BOARD OF STUDIES
NEW SOUTH WALES

HIGHER SCHOOL CERTIFICATE EXAMINATION

1996
MATHEMATICS
2/3 UNIT (COMMON)

Time allowed—Three hours
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on page 12.
- Board-approved calculators may be used.
- Answer each question in a *separate* Writing Booklet.
- You may ask for extra Writing Booklets if you need them.

QUESTION 1. Use a *separate* Writing Booklet.

Marks

(a) Find the value of $13^{-1 \cdot 2}$ correct to two significant figures. **2**

(b) Factorise $2x^2 + 3x - 2$. **2**

(c) Rationalise the denominator of $\dfrac{4}{\sqrt{5}+2}$. **2**

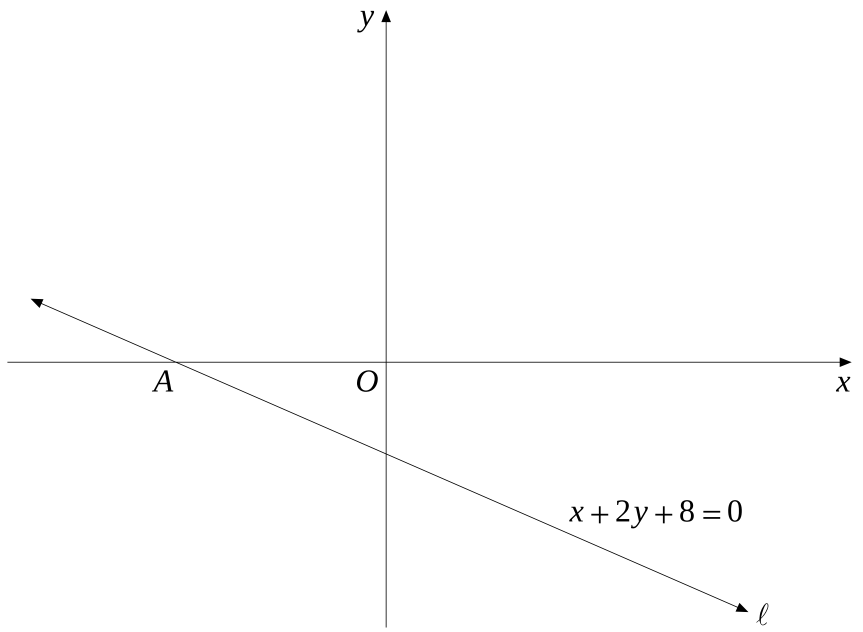
(d) Simplify $\dfrac{2}{3} - \dfrac{x-1}{4}$. **2**

(e) Find a primitive function of $6 - x^{-3}$. **2**

(f) Graph the solution of $|x+2| \le 3$ on a number line. **2**

QUESTION 2. Use a *separate* Writing Booklet.

Marks



The line ℓ is shown in the diagram. It has equation $x + 2y + 8 = 0$ and cuts the x axis at A.

The line k has equation $y = -\frac{1}{2}x + 6$, and is not shown on the diagram.

Copy or trace the diagram into your Writing Booklet.

(a) Find the coordinates of A. **1**

(b) Explain why k is parallel to ℓ. **1**

(c) Draw the graph of k on your diagram, indicating where it cuts the axes. **2**

(d) Shade the region $x + 2y + 8 \le 0$ on your diagram. **1**

(e) Write down a pair of inequalities which define the region between k and ℓ. **2**

(f) Show that $P(8, 2)$ lies on k. **1**

(g) Find the perpendicular distance from P to ℓ. **2**

(h) $Q(4, -6)$ lies on ℓ. Show that Q is the point on ℓ which is closest to P. **2**

QUESTION 3. Use a *separate* Writing Booklet.

Marks

(a) Differentiate: **5**

(i) $\sqrt{x}$

(ii) xe^{2x}

(iii) $\cos^2 x$.

(b) A layer of plastic cuts out 15% of the light and lets through the remaining 85%. **3**

(i) Show that two layers of the plastic let through 72·25% of the light.

(ii) How many layers of the plastic are required to cut out at least 90% of the light?

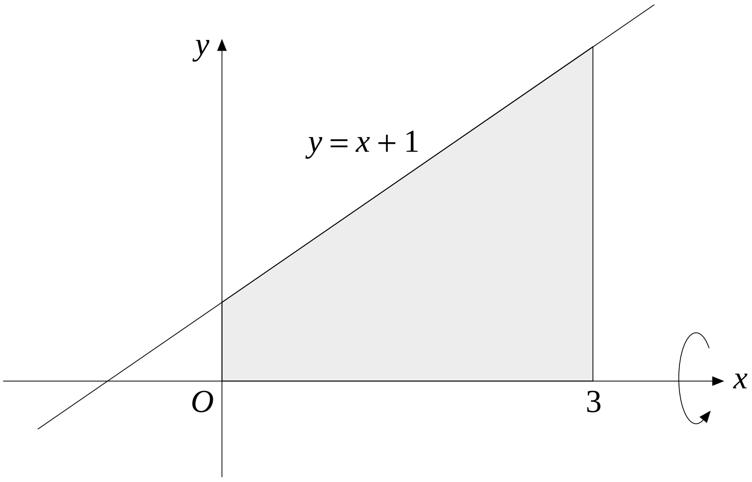
(c) Find $\int \sec^2 6x \, dx$. **1**

(d) Evaluate $\int_1^{e^3} \frac{5}{x} \, dx$. **3**

QUESTION 4. Use a *separate* Writing Booklet.

Marks

(a) **3**



The region which lies between the x axis and the line $y = x + 1$ from $x = 0$ to $x = 3$ is rotated about the x axis to form a solid. Find the volume of the solid.

(b) Let $f(x) = \sqrt{25 - x^2}$. **9**

(i) Copy the following table of values into your Writing Booklet and supply the missing values.

x	0	1	2	3	4	5
$f(x)$	5·000		4·583			0·000

(ii) Use these six values of the function and the trapezoidal rule to find the approximate value of

$$\int_0^5 \sqrt{25 - x^2}\, dx.$$

(iii) Draw the graph of $x^2 + y^2 = 25$ and shade the region whose area is represented by the integral

$$\int_0^5 \sqrt{25 - x^2}\, dx.$$

(iv) Use your answer to part (iii) to explain why the exact value of the integral is $\dfrac{25\pi}{4}$.

(v) Use your answers to part (ii) and part (iv) to find an approximate value for π.

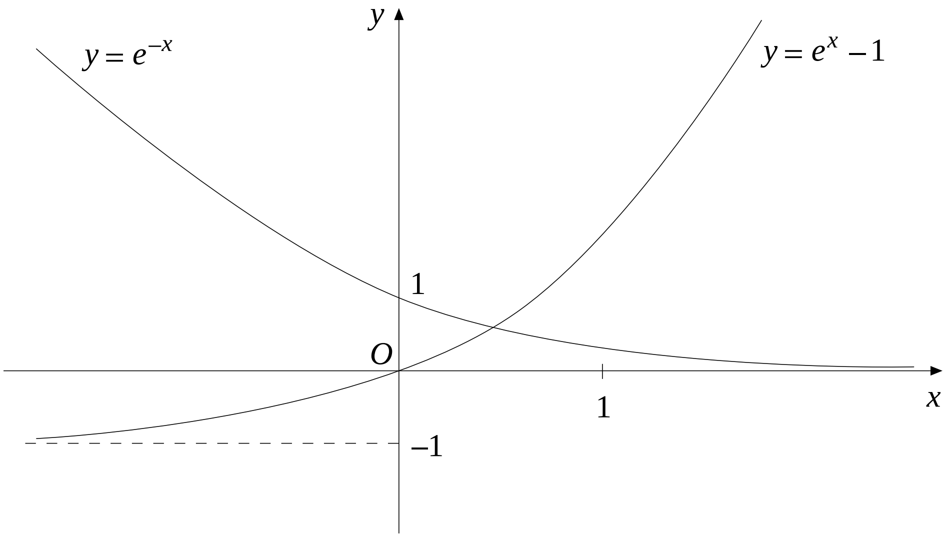
QUESTION 5. Use a *separate* Writing Booklet.

Marks

(a) The graph of $y = f(x)$ passes through the point (1, 3), and $f'(x) = 3x^2 - 2$. Find $f(x)$. **2**

(b) Solve the equation $u^2 - u - 1 = 0$, correct to three decimal places. **2**

(c) **6**



The diagram shows the graphs of $y = e^x - 1$ and $y = e^{-x}$.

(i) Find the area between the curves from $x = 1$ to $x = 2$. Leave your answer in terms of e.

(ii) Show that the curves intersect when

$$e^{2x} - e^x - 1 = 0.$$

(iii) Use the results of part (b) with $u = e^x$ to show that the x coordinate of the point of intersection of the curves is approximately 0·481.

(d) For all x in the domain $0 \le x \le 4$, a function $g(x)$ satisfies $g'(x) < 0$ and $g''(x) < 0$. **2**

Sketch a possible graph of $y = g(x)$ in this domain.

QUESTION 6. Use a *separate* Writing Booklet.

Marks

(a) Lee takes some medicine. The amount, M, of medicine present in Lee's bloodstream t hours later is given by **7**

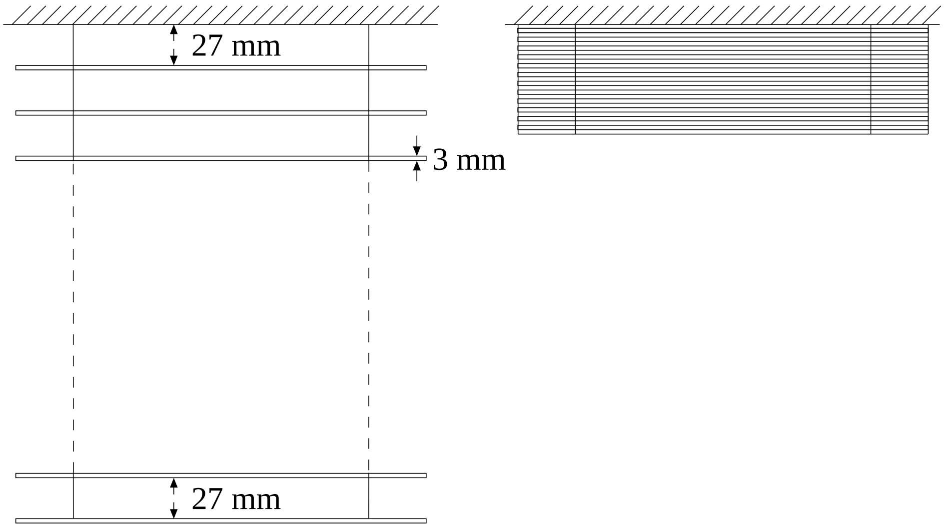
$$M = 4t^2 - t^3, \text{ for } 0 \le t \le 3.$$

(i) Sketch the curve $M = 4t^2 - t^3$ for $0 \le t \le 3$ showing any stationary points.

(ii) At what time is the amount of medicine in Lee's bloodstream a maximum?

(iii) When is the amount present increasing most rapidly?

(b) **5**



A venetian blind consists of twenty-five slats, each 3 mm thick. When the blind is down, the gap between the top slat and the top of the blind is 27 mm and the gap between adjacent slats is also 27 mm, as shown in the first diagram.

When the blind is up, all the slats are stacked at the top with no gaps, as shown in the second diagram.

(i) Show that when the blind is raised, the bottom slat rises 675 mm.

(ii) How far does the next slat rise?

(iii) Explain briefly why the distances the slats rise form an arithmetic sequence.

(iv) Find the sum of all the distances that the slats rise when the blind is raised.

QUESTION 7. Use a *separate* Writing Booklet.

Marks

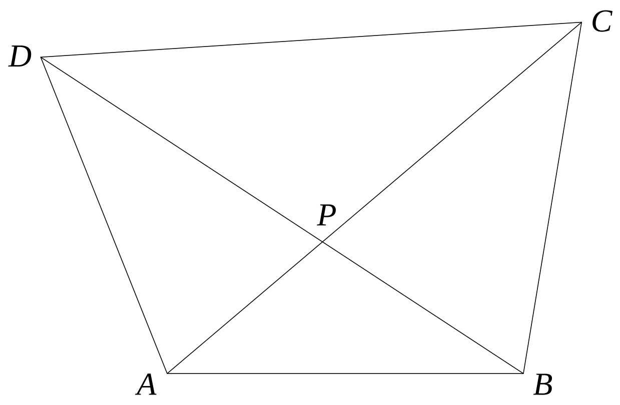
(a) (i) Sketch the graph of $y = 2\cos x$ for $0 \le x \le 2\pi$. **5**

(ii) On the same set of axes, sketch the graph of

$$y = 2\cos x - 1 \text{ for } 0 \le x \le 2\pi.$$

(iii) Find the exact values of the x coordinates of the points where the graph of $y = 2\cos x - 1$ crosses the x axis in the domain $0 \le x \le 2\pi$.

(b) **7**



NOT TO SCALE

ABCD is a quadrilateral. The diagonals *AC* and *BD* intersect at *P*.
$AD = BC$ and $AC = BD$.

Copy the diagram into your Writing Booklet.

(i) Show that triangles *ABC* and *ABD* are congruent.

(ii) Show that triangle *ABP* is isosceles.

(iii) Hence show that triangle *CDP* is isosceles.

(iv) Show that *AB* is parallel to *CD*.

QUESTION 8. Use a *separate* Writing Booklet.

Marks

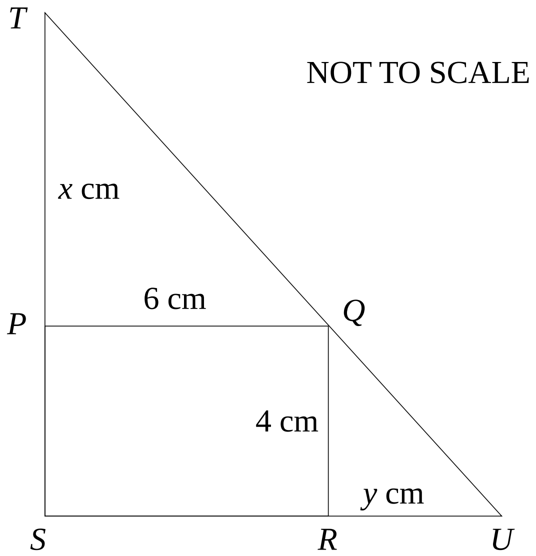
(a) Students studying at least one of the languages, French and Japanese, attend a meeting. Of the 28 students present, 18 study French and 22 study Japanese. **4**

(i) What is the probability that a randomly chosen student studies French?

(ii) What is the probability that two randomly chosen students both study French?

(iii) What is the probability that a randomly chosen student studies both languages?

(b) **8**



PQRS is a rectangle with $PQ = 6$ cm and $QR = 4$ cm. *T* and *U* lie on the lines *SP* and *SR* respectively, so that *T*, *Q*, and *U* are collinear, as shown in the diagram. Let $PT = x$ cm and $RU = y$ cm.

(i) Show that triangles *TPQ* and *QRU* are similar.

(ii) Show that $xy = 24$.

(iii) Show that the area, *A*, of triangle *TSU* is given by

$$A = 24 + 3x + \frac{48}{x}.$$

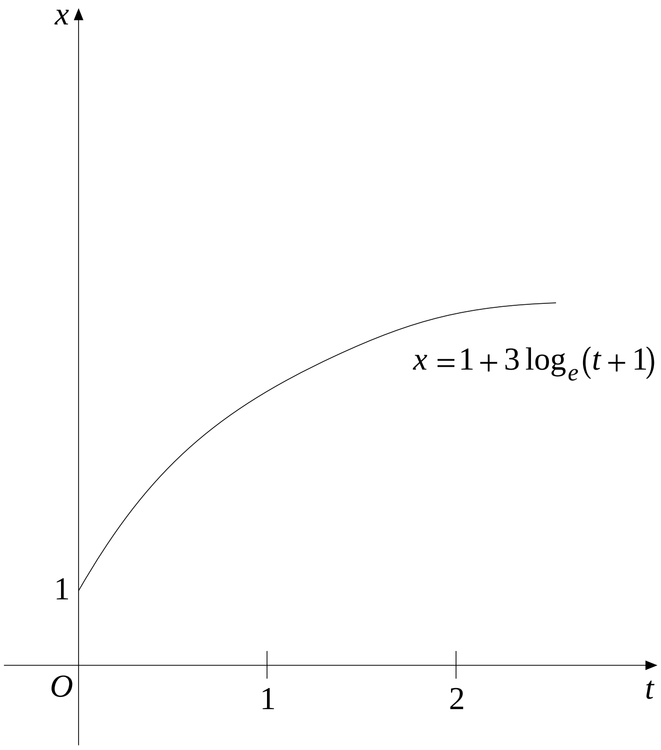
(iv) Find the height and base of the triangle *TSU* with minimum area. Justify your answer.

QUESTION 9. Use a *separate* Writing Booklet.

Marks

Two particles P and Q start moving along the x axis at time $t = 0$ and never meet. Particle P is initially at $x = 4$ and its velocity v at time t is given by $v = 2t + 4$.

The position of particle Q is given by $x = 1 + 3\log_e(t+1)$. The diagram shows the graph of $x = 1 + 3\log_e(t+1)$.



(a) Find an expression for the position of P at time t. **2**

(b) Copy the diagram into your Writing Booklet and, on the same set of axes, draw the graph of the function found in part (a). **2**

(c) P and Q are joined by an elastic string and M is the midpoint of the string. Show that the position of M at time t is given by **1**

$$x = \tfrac{1}{2}\left[t^2 + 4t + 3\log_e(t+1) + 5\right].$$

(d) Find the time at which the acceleration of M is zero. **3**

(e) Find the minimum distance between P and Q. **4**

QUESTION 10. Use a *separate* Writing Booklet.

Marks

(a) (i) On the same set of axes, accurately draw the graphs of **5**

$$y = \sin x \text{ and } y = \tfrac{2}{3}x \quad \text{for } 0 \le x \le \pi.$$

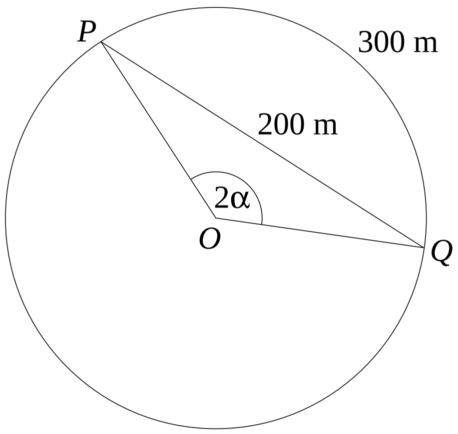
(ii) Find the gradient of the tangent to $y = \sin x$ at the origin.

(iii) For what values of m does the equation

$$\sin x = mx$$

have a solution in the domain $0 < x < \pi$?

(b) **5**



NOT TO SCALE

The diagram shows a circle centre O with points P and Q on the circle. The angle $POQ = 2\alpha$, the length of the chord PQ is 200 metres, and the length of the arc PQ is 300 metres as shown.

(i) Show that $\sin\alpha = \dfrac{2\alpha}{3}$.

(ii) Use your graphs from part (a) (i) to find an approximate value for α.

(iii) Hence find the size of $\angle POQ$, and also find the radius of the circle.

(c) A and B are two points 200 metres apart. For what values of ℓ is it possible to find a circular arc AB of length ℓ metres? Justify your answer. You may use the results from parts (a) and (b). **2**

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$