation although not required to do so, thus wasting valuable time. Some attempted to use integration to solve the differential equation, evaluating the constants from the given conditions. Whilst the best (presumably 4 unit) students were successful, the remainder disadvantaged themselves by the difficulty of this approach. A common error was to confuse the constants, eg $70 - 20e^{kt}$ or $-20 + 100e^{kt}$. Use of e^{-kt} instead of e^{kt} (obtaining a positive value for k) reflected differences in teaching style and was accepted. Overall the standard of arithmetic was very poor and even though the conversion of an exponential equation to a logarithm was generally well done errors such as '$70 = -20 + 120e^{3k}$, therefore $50 = 120e^{3k}$' were quite common.

(ii) (2 marks)

The student was required to use the same model with different initial conditions and evaluate the temperature at a later time. The first mark was awarded for an exponential formula with the correct value of A (namely 50). Most students failed to score this mark. It was more common to score the second mark which was for a correct numerical evaluation of the temperature based on their previous work although this sometimes led to some very unrealistic temperatures.

(b) (2 marks)

The student had to find the acceleration from a formula for velocity in terms of x. Quoting the formula $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ was not by itself enough to score marks here as some candidates quoted the rule and then proceeded to do something else. A large number merely differentiated v with respect to x. This approach yielded zero marks unless it was used to find $v\frac{dv}{dx}$. Once again there were many careless errors in the simple differentiation and substitution.

(c) This question concerned the probability of mice choosing exits from a maze. The four parts were linked, with answers to later parts following on from their previous answers.

Overall this question was very poorly done with very few candidates scoring full marks. It would seem that many students try to do probability by a formulaic approach rather than by actually thinking about the situation and using simple counting techniques. It also seems that even for 3 and 4 unit students probability is something of a mystery.

(i) (1 mark)

Most students obtained $\frac{1}{625}$ instead of $\frac{1}{125}$, forgetting that the first mouse has a choice of 5 exits.

(ii) (1 mark)

Most students made a similar error to that made in (i). The marking scale endeavoured not to doubly penalise this.

(iii) (1 mark)

The many students who correctly realised that this answer should be four times their answer to (ii) were awarded the mark.

(iv) (2 marks)

Full marks were awarded for subtracting their answers to (i) and (iii) from 1. Only a minority successfully did this although some did score 1 mark for an unsuccessful attempt (such as for subtracting the wrong pair of answers from 1). Most students failed to consider the use of complementary events. The alternate method of splitting into mutually

exclusive cases and adding was extremely difficult and whilst the majority tried such an approach virtually none at all succeeded. Most seemed to think that this was a binomial probability question and proceeded to split into cases not all of which were meaningful,
eg P(≤2 use same exit) = P(0 same, 4 different) + P(1 same, 3 different) + P(2 same, 2 different) .

Question 6

This question contained five parts taken primarily from two areas of the syllabus, namely trigonometric ratios/trigonometric functions and applications of calculus to the physical world, specifically on simple harmonic motion.

(a) (2 marks)
This was generally well answered. There were, however, a number of students who needed to differentiate the given function (to write $\ddot{x} = -n^2 x$) to find n and hence find the period. A number of students could not find the correct amplitude and/or period, yet these features appeared correctly on the graph drawn in part (b). Similarly, quite a number of students had the correct amplitude and period in part (a), but did not translate these to their graph.

(b) (2 marks)
Generally, students appear to have very poor graphing skills. Many graphs had one or two features correct, but the other features were inconsistent. Among the many inconsistencies/errors were:

- inconsistent scale used, particularly on the t-axis
- a graph of $x = 6\sin 2t$ (ie no regard for the effect of the $\frac{\pi}{4}$ in the equation)
- the phase shift was often thought to be $\pm \frac{\pi}{4}$
- some marked the t- intercepts correctly, but the turning points were not correctly graphed
- many did not consider the end-points
- a number graphed a function for $0 \le t \le \pi$.

(c) (2 marks)

Although many students knew that to find the velocity they needed to find the derivative, this operation was not always done correctly. Some students wanted to quote formulae such as $\dot{x} = an\cos(nt + \alpha)$ blindly and often incorrectly. More commonly, students attempts included the following:

$\frac{dx}{dt} = -12\cos\left(2t + \frac{\pi}{4}\right)$

or, $\frac{6}{2}\cos\left(2t + \frac{\pi}{4}\right)$

or, $12t\cos\left(2t + \frac{\pi}{4}\right)$

or, $12\sin\left(2t + \frac{\pi}{4}\right)$

The most disturbing attempt was to write $x = 6\sin 2t + 6\sin\frac{\pi}{4}$ before differentiating.

(d) (2 marks)

Some students thought that $x = 3$ was the centre of motion, and therefore solved $\dot{x} = 0$. For those who attempted to solve the correct equation, many considered only one solution, namely $2t + \frac{\pi}{4} = \frac{\pi}{6}$, but either just dropped the negative in their answer, or did not bother to address the sign at all. Others had a series of possible solutions, some of which were incorrect. A surprising number of students had:

$2t + \frac{\pi}{4} = \frac{5\pi}{6}$	or	$2t + \frac{\pi}{4} = \frac{5\pi}{6}$
$2t = \frac{7\pi}{12}$		$t = \frac{7}{24}$
$t = \frac{7\pi}{6}$		

In the second case, students probably used the calculator to deal with the fractions, omitting π from their answer.

Quite a number of students solved the equation, giving the answer in degrees.

(e) (i) (2 marks)

Many correctly found the second derivative of the given function but seemed to have difficulty in factorising it and recognising that $\ddot{y} = -n^2 y$. Students were not comfortable with the variables used and interchanged them. A number of students failed to deal with the given equation, and showed that $w = \sin 2t$ represented a SHM function.

Some students found the derivative of $\sin 2t$ correctly, but not of $\sin\left(2t+\frac{\pi}{4}\right)$, and vice versa.

(ii) (2 marks)
This was difficult for many students. The two most common incorrect answers were: amplitude = 7, and amplitude = $6+\frac{1}{\sqrt{2}}$ (ie at $x = \frac{\pi}{8}$).

Some had the correct approach, but they seemed to lose confidence along the way and reverted to one of these two common incorrect answers.

Question 7

(a) (2 marks)
Students were asked to use the fact that $(1+x)^4(1+x)^9 = (1+x)^{13}$ to prove an identity involving binomial coefficients. Students could gain one mark for demonstrating a knowledge of the binomial theorem, or for attempting to equate coefficients of x^4 (or x^9). To gain the second mark they needed to correctly complete the argument.

No marks were awarded to attempts which demonstrated the result by simply evaluating both sides.

This part of the question was quite well done, with an average mark of around 1 out of 2, but it was still disappointing to see the number of students (over 10%) who did not attempt it, even though they went on to attempt later parts of the question.

(b) Part (b) consisted of six linked sub-parts relating to the quadratic function $f(x)=\frac{1}{4}\left[(x-1)^2+7\right]$ and its inverse.

(i) (2 marks)

Students were asked to sketch the parabola $y = f(x)$. They were told to use the same scale on both axes, as a good sketch here was useful in later parts. In the marking, no marks were deducted for different scales. Full marks were awarded for a vaguely parabolic curve with a correct y-intercept and vertex.

One mark was awarded for a variety of partially correct curves, including concave-up parabolas with a correct y-intercept or axis, or for correctly finding the stationary point.

Despite the fact that the equation of the curve was written in a way to assist graphing it, many students took a number of pages of algebra before they were able to draw a graph.

(ii) (1 mark)

1 mark was awarded for '$x \geq 1$', which was the largest (connected) domain containing $x = 3$ for which the function has an inverse. Students could also gain this mark by writing the correct domain for their (incorrect) graph.

(iii) (2 marks)

To graph the inverse function, the easiest way was to reflect the appropriate part of the original curve in the line $y = x$. One mark was awarded for an attempt to do this, but there were some very odd curves and it was difficult to determine whether or not this was what was being attempted. It made it easier to award the mark if the line $y = x$ was shown.

The most common error was to reflect the complete parabola rather than the right-hand side only (1 mark). Another very common error was to reflect the curve in a line parallel to $y = x$, but passing through the vertex of the parabola. This was awarded no marks.

A large number of students found the equation of the inverse function before sketching it. Most could find the equation successfully, but not many were able to graph it sufficiently accurately to score full marks. However the equation was useful in the next part.

(iv) (1 mark)

Students were asked for the domain of the inverse function. One mark was awarded for the correct answer, or for an answer which was correctly derived from incorrect answers from previous parts. This was done surprisingly well – the mean was higher than for part (ii). It was also surprising how many candidates were not put out by the fact that their (correct) answer for this part bore no resemblance to their graph in the previous part. Many appeared to obtain their domain from their equation for the inverse from the previous part, or from the range of their parabola in part (i), rather than directly from their graph in part (iii).

(v) (2 marks)

This part asked for the value of $f^{-1}\left(f(a)\right)$, where *a* was a number not in the domain where the function had an inverse. This was difficult, and was only completed successfully by a very small percentage of candidates. Most did not attempt it, and the average mark was around 0.1.

Of those who attempted it, the most common approach was to substitute *f*(*a*) for *x* in the equation for $f^{-1}(x)$. A correct substitution was awarded 1 mark, but careful algebra was necessary to gain full marks. In particular, it was necessary to state that, since $a < 1$, $\sqrt{(a-1)^2} = (1 - a)$. Few realised this.

The most elegant approach was to consider the graph, and to realise that the answer was the value (say *b*) in the domain such that $f(b) = f(a)$. By symmetry, *b* is the same distance as *a* from the axis, but on the other side. One mark was awarded for some demonstration of the understanding of this idea; for full marks, a clear argument was required.

(vi) (2 marks)

In this part, students were asked to find any points of intersection of the original curve with its inverse. It was well done compared to the previous part, with an average mark of around 0.4. It helped that the points had integer coordinates, so could be fairly easily found by trial and error, or a variety of other methods.

The key, of course, was to realise that the curves intersect on the line $y = x$. For those who realised this, and who had drawn good graphs (or who had plotted points) in parts (i) and (iii), it was clear that (2, 2) was one point of intersection. Many could also see that (4, 4) was the other.

For those who attempted to find the points algebraically, one mark was awarded for a correct equation in x or y, arising from simultaneous solution of any two of $y = x$, $y = f(x)$, or $y = f^{-1}(x)$. The second mark was awarded for correctly finding both points.

4 Unit (Additional)

Question 1

This question, on the topic of integration was generally very well done. If the students knew their work well, then full marks were easily obtained with minimum effort and minimum time. On the other hand, the amount of time (and paper) spent by some candidates on this question caused concern, especially when they still had another seven questions to do! Many students lost marks because of deficiencies in their arithmetic and algebraic skills.

(a) Evaluate $\int_1^3 \frac{4}{(2+x)^2}\,dx$.

This question was not as well done as the examiners might have anticipated. Successful candidates used one of the following two methods:

(i) $\int 4(2+x)^{-2}\,dx$, applying the formula for $\int (ax+b)^n\,dx$, followed by evaluation;

(ii) use of the substitution $u = 2 + x$, so that the question then became $\int_3^5 \frac{4}{u^2}\,du$.

Many unsuccessful candidates spent pages in trying to transform the integrand using partial fractions. As already mentioned, for some, their arithmetical skills let them down when it came to the evaluation of $-4\left[\frac{1}{2+x}\right]_1^3$.

Students were not penalised for transcription errors.

(b) Find $\int \sec^2\theta \tan\theta\, d\theta$.

This part was very well done by most candidates, with the majority using the substitution (sometimes implied) $u = \tan\theta$, others using $u = \sec\theta$. The other successful students used integration by parts. Unsuccessful candidates attempted to expand the expression, using $1 + \tan^2\theta = \sec^2\theta$, but were then unable to integrate $\tan^3\theta$, so gave up! Some appeared to be already panicking about time spent on this question, and so stopped halfway through their calculations. Some students, and not just a few, wrote down $\frac{d}{dx}(\sec^2\theta) = \tan\theta$, followed by the substitution $u = \sec^2\theta$. If they then successfully obtained their solution

$\frac{1}{2}\sec^4\theta + C$), they were awarded 1 mark. The constant of integration was quite often omitted.

(c) Find $\int \frac{5t^2+3}{t(t^2+1)}dt$.

This part was not done as well done as would have been expected, with algebra and arithmetic, not integration, being the problem. There were at least 10 different successful ways of doing this question, most of these using partial fractions at some stage.

An easy 3 marks was obtained by those who proceeded as follows:

$$\int \frac{5t^2+3}{t(t^2+1)}dt = \int \frac{3t^2+1}{t^3+t}dt + \int \frac{2t^2+2}{t^3+t}dt$$

$$= \int \frac{3t^2+1}{t^3+t}dt + \int \frac{2(t^2+1)}{t(t^2+1)}dt$$

$$= \int \frac{3t^2+1}{t^3+t}dt + \int \frac{2}{t}dt$$

$$= \log_e(t^2+1) + 2\log_e|t| + C.$$

A well-earned 3 marks were obtained by the handful who used the substitution $t = \tan\theta$, as follows:

Put $t = \tan\theta$ so that $\frac{dt}{d\theta} = \sec^2\theta$

$$\therefore \int \frac{5t^2+3}{t(t^2+1)}dt = \int \frac{(5\tan^2\theta+3)\sec^2\theta}{\tan\theta(\tan^2\theta+1)}d\theta$$

$$= \int (5\tan\theta + 3\sec\theta)d\theta$$

$$= -5\log_e\cos\theta + 3\log_e\sin\theta + C$$

$$= -5\log_e\frac{1}{\sqrt{1+t^2}} + 3\log_e\frac{t}{\sqrt{1+t^2}} + C$$

$$= \tfrac{5}{2}\log_e(1+t^2) + 3\log_e t - \tfrac{3}{2}\log_e(1+t^2) + C$$

$$= \log_e(1+t^2) + 3\log_e t + C.$$

Students were not penalised for omitting absolute value signs, no matter which method was used.

Overall, students lost their marks in one of the following ways:

(i) by using $\frac{5t^2+3}{t(t^2+1)} \equiv \frac{a}{t} + \frac{b}{t^2+1}$;

(ii) after correctly solving, for *a*, *b* and *c*, the identity $\frac{5t^2+3}{t(t^2+1)} \equiv \frac{a}{t} + \frac{bt+c}{t^2+1}$ and obtaining $a = 3$, $b = 2$, $c = 0$, they then incorrectly stated $\frac{5t^2+3}{t(t^2+1)} = \frac{3}{t} + \frac{2}{t^2+1}$, then integrated their expression correctly.

(d) Using integration by parts, or otherwise, find $\int x \tan^{-1} x dx$.

Many students obtained 3 marks for this part, with very few non-attempts being encountered. Surprisingly, while their algebra let them down in other parts of this question, most students successfully showed that $\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$.

2 marks were awarded for the following (very common) response:

$u = x,\ \frac{dv}{dx} = \tan^{-1} x$

$\frac{du}{dx} = 1\ v = \frac{1}{1+x^2}$.

$\therefore \int x \tan^{-1} x dx = \frac{1}{1+x^2} - \int \frac{1}{1+x^2} dx$

$= \frac{1}{1+x^2} - \tan^{-1} x + C.$

The only successful 'or otherwise' responses (and there were only two of these) involved use of the substitution $u = \tan^{-1} x$, but then the students still needed to use integration by parts, later on, to find $\int u \tan u \sec^2 u du$ which, surprisingly, then used the answer to (b)!

(e) Using the substitution $x = 2\sin\theta$, or otherwise, calculate $\int_{-1}^{\sqrt{3}} \frac{x^2}{\sqrt{4-x^2}}$

Only one successful 'or otherwise' solution was encountered, and this involved finding the area bounded by $y = \sqrt{4-x^2}$, $x = -1$, $x = \sqrt{3}$ and the *x* axis.

Many students misread the question as $\int_1^{\sqrt{3}} \frac{x^2}{\sqrt{4-x^2}}$ and, if done correctly, had as their solution $\frac{\pi}{3}$ Solution of this definite integral did not require the same standard of trigonometric substitution as the correct solution, so these students received one mark less than those who answered correctly.

Students who correctly evaluated either $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin^2\theta\, d\theta$ or $\int_{-\frac{\pi}{2}}^{\frac{\pi}{3}} \sin^2\theta\, d\theta$ were also awarded 4 marks (out of a possible 5 marks).

Throughout (e), students lost valuable marks because of their poor arithmetic and algebraic skills, especially in the final few lines where they were required to evaluate $[2\theta - \sin 2\theta]_{-\frac{\pi}{6}}^{\frac{\pi}{3}}$. Finally, it is important to mention the large number of candidates who did not substitute, at all, for dx (using $dx = 2\cos\theta d\theta$).

Question 2

This question was well done. There were only two non attempts and quite a few candidates obtained full marks. The question was in four sections. Generally the first three were well done, with many candidates obtaining the full 11 marks for these sections. The last section, worth 4 marks, posed some problems with many candidates omitting it altogether or making poor attempts. It was pleasing to note that many candidates who attempted (d)(i) could give clear, logical and well documented explanations. It was also pleasing to see so many well prepared, talented students who write their mathematics clearly and confidently.

(a) Well done. Most students obtained full marks. The main errors were with signs. Students were awarded one mark for each correct part.

(b) Well done with (mostly) clearly labelled diagrams. Some candidates left it to the imagination of the marker to decide which region they meant as the answer and lost unnecessary marks as a result. There were many errors with the centre of the circle (–4, 5), (4, 5) and (–4, –5). If the region was otherwise correct, candidates were awarded 1 mark. Many candidates wasted time on algebra for this small question.

(c) (i) Well done. The main loss of marks was due to the use of de Moivre's theorem in proving the result or due to the conclusion being absent or inadequate (eg $n \geq 1$). Some candidates took two pages to prove this result which may have prejudiced their later questions.

(ii) Unfortunately some candidates omitted this section, maybe due to the length of time they had spent on (i) or just carelessness. The question was very easy! One mark was awarded for the correct argument and one for the correct modulus. Most students gained full marks though there was a surprising array of arguments!

(iii) Well done. Full marks were awarded for those who correctly followed through from their incorrect answer to (ii). One mark was awarded for the correct mod-arg form and one for the correct $x + iy$ form. Candidates who found the correct answer without following the 'hence' instruction were awarded 1 mark.

(d) (i) There were many answers to this section. The most common answer used the external angle. The next most common answer used the construction of a parallel line at (–1, 0) or (3, 0) or at z (parallel to x–axis) and then used alternate or corresponding angles in their explanation. There were many well-drawn diagrams with angles clearly marked. Candidates were awarded 1 mark for clearly identifying, either on a diagram or by words, the two angles $\arg(z - 3)$ and $\arg(z + 1)$ and a further mark for a clear reason for $\theta = \pi/3$.

Reasons such as '$\theta = \pi/3$ because it is the angle between the lines' were not considered sufficient without other evidence. Similar statements were '$\arg(z + 1)$ rotated by $\pi/3$ equals $\arg(z - 3)$, therefore $\theta = \pi/3$.'

Problems: Many candidates were confused between vectors, lines, and arguments, eg 'the line $\arg(z - 3)$'. Some used circuitous arguments.

Some made irrelevant statements such as 'angles standing on the same arc are equal, therefore $\theta = \pi/3$.' Generally candidates with clear diagrams with angles marked correctly received at least 1 out of 2.

(ii) Again there are many ways to answer this question. Unfortunately many chose long and difficult algebraic methods and wasted valuable time. The quickest and easiest methods involved geometry. The most common answer involved using the angle at the centre of the circle being equal to 2θ and the fact that the centre lies on the perpendicular bisector of the chord between (–1, 0) and (3, 0). The answer took two lines! Other candidates placed z above the centre and dropped a perpendicular to the chord between (–1, 0) and (3, 0). Still others placed z so that the line between (–1, 0) and z became the diameter and used the angle in the semi circle is a right angle. The main successful algebraic method was using the tan(A + B) formulas. One student found the equation of the perpendicular bisectors of two sides of the triangle and solved these simultaneously.

Many students received 1 mark for the x coordinate of the centre by noticing it was the bisector of the chord. Many candidates made errors such as $x = (3 - -1)/2 = 2$ therefore the x coordinate of the centre equals 2. A few candidates had the correct answer mixed in with other working but did not identify it as the answer, so it was not possible to award marks.

Again, candidates should be encouraged to draw diagrams where possible to help them with their solutions.

Question 3

This question consisted of three parts. The first part related to volume via the rotation of an area about a vertical axis. When taking a slice perpendicular to the axis of rotation the cross sectional area was an annulus. The second part was on integration via 'show that', using binomial expansion and/or sigma notation. The last part was an application on the interpretation of a velocity-time sketch.

This question was reasonably well attempted by the great majority of the candidates, with many gaining full marks. That being said, there were a substantial number who did not perform as well as one might have expected for question 3 on this paper, and the standard, presentation and skills demonstrated by candidates was less than expected.

Many candidates lost marks due to unwarranted carelessness in their setting out, basic numerical/algebraic errors, not using the given information and fudging their result. As has often been observed, candidates still have particular problems with the 'show that' type question.

(a) Some candidates appeared inexperienced and/or unskilled in finding an area. Frequently students could not find the correct expression for the outer radius of the annulus. Many others had no idea on part (i) but could do part (ii).

(i) (2 marks)
1 mark was awarded for obtaining a correct expression for the area of the annulus pre-expansion in terms of *x* or *y* or *S* (ie identifying the correct radii in $\pi[R^2 - r^2]$).

The second mark was awarded for correctly gaining the required result.

It was noticeable that some candidates took a few attempts at this part before they could show that the area of the annulus was $\pi(y^4 + 8y^2 + 7)$.

There was much fudging with the expression for the outer radius of $4 - x$ to get $4 + y^2$ to gain the correct result of $\pi(y^4 + 8y^2 + 7)$. Candidates were uncertain about the *x*-value which was in the 2nd quadrant and so negative. Nevertheless, they tried to cater for it!!.

(ii) (3 marks)
1 mark was given for correct upper & lower bound for *y* and *dy*,
1 mark for a correct primitive of *A(y)* (with three terms), and
1 mark for a correct evaluation of the primitive.

Careless errors with algebra and numerical handling were identified in this section. Many did not use the stated result from part (i) even when theirs was not the same!!

Many could only do this part and so gain 3 out of 5 marks for part (a).

(b) The candidates had a lot of trouble with this part and appeared to lose their way. Parts (i) and (ii) were the parts which were done best.

(i) (1 mark)
The mark was awarded gaining a correct primitive via the chain rule or by substitution without any subsequent error to show the required result.

This was well done. Many showed the result by integration by parts. A few tried mathematical induction!

(ii) (4 marks)
Basically the marking scheme awarded:
1 mark for correct 1st 3 terms of the binomial expansion of or $(1-\sin^2 x)^n$ or $(1-u^2)^n$;
1 mark for the correct last term of the binomial or the general term: $(k+1)$ th;
1 mark for identifying clearly the $(-1)^k$ in sigma notation or expansion with $\sin^{2k} x \cos x$; and
1 mark for producing the correct expression via b(i) and or evaluating the definite integral.

This was not well attempted. It did discriminate between the candidates. The ones with skill and experience with $\sum$ notation made it very easy. Many stopped at $\int_0^1 (1-u^2)^n \, du$ and could not connect it with expansion and $\sum$ notation.

(iii) (1 mark)
1 mark for gaining $\frac{8}{15}$ or the correct numerical equivalent of $1-\frac{2}{3}+\frac{1}{5}$.

This was reasonably well done by the candidates. However, it was surprising that many who gained the marks in part (ii) did not gain the mark for this part. Many used $n=5$ instead of $n=2$, unfortunately.

Many did not use the result given in part (ii) ie the 'hence'. There were many who used the 'otherwise' approach.

Eg I:

$$I = \int_0^{\frac{\pi}{2}} \cos^5 xdx = \int \cos^4 \cos xdx = \int (1-\sin^2 x)^2 \cos xdx \equiv \int_0^1 (1-u^2)^2 du$$
$$= \int \cos x - 2\sin^2 x \cos x + \sin^2 x \cos x + \sin^4 \cos xdx \equiv \int 1 - 2u^2 + u^4 du$$
$$= [\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x]_0^{\pi/2} \equiv [u - \frac{2}{3}u^3 + \frac{1}{5}u^5]_0^1$$
$$= [(1 - \frac{2}{3} + \frac{1}{5}) - (0)] = \frac{8}{15}.$$

This approach gained 1 mark for part (iii) and students who had otherwise scored 0 for part (ii) could gain 1 mark for that part here, having displayed a correct method and expressions.

Eg II: Candidates used reduction formulae either quoted or mainly derived:

$$I_n = \int \cos^{2n+1} dx = \frac{2n}{2n+1} I_{n-1}.$$

So $I_2 = \frac{4}{5} I_1 = \frac{4}{5}\frac{2}{3} I_0 = \frac{4}{5}\frac{2}{3}\int_0^{\frac{\pi}{2}} \cos xdx = \frac{4}{5}\frac{2}{3} 1 = \frac{8}{15}$

or $I_n = \int_0^{\frac{\pi}{2}} \cos^N xdx = \frac{N-1}{N} I_{N-2}.$

So, $I_5 = \frac{4}{5} I_3 = \frac{4}{5}\frac{2}{3} I_1 = \frac{4}{5}\frac{2}{3}\int_0^{\frac{\pi}{2}} \cos xdx = \frac{4}{5}\frac{2}{3} 1 = \frac{8}{15}.$

Both of these approaches gained the mark.

Eg III: Some candidates tried to use De Moivre's Theorem via:

$z = cisx$, so that $z + \frac{1}{z} = 2cisx$, and $z^n + \frac{1}{z^n} = 2cisnx$;

$\therefore \cos^5 x = \frac{1}{32}\left(z + \frac{1}{z}\right)^5 = \ldots = \frac{1}{32}\left(\left(z^5 + \frac{1}{z^5}\right) + 5\left(z^3 + \frac{1}{z^3}\right) + 10\left(z + \frac{1}{z}\right)\right).$

This led to

$$\int_0^{\frac{\pi}{2}} \cos^5 x\, dx = \frac{1}{32}\int_0^{\frac{\pi}{2}} 2(\cos 5x + 5\cos 3x + 10\cos x)\,dx = \ldots = \frac{8}{15},$$

and gained the mark.

(c) This part was well attempted and frequently done first. It was a good idea for the candidates to make a copy of the sketch as it helped many explain their response to all three parts.

(i) (1 mark)
The mark was given for a value of t in the range $6.5 \le t < 7$ and a correct explanation. This was basically: as acceleration is the gradient function of the velocity-time graph then the greatest acceleration occurs at the steepest slope (of the tangent) on this curve, or words to this effect.

Most gained their mark for this part. However, many others stated that the greatest acceleration occurs when $v = 0$, so $t = 0$ or just $t = 7$!! and so did not gain the mark.

(ii) (1 mark)
The mark was given for $t = 4$ and a correct explanation. This was basically: As displacement is the area enclosed by the velocity curve and the t axis, then when $x = 0$, $t = 4$ The candidates found many different ways in which to express the essence of this idea. Others based correct explanations on the fact that acceleration was uniform over $0 \le t \le 5$.

Some candidates thought that in parts (i) and (ii) the question had something to do with simple harmonic motion or, occasionally, projectile motion. This was possibly due to acceleration being constant.

(iii) (2 marks)
1 mark was awarded for a curve through (0,0) and (part (ii),0) and showing 2 turning points [min at $t = 2$ and max at $t = 7$].

The other mark was for indicating $x(9) < 0$ in their displacement graph. Many gained this second mark but many others had lost the thread of the plot by now or did not realise the significance of the attention given to (signed) areas under the curve part (ii) had for answering part (iii).

Question 4

This question had three main parts. Only 1% of the candidates made no attempt at all at this question, but even this low rate was disappointing since most candidates who attempted the question gained at least one mark. The most pleasing aspect of the responses was generally large clear sketches of the graphs, while the most disappointing aspect was the poor understanding of probability.

It needs to be emphasised that candidates should attempt every part of every question. Since it is relatively easy to gain the first mark or two, even partial completion of a correct method is usually sufficient to enable the candidate to score some marks.

(a) (3 marks)
By differentiating both sides of the formula

$$1 + x + x^2 + x^3 + \cdots + x^n = \frac{x^{n+1} - 1}{x - 1},$$

find an expression for

$$1 + 2 \times 2 + 3 \times 4 + 4 \times 8 + \cdots n2^{n-1}.$$

A disappointing number of students failed to differentiate the polynomial expression correctly, omitting indices or introducing a minus sign. The expression on the right also caused a number of problems, with candidates frequently failing to write the denominator $(x-1)^2$. Candidates who failed to do so deprived themselves of two marks, since it yielded a much easier expression to simplify in the subsequent work for this part. Manipulation of indices, removal of parentheses and factorisation were carelessly done. For instance, $n(2^n - 1)$ became $(2n)^n - 1$ and $-(2^{n+1} - 1)$ became $-2^{n-1} - 1$.

(b) (i) (2 marks)

On the same set of axes, sketch and label clearly the graphs of the functions $y = x^{\frac{1}{3}} e^x$ and $y = e^x$.

Candidates generally drew the graphs correctly although mistakes were often made in the nature of the concavity of the second graph.

(ii) (3 marks)

Hence, on a different set of axes, without using calculus, sketch and label clearly the graph of the function $y = x^{\frac{1}{3}} e^x$.

While most candidates recognised that the graph would pass through the origin and that y and x would both approach infinity, about half did not realise that y would approach zero as x approached negative infinity. Some candidates did not take sufficient care in clearly showing not only a minimum turning point in the third quadrant but also the curve approaching the x axis and so failed to gain full marks. A significant number of candidates drew the three graphs of parts (i) and (ii) all on the same set of axes, making it difficult to distinguish between them.

Many candidates did use calculus, either to draw their graph or to convince themselves that their graph was in fact correct. Many of these (unnecessarily) attempted to hide the fact by performing the computations in the writing booklet for another question. This only succeeded in making more work for clerical staff and markers who make every effort to assemble all the relevant work done by candidates for each question so that the proper mark can be determined.

(iii) (1 mark)

Use your sketch to determine for which values of m the equation $x^{\frac{1}{3}} e^x = mx + 1$ has exactly one solution.

At least half the candidates failed to attempt this part, and of those who did, half failed to appreciate that a graphical approach would quickly yield the answer $m < 0$.

(c) Consider a lotto-style game with a barrel containing twenty similar balls numbered 1 to 20. In each game, four balls are drawn, without replacement, from the twenty balls in the barrel.

The probability that any particular number is drawn in any game is 0·2.

(i) (1 mark)
Find the probability that the number 20 is drawn in *exactly* two of the next five games played.

A surprising number failed to recognise that this was a question involving the binomial distribution. The values assigned to *n, p, q* and *r* in $^{n}C_{r}p^{n-r}q^{r}$ were many and varied. The numerical answer was often calculated incorrectly even when the correct values were assigned to the pronumerals.

(ii) (2 marks)
Find the probability that the number 20 is drawn in *at least* two of the next five games played.

Here many candidates were confused as to whether to include the case of two successful selections of the number 20. Even those who attempted to add the probabilities of the four favourable outcomes often omitted the binomial coefficients. One disturbing aspect was that candidates happily accepted probabilities of more than one with no attempt to query the basis for their calculations.

(iii) (2 marks)
Let *j* be an integer, with $4 \le j \le 20$.

Write down the probability that, in any one game, all four selected numbers are less than or equal to *j*.

Very few candidates received full marks for this part, although many did calculate the size of the sample space correctly as $^{20}C_{4}$. A few candidates correctly used an inductive approach to arrive at the correct answer.

Another successful group clearly attempted to visualise the process of selection. This part and the next were the parts with the most non-attempts.

(iv) (1 mark)

Show that the probability that, in any one game, j is the *largest* of the four numbers drawn is

$$\frac{\binom{j-1}{3}}{\binom{20}{4}}.$$

Because the answer was given, those candidates who attempted this part resorted to all sorts of desperate arguments to justify the required result. The (relatively few) successful ones simply stated that there were $(j-1)$ numbers left, of which three had to be selected.

A few general comments about the question are in order. Overall, there appeared to be no problems in interpreting the question: it was set in clear and unambiguous terms. In (c)(ii) some candidates may have earned more marks if it had been made even clearer that the number of correct draws had to be 2, 3, 4 or 5. These candidates appeared to have lost marks because they failed to understand the meaning of ‘at least’ and their other work suggested that this was all that prevented them from earning the marks. On the other hand, it must be said that the language of probability, including such phrases as ‘at least’, ‘more than’ etc. is an important part of this area of the syllabus.

It has also been suggested that the ‘show’ part of (c), that is (iv), would have been better set in (iii) to act as an internal check for the candidate. This would have helped the ones successful in the revised (iii) to deduce the result in the new (iv).

In conclusion, candidates need to proceed carefully through routine calculations so that easy marks are not sacrificed needlessly. Furthermore, in questions structured like this one, errors in the early parts often make subsequent parts more difficult and complicated than they are intended to be, and so make it more difficult for the student to earn the later marks. If, as in part (a), errors happen to make the question simpler, students will not have the opportunity to show the level of skill and sophistication to gain the later part marks.

Question 5

If a student persevered with this question, there were marks to be gained at frequent intervals. Perhaps the major comment to make is the weakness that appeared in the manipulation of algebraic fractions. One wonders whether this caused by the excessive use of calculators in the years in which students learn basic arithmetic skills.

Part (a) required careful reading to ensure correct interpretation. Students who sketched the graphs tended not to confuse the situation involving the two hyperbolas.

(a) (i) (2 marks)

Most students applied the mid-point formula correctly but many found difficulty when attempting to simplify $\frac{1}{3t} + \frac{1}{t}$ or $\frac{4/3t}{2}$.

The words ‘straight line’ were sighted by some who proceeded to find the equation not required until section (ii). The elimination of the parameter was well done by candidates who understood the word locus.

(ii) (2 marks)

This section should have been answered far better than it was. Simplifying $\dfrac{\frac{1}{3t} - \frac{1}{t}}{3t - t}$ caused great concern. A very common error was, for example, $1/3t$ becoming $\frac{1}{3}t$.

Students tended to compare equations of lines rather than compare gradients. The curve $y = 1/x$ was frequently used as ‘... the locus in part (i)’. Maybe the question could have read ‘... is tangent to the curve $3xy = 4$’.

Apart from using gradients, one of the most efficient methods used was to find the equation of the tangent and to show that the two points satisfied this equation.

(iii) (2 marks)

This section was extremely well done by nearly all the candidates.

(iv) (2 marks)

Substituting $R(0, h)$ in $t^4 - t^3x + ty - 1 = 0$ to obtain $t^4 + th - 1 = 0$ was well done. Showing that this equation had exactly two solutions caused difficulty. There were two methods used by candidates.

Method 1 required three facts about $f(t) = t^4 + th - 1$:

a) one stationary point

b) always concave upwards

and c) f(0) = –1

Method 2 involved graphing ingredients of $f(t) = t^4 + th - 1$, such as $f(t) = t^4$ and $f(t) = 1 - th$, and showing these graphs intersect exactly twice. This method was more efficient.

Most students, however, were unsure of a path to follow after the substitution.

(b) (i) (1 mark)

The word 'real' applying to the coefficients of $P(x)$ had to appear in a meaningful context to gain the mark.

(ii) and (iii) (2 marks each)

There was a degree of dependence between these two parts. Rarely did a student obtain a correct solution for (iii) and not (ii). Four methods were mainly used.

Method 1: Finding $\Sigma\alpha$, $\Sigma\alpha\beta$, $\Sigma\alpha\beta\gamma$, $\Sigma\alpha\beta\gamma\delta$

A mark was lost in part (ii) for a sign error for $\Sigma\alpha$ and $\Sigma\alpha\beta\gamma$. Successful algebraic manipulations gained the final marks.

Method 2: Substituting ki, $-ki$ in $P(x)$ then either solving simultaneously or equating real and imaginary parts. This method was well done if the expansions containing 'i' were successful.

Method 3: Using the division algorithm and showing the remainder to be zero. Usually $(x^2 + k^2)$ was the divisor. There was plenty of room for algebraic error in this process.

Method 4: Equating coefficients. The student had to be sure to give different coefficients to $(x^2 + k^2)(x^2 + Ax + B)$ than $x^4 + ax^3 + bx^2 + cx + d = 0$ and explain relationships, eg coefficients of x^3, gives $A = a$.

(iv) (2 marks)

Many candidates substituted $x = 2$ and $b = 0$ and then stopped. Some went on to form a complicated quadratic in C with only one other pronumeral. This earned the first mark and led towards the second.

The simpler approach, taken by many who had summed the roots in the earlier section, was to substitute 2 for β in $\Sigma\alpha$ and 0 for b in $\Sigma\alpha\beta$. Two more simple substitutions $[(k^2 = c/a)$ and $\alpha = -(a + 2)]$, given earlier, led to the answer. However many students asserted that α (or β) was an integer as part of their reasoning to obtain the answer. This is not obvious, although it is in fact true that if 2 is a root of the polynomial, the other real root is also an integer. To 'show' c was even required students to obtain $c = 2a(a + 2)$, and then use the fact that the question stated that 'a' was an integer.

Another approach was to divide $(x - 2)$ into the quotient from Method 3 in parts (ii) and (iii). This yielded a remainder $k^2 - 2(a + 2)$ which finally gave the answer when equated to 0.

In summary, students who used gradients in part (a) when equations of lines where not essential and the sum of the roots in part (b) were more successful in answering the complete question.

Question 6

(a) The clearest approach is to sketch the two curves $y = 3x^2 - 2x - 2$ and $y = |3x|$ on one set of axes, then to find their two points of intersection by solving the two quadratics:

$$3x^2 - 2x - 2 = 3x \text{ and } 3x^2 - 2x - 2 = -3x,$$

and then to read the solution to the inequation off the graph. It was disappointing that few drew a sketch, and that about half of these sketches failed to show one or both points of intersection. With or without sketch, nearly everyone used inequations rather than equations (many thinking only one was involved) and the manipulation of inequality signs was often wrong. There was even considerable difficulty solving the quadratics accurately, with mistakes common in adding $3x$ to $-2x$, in factoring, in use of formula, in choice of signs, and in thinking $x = \frac{3}{2}$ solves $3x-2=0$.

Once the quadratics were solved, a minority explained carefully that one argument was valid only when $x \geq 0$ and the other when $x \leq 0$, but most seemed to have little idea how to proceed to the final solution. Most who solved the quadratic inequalities and found two intervals then took their intersection, and many correct solutions seemed only the result of good luck (the union of the intervals is the solution, but only because each interval happens to overlap $x = 0$). It is quite clear that there is little understanding amongst candidates of the words ‘and’ and ‘or’.

Some squared both sides, which is a valid method of solving the equation, but is invalid for the inequation. Difference of squares makes this method quick and attractive, but most who squared got nowhere, and the few who did manage to find all four roots used long division.

Most solutions were poorly set out, with few indicating clearly how the logic proceeded, what the two cases were, or where the divisions fell between the first case, the second case and the conclusion.

(b) This question was poorly done. The great majority who attempted it ignored the clear instructions and resolved horizontally and vertically, and of these only a handful realised that the acceleration then needed to be resolved as well as the forces. Those who resolved forces as instructed made many mistakes in sign and in the choice of trigonometric function. There was a reluctance to write down resolutions according to Newton's second law:

Force = Mass x Acceleration.

Questionable ideas like 'centripetal force' and 'centrifugal force' were common, and there was a general blurring of the distinction between forces and accelerations, both on the diagram and in the equations.

Many solutions were poorly presented and extremely difficult to read, with no indication of the flow of the logic.

(c) Most candidates managed reasonable progress with this question, but less than a dozen scored full marks on (v). The reasoning required in this part was unexpected, being the reverse of the usual argument about how the parabola focuses light at the focus, and many candidates seemed to be answering individual parts with no understanding of how the question held together.

Many candidates seriously misinterpreted the language and logic of the question. Some assumed in (i), (ii) or (iii) that Q was the focus, not realising that this was to be proven in (iv). Some assumed in (i) that $TQ = PQ$ (sometimes claiming this as a parabola's definition), or assumed from the diagram that $PQ = PL$ or that $QL \parallel TP$. Many misunderstood the grammar of (iv), where the examiners placed the information that $(a,0)$ is the focus in a position to the complement in the statement to be proven. Some interpreted the word 'path' in (v) to mean 'path of light', correctly pointing out that only light parallel to the axis of symmetry is reflected through the focus, and thus failing to answer the question. A very few were rightly concerned whether L was above, below or on the x-axis, and therefore whether t or θ or $\tan\theta$ could be negative or zero, or even whether the point Q was well defined.

The question involved writing five successive proofs, and unfortunately many candidates did not seem to understand that in such questions they absolutely cannot omit the smaller details of their working.

Part (i) required a minimum of geometrical reasoning. Although most scored well, the argument was often poorly set out, with answers made difficult to read by omission of reasons, slips in naming vertices, irrelevant digressions in logic, failure to describe or to draw a construction, or poor handwriting. A large number of candidates were not able to use the words ‘alternate’, ‘corresponding’ and ‘co-interior’ correctly. Many were unaware that the word ‘supplementary’ means only that two angles add to 180^{o}, and is not a geometrical reason why they do so. For example, opposite angles in a cyclic quadrilateral are supplementary.

Part (ii) was generally well done, and it was encouraging to see parametric or implicit differentiation freely used to avoid fractional indices. Setting out, logic and handwriting were often poor.

Part (iii) was less well done, but most who tried could find tan with a sizeable minority however believing that $\tan\theta = 2\tan\theta$ or otherwise getting the formula wrong.

Part (iv) was easily done by most candidates. However the logic was poorly expressed, and few wrote a clear conclusion. Many claimed they had proven that $(a, 0)$ was the focus.

Attempts to answer (v) were not common and only rarely proceeded beyond a correct or incorrect statement of the focus-directrix property, sometimes followed hopefully by a conclusion. Many of those who did begin to argue wrote such poorly composed or badly handwritten sentences that it was difficult to make out what they were trying to say. Only a bare handful either regarded *P* as a variable point, or better, placed another point *P’* on the curve and argued that the path via this point was longer. The subsequent geometric arguments are straightforward, and although algebraic methods using the distance formula are possible, none who tried them were able to complete them.

Question 7

Part (a) of this question, on the motion of a particle along the *x*-axis, contained three sub-parts, and was worth 6 marks.

Part (b) contained five sub-parts, and was worth 9 marks. The question concerned various properties of the logarithm function, and led students to the derivation of an expression similar to Stirling's formula.

Most candidates made a reasonable attempt at part (a), and at (b)(i) and (b)(iii). The majority did not attempt (b)(v), and there were very few full marks.

Of the eight sub-parts of the whole question, seven required students to show a given result. The advantages of such questions are clear. Students can attempt later parts, even if they have been unable to do previous parts, and the question can include interesting results, as this one did. Students must, however, be able to convince the marker that they have, in fact, *shown* the required result, and not just lifted it off the paper. Students lose marks in these types of question because of their traditional reluctance to write sensible sentences.

Unfortunately, good students are often disadvantaged by these questions, since they will easily see how the result follows, and may not write sufficient reasons. (The more obvious the result, the more this applies.)

(a) Students were given the acceleration, $\frac{d^2x}{dt^2}$, as a function of *x*, of a particle starting from rest at $x = 1$.

(i) (1 mark)
Students were asked to show that the particle starts moving in the positive *x*-direction. The mark was awarded for substituting $x = 1$ into $\frac{d^2x}{dt^2}$, and making some comment. Responses such as $a>0$ (and nothing else) did not score the mark.

(ii) (4 marks)
Students were asked to derive a given expression for the velocity of the particle.

1 mark was awarded for knowing that $\frac{1}{2}v^2 = \int \frac{d^2x}{dt^2}dx,$ 1 for the integration, 1 for using the initial conditions to obtain the constant of

integration, and the final mark for observing that the positive square root is taken, for $x \geq 1.$ (The fact that the particle starts with positive velocity does *not* guarantee that it will remain positive for $x \geq 1.$, but the final mark was awarded even if the correct reason for $v > 0$ when $x \geq 1.$ was not given.) Most students scored at least 3 marks. Unsuccessful attempts included those who wrote $v = \int \frac{d^2x}{dt^2} dx$. Happily, only a handful of students integrated with respect to *t*.

(iii) (1 mark)

Students were asked to describe the behaviour of the velocity of the particle after it passed a certain point. The examiners probably had in mind, a description of the long-term behaviour. However, in marking the question, the view was taken that the question could legitimately be interpreted as meaning the behaviour *just after* the point mentioned (where the acceleration became negative), and so students were awarded the mark for saying that the velocity decreased (or, of course, for a correct description of the eventual behaviour).

(b) (i) (2 marks)

This very simple part asked students to show that a function has a single maximum turning point. Because the question is so easy, marks were awarded only if the result was well and truly shown. Most students scored the first mark, for the solution $x = \frac{1}{a}$ to $f'(x) = 0$.

Lamentably few students commented that the equation had one solution only. Many students failed to score the second mark, which was awarded for showing that the turning point is a maximum. Students who found the second derivative were generally okay, although a significant number simply wrote down $f''(x) = -\frac{1}{x^2}$ and claimed 'therefore maximum'. (They did not score the mark.) Students who stated that $f(x)$ was positive for $x < \frac{1}{a}$ and negative for $x > \frac{1}{a}$, or who drew little boxes such as:

x	$\frac{1}{a}^-$	$\frac{1}{a}$	$\frac{1}{a}^+$
$\frac{dy}{dx}$	+	0	–

were awarded the mark only if there was some work shown to justify the statements. Very few students scored the mark this way.

(ii) (1 mark)

This part required students to show that any chord joining two points on the curve $y = \ln x$ lies below the curve, using part (i) or otherwise. The majority did not use part (i), and those who did were rarely successful. To score the mark (using 'otherwise'), the fact that the curve $y = \ln x$ is always concave down had to be mentioned. Unsuccessful attempts included students who wrote statements in which they confused $\ln x$ with $f(x)$ (from part (i)).

(iii) (1 mark)

Almost all students who attempted this part gained the one mark for using integration by parts to show that $\int_1^k \ln x\,dx = k\ln k - k + 1$.

(iv) (2 marks)

Students were asked to use the trapezoidal rule to show that the integral in part (iii) is approximately equal to $\frac{1}{2}\ln k + \ln[(k-1)!]$ The first mark was awarded for a correct expression obtained by using the trapezoidal rule, and the second for a clear indication that $\ln 2 + \ln 3 + \cdots + \ln(k-1)$ can be written as $\ln\{(k-1)!]$. I would guess that about half the candidature attempted this part, and only about half of those scored 2 marks. The trapezoidal rule is a simple concept, and students are bound to fare better if they understand the idea, rather than just know a formula. It was clear that quite a few students are unaware of the fact that the trapezoidal rule involves areas of trapeziums. A surprising number of students used a wrong formula, or were unable to apply a correct formula. (For example, some students wrote down a formula involving 'n', but had no idea what this represented.) Many students did not even draw a diagram!

(v) (3 marks)

This part asked students to use parts (ii), (iii) and (iv) to derive an upper bound for $k!$.

The first mark was awarded for a statement that the result in part (iv) is less than the result in part (iii). Since only a small minority attempted this part, this first mark was awarded even if no reason was given. Only a very few students appeared to realise that part (ii) had anything to do with this. The second mark was awarded for correctly collecting the logarithms together and removing them, and the third mark for introducing $k!$ correctly (or vice versa). Of those who attempted this part, a reasonable number scored 3 marks, although many were unable to progress from the initial statement.

Question 8

Most students attempted this question, but it was noticeable that some students were more effective time managers than others. While it was true that there were some easy marks to be picked up on the question, it was equally true that it was easier to pick up marks on the earlier questions, and so students should not have spent a lot of time on this question unless they were confident of having answered the questions on the first half of the paper to the best of their ability. It was distressing that a large number of students spent a lot of time on this question for very little reward.

Many students who tackled this question were obviously rushing, and there were many careless errors which could be attributed to this. Unfortunately, many students could not read their own handwriting, and θ become zero, π (written r) become n, and 9 became a. In many cases this led to incorrect or even absurd answers.

(a) Let $w = \cos\frac{2\pi}{9} + i\sin\frac{2\pi}{9}$

(i) Show that w^k is a solution of $z^9 - 1 = 0$ where k is an integer.

(ii) Prove that $w + w^2 + w^3 + w^4 + w^5 + w^6 + w^7 + w^8 = -1$

(iii) Hence show that $\cos\left(\frac{\pi}{9}\right)\cos\left(\frac{2\pi}{9}\right)\cos\left(\frac{4\pi}{9}\right) = \frac{1}{8}$

Part (i) was generally done successfully, though frequently by the inefficient method of finding all the solutions of the equation then checking that w^k was one of these. On the whole, checking that $(w^k)^9 - 1 = 0$ using de Moivre's theorem is a quicker, easier, and neater approach.

Part (ii) was frequently attempted, either by considering the sum of the roots of the equation $z^9 - 1 = 0$, by using the formula $z^9 - 1 = (z-1)(z^8 + z^7 + \cdots + z + 1)$, or by summing a GP. Many students lost marks here by not explaining what they were doing – all too many wrote just:

$$1 + w + w^2 + w^3 + w^4 + w^5 + w^6 + w^7 + w^8 = 0$$
$$\therefore w + w^2 + w^3 + w^4 + w^5 + w^6 + w^7 + w^8 = -1$$

which was awarded no marks. Some mention of the sum of the roots would have produced a different outcome for these students.

Part (iii) was very hard, and only a handful of students were successful. One approach was to use (ii) to write

$$2\cos\left(\tfrac{2\pi}{9}\right) + 2\cos\left(\tfrac{4\pi}{9}\right) + 2\cos\left(\tfrac{6\pi}{9}\right) + 2\cos\left(\tfrac{8\pi}{9}\right) = -1$$

and then to use the formula $\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ several times. Another was to use first the fact that $\cos\left(\frac{8\pi}{9}\right) = -\cos\left(\frac{\pi}{9}\right)$ to write the product of cosines as $\left[\frac{1}{2}(w + w^8)\right]\left[\frac{1}{2}(w^2 + w^7)\right]\left[\frac{-1}{2}(w^4 + w^5)\right]$, then to expand out and simplify (using $w^9 = 1$). Some students attempted to use sums of products of roots with the factorisation

$z8 + z7 + \cdots + z + 1$

$= (z^2 - 2\cos\frac{2\pi}{9} z + 1)(z^2 - 2\cos\frac{4\pi}{9} z + 1)(z^2 - 2\cos\frac{6\pi}{9} z + 1)(z^2 - 2\cos\frac{8\pi}{9} + 1)$

but many omitted the term with $\cos\frac{6\pi}{9}$ in it, and in any case this is difficult unless the trick substitution $z = i$ is used.

(b) The points P, Q, R lie on a straight line, in that order, and T is any point not on the line. Using the fact that $PR - PQ = QR$, show that $QT - QP > RT - RP$.

This question used the given fact and the triangle inequality in triangle QRT. In this triangle, there are three possible inequalities:

$QR < RT + TQ$, $RT < TQ + QR$, and $TQ < QR + RT$.

The middle one is relevant for this problem. It was disappointing that many students thought that only the first of these triangle inequality holds. Most students were reluctant to use the strategy of reducing the complicated expression to be proved to an equivalent simpler expression (using the stated fact). Those who did solved the question in three lines, while regrettably others filled pages without success. A number of students tried an ‘algebraic’ approach, considering the inequality $(Q - R)\ (T - P)\ .\ 0$ to no avail.

(c) (i) $ABCD$ is a quadrilateral, and the sides of $ABCD$ are tangent to a circle at points K, L, M and N, as in the diagram. Show that

$$AB + CD = AD + BC.$$

(ii) $ABCD$ is a quadrilateral, with all angles less than 180°. Let X be the point of intersection of the angle bisectors of $\angle ABC$ and of $\angle BCD$. Prove that X is the centre of a circle to which $AB, BC,$ and CD are tangent.

(iii) $ABCD$ is a quadrilateral, with all angles less than 180°. Given that

$$AB + CD = AD + BC,$$

show that there exists a circle to which all sides of $ABCD$ are tangent.

You may use the result of part (b).

Part (i) was very accessible: it relies on the theorem that ‘tangents from an external point are equal’, ie $AK = AN$, $BK = BL, CM = CL$, and $DM = DN$. Adding these formulae then yields the result. Many students gave this proof. Many found the bookkeeping easier when the segments were labelled differently, eg $AK = x$, $BL = y$, etc. When the labelling system was logical, eg $AK = a$, $BL = b$, etc, then the bookkeeping is even easier. It was disappointing that students frequently failed to justify their arguments adequately.

Part (ii) was attempted by comparatively fewer students, and unfortunately some of these took *ABCD* to be the same quadrilateral as in (i). However, quite a few students did take the correct approach: drop perpendicular bisectors from *X* to *AB*, *BC*, and *CD*, and then find many congruent triangles (as in the incentre construction), and a significant number obtained full marks.

Part (iii) was hard, and only a handful of students managed to do it. The best approach is to take E on CD such that AE is also tangent to the circle constructed in (ii). Then $AB + CE = AE + BC$ (by part (i)), while $AB + CD = AD + BC$ is given. Thus $AE - CE = AD - CD$. This then implies that $D = E$ (for instance by applying part (b) with $A = T$, $C = P$, and Q and R equal to D and E (or E and D). This proof mimics the proof that if $ABCD$ is a quadrilateral and the sum of the angles A and C is π, then the quadrilateral is cyclic, with the role of the sides and angles interchanged.