BOARD OF STUDIES
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1998 HSC

EXAMINATION REPORT

Mathematics

Including:

- **Marking criteria**
- **Sample responses**
- **Examiners' comments**

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1998 HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS

ENHANCED EXAMINATION REPORT

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Mathematics 3 Unit (Additional) and 3/4 Unit (Common)

Mathematics 4 Unit (Additional)

General Comments

The five mathematics papers in 1998 were of a similar standard to the corresponding examinations in earlier years, and the distribution of raw marks obtained by the various candidatures closely followed the pattern which has been established over the years. Details of the way in which raw marks are combined to form a total raw score, and in particular of how the option questions are dealt with in Mathematics in Society, can be found in the introductory comments for the 1997 Mathematics examination report.

The comments in this report are compiled from information supplied by examiners involved in marking each individual question. While they do provide an overview of performance on the 1998 examinations, their main purpose is to assist candidates and their teachers to prepare for future examinations by providing guidance as to the expected standard, highlighting common deficiencies and, in the process, explaining in some detail the criteria which were used in the marks for each part of each question. Where appropriate, the method of solution is outlined and the merits of different approaches to the question are discussed. It should be noted that sometimes these comments take different views of the same phenomenon. For example, the examiners marking some questions in the 2/3 unit (common) paper were encouraged by the number of candidates who knew that $a^0 = 1$, while those marking other questions were clearly disappointed by the number who did not.

Candidates should be aware of the fact that it is their responsibility to indicate the process by which they have obtained their answer to the examiners. In marking, each individual mark is allocated to a step or process which is essential to a correct solution of that question. Those who provide sufficient evidence that the appropriate step, or its equivalent, has been completed are awarded the mark, which then cannot be lost for a subsequent error. Candidates who give only a single word or figure as their response forego any possibility of earning any marks unless their answer is completely correct. Sometimes, in cases where examiners believe that the correct answer is easily guessed without doing the work required to establish the result, a mere correct answer without any supporting justification may not earn all of the available marks.

It is very important that candidates record their working in the same writing booklet as their answer, even if it is experimental work done to develop an approach to the question. Examiners read everything written by the candidate in an attempt to find evidence which will justify the awarding of a mark. This includes work which the candidate has crossed out, or explicitly requested the examiner not to mark. This is always to the candidate's advantage, as marks are awarded for elements of the solution which are correct and are not deducted for errors which have been made. This means that candidates should take care to make sure that work which has been crossed out is still legible, and should not, in any circumstances, use correction fluid or an eraser. Candidates who wish to distinguish their rough work from their considered answers should use the unruled left hand pages for such work.

Candidates who accidentally answer part of one question in the wrong writing booklet should not waste valuable time transcribing their work from one booklet to another. Instead, they should make a clear note on the cover of both the writing booklets to the effect that part of the answer to Question 7 is included in the booklet for Question 5. There are procedures in place at the marking centre to ensure that such misplaced material is brought to the attention of the examiner marking the appropriate question, and no marks are ever deducted for such slips.

Examiners greatly appreciate work which is clearly presented, in which the order of a candidate's work is readily apparent. In particular, candidates are encouraged to avoid setting their work out in two or more columns per page, and to make certain that the parts and subparts of questions are appropriately labelled. It is not essential for the parts within a question to be presented in the same order as they appear in the examination question, but departures from the original order make careful labelling of the responses even more important.

Examiners frequently comment on the need for candidates to provide clearly labelled, reasonably sized and well executed graphs and diagrams. Appropriate use of a ruler and other mathematical instruments is essential to obtain a diagram of the appropriate standard. In making these comments, examiners are motivated by the assistance such graphs and diagrams provide candidates in the process of answering the question. In particular, candidates ought to realise that instructions on the examination paper asking candidates to reproduce a diagram in their writing booklet are invariably given because the diagram is likely to assist the candidate to solve the problem or provide a means for them explain their solution.

Finally, candidates for the related courses are reminded that a table of standard integrals appears on the back page of each examination paper. Candidates should become familiar with this table, and be aware of its usefulness for both integration and differentiation.

Mathematics in Practice

Question 31 The Consumer

(a) (i) (1 mark)
This question required the extraction of a repayment figure from a table for a car loan, after subtraction of the deposit paid. Quite a high percentage of candidates were able to do this correctly. The common error was the failure to subtract the deposit.

(ii) (1 mark)
Candidates were then asked to calculate the total repayments over two years. While many were able to do this correctly, a substantial number used 48 fortnights. Some added the deposit, sometimes even when they had not subtracted it in the first place.

The number of candidates offering figures which were either too big or too small to be sensible raises some concerns. For example, some calculated total repayments of $162 000 000 for a $13 500 car.

(iii) (1 mark)
In order to calculate total interest paid, candidates had to subtract the principal from the previous answer. While this was managed fairly well, many candidates incorrectly included the deposit somewhere in their calculation.

(iv) (2 marks)
When calculating the extra cost of repaying the loan over three years, the number of candidates able to account for all the required steps was quite high. However, there were many who scored either one or zero marks on this part, making this a good question for discrimination purposes.

(b) (i) (1 mark)
Using figures provided directly on a graph, candidates were asked to calculate the increase in daily water usage between two billing periods. While there were many exceptions, this was generally handled very well.

(ii) (1 mark)
Candidates were asked to use their answers from part (i) to calculate the percentage increase in average daily water usage between the two billing periods. Few candidates used the correct figures to construct the percentage. It seemed that there was a general understanding of the process of calculating a percentage, but the choice of which

figures to use when forming percentages for comparisons was poorly understood.

(iii) (2 marks)
In this question, few scored two marks. Candidates were required to find the amount which would have been due if 30% less water had been used. Many simply subtracted 30% of the total bill.

(c) (i) (1 mark)
Most candidates were able to calculate the amount required for a one-third deposit. However, there were many unreasonable answers. For instance, a common answer was $4.23 as a one third deposit on $1269.

(ii) (1 mark)
Candidates were generally able to calculate the 12 equal monthly instalments required to pay for the balance of the purchase.

(iii) (1 mark)
Candidates were asked to find the interest charged on the final payment, which was 10 days late, at a rate of 0.05% per day. Most candidates' responses were incorrect, but there were a substantial number who succeeded in finding the correct answer which was $0.35.

Question 32 Travel

(a) Candidates had to answer questions relating to a train timetable for both slow and fast trains travelling between Penrith and Central.

(i) (1 mark)
A subtraction of times was required. Most candidates did this very well.

(ii) (1 mark)
Candidates were asked to find the difference in the travel times taken by the slow and fast trains. It was reasonably well done. The most common error resulted from misreading the question and using the wrong stations.

(b) Candidates were given a map of the Pacific Highway and New England Highway between Sydney and Brisbane. The distance between Sydney and each of the major towns and cities along both routes was indicated.

(i) (1 mark)
Candidates were asked to find the total distance travelled. This was quite well done. The most common error was to find the sum of the distances given on the map. Candidates making this error were not awarded the mark.

(ii) (1 mark)
Candidates had to find the distance between two cities (by subtraction). The majority answered this correctly.

(iii) (2 marks)
Given a rate of petrol consumption, and the cost per litre, candidates were asked to calculate the cost of the petrol for the trip from Sydney to Brisbane. This question caused a lot of problems for candidates. A common occurrence was inconsistency in rounding and rounding at inappropriate times during the calculation. Frequent errors also occurred because of misinterpretation of the question. For instance, many candidates calculated the cost for the wrong highway, or used the round trip distance found in part (i). Calculations which were correct but for one of these errors were awarded one mark.

(c) (i) (1 mark)
A map of the area around Cairns was supplied, and candidates were asked for the distance travelled by tourists on a round trip through certain towns.

This was well done by most candidates. Errors occurred when candidates did not read the question carefully and simply added every distance that appeared on the map.

(ii) (2 marks)
This part asked for the the cost of car hire for the journey, given the rate per day and the rate per km. It was well done. The most common errors were to compute either $(55 \times 3) + (40 \times 315)$ or $55 + 40 \times 315$. Candidates with these errors were awarded one mark.

(d) A table was provided showing prices per person for a tour.

(i) (1 mark)
The total cost for two people taking the tour in winter had to be found. Errors occurred when candidates misread the table or neglected to double the cost. However, candidates had reasonable success in answering this question.

(ii) (1 mark)
Candidates were asked to find the difference in price if the tour was taken during the high season. This was well done.

(iii) (1 mark)
Candidates had to perform an exchange rate calculation. This was well done. The most common error was to divide \$2500 by 1.15, rather than calculating $\$2500 \times 1.15$.

Question 33 Accommodation

(a) Candidates were given a table detailing properties offered for auction. The information gave the number of bedrooms, the highest bid and stated whether the property was sold or passed in. The question was very well done by the majority of candidates.

(i) (1 mark)
This part asked for the names of the suburbs in which the unsold properties were situated. It was well done.

(ii) (1 mark)
From the properties listed, candidates had to find the highest price paid for a 3 bedroom house. Most answered this correctly. A common incorrect answer was $585 000, the price of one of the 3 bedroom houses that had been passed in.

(iii) (2 marks)
Finding the median price of the properties sold was poorly done. The majority of candidates found the average price instead. Two common errors were neglecting to rank the prices in ascending or descending order before finding the median, resulting in an answer of $690 000, and finding the median of all properties listed which was $427 500. Candidates making either of these errors were awarded one mark.

(iv) (1 mark)
Candidates were asked to find what the value of a house would be if house values increased by 5%. Most were able to answer correctly. A common error was neglecting to add on the 5% increase.

(b) (i) (1 mark)
Candidates were asked how many hinged doors were indicated on a house plan. This was reasonably well done. However, the word 'indicated' caused problems for a few candidates.

(ii) (2 marks)
The house floor plan showed various types of doors. Candidates were told the price of each type of door and were asked to find the total cost of supplying all the doors for the house. This was reasonably well done. Errors commonly arose when candidates did not correctly distinguish between interior and exterior hinged doors. One mark was awarded to those who were able to correctly calculate the subtotals for three of the four types of doors involved.

(iii) (2 marks)
This part provided a breakdown of the building costs for the house and asked candidates to find the percentage due to labour costs. The

majority answered this well. One mark was awarded if the correct fraction, $\frac{21580}{64000}$, was found or if $\frac{}{64000} \times 100$ appeared in the working.

(iv) (1 mark)
Candidates had to work out the area of the family room. The dimensions were shown on the floor plan as 3.4×3.0. This part was well done. The most common error was to leave the answer as 3.4×3.0 without any further calculation. Candidates who did this received no marks for this part.

(v) (1 mark)
Having found the area in (iv), this part asked for the cost of a wooden floor for the family room based on a price of $96 per square metre. It was very well answered.

Question 34 Design

(a) A diagram showing a section of a tessellation was provided.

(i) (1 mark)
Candidates had to name the shape of a shaded tile in the tessellation. This was well done. The desired answer was 'rhombus,' but the answers 'parallelogram' and 'quadrilateral' were also accepted.

(ii) (1 mark)
Candidates had to complete the tessellation. As is the case every year, this caused problems. Too many candidates still attempt it freehand despite the instruction to use a ruler. Those who did use a ruler still had great difficulty completing it successfully. The mark was awarded provided there were enough correct elements in the continuation of the pattern.

(b) (i) (1 mark)
Candidates were asked to calculate the area of a logo. This was very poorly done. Although most attempted the question, many could not divide the area into disjoint regions to obtain the correct answer.

(ii) (1 mark)
This part asked how much paint was needed to cover the area in (i). Most candidates were able to provide a correct answer based on their earlier response.

(iii) (2 marks)
Candidates were then asked to find the cost of purchasing the paint required. There were a few steps involved in obtaining the correct answer. Problems occurred when candidates forgot to divide by 4 to find the number of tins required, or rounded incorrectly. Candidates

whose answers were correct but for one of these errors were awarded one of the two marks available.

(c) (2 marks)
Candidates were asked to enlarge a design. It was quite well done, considering the level of difficulty. One mark was awarded if the candidate was able to locate at least six of the vertices on the circumference to sufficient accuracy or if the candidate drew the inner circle correctly and showed the correct location for four vertices. Many provided freehand guesses for the location of the vertices on the circumference and then tried to complete the diagram with their pair of compasses. Such responses invariably received no marks.

(d) (i) (1 mark)
This part asked candidates to calculate the area of a shaded region. This was poorly done. The majority of the candidates computed the area of a rectangle 1×5 instead of the triangle $\frac{1}{2} \times 1 \times 5 = 2.5\,\text{m}^2$.

(ii) (2 marks)
Candidates had to find what fraction of the total area was shaded. This was poorly done as most were unable to find the total area, which was $10\,\text{m}^2$. Over half of the candidates incorrectly found the total area to be $3 \times 5 = 15\,\text{m}^2$. Such candidates who then expressed their part (i) answer as a fraction of 15 were awarded one mark. Another common incorrect answer was based on the calculation of the total area as $\frac{1}{2} \times 3 \times 5 = 7.5\,\text{m}^2$. The corresponding answer of $\frac{\text{part (i) answer}}{7.5}$ was also awarded one mark.

(e) (1 mark)
The question showed a hexagonal prism and asked for its net to be drawn. Those who knew what a 'net' was were able to draw it reasonably well. Most candidates knew there had to be six rectangles, but some then drew octagons or pentagons for the remaining two faces and so did not receive the mark. Another common error was to produce an enlargement of the diagram of the solid. Clearly, these candidates either did not know what a 'net' was, or had misread the question.

Question 35 Social Issues

(a) (2 marks)
This question required the construction of a pie chart from data provided as percentages. Some candidates were able to make the calculations and construct the graph with reasonable accuracy, but a substantial percentage were unable to complete the task accurately.

(b) (i) (1 mark)
Candidates were told that a newspaper page was 27 cm wide and had six columns. They were asked to calculate the width of each column. While this was well done, there were still many who went astray in the calculation.

(ii) (1 mark)
An advertisement was shown at the actual size. Candidates had to work out how many columns it would span. The answer was two. Many candidates did not gain this mark.

(iii) (1 mark)
In this question, candidates were asked to calculate the size of the advertisement in cmcols, a unit of measurement which had previously been explained. This proved to be very difficult, with many candidates simply measuring the dimensions of the advertisement in cm and multiplying to find the area in cm^2.

(iv) (1 mark)
The price of the advertisement was then to be calculated using a price per cmcol. While this was reasonably easy, many candidates still failed to get the mark, mostly because they seemed to have not grasped the idea of cmcol.

(c) (2 marks)
Candidates were required to calculate the agreed level of CO_2 emissions for 2012, based on figures provided in a table. They were also to calculate the emission rate per person using a population figure also supplied in the table. The latter calculation seemed to be the better done.

(d) (2 marks)
This question consisted of asking the candidates to calculate a percentage from the absolute figures provided in various sectors of a pie chart. The concept of comparing the appropriate figure with the sum of the sector values seemed to be well understood, but there was great variety in calculating the sum of the sector values, with omissions being common.

(e) (2 marks)
The preferences of the third placed candidate in an election were to be distributed between the first two candidates in the ratio 70 : 30. This was well attempted, with many getting both figures correct.

Mathematics in Society

Question 21

(a) (i) (1 mark)
This was an easy question and nearly all candidates answered correctly. Those who did not get it correct often found 0.2, but then did something else to it.

(ii) (1 mark)
Just under half the candidates answered this correctly. Quite a few drew the probability tree and highlighted the correct row. Some common errors included writing the probabilities of snow on the first, second and third days but not continuing any further, merely finding SNN on the probability tree and so giving the probability as $\frac{1}{8}$, and adding the three correct probabilities rather than multiplying them.

(iii) (2 marks)
This was poorly done. Few candidates used the complementary method of calculation and it was clear that many did not understand the meaning of 'at least.' Many candidates used a tree diagram but common errors involved not including all branches, with three branches being the most common occurrence. Other common incorrect answers were $\frac{7}{8}$ and $(0.8)^3 = 0.512$.

(b) (i) (2 marks)
The required conversion of units was overlooked by a large number of candidates. Many did not convert 15 000 L to m^3 and so gave an answer somewhere between 48 and 49. The manipulation of the equation to make r the subject was not handled well. Candidates who got to $15 = \pi r^2 \times 2$ often simply subtracted π or 2π and then found the square root of that answer. Others seemed to think that the way to find the square root was to divide by two.

(ii) (2 marks)
Many candidates showed a poor knowledge of the process of converting mL to L. As a result, 50 minutes was a common answer. Candidates were generally unable to convert 500 min, $8.\dot{3}$ hr or $8\frac{1}{3}$ hr to hours and minutes successfully. Some did not understand the question and computed the time taken to empty the 15 000 L tank rather than that required to fill the 10 L tank. Others attempted to use a rates method with limited success.

(c) (i) (1 mark)
This part was well answered with most candidates using the multiplication principle. There were a large number who could not multiply fractions or the decimal equivalent to give the correct answer.

(ii) (2 marks)
Most candidates responses were one of 2, 2/4 , 50% or 2 right 2 wrong but few could adequately explain the answer. An answer of $\frac{1}{2}$ received no marks.

Most arrived at 2 by halving 4 rather than by counting the numbers of outcomes with 0, 1, 2, 3 and 4 answers correct on the probability tree. The most common incorrect answer stated that there were four questions and each question had a probability of $\frac{1}{2}$. Candidates then claimed that half of the answers would therefore be guessed correctly, resulting in a mark of 2.

(iii) (1 mark)
This was answered reasonably well. However, many gave an answer of $\frac{1}{16}$ because they could not see the difference between the questions in (c) (i) and (c) (iii) . Others knew that the answer should be $1 \times 1 \times \frac{1}{2} \times \frac{1}{2}$ but could not simplify this expression correctly to $\frac{1}{4}$ or 0.25 or 25%. Often this expression was evaluated as $2\frac{1}{2}$, $1\frac{1}{4}$ or 3.

Another common response was $\frac{1}{16}$ arising from the branch of the tree diagram corresponding to all four answers being correct.

Question 22

(a) (2 marks)
This question required candidates to solve an equation with the unknown in the denominator. It was not well done. Many candidates were unable to cross multiply, apply the distributive law or transpose terms.

(b) This question could be answered by applying a knowledge of right angle trigonometry. Part (ii) could also be answered through an application of Pythagoras' theorem.

(i) (1 mark)
Candidates were asked to find the size of an angle given a diagram of a right triangle. It was generally well done. However, many candidates were not able to correctly find the size of the angle from a correct trigonometric ratio, while others were unable to choose the appropriate trigonometric ratio. A large number of candidates used their answer to part (ii) and the sine rule to do this part of the question. Unfortunately, the majority of these candidates did not score the mark here because they failed to subtract the 1.2 m from their part (ii) answer.

(ii) (2 marks)
Candidates were asked to find the height of a kite above the ground given a sketch that showed a right triangle 1.2 m above ground level. Unfortunately, most candidates did not add the 1.2 m after they calculated the side of the right triangle. A large number applied the sine or cosine rule to the question instead of using simpler right angle trigonometry, thus increasing their chance of error.

(c) (i) (1 mark)
Candidates were asked to explain why an angle on the given diagram was 45°. The vast majority of the candidates gave a correct explanation, such as $\frac{1}{8}$ of a circle, $\frac{1}{4}$ of a straight angle or $\frac{1}{2}$ a right angle.

(ii) (2 marks)
Candidates were asked to find the length of an interval from a sketch. This was extremely well done. However, some candidates who correctly substituted into the cosine rule were unable to perform the calculation correctly. By far the most common error was to neglect taking the square root.

(d) (i) (2 marks)
Candidates were required to sketch the course of a yacht and to label the sketch. This was poorly done due to a lack of understanding of compass bearings and an inability to label diagrams correctly. A common error was to correctly place the 20 km (XY) part of the course on a compass rose but to then join Y to that part of the compass rose labelled W.

(ii) (2 marks)
Candidates were asked to find one of the angles in the sketch they had drawn for part (i). This was poorly done as many candidates were unable to substitute the information correctly into the sine rule. Many of those who substituted correctly were then unable to perform the calculation correctly. Many candidates realised that there was insufficient data on their sketch to do the question and either added wrong data or made use of the given data incorrectly. The most common error was the linking of the 34° angle with the 40 km side.

Question 23

The first part of this question involved graph reading and interpretation, while the second part required a knowledge of statistics.

(a) (i) (1 mark)
This part was generally well done. Most candidates were able to plot at least four of the five points correctly according to the scale.

(ii) (1 mark)
Again, this was generally well done. However, there was some confusion between 'curve of best fit' and 'line of best fit'. Candidates were also unsure of what happens to the curve beyond the given values, with many apparently feeling the need to ensure that the curve began at the origin.

(iii) (1 mark)
Reading the scale on both the horizontal and the vertical axes was a problem. A common assumption was that both axes had the same scale. A point was often marked on the curve without an answer to the question being stated.

(iv) (1 mark)
This was poorly done. Most candidates did not appear to be familiar with the term 'tangent'.

(v) (1 mark)
The poor response to (iv) meant that this was the least attempted part of the whole question. For those who attempted to find the gradient, the problem of reading both the horizontal and vertical scales again proved difficult. Hardly any candidates realised that the gradient should be negative. Few candidates made the final connection between 'gradient of the tangent' and 'rate of change' to provide an explicit estimate of the rate of change of the intensity.

(b) (i) (1 mark)
The calculation of the value given as A in the table was well done.

(ii) (1 mark)
The computation of $\sum fx$ was also well done.

(iii) (1 mark)
While this was well done, a common mistake was to divide by 6 rather than 70.

(iv) (1 mark)
Quite a number of candidates failed to distinguish between the median requested here and the mean in part (iii), giving 30 000 – 39 999 as their answer. It was perhaps unfortunate that the answer was also the modal class, as some candidates may well have given the correct answer for the wrong reason.

(v) (1 mark)
There were a great variety of answers. Some came from incorrect use of the calculator, while others resulted from a lack of knowledge of standard deviation. A common error was taking the standard deviation of the numbers in the fx column.

(vi) (2 marks)
There was a lack of clear understanding of changes to the standard deviation. Many candidates calculated the new value for the standard deviation and argued from the two values. These candidates could usually earn at most one of the two marks.

The range of vague answers that were given made it difficult to decide which answers merited the mark which was awarded for an appropriate reason.

Question 24 Space Mathematics

This question contained parts dealing with units of measurement, space travel, lengths of orbits, the speed of a satellite, the shape of an orbit, the distance between planets, the diameter of a planet and some harder numeration work involving scientific notation.

The majority of candidates handled this question well, with a large number scoring at least ten marks. The main sources of loss of marks were poor handling of appropriate formulae, an inability to convert from one unit of measurement to another (especially involving units of time) and inaccurate procedures for calculator work with large numbers.

(a) (1 mark)
Only 30% of the candidature were awarded the mark for this part. A common response was $2.3 \times 3 \times 10^5 = 690\ 000$. Some candidates did not understand the concept of a light year, while many had difficulties converting years to seconds. A number of candidates used the result 1 light year $= 9.46 \times 10^{12}$ km.

(b) (i) (1 mark)
This part was generally well done, although a number of candidates did not read the question carefully or did not study the diagram sufficiently closely to appreciate that the radius of the orbit is 2000 km greater than that of the Earth.

(ii) (1 mark)
Approximately 70% of the candidates were awarded the mark. Common errors included mistaking the length of the orbit for the radius of the orbit, using the formula for the area of a circle rather than the circumference of a circle, using an incorrect formula for the circumference of a circle such as $C = \pi r$ or $C = 2\pi d$ and using either 6400 km or 2000 km as the radius of the orbit.

(iii) (1 mark)
This part was done well although an incorrect quoting of the distance, speed and time relationship was fairly common.

(iv) (1 mark)
Again, this part was not well done with only about 30% of the candidature scoring the mark. Candidates who understood the concept of zero eccentricity for a circle found this an easy mark. A common error was to use the radius of Earth as a or b in the formula. A number of candidates were unable to attempt this part because they believed that insufficient information about the radii had been given.

(c) (i) (1 mark)
Many candidates confused the problem here with the instance of measuring the elapsed time for a signal to travel from Earth to an object in space and return and so used the formula $d = \frac{1}{2}ct$. A common incorrect response for the distance was $8.5 \times 3 \times 10^5 = 2\ 550\ 000$ km resulting from a failure to convert minutes to seconds.

(ii) (2 marks)
Most candidates used trigonometry for this part with a high degree of success. Many who lost marks did so because they had drawn a poor diagram. A common error was to use an incorrect angle for their method, usually 0.0026° instead of 0.0013°. A small number of candidates treated the diameter of Mars as an arc of a circle with centre at Earth, using the formula arc length $= \dfrac{\theta}{360} \times 2\pi R$.

(d) (i) (2 marks)
This part was relatively badly done, with most candidates unable to get the correct expression for the conversion from km to AU. Common responses were $1.49 \times 10^8 \times 384\ 000 = 5.7216 \times 10^{13}$ and $\dfrac{1.49 \times 10^8}{384\ 000} = 388.02$. Lack of proper understanding of significant figures accounted for many errors.

(ii) (2 marks)
The most common error was an incorrect substitution into the given formula, especially for the radius. Even though the question clearly stated that the answer from (d) (i) was to be used, many candidates used some other value for the radius, often with no relevance to the question. Many lost a mark because they did not know how to use the EXP key on their calculator.

Question 25 Mathematics of Chance and Gambling

This question contained parts dealing with probability, the language of chance, counting techniques, mathematical expectation and fairness.

Generally, candidates answered this question very poorly, with the majority displaying a lack of understanding of the concepts that were tested in the question.

(a) (i) (1 mark)
This part was very well done with about 90% of the candidates scoring the mark.

(ii) (2 marks)
This part was poorly done. Most did not see the connection between the outcomes of the first spin and the outcome of the second spin. A very high percentage of candidates just had the answer $\frac{2}{3}$.

(b) (i) (1 mark)
This part showed that most candidates do not understand the concept of 'odds' in gambling. Less than 30% were awarded the mark. The most common response was $\frac{2}{7}$.

(ii) (1 mark)
The responses to this part were poor. Most candidates gave the incorrect answer \$20 which was obtained from $180 \div 9$.

(iii) (1 mark)
This part also showed that candidates have difficulty understanding the language of chance. Most still do not understand the term 'odds on.' A very common response was $2 \times 10 = \$20$. A number of candidates obtained the answer \$5 for the winnings but then failed to add this to \$10 to find the amount collected from the bookmaker.

(iv) (1 mark)
This part was also poorly done. The most common responses were 7×3, $7 \times 7 \times 7$, 7×6 and even $7 \times 6 \times 5 \times \cdots \times 1$.

(c) (i) (1 mark)
Less than half of the candidates were awarded the mark for this part. Common answers were $\frac{1}{6}$, $\frac{1}{12}$ and $\frac{2}{12}$.

(ii) (2 marks)
This was a very badly done part of the question. From the responses in the examination, it was obvious that the large majority of the candidature had little or no understanding of the concept of expected return. Less than 1% of candidates were awarded 2 marks.

(iii) (2 marks)
This part was also badly done with most candidates having no understanding of the concept of a 'fair' game.

Question 26 Land and Time Measurement

The majority of the candidature answered most parts well. The exception was (c) (v) which was poorly understood.

One clear lesson from this question is the need to avoid rounding values during the course of calculations. Candidates should only round off at the end of the calculation, and then only after recording the full calculator display. The impact of using rounded numbers in a calculation was obviously not understood and this partly accounted for the very poor response to (c) (v).

The order of operations caused problems for many candidates in (a) and (c) (iv).

(a) (2 marks)
The question did not specify the number applications of Simpson's rule to use, and candidates attempted this part with either one or two applications. Common errors were the use of an incorrect value for h, typically 0.4 or 0.8, and rounding off numbers incorrectly during or at the end of the calculation. A significant number of candidates failed to multiply by $\frac{0.2}{3}$ and a substantial number were unable to correctly use their calculator to evaluate their expression.

(b) (i) (1 mark)
Many candidates did not know the difference between latitude and longitude. Many gave the incorrect response of 0° for (i) but then used the correct value of 72° in the course of answering (ii). A significant number subtracted the latitudes to give an answer of 8°.

(ii) (1 mark)
A great number of candidates successfully used the arc length formula, perhaps taking the radius of the Earth from Question 24 (b). Candidates who correctly applied this method using values for the radius of the Earth between 6300 km and 6500 km were awarded the mark. However, many candidates using this method then went on to do something else so as to make use of the information that 1° subtends 60 nautical miles and that 1 nautical mile = 1.852 km.

The most common error in the intended method of solution was to divide by 1.852 instead of multiplying.

(iii) (1 mark)
The majority of those attempting this part obtained the correct answer. However, many others added the eleven hours of travel time correctly but then tried to make an adjustment for time differences. There were also quite a few incorrect responses of 9:30 pm.

(iv) (1 mark)
Many candidates did not attempt this part. It is not clear whether

this was due to its position on the page or the fact that they did not know about Greenwich Mean Time.

The majority of those who did attempt it were able to give the correct answer. Many candidates gave approximations for the answer when it was just as easy to write down the correct answer. Others thought that 7.73 hr = 8 hr 13 min.

Some candidates misunderstood the intent of the question and gave an explanation of the difference between Greenwich Mean Time and Beijing time.

(c) (i) (2 marks)
A large number of candidates were unable to successfully draw a neat sketch of the playground while many others drew a scale drawing even though it was not asked for in the question. The major error was to not mark in all the interval measurements on the diagram. All internal measurements were required in order to earn the second mark.

Markers were disappointed to see that many candidates did not use a ruler despite the instruction to draw a neat sketch.

(ii) (1 mark)
Most candidates were able to find the distance between two points in the playground using Pythagoras' theorem, although a significant number were unable to use their calculators correctly when evaluating.

(iii) (1 mark)
This part was answered well. The most common error was to add 113° and 28° when the difference was required. Some candidates misread the question as find the size of $\triangle BOC$.

(iv) (1 mark)
The arithmetic calculation after substitution into the cosine rule was performed satisfactorily. Common errors included rounding early in the calculation and failing to perform the final step of finding the square root to give the required answer. Performing operations in the incorrect order and the use of the wrong angle were also common.

(v) (1 mark)
This part was very poorly done. The most common responses were that the slight difference was due to the techniques being different or the radial survey being more accurate than the traverse survey. Many thought human error caused the difference, but most believed the large number of decimal places arising in the cosine rule from the evaluation of $\cos C$ made that the more accurate

method. Spelling, grammar and sentence structure were extremely poor. Very few candidates were able to demonstrate that they understood that the recorded measurements of distances and bearings were approximations.

Question 27 Personal Finance

Most parts of this question required candidates to read and interpret tables of information and to use their calculator to manipulate figures. In general, those who had some grasp of how to use the tables were able to score at least some of the available marks. Candidates need to ensure that working is shown, partly because transcribing incorrectly from the calculator is so common, but also because showing working seems to assist in seeing how to approach questions.

(a) This part involved calculating interest from a credit card account.

(i) (1 mark)
This was generally well done by the candidates, although many did not know the number of days in a year. Common incorrect responses were 0.0005% and 5%.

(ii) (1 mark)
A number of candidates had difficulty calculating the interest, although those with an answer in part (i) were generally able to score a mark. Common mistakes included

$$0.05\% \times 975 = 0.4875 \text{ (not multiplying by 24)}$$

or an answer of \$1170. The correct answer was \$11.70.

(b) This part involved interpreting a timesheet to calculate wages.

(i) (1 mark)
Messy and illogical addition of hours led to errors, as did ignoring or not understanding the given information about meal breaks. Many did not understand the meaning of working at a normal rate of pay.

(ii) (1 mark)
This was an easy mark as most candidates could multiply their answer for part (i) by \$9.80.

(iii) (1 mark)
This part was generally quite poorly done. Many candidates had difficulty calculating the total number of hours worked at penalty rates, with many ignoring the penalty rates which applied for part of Thursday. Some candidates also had difficulty working out the time-and-a-half calculations.

(iv) (2 marks)
Most candidates were able to use their previous answers correctly to calculate the percentage of the total pay earned at penalty rates. A common mistake was to find the pay for the time worked at penalty rates as a percentage of pay for time worked at normal rates.

(c) (i) (1 mark)
The majority of the candidature were able to find the appropriate figure of \$7.72 per month in the table. A common mistake was not realising that this figure represented the payment for each \$1000 borrowed, so many candidates did not multiply by 150. Others mistakenly multiplied by 15.

(ii) (1 mark)
Most candidates were able to multiply the answer in part (i) by 300 to obtain the answer. However, many then added the principal of \$150 000 to this amount and so did not earn the mark.

(iii) (1 mark)
Candidates who had correctly answered part (i) were able to use the same process to score a mark in this part. Those who did not get the mark in (i) generally did not score here either. A repetition of the error from part (i) meant that a common answer was $\$8.36 - \$7.72 = \$0.64$.

(iv) (1 mark)
Less than half of the candidature were able to calculate the amount Yang Yang will borrow. The most common error was to multiply \$1195 by 12 and by 15 to get \$215 100. Many candidates did not seem to realise that answers greater than \$150 000 were clearly incorrect.

(v) (1 mark)
This was an easy mark, as most were able to work out the deposit by subtracting their previous answer from \$150 000. However, those candidates with answers greater than \$150 000 in (iv) were generally unable to score the mark for this part.

Question 28 Mathematics in Construction

This was a question which should not have caused too many difficulties. However, the question was poorly done, with the quality of the responses below the standard of some of the other option questions.

Poor knowledge of the technical terms used in this topic and the differing usage of the word ‘step’ in parts (e), where it referred to the number of treads in the staircase, and (f), where it related to the number of risers in the staircase, caused confusion.

The quality of the plan provided in the question was excellent for examination purposes, as it gave minimal reason for inaccurate measurements to be used. On the other hand, the quality of the diagrams provided by candidates in their responses to part (g) was disappointing and this area needs emphasis in the teaching of this option.

(a) (1 mark)
This part was not well answered. Candidates were confused by the use of the word ‘shape’ in the question, with triangular prism a common answer. Others answered rectangle, perhaps referring to the ceiling and not the roof. This indicates a poor understanding of the technical terms used in this topic and it seems that this aspect needs greater emphasis in the teaching of the topic. It is possible that a reference to roof line type in the question may have assisted interpretation and elicited better responses.

(b) (1 mark)
By contrast, this part was reasonably well answered. Many scored zero because they gave the scale in reverse order as 100 : 1. Others, whose working appeared to show that they knew that the scale was 1 cm : 1 m, gave their answer as 1 : 1 and so did not receive the mark.

(c) (2 marks)
Many candidates found it difficult to measure accurately from the plan and apply the scale. Common incorrect measurements were 9.1 m $\times$1.6 m etc. Examiners allowed a tolerance of 1 mm for measurement on the plan, which corresponds to 0.1 m after applying the scale. Most candidates still earned the mark for the calculation of the area even if their length measurements were incorrect. However, those who did the calculation in square centimetres or square millimetres were often unable to convert their answer to square metres.

(d) (2 marks)
This part was very badly answered. Candidates frequently calculated the area of the two rooms and used this as the basis of their cost calculation. Very few understood that carpet is sold by the lineal metre, with many thinking that the price was $120 per square metre. Some who calculated the area divided by the width of the carpet and so found the correct cost in this instance because the width of the rooms and the width of the carpet were identical.

(e) (1 mark)
This was also badly answered. As mentioned earlier, candidates had difficulty with the wording of the question. Most candidates correctly counted 14 steps but some other answers included 24 (10 + 14 from the 2 sets of stairs shown on the plan), 9, 11 and 12. Some candidates clearly arrived at an answer of 15 by counting the number of steps they would take to get to the first floor, thereby including 14 steps on the staircase and a final step to arrive on the first floor.

(f) (1 mark)
Amongst many parts which were badly answered, this was clearly the worst. Very few candidates understood the concept being tested here and simply divided the 3.5 metres by the number of steps that they found in part (e). It was extremely rare to find a candidate correctly dividing by a number which is one more than their answer in part (e). Those who did get marks in this part usually obtained it because their answer happened to be the correct answer to the question, having incorrectly answered 15 for part (e).

(g) (2 marks)
The quality of the answers for this part ranged from beautifully ruled diagrams, often drawn to scale, to extremely basic, small diagrams that were quickly drawn with little care. The allocation of marks to this question often became quite subjective because of the poor quality of the sketch drawn. The first mark was awarded for a reasonable depiction of the shape of the roof line. For the second mark, examiners looked for a wide window placed in the first quarter of the wall and a narrow window placed in the third quarter of the wall

Candidates should take more care when asked to draw a sketch of an elevation from the floor plan provided. Many drew the windows but left off the roof line, while poor representations of relative window size and position were common.

(h) (2 marks)
The answers given for this part revealed a poor understanding of the concept of pitch. Incorrect placement of the angle was common. Poor measurement skills were also evident from attempts to determine the length of the adjacent side. Measurements taken to determine this side included 4.6 m (to the outer edge of the external wall), 4.3 m (to the inner edge of the external wall) as well as the use of half the length of the house or even the full length of the house. Those who placed the angle at the top of the triangle found the size of their nominated angle correctly but then doubled it.

Many candidates were able to find the length of the adjacent side by measurement but did not realise that they then had sufficient information to find the required angle by using the tangent ratio. Instead they applied Pythagoras' theorem to find the hypotenuse and proceeded to use either the sine ratio or the cosine ratio to find the angle. Some complicated the process even further by using the sine rule with an angle of 90° and the hypotenuse. Given the time that this would have taken, it was fortunate that those who did this were usually successful.

Mathematics 2/3 Unit (Common)

Question 1

This question consisted of seven parts taken from four separate areas of the syllabus, namely arithmetic (recurring decimals, absolute value, rationalisation of denominators and percentages), trigonometry (exact trigonometric ratios), calculus (primitive functions) and probability.

On the whole, the question was extremely well done, with the majority of candidates scoring at least ten marks out of a possible twelve. Candidate's responses were usually well set out with working shown sequentially in each step.

(a) (1 mark)
This was generally done very well although a significant number of candidates unnecessarily went on to try and prove that $0.\dot{2}\dot{7} = \frac{27}{99} = \frac{3}{11}$, indicating that they were anticipating a question requiring the conversion of a recurring decimal to a fraction. The marking scheme insisted that the candidate's response indicate the 'recurring' nature of the decimal and answers such as 0.27272727 received zero marks. However, an answer of $0.2727\ldots$ did receive the mark.

(b) (1 mark)
The majority of the candidature answered correctly. However, multiple answers, in particular ± 3, were frequent. These were given no marks. Many candidates thought that their answer had to be positive because there were absolute value signs in the question, while others were under the impression that $|-5| = \pm 5$ and $|8| = \pm 8$.

(c) (2 marks)
This was well done with many candidates scoring 2 marks. There was some confusion over the meaning of the phrase 'heads appear every time.' A number of candidates interpreted the question as asking them to calculate the probability that at least one head occurs in a set of three throws, while most thought that the question asked for the probability that three heads occur in a set of three throws.

A tree diagram approach was often used. An incorrect answer accompanied by a tree diagram with eight branches and having one branch correctly labelled with three heads received one mark. Mistakes frequently appeared in fraction manipulation such as $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$ and $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{6}$. Other candidates thought that the probability of the independent events should be added.

(d) (2 marks)
This part was answered correctly by an overwhelming majority of the candidature. By far the most common error was to differentiate rather

than integrate. Those making this error presumably did not understand the meaning of the word primitive. The marking scheme awarded one mark for the correct primitive of each term.

(e) (2 marks)
This was by far the worst answered part of the question. The most common incorrect response was to use a calculator and round off the decimal approximation to give an 'exact' answer. Some candidates thought it sufficient to convert the $\frac{\pi}{4}$ and $\frac{2\pi}{3}$ to 45° and 120°, without finding the value of the trigonometric ratios.

Many had trouble with $\frac{2\pi}{3}$ because it wasn't one of the angles in the right triangles they had drawn to help them find exact values. On many occasions this led to candidates writing $\sin\frac{2\pi}{3} = 2\sin\frac{\pi}{3} = 2\times\frac{\sqrt{3}}{2}$. Others added the angles together stating $\sin\frac{\pi}{4} + \sin\frac{2\pi}{3} = \sin\frac{11\pi}{12}$ and then went on to state that an exact value did not exist for this angle.

One mark was awarded for obtaining $\frac{1}{\sqrt{2}}$ and one mark was awarded for obtaining $\frac{\sqrt{3}}{2}$. Many candidates went on to simplify $\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}$ but this was not required to obtain full marks, and errors in this process were ignored.

(f) (2 marks)
There were two common approaches by candidates attempting to answer this question. The first involved rationalising the denominator in each fraction separately, while the second treated the entire expression as an addition of two fractions. While most candidates showed a good understanding of their chosen technique, many mechanical errors were made in manipulating fractions. Incorrect attempts to rationalise denominators included such things as

$$\frac{1}{3-\sqrt{2}}\times\frac{3-\sqrt{2}}{3-\sqrt{2}},\quad \frac{1}{3-\sqrt{2}}\times\frac{3-\sqrt{2}}{3+\sqrt{2}},$$
$$\frac{1}{3-\sqrt{2}}\times\frac{3+\sqrt{2}}{3-\sqrt{2}}\quad\text{and}\quad\frac{1}{3-\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

The first mark was awarded if candidates had written

$$\frac{1}{3-\sqrt{2}}\times\frac{3+\sqrt{2}}{3+\sqrt{2}}+\frac{1}{3+\sqrt{2}}\times\frac{3-\sqrt{2}}{3-\sqrt{2}}$$

and the second mark was awarded for the simplification of this expression. Arithmetic errors were ignored if they occurred after the numerators and denominators had been expanded and all brackets had been removed.

Working was not shown on a significant number of occasions. Candidates with incorrect answers cannot be awarded part marks for the steps they have done correctly if there is no evidence of these steps in their writing booklet.

(g) (2 marks)
The question did not specify whether the profit was 37.5% of the wholesale price or 37.5% of the selling price, and both interpretations were accepted. One mark was awarded for part solutions which included writing the statement '137.5% of wholesale price = \$3.08,' or for correctly evaluating 37.5% of \$3.08. Some of the common mistakes were adding 37.5% to the selling price rather than subtracting it, treating 37.5% as 37.5 cents, and thinking that 37.5% of the wholesale price was the selling price. A number of candidates failed to give adequate explanations of their working, making the awarding of part marks difficult.

Question 2

This question was based on differentiation and integration. Part (a) asked candidates to differentiate three functions. The first involved the chain rule, the second required the product rule and the third needed the quotient rule. In part (b) candidates were asked to find two definite integrals while in part (c) they were required to find an integral of the form $\int \frac{f'(x)}{f(x)}\,dx$ by inspection.

Most candidates handled the question well, with a large number scoring full marks and with many of the remaining candidates losing only one or two marks. Part (c) caused most difficulties for the candidates and there were fewer attempts at this part. Only a minuscule number of candidates did not attempt any part of the question.

(a) (i) (2 marks)
The majority of the candidature recognised the need to use the chain rule and applied it correctly. Almost all candidates received at least one mark. Common incorrect responses included $5(3x^2+4)\cdot 6x$, $5(3x^2+4)^5\cdot 6x$, $4(3x^2+4)^4\cdot 6x$ and $30(3x^2+4)^4$. Only a few candidates failed to differentiate the function $3x^2+4$.

(ii) (2 marks)
In this part, many candidates correctly stated the product rule but then showed their lack of mathematical understanding by incorrectly writing the two elements of the product as $x \sin$ and $x+1$. Some who correctly stated that $u = x$ and $v = \sin(x+1)$ and arrived at the correct answer, then tried to take out a common factor with their final answer being $(x+1)(x\cos + \sin)$. Another indicator of confusion was the omission of parentheses in candidates' responses.

Only a few candidates incorrectly differentiated $\sin(x+1)$ to get $-\cos(x+1)$.

(iii) (2 marks)
Most candidates correctly wrote down the quotient rule. Nearly all were then able to apply it without error to the given function and most knew the derivative of $\tan x$.

Some with correct answers then incorrectly simplified $\dfrac{x\sec^2 x - \tan x}{x^2}$ and gave $\dfrac{\sec^2 x - \tan x}{x}$ as their final answer.

Candidates who used the product rule in preference to the quotient rule were generally successful in gaining full marks for this part.

(b) (i) (2 marks)
A significant number of candidates thought the answer to the integral should be $[\log x^2]_1^2$. This included many candidates who were correct in every other part of the question. Some differentiated instead of integrating the function. In general the substitution and the relatively difficult final calculation were done well.

(ii) (2 marks)
Although this part involved a standard integral supplied in the table on the back page of the paper, many candidates appeared to be unaware of the existence of the table. Others, who did quote $\int e^{ax}dx = \frac{1}{a}e^{ax}, \quad a \neq 0$ from the table, showed that they were unable to transfer this information to the question and subsequently wrote $\int e^{4x}dx = \frac{1}{5}e^{5x}$.

It was pleasing to see that so many candidates knew that $e^0 = 1$.

(c) (2 marks)
Properly prepared candidates should be able to integrate $\dfrac{f'(x)}{f(x)}$ by inspection, without formal change of variable. Most candidates who, before integrating, wrote $\dfrac{1}{2}\displaystyle\int \frac{2x}{x^2+3}\,dx$ gained full marks. Those who did not show this step often obtained either $2\log(x^2+3)$ or $\log(x^2+3)$ as the primitive and received only one mark. A few candidates who recognised that the answer involved a log function lost marks because they either integrated or differentiated the function x^2+3.

Candidates from centres using the technique $\frac{x}{2x}\log(x^2+3)$ almost invariably failed to score full marks. Others attempted to integrate the numerator and denominator separately or worked the inverse tangent function into their answer. Naturally, they did not earn any marks.

Question 3

This question contained eight parts on coordinate geometry. It presented candidates with a diagram which showed three lines on a coordinate plane and required them to use their knowledge of coordinate geometry and trigonometry to establish relationships between the sides of the resulting triangle. While the diagram included the statement 'not to scale,' in fact it was to scale, and so hinted that the point that needed to be found in part (g), the culmination of the question, was situated on the y axis.

Another feature of the question was the number of instructions to show something. The first four parts were of this nature.

Very few candidates made no attempt at this question. Most candidates gained at least eight marks and many gained ten or more marks.

(a) (2 marks)
This question simply asked candidates to show that two points were on the line with a given equation. Most candidates used the substitution of both coordinates of the points into the equation successfully, while a large proportion found the gradient and used the point-gradient form of the straight line to show that they obtained the same equation.

Of the candidates who substituted, most did not use the 'LHS = , RHS = ' form of argument, and instead made statements like $3 + 0 = 3$ and $-3 + 6 = 6$, or even $3 = 3$ on some occasions. The examiners looked for evidence of correct substitution to award the marks.

It was interesting to note that a good number of candidates did not realise when they had given enough information, wasting time by substituting x to find y and then substituting y to find x, or finding the equation using the gradient and one point and then finding the same equation using the gradient and the other point.

(b) (1 mark)
This question asked candidates to show that the gradient of the line joining two points was $\frac{1}{3}$. The vast majority had no trouble with this, although a small number were not awarded the mark because of a failure to show any connection between their stated formula and the result. Some used the two point form to find the equation of the line and then gave the gradient.

(c) (1 mark)
This was probably the easiest part of the question. Candidates were asked to show that the distance between two points was $\sqrt{10}$. Most were able to use the distance formula correctly, or at least work backwards from their answer to find their mistake. Others used Pythagoras' theorem.

(d) (1 mark)
Candidates needed to show that two lines were perpendicular. Most did

so by stating the two gradients and showing that their product equalled -1, or else mentioned negative reciprocals. Those candidates who used $m_2 = \dfrac{-1}{m_1}$ often made mistakes when substituting in their gradients, making statements like $\frac{1}{3} = -3$.

(e) (2 marks)
This part asked candidates to find $\tan\theta$, where θ was the angle between two lines marked on the diagram. Since part (d) had established that the three lines in the question formed a right triangle, the use of 'opposite over adjacent' in this triangle was the most common method. Many candidates correctly obtained $\dfrac{\sqrt{40}}{\sqrt{10}}$ for this ratio. However, many did not simplify this to 2, thus making part (g) more difficult for themselves.

Some used a geometrical argument to find the angle θ, after first using a gradient to find the angle between a line and the horizontal. Many, who had studied the 3 unit course, undertook the more difficult method of using the formula for the angle between two lines with varying degrees of success. It was clear that many could not remember where the $+$ and $-$ went in the formula, and many also forgot to take the absolute value.

Once again, many candidates did not know when they were finished and went on to find the angle θ to the nearest minute. It was disturbing to see the poor notation used here. For example $\tan\theta = 2 = 63°26'$, $\tan\theta = 2°$, and $\tan \frac{\sqrt{40}}{\sqrt{10}}$ were relatively common.

It should also be mentioned that a number of candidates apparently thought that the angle θ was the angle between the line and the x axis and found θ to be $\tan^{-1}\frac{1}{3}$.

(f) (2 marks)
This part required candidates to find the equation of the circle centred at $A = (1, 0)$ passing through B. The fact that $AB = \sqrt{10}$ had been given in part (c). While one would have anticipated that this would be an easy two marks for candidates, too many candidates did not gain full marks here.

Common errors involved using the wrong centre or radius, writing r instead of r^2 in the equation of a circle, using an equation of a straight line or parabola and the omission of the $+$ between $(x-1)^2$ and y^2.

(g) (2 marks)
This part asked candidates to find the point D. This point was described as being between A and C on the line joining those two points and placed so that both D and B are the same distance from A.

This part presented the greatest difficulty of all parts in the question. Candidates who had found that $\tan\theta = 2$ in part (e) were more likely to see that the required point D was the midpoint of the interval AC, and this led to a very simple calculation and full marks. A number of candidates thought that D was $\frac{1}{4}$ of the way along AC because they did not simplify their result in part (e) correctly.

Many candidates tried to make use of the fact that D was on the circle whose equation had been found in part (f), and attempted to solve this equation simultaneously with the equation of the straight line given. There were varying degrees of success. This method exposed many errors, particularly in the expansion of $(3-3x)^2$. Some candidates, who had not been able to find the equation of the circle when asked to do so in part (f), wanted to make use of the fact that the distance from $D=(x,y)$ to $(1,0)$ was $\sqrt{10}$ and so effectively found the equation of the circle in the course of answering this part.

It was unfortunate that the point D was also the y-intercept of the line AC, since it was obvious that many candidates who had not done well in earlier parts of the question merely guessed the correct answer, and then showed that it was D by finding the length of the interval AD.

(h) (1 mark)
This part asked candidates to shade the region $3x+y \leq 3$. Note that they had already shown that the line $3x+y=3$ was the line AC in the diagram. Most candidates knew to test a point and so decide on the correct side of the line. However, they then used almost any of the five intersecting lines in the diagram as boundaries for their region. Some were very inexact in their shading, making it difficult for the examiners to judge which region was intended. A considerable number bounded their region by the horizontal line $y=3$, perhaps trying to link this part with part (g). Some candidates drew a completely new line on their diagram, and others spent a considerable amount of time testing points from every region formed by intersecting lines on the diagram.

Question 4

The four parts of this question involved finding approximations for a definite integral using both Simpson's rule and the trapezoidal rule, finding the common difference and the sum of ten terms of an arithmetic series, finding the common ratio and the limiting sum of a geometric series, and determining the vertex and equation of the directrix given the equation of a parabola.

Many candidates were distracted by the close proximity of the different series type questions, with many good candidates omitting part (c). Poorly prepared candidates often found answers to both parts (b) and (c) by trial-and-error methods, helped by the fact that the common ratio in the latter part was the given fourth term. Atrocious algebraic manipulation skills cost many candidates at least one mark in part (d), and sloppy calculator work did not aid the gaining of marks in parts (a) and (b).

Candidates who appeared to be well prepared usually earned close to full marks. The marks which such candidates were most likely to have missed were the mark for h in (a) (ii), and the mark for the calculation of the directrix in (d) (ii).

(a) (i) (2 marks) and (ii) (2 marks)

In each part one mark was awarded for establishing the width of each strip and arriving at the correct multiplier ($\frac{1}{3}$ for Simpson's rule, $\frac{1}{2}$ for the trapezoidal rule) and one mark for adding the three given ordinate values with the correct weightings. Those candidates who used $\frac{b-a}{n}$ for h, rather than the difference of successive x-values, often substituted $n = 3$ (the number of ordinate values) rather than $n = 2$ (the number of strips or subintervals).

While many candidates substituted successfully into a variety of learned formulae, others either could not place the given numbers in the correct positions (usually because of confusion over 'even' and 'odd' x-values) or used the x-values rather than the values of $f(x)$. This latter mistake was endemic to those less well-prepared candidates using a formula which contained function notation. Such candidates were often mystified by the expression $f(\frac{a+b}{2})$ in their formula. The most successful candidates were those who used a table of the supplied values with the weights added in an extra column. For more details of this method, see the Examiners' comments on 2/3U Question 4, part (c) in 1995.

Many candidates misnamed the two rules, put the wrong multiplier with the weighted ordinate values, used the same weighted ordinate values with the two different multipliers, or had learnt only one rule. A few candidates tried to create 5 function values for Simpson's rule, while others used only the end-point values in their attempt at the trapezoidal rule.

(b) (i) (1 mark)

Most candidates used $T_n = a + (n-1)d$ to establish two equations involving the first term and the common difference and usually proceeded to $3d = -15$ or $3a = 126$. A common error was to write $17 = a + 16d$ for T_6.

The equation $3d = -15$ appears to have 4 solutions, -5, 5, -3 and 3, but only -5 gained the mark! Good candidates shortened the simultaneous equation process by substituting $a = 32$, $n = 4$ and $T_4 = 17$, since that series has the same common difference. Those candidates whose working was more intuitive often wrote that the common difference was 5, failing to recognise the significance of the negative sign in -5.

(ii) (2 marks)
The majority of those candidates who worked intuitively in (i) listed the ten terms in their series and summed them on a calculator and such answers which were consistent with the candidates value for d in part (i) received full marks.

A large section of the candidature used either $\frac{n}{2}(2a + (n-1)d)$ or $\frac{n}{2}(a+l)$ to find the sum. Many who had stated that $d = 5$ in (i) used the correct value, $d = -5$, in (ii). No penalty was invoked for this discrepancy. However, those candidates who changed a negative d in (i) into a positive d in (ii) received at most one mark for part (ii). The most common error in finding a was to count back three terms, rather than two terms, from 32. Disappointingly, not one candidate used $S_n = \frac{n}{2}(\text{5th term} + \text{6th term})$.

A small proportion of the candidature had no formula, or an incorrect formula, for S_n. A small number of candidates believed the question asked for T_{10}. Others used the formula for the sum of a geometric series.

(c) (i) (1 mark) and (ii) (1 mark)
This was not as well done as part (b), with many candidates not knowing or unable to use the general formula $T_n = ar^{n-1}$ to arrive at $a = 16$ and $ar^3 = \frac{1}{4}$. Reaching this stage was sufficient to earn the mark in part (i). The appearance of r^0 in the formula for T_1 created a problem for a significant proportion of the candidature.

Even amongst those candidates who wrote down $a = 16$ and $ar^3 = \frac{1}{4}$, some proceeded to $r^3 = -15\frac{3}{4}$ while others deduced that $r = 4$ from $r^3 = \frac{1}{64}$. A noticeable number used trial-and-error methods, often after much crossed-out 'working'. Some were able to see that the series was 16, 4, 1, $\frac{1}{4}$, ... and were awarded the mark for part (i), but went on to write $r = 4$, or that the common ratio was 'division by 4'.

A high proportion of the candidature knew that the formula for the limiting sum was $\frac{a}{1-r}$, though some could not substitute $a = 16$ to gain the mark for part (ii). Most of those candidates with $|r| > 1$ in their part (i) answer made no mention of the restriction $|r| < 1$ in the calculation of their limiting sum. The very small number who did state that there would not be a limiting sum because this restriction was violated were also awarded the part (ii) mark.

The candidates who believed $T_n = ar^n$ often gained the mark in part (i), but did not gain the mark in part (ii) because they would calculate that $a = 64$. Those who tried to use arithmetic series formulae had no success in either part.

(d) Substantially the same team of examiners marked the similar question,

Question 2 (d), in the 1997 examination paper, and the comments made on that occasion are still pertinent.

Although this was the least well-done part, there were signs that many centres had worked through the previous examination paper. It still appears that the work examined in part (d) has hardly been extended beyond the standard parabola $x^2 = 4ay$ in many centres. Many candidates also have little idea of the significance of the vertex, focus and directrix. It seems that few have ever drawn the locus of a point equidistant from a fixed point and a fixed line.

Well-prepared candidates often simply wrote down the correct answers to (i) and (ii) and gained full marks.

(i) (1 mark)
The mark was awarded only when both coordinates ($x = 0$, $y = -3$) of the vertex were correctly stated.

Most of the candidates who compared the equation in the question with $(x-h)^2 = 4A(y-k)$ gained the mark. On the other hand, those who used $(x+h)^2 = 4A(y+k)$ were rarely awarded the mark.

Candidates who made y the subject of the formula often made errors in this process. They typically used calculus to find the turning point, or vertex, and usually correctly found that $x = 0$. However, most would then substitute back into their incorrect equation for y and so obtain the wrong second coordinate.

Many tried to use a formula for the axis of symmetry to find the point required, but did not possess the algebraic skills needed to proceed to the correct answer. All too often these candidates believed $x^2-8y-24$ to be a quadratic in x. Many poorly prepared candidates believed that the coordinates of the vertex are obtained from the non-zero values of the axial intercepts, which meant that $(\sqrt{24}, -3)$ was a fairly common answer.

Candidates often confused the vertex with the focus, and even the directrix, as happened in the previous year.

(ii) (2 marks)
To gain the first mark, candidates needed to indicate that the focal length of the parabola was 2. For the second mark, they were required to either establish the equation of the directrix appropriate for their vertex and focal length or suitably label a sketch of the directrix in relation to the parabola. Many candidates did draw clear sketches which aided the awarding of these marks.

The main errors in the calculation for the focal length were concluding that $A = 4$ from $4A = 8$, setting $2A = 8$, and writing $4ay = 8y + 24$. This last error led to a great variety of answers. Many candidates either believed that the directrix is always $y = -A$ or that the focus is always $(0, A)$. In the latter case, the focal length was usually obtained by subtracting the vertex's ordinate value from A.

A significant group of candidates believed that the directrix is a point. Another group found the correct focal length but could not proceed to find the directrix.

Question 5

In this question, part (a) dealt with plane geometry and involved congruence tests, angle sums and the computation of the area of triangles using trigonometry. Part (b) was about exponential growth, and involved computations using logarithmic and exponential functions. The average mark scored on this question was almost 8.5 and approximately 15% of candidates obtained full marks.

(a) It was pleasing to see that most candidates followed the direction to copy the diagram into their writing booklet. This should, in any case, be an automatic step in any geometry problem. Most candidates then made good use of their diagram by adding information to it as they worked through the question.

(i) (2 marks)
This was generally well done. However, the simple fact that the triangle is isosceles because it has two equal sides — the response '$CB = CD$ (equal sides of regular pentagon)' was all that was required — was missed by many candidates and many more failed to answer the second part of the question which asked them to find $\angle CBD$.

Common errors included working with the wrong triangle, typically proving that $\triangle ABD$ was isosceles, and use of various equalities such as $\angle EDA = \angle CDB$ which had not been established.

(ii) (2 marks)
This required writing a congruence proof with reasons for each statement. In general, it was attempted well, although there frequently was a lack of understanding and incorrect usage of congruence tests. For example, use of the abbreviation ASS does not indicate that the candidate understands the true significance of an 'included angle'. It should be noted that geometric proofs require reasons either indicating that the 'fact' was given or has been proved in the question. Candidates often quoted 'facts' that actually required proof or some justification. For example, $AD = DB$ was often used in SSS tests without any justification.

Common errors included the use of SAS where the angle was not the 'included' one, failure to link facts in the two triangles, failure to state the congruence test used, and inclusion of correct but superfluous information that inevitably led to confusion. Others made use of AAA as a 'congruence' test. Symbols were frequently misused. Many used $|||$ instead of $\equiv$ for 'is congruent to' and others used $||$ instead of $=$.

(iii) (1 mark)
This required the calculation of the size of $\angle ABD$ which is $36°$. This was probably the most successfully answered part in the whole question.

The most common error was the assumption that AD and BD 'bisected' the angles at A and B, giving base angles of $54°$.

(iv) (3 marks)
Finding the area of the pentagon required the calculation of the areas of at least two shapes. These, in turn, involved the use of trigonometry to find one (or more) of the sides.

This part of the question was certainly a discriminator. While almost half of the candidates either failed to attempt it or scored zero, those who did attempt it demonstrated a range of understanding of the trigonometry involved. It was encouraging to see the variety of correct solutions produced here.

An amazingly common algebraic error was that the square of an expression or term was simply equal to double the term. For example, many candidates wrote $\frac{1}{2} \cdot x \cdot x \sin 108° = \frac{1}{2} \cdot 2x \cdot \sin 108°$. Many quoted the cosine rule without the side squared, writing $ED = \ldots$ instead of $ED^2 = \ldots$ Others who used the cosine rule simplified incorrectly, claiming that $x^2 + x^2 - 2x^2 \cos 108° = \cos 108°$. Many did not use brackets correctly and wrote such things as $\frac{1}{2}(2x^2 - 2x^2 \cos 108°) \sin 36° = x^2 - x^2 \cos 108° \sin 36°$.

(b) Candidates had a high degree of success with this part. The vast majority scored full marks.

(i) (1 mark)
This only required a statement of the fact that $A = 1\ 000\ 000$. Many of those who attempted to calculate the value of A got into trouble with their usage or understanding of e^0.

(ii) (2 marks)
A correct substitution of $t = 2$ and of their value of A from part (i) was sufficient to receive one of the marks here. Candidates were generally able to go on to deduce that $k = \frac{1}{2} \log_e 1.075$ or get $k = 0.034\ 996\ldots$

Most common errors were the use of $\log_{10}$ instead of $\log_e$, the use of e^x instead of $\log_e$ and incorrectly rewriting $\log 1\ 000\ 000e^{2k}$ as

$2k \log 1\ 000\ 000$ or $\log(\frac{a}{b})$ as $\frac{\log a}{b}$. Transcription errors, particularly writing 100 000 in place of 1 000 000, were also common.

(iii) (1 mark)
This part was also well done. It required the calculation of the time taken to reach a population of 2 million using the value of k obtained in part (ii) and the value of A found in part (i).

Question 6

Part (a) of this question required candidates to interpret a displacement-time graph, while part (b) involved the application of given derivatives to sketching a function involving exponentials. A small number of candidates did not attempt this question. Unfortunately, however, a significant proportion of the candidature scored zero. Many candidates attempted only one of the two parts.

(a) It was apparent that many candidates had not previously encountered a question asked in this style and had difficulty in answering any part. Others quoted facts about velocity and acceleration, but were not able to apply them to the diagram. Incorrect units were ignored in the marking of this question.

(i) (2 marks)
A significant number of candidates had difficulty with this part. Many candidates simply found where the graph cut the t-axis, possibly thinking that the graph was a velocity-time graph. Candidates often missed the second solution which was between 5 and 6.

(ii) (1 mark)
About half of the candidates gave the correct answer to this part. Of those who were incorrect, many gave more than one answer, even though the question made it clear that there was only one answer.

(iii) (1 mark)
A number of candidates incorrectly chose the time when $t = 1$ as the answer, only considering when the particle was to the right of its initial position. Any value in the range $5 - 6$ seconds was acceptable, but many candidates had difficulty in determining the value from the graph to even this level of accuracy.

(iv) (1 mark)
A large number of candidates wrote $t = 3$. Many of these were clearly trying to find when the acceleration was 0, rather than looking for the steepest gradient which occurred at $t = 8$. Those who scored 11 out of 12 often lost the mark here.

(b) (i) (1 mark)

Many candidates correctly determined that it was necessary to find when $f'(x) = 0$. However, of these, a significant number incorrectly solved as follows:

$$\begin{aligned}
&e^{-2x} - 2xe^{-2x} = 0 \\
&e^{-2x}(1 - 2x) = 0 \\
&e^{-2x} = 0 \text{ or } 1 - 2x = 0 \\
&\therefore x = 0 \text{ or } x = 0.5
\end{aligned}$$

Candidates who attempted to use logarithms to solve this equation were almost always wrong. A small number found when $f''(x) = 0$ and used this as their stationary point while a similar number simply chose $x = 0$ as their solution.

(ii) (1 mark)

Many candidates incorrectly stated that 'increasing means $f''(x) > 0$', and so found that $f(x)$ is increasing when $x > 1$. Even more worrying was the fact that a significant number of candidates in this part simply evaluated the y-coordinate of the stationary point and determined its nature.

Candidates used a number of methods. Some constructed a table of values, others argued from the nature of the stationary point, but the most common approach involved solving $f'(x) > 0$.

Very few candidates remarked that e^{-2x} is always positive, though many simply ignored the possibility that the e^{-2x} could have any impact on the solution. A significant number divided by a negative number in the course of attempting to solve $e^{-2x} - 2xe^{-2x} > 0$ and failed to reverse the direction of the inequality.

(iii) (2 marks)

Many candidates realised that the point of inflection occurs where $f''(x) = 0$. A few then incorrectly factorised as $4e^{-2x}(4x - 1) = 0$ or $e^{-2x}(x - 4) = 0$

As in part (i), a high percentage of the candidature attempted to solve $e^{-2x} = 0$, inevitably finding $x = 0$ as one of their solutions.

Many candidates did not proceed to attempt to find where the graph is concave up. Of those who did, many simply found that $f''(x)$ was negative at the stationary point and then stated that the curve was concave down at $x = 0.5$. Of those who correctly realised the need to solve $f''(x) > 0$, a large percentage were able to correctly determine that the solution was $x > 1$. Others tried to solve $e^{-2x} > 0$, usually claiming that the solution was $x > 0$.

Of those who did check that the concavity changed on either side of $x = 1$, many were either not able to draw the conclusion about when the curve was concave up or simply left it out.

A smaller number of candidates did not understand the meaning of concave up, and believed that they were being asked to find where $f(x) > 0$. These candidates then attempted to solve $xe^{-2x} + 1 > 0$ and a surprising number, by methods known only to them, arrived at the correct answer for this part. Needless to say, they did not receive any marks for this unless their answer to this part was actually the result of a correct graph in part (iv). (A similar number had also believed that increasing meant $f(x) > 0$ in part (i), often claiming that the solution to $xe^{-2x} + 1 > 0$ was $x < 0.5$.)

(iv) (2 marks)
Generally, this part was poorly done. Candidates usually failed to relate their answers for the previous parts to their sketch. Many candidates simply 'plotted points' and missed the stationary point at $x = 0.5$, thereby drawing the incorrect graph. Candidates experienced difficulty in using their calculator to determine the y-coordinates of the important points, and often used an inconsistent scale on the vertical axis. This was not helped by the small variation in the y-coordinates which led to rather compressed looking graphs.

Some candidates thought that the point of inflection had to be horizontal, which created further difficulties. Those who did this part correctly, generally produced graphs of a very high standard. However, candidates should be encouraged to label their graphs clearly, as this caused some problems.

(v) (1 mark)
Many candidates correctly stated that the graph would tend towards 1, or used words that implied the same. Of those who answered incorrectly, many assumed that the graph would 'get closer and closer to the x-axis', or 'get bigger and bigger.'

Question 7

The question contained three parts. Part (a) involved knowledge of the term discriminant and the condition for a quadratic equation to have real roots. Part (b) required finding the equation of a tangent to a trigonometric function and the maximum and minimum values of that function in a given domain.

The final part, part (c), led to an optimisation problem with a restriction on the domain. Although the function was given, candidates required a knowledge of arc length and the area of a sector of a circle to answer the lead-in parts.

Candidates responses were varied, with candidates often being let down by poor basic skills in solving linear inequalities, evaluating surds, solving trigonometric equations and using the quotient rule to differentiate. A common error was the use of degrees in a question which clearly involved radian measure.

In parts (b) and (c) candidates often wrote down a correct process without attempting to implement the process. This used time but did not gain any marks.

(a) (i) (1 mark) and (ii) (1 mark)

Overall this part was poorly done with only 65% of the candidature being able to write down the discriminant in part (i). Many of those then made mistakes while simplifying their expression or changed their expression for the discriminant into an equation. In part (ii), less than half of the candidates were able to write down the condition for the quadratic to have real roots and solve the inequality correctly.

Some candidates ignored the separation of the question into two parts and began the question by solving $2^2 - 4 \times 3 \times k \geq 0$. Others confused 'discriminant' with 'differentiate'.

In part (i), a common mistake was to claim that $\Delta = 4 + 12k$ or $\Delta = 16 - 12k$. In part (ii), many believed that $\Delta = 0$ or $\Delta \leq 0$ was the condition for a quadratic to have real roots, while many thought that the solution to $-12k \geq -4$ was either $k \geq \frac{1}{3}$ or $k \leq 3$.

Answers such as $k < \frac{1}{3}$, $\frac{1}{3} \geq k$ and $\frac{1}{3} > k$ all received the mark for part (ii).

(b) (i) (2 marks)

Roughly equal numbers scored zero, one and two marks in this part. Most candidates realised that a derivative was required, though some believed that the derivative itself was the equation of the tangent. Others did not attempt to substitute $x = \frac{5\pi}{6}$ into their derivative. Many candidates were unable to evaluate the y coordinate correctly, making errors in manipulating surds or using $\cos \frac{5\pi}{6} = \frac{1}{2}$.

Common mistakes in the course of finding the derivative included $\frac{d}{dx}\sqrt{3}\sin x = \cos x$, $\frac{d}{dx}\cos x = \sin x$ and $\frac{d}{dx}1 = 1$.

(ii) (3 marks)

About half the candidates either did not attempt this part or attempted it but gained no marks.

Most of the remainder started by equating their derivative to 0. Unfortunately many then found either one or four solutions for the required domain. Examiners were disappointed by the fact that many strong candidates stated the values of x which produced maximum and minimum values, providing appropriate justification, but did not attempt to find these values.

Very few candidates attempted to put $1+\sqrt{3}\sin x+\cos x$ in the form $1+2\sin(x+\alpha)$. However, many more rewrote the derivative in this form, or used the t method, in their attempt to find the stationary points. Attempts at graphical solutions were usually very poor, with many candidates concluding from their graphs that the maximum and minimum values were $\pm(1+\sqrt{3})$.

(c) Most candidates had difficulties with this part and only a small proportion were able to gain 3 or more marks.

(i) (1 mark)
About one third of the candidature gained the mark. The most common mistakes were to only include the arc length in the perimeter leading to $r=\dfrac{8}{\theta}$, being unable to make r the subject of the equation $2r+r\theta=8$, or finding an expression for θ in terms of r.

(ii) (1 mark)
Approximately half of the candidates gained this mark by correctly substituting their expression for r into the correct formula. Even if the correct answer cannot be gained, candidates should at least show their substitution line before they attempt to 'fudge' the correct answer. This can often, as in this case, earn the mark. However, if no evidence of a correct substitution is found in the candidate's writing booklet, the candidate will not receive any marks.

Similarly, candidates should not completely obliterate all their work if they cannot reach the correct answer. A single line through the work, or even a comment to the effect that this is clearly wrong, greatly assists the examiner to find each mark earned by the candidate.

(iii) (3 marks)
Only two-thirds of the candidature attempted this part. Most, even if they were able to differentiate, were then unable to solve the resulting equation correctly. Among the better candidates, who found that there was a stationary point at $\theta=2$, very few realised that this solution was outside the required domain. Those who did often did not make any comment to this effect. Instead, they crossed out all their working and went on to find the area for $\theta=\frac{\pi}{2}$. While these candidates were given the benefit of the doubt in this instance, it is the candidate's responsibility to clearly indicate to the examiner the reasons for their answers.

As in part (b) (ii), many found a value for θ, but did not compute the value of the area corresponding to this value for θ. Many spent much time and effort finding the value of the second derivative at $\theta=2$.

Testing their first derivative at values on either side would have been much simpler.

Weaker candidates who just tested the end points of the domain often then incorrectly found an 'area' by changing to degrees. Candidates who used the product rule with negative indices were usually unsuccessful, usually making errors while attempting to find when their derivative vanished.

Question 8

The first part of this question was an application of calculus based on an equation giving the rate, R, at which sand was being tipped from a truck. Many candidates predictably failed to realise that R was already the rate, and not the amount which had been tipped.

The second part of the question involved finding the volume of revolution when part of the curve $y = \log_2 x$ was rotated about the y-axis.

Both parts were quite demanding. However, the examiners were pleased with the perseverance shown by the vast majority of candidates. Non-attempts were extremely rare. There was a clear discrimination between average candidates, who usually scored between one and three marks and the really capable candidates who scored eight or more marks.

Confused labelling of parts in the answers to part (a), or a complete absence of part numbers, caused significant problems for the examiners.

(a) (i) (1 mark)
This required a simple substitution, and more than 90% of the candidature scored this mark.

(ii) (1 mark)
This part required an understanding that the flow will stop when $R = 0$, leading to the solution $T = 10$ or $T < 10$. The restrictions in the introduction to the question meant that the other solutions to $R = 0$, namely $t = -10$ and $t = 0$, were not relevant to this part.

Many candidates seemed to think time was discrete, and so thought that $T = 9$ was the only possibility for $T < 10$. Others tried to solve $\frac{dR}{dt} = 0$ here rather than in part (iii).

(iii) (2 marks)
Candidates had to find the maximum value for R. This required recognition of the need to solve $\frac{dR}{dt} = 0$ which leads to the answer $R = 384.9$.

A substantial number used a method of trial-and-error, substituting different values for t. This usually resulted in candidates believing that the maximum occurred when $t = 6$, leading to a maximum value of 384. This was awarded one of the two available marks.

Careless differentiation cost some candidates marks, with many ending up solving $100 - 2t^2 = 0$. A failure to substitute the value found for t in order to actually find the maximum value for R was another common error.

(iv) (2 marks)
The key to this part was an understanding that the amount of sand could be found by computing the integral of R with respect to t. If done correctly, this yields $A = 50t^2 - t^4/4 + 300$.

Candidates often omitted the constant of integration, which was required for the second mark. Others were confused as to where to place the 300, and gave answers such as $300 = 50t^2 - t^4/4$. Other variations included exponential growth, summing a series and simply multiplying R by t, as if R were constant.

(v) (1 mark)
Substituting $t = 8$ in the answer for part (iv) and then subtracting 300, led to the correct answer of 2176 kg. The most common error, made by more than half the candidates, was to forget to subtract 300. Another common incorrect answer was 2304, which was obtained by summing the value of R for $t = 1, 2, \ldots 8$, again reflecting candidates' belief that time is discrete.

(b) (i) (3 marks)
With an answer provided in the question, the onus is clearly on the candidate to show why $x^2 = e^{y \ln 4}$. There are two main pathways to this. The first is to write $x = 2^y$, which means that $x^2 = 2^{2y}$, which in turn means that $x^2 = e^{2y \ln 2}$. The second argument is to use the fact that $y = \frac{\ln x}{\ln 2}$ to deduce that $x = e^{y \ln 2}$ and so $x^2 = e^{2y \ln 2}$.

Many candidates omitted the third step of their argument, going straight to $e^{y \ln 4}$, which was not convincing in the context of the question. A poor knowledge of logarithms and exponentials was evident in the work of many candidates, and there was much obvious fudging by working backwards.

(ii) (2 marks)
The first mark was awarded for a correct primitive and the second mark was given for calculating the volume as either $63\pi/\ln 4$ or 142.8. Candidates who did not include the π in their final computation of the volume did not receive the second mark.

Candidates whose primitive was incorrect had to show they knew that $e^0 = 1$ in the course of their working in order to get a mark.

It was fairly common for a candidate's primitive to have y in the denominator, which led to division by zero in the attempt to evaluate the volume. This made it difficult for such a candidate to earn the second mark.

Question 9

There were three parts to this question. The first required the solution of a logarithmic equation, the second asked for the calculation of the area between two curves and the third began with a proof involving similar triangles. Nearly all of the candidates attempted this question, with the majority gaining at least some marks.

(a) (2 marks)
This question asked candidates to solve the equation $\ln(7x - 12) = 2 \ln x$. Those who realised that $2 \ln x$ could be rewritten as $\ln x^2$ were generally able to establish that $7x - 12 = x^2$ and proceed to the solution of the quadratic. However, a surprising number made mistakes in this last step.

A large number of candidates appeared to have little or no knowledge of logarithmic laws. Common errors included writing $\ln(7x-12)$ as $\ln 7x - \ln 12$ and work which can only be described as dividing both sides by ln.

(b) (4 marks)
Candidates were required to calculate the area of the shaded region between two trigonometric curves. Those who used the upper curve minus lower curve approach, and so immediately wrote down $\int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} \cos 2x - \sin x \, dx$, often gained full marks.

The candidates who attempted to divide the area up into sections made the question much more difficult and created many more opportunities to make mistakes. Candidates often used absolute value signs inappropriately in this question, and were careless in the writing of the limits of integration. Negative signs appeared and disappeared frequently.

Many candidates failed to gain the mark which was awarded for the correct substitution of the limits and evaluation of the resulting expression because their calculators were not correctly set in radian mode. A number of candidates interchanged the signs associated with the integrals of $\sin x$ and $\cos 2x$ or were unable to handle the 2 appearing in $\int \cos 2x \, dx$.

(c) (i) (2 marks)
This part asked for a proof that the two triangles were similar. Many candidates failed to earn marks through careless inaccuracies, by not giving reasons or by failing to state the test used.

On the other hand, some candidates who earned no marks elsewhere in Question 9 managed to score marks in this part.

Candidates should be advised to reproduce the diagram for a geometry proof in their writing booklet, especially as marks are sometimes awarded for reasons which are apparent from markings on the diagram, even if they are not present in the text of the candidate's answer. Obviously, such marks cannot be awarded if the markings are only made on the diagram provided in the question paper.

(ii) (2 marks)
Many candidates did not attempt this part. Others did not make use of enough of the information provided in the question to be able to prove the required result.

Common errors were to use measurements from the diagram to demonstrate the relationship or to use the result to be proved in the proof. A number of candidates also demonstrated a lack of understanding of ratios, treating the numbers involved as though they were lengths. A valid proof can be provided in this way, but only by explicitly choosing a unit of measurement with a statement such as 'choose units so that the length of AD is 1.'

(iii) (2 marks)
There were fewer non-attempts for this part than for part (ii). Quite a number of candidates realised the connection with Pythagoras' theorem, and so scored at least one mark. Many candidates were unable to handle the squaring of expressions involved and so were unable to earn the second mark. For example, $(2AB)^2$ often became $2AB^2$ rather than $4AB^2$.

Question 10

Part (a) of this question dealt with probability while part (b) was an application of series. Nearly all of the candidates attempted this question, with very few non-attempts. Candidates found it easy to get some marks, but even amongst the better candidates, very few were able to earn full marks. Good attempts in part (a) were sometimes followed by poor attempts in part (b) and vice-versa.

(a) This probability question concerned the rolling of two dice. A pink die, with faces numbered 2, 3, 5, 7, 11 and 13, and a blue one with faces numbered 4, 6, 8, 9, 10 and 12. A win occurred if the pink die displayed the larger number when both dice were rolled.

(i) (3 marks)
The candidates were asked to determine the probability of a win. The question suggested that this could be done by drawing up a table of possible outcomes, but did not insist on this method.

This was generally well done, particularly by those who could draw a 6×6 table to get $\frac{14}{36}$. There were two common errors. Some candidates miscounted the number of favourable outcomes. Others omitted one of the faces and drew a 6×5 table.

Candidates who tried to use tree diagrams or who listed outcomes were often successful, but were more likely to make mistakes than those using the suggested method. Candidates who interchanged the colours were still able to earn full marks if it was clear that this is what they had done.

(ii) (2 marks)
This part asked for the probability of at least one win in two throws of the dice. The answer, which was $\frac{203}{324}$, could be obtained by evaluating $1 - p(\text{LL})$ or $p(\text{WL, LW, WW})$. Attempts using the latter method were usually followed by a tree diagram.

This question was not well answered. Many gave their response as $p(\text{WW})$, $1-p(\text{L})$, or $1-p(\text{WW})$. Candidates who earned three marks in (i) often earned no marks here.

Of course, candidates could earn full marks here by basing their answer on an incorrect response to part (i).

(b) This testing question required candidates to apply methods which are usually taught in the context of loan repayment schedules to the management of a fish farm.

(i) (1 mark)
Candidates had to find the number of fish remaining just after the second harvest. This could be done by evaluating $100\ 000 \times 1.1^2 - 15\ 400 \times 1.1 - 15\ 400$ or by substitution into the formula for F_2 given in part (ii) to get 88 660. This was generally well done.

(ii) (2 marks)
Candidates were asked to show that $F_n = 154\ 000 - 15\ 400(1.1)^n$. This was not answered well. Candidates who gained both marks overwhelmingly started with $F_n = 100\ 000 \times 1.1^n - 15\ 400(1.1^{n-1} + \cdots + 1.1 + 1)$ and then clearly proceeded to the required formula. Many made minor mistakes in the powers of 1.1.

A great many incorrectly thought that it was sufficient to substitute $n = 2$ into the given formula for F_n and compare this answer with their answer in part (i).

(iii) (2 marks)
This part asked candidates to find when all the fish would be sold and to evaluate the total income at this time. On the whole this part was handled better than part (ii).

Many could recognise the need to solve $F_n = 0$, but could not proceed past the point in the calculation at which this had been reduced to $1.1^n = \frac{154}{54}$ or $1.1^n = 2.851\ 85$. Some were able to see that by taking logarithms of both sides, one could evaluate n to be 10.995.

Another frequent method involved evaluation of F_n using a calculator to find the first value of n for which $F_n < 0$. Candidates using this method were equally likely to give $n = 10$ or 11 as their answer, and both were acceptable for the first mark.

Even after a value for n had been obtained, many candidates could not multiply by $\$10 \times 15\ 400$ to get the total income.

(iv) (2 marks)
Given a persistent offer to buy the business on certain terms, candidates were asked to determine when the farmer should sell in order to maximise his total income. Many candidates did not attempt this part or provided poor answers.

There were two possibilities for a correct formula for S_n, the total income, depending on how the question was interpreted. (One could work on the assumption that the offer was made either before or after the annual contract had been fulfilled.) A small number of candidates derived or stated one of the formulae, but most of these were unable to use it correctly to find the answer to the question.

Many candidates did not attempt to find a formula. Instead, they used their calculators, with varying levels of success, to evaluate S_n for a variety of values of n. This was usually the most successful procedure.

Some appeared to think that they needed to maximise F_n, and attempted to differentiate the expression given in part (ii). The claimed derivative was usually incorrect.

Mathematics 3 Unit (Additional) and 3/4 Unit (Common)

Question 1

This question consisted of six unrelated parts. It was generally well done, with twelve the most common mark, but many candidates lost marks through errors which could have been avoided by clearly showing each formula and step. Parts (c), (d) and (e) exhibited the poorest attempts.

(a) (2 marks)
Candidates were required to differentiate a product containing an inverse trigonometric function. Most candidates easily gained both marks. Common errors involved either omitting the x in one term of the answer or including an additional x in the other term. Candidates who misunderstood the notation $\tan^{-1} x$, mistaking it for $(\tan x)^{-1}$ often used the quotient rule and were awarded zero marks. However, it was still possible to earn one mark with this misunderstanding if the product rule was used and the answer contained the term $2\tan^{-1} x$.

(b) (2 marks)
This part required candidates to find two gradients and use them to calculate the acute angle between two given lines, which was 45°. Candidates could either use a formula or find the difference between $\tan^{-1} 5$ and $\tan^{-1} \frac{2}{3}$. Common errors included results which gave the obtuse angle, a negative angle or more than one angle. Such answers received one mark if they were related to the correct answer.

The formula for the tangent of the angle between two lines was often misquoted, with $+$ and $-$ signs interchanged. Candidates who quoted their formula before substituting were sometimes able to gain one mark despite this error.

Some candidates attempted to use the methods of coordinate geometry to find the point of intersection and then attempted to find a right triangle involving the angle at the point of intersection in order to apply trigonometric ratios. In almost all of these cases, the candidates attempts merely wasted valuable time, often filling two or more pages of a writing booklet. With errors so easy to make, it was very rare for full marks to result from this process.

(c) (1 mark)
The technique required to evaluate $\lim_{x\to 0} \dfrac{\sin 3x}{5x}$ was generally not clearly understood. Most candidates attempted to 'juggle' the numbers 3 and 5, commonly arriving at $\frac{1}{5}$ or $\frac{5}{3}$ instead of the correct answer, $\frac{3}{5}$. Those who used L'Hôpital's rule usually found the correct answer.

(d) (1 mark)
This part examined the candidates' ability to manipulate logarithms and make use of a given value. Many candidates did not see that $\log_2 14 = \log_2 7 + \log_2 2$. A common error was $\log_2 14 = 2\log_2 7 = 2 \times 2.807 = 5.614$, which is actually the value of $\log_2 49$. The correct answer was, of course, $2.807 + 1 = 3.807$. Candidates who avoided the use of the information that $\log_2 7 = 2.807$ and used the change of base rule generally arrived at the correct answer and were awarded the mark. Candidates would benefit from more attention being given to the manipulation of logarithms.

(e) (2 marks)
Although a simple use of the formula $\alpha\beta\gamma = -d/a$ for the product of the roots of $ax^3 + bx^2 + cx + d = 0$ would provide the answer $\frac{1}{2}$, candidates often managed to use the wrong formula, with d/a, $\pm c/a$ and $\pm b/a$ being common substitutes. Some read the incorrect value for d and candidates all too frequently assumed that $a = 1$, perhaps being regularly exposed to examples involving monic polynomials.

(f) (4 marks)
This part, involving the use of a double angle trigonometric identity to evaluate a definite integral, was a direct application of a specific case mentioned in the syllabus. Candidates usually recognised the use of $\cos 2x$ was necessary, but often confused the form of the identity to be used. Common initial errors were omitting the $\frac{1}{2}$, using $1 + \cos 2x$ instead of $1 - \cos 2x$ and writing $\cos x$ in place of $\cos 2x$.

Candidates must be encouraged to show every step. It was difficult to see whether a candidate had integrated correctly if the identity had not been quoted. It is also important to show the substitution of limits into the expression because too many errors occur when candidates attempt their evaluation at the same time. The examiner is then unable to find evidence that either of the steps have been carried out correctly and so cannot award either part mark.

Most candidates who used the correct identity ultimately attained full marks for part (f).

Question 2

While the first two parts of this question were generally well done, the last part caused almost all the candidates a lot of trouble.

(a) (3 marks)
Most candidates could at least start this part, with most of those going on to find that

$$x^4 - x^2 + 1 = (x^2 + 1)(x^2 - 2) + 3\,,$$

or some equivalent statement. The most common mistake, apart from arithmetic errors in the long division, was confusion as to the identity of the quotient and remainder. Many candidates claimed that $Q(x) = (x^2+1)(x^2-2)$ or that $R(x) = \frac{3}{x^2+1}$, while others didn't specify values for either.

(b) (4 marks)
Very few candidates just gave a single number as an answer which was fortunate, as most of these were wrong. A larger number merely wrote down some sort of formula and proceeded to substitute some numbers to get an answer. About half of these were correct.

The majority attempted to derive the answer, in keeping with the syllabus, using one of two methods. One was to work out that the initial \$500 will grow to $\$500(1.08)^{40}$ by the time Paul reaches 65, the next will grow to $\$500(1.08)^{39}$ and so on. The other method calculated the amount in the fund at each birthday, namely \$500, $\$500(1.08) + 500, \ldots$ Both methods lead to the same geometric series which must be evaluated.

Common mistakes included calculating the interest incorrectly by using $r = 1.008$, 1.8 or even 1.18, starting with \$500(1.08) in the fund at the age of 25, and adding, without comment, a further \$500 at age 65. Many summed the geometric series incorrectly, such as

$$ar + ar^2 + \cdots + ar^n = \begin{cases} \dfrac{a(r^n-1)}{(r-1)} & \text{or} \\ \dfrac{ar(r^{n+1}-1)}{(r-1)} \end{cases}$$

$$a + ar + \cdots + ar^n = \begin{cases} \dfrac{a(r^n-1)}{(r-1)} & \text{or} \\ \dfrac{ar^n(r^n-1)}{(r-1)} \end{cases}$$

and other even less likely answers.

Candidates who tried to derive the answer often confused themselves as to exactly how many terms were required. This was less common among those quoting a formula.

(c) Very few candidates even claimed to have finished this part. Most easily disposed of the first two subsections and managed to deduce that $a\cos B = b\cos A$ in (iii) but then ground to a halt.

(i) (1 mark)
About half the candidature used the sine rule while the others showed that both sides are equal to the length of CD. The only common mistakes were to claim that $\sin A = \frac{a}{c}$ and similarly that $\sin B = \frac{b}{c}$.

Some candidates answered this twice, first using the sine rule and then, perhaps feeling this was too easy, showing that $a\sin B = CD =$

$b \sin A$. This might have cost them some time but it also provided a hint for part (ii).

(ii) (1 mark)
The level of difficulty of this part depended upon the approach used in part (i). Some candidates who had used the sine rule in (i) assumed that they needed to use the cosine rule here and began endless calculations. Only a very small number managed to succeed with this approach.

The rest merely showed that $a \cos B = DB$ and $b \cos A = AD$ and noted that $AD + DB = c$.

Some candidates let $AD = x$ and proceeded to show that $a \cos B = c - x$. This approach led to the other common problem. Some candidates showed that $a \cos B = c - AD$ and that $b \cos A = c - BD$ but were unable to finish as they had not noticed that $AD + BD = c$.

(iii) (3 marks)
There were three common successful approaches to this part. The first two began by showing that $a \cos B = b \cos A$, using the result from (ii). This could be done by expanding $c^2 = (a \cos B + b \cos A)^2$ correctly and equating the result with $4ab \cos A \cos B$. This leads to the equation $(a \cos B - b \cos A)^2 = 0$. From this point it is easy to deduce that $a \cos B = b \cos A$.

The first approach to finishing from this beginning was to make use of the fact that $a \sin B = b \sin A$. One could either square and add to show $a^2 = b^2$, or divide to show $\tan A = \tan B$, leading to $A = B$ or $AD = BD$. Each of these cases clearly shows that the triangle is isosceles.

The second approach using $a \cos B = b \cos A$ involved noting that this means

$$\frac{\cos A}{\cos B} = \frac{\sin A}{\sin B}$$

and so, after some algebra, $\sin(A - B) = 0$. Thus $A = B$ and the triangle is isosceles.

The third approach to this part was a self-contained application of the cosine rule. This involved noting that $\cos A = \frac{b^2+c^2-a^2}{2bc}$ and similarly for $\cos B$. Putting these in the equation $c^2 = 4ab \cos A \cos B$ and clearing denominators leaves the equation

$$a^4 - 2a^2b^2 + b^4 = 0$$

and hence $(a^2 - b^2)^2 = 0$, so $a^2 = b^2$ and finally $a = b$.

No matter which approach was used, the examiners did not require candidates to deal with degenerate triangles or note that lengths are non-negative in order to dismiss the possibility that $a = -b$. It is worth noting that most of those using the third method who got that far did make mention of the fact that a and b are both positive.

Over half the candidates used the result from (ii) to show that

$$4ab\cos A\cos B = c^2 = a^2\cos^2 B + 2ab\cos A\cos B + b^2\cos^2 A$$

but stopped there. Some others managed to deduce that $a\cos B = b\cos A$ and but at this point almost all candidates stopped, or simply asserted that 'therefore $a = b$.'

There were several less common solutions. For example, some rewrote

$$4ab\cos A\cos B = c^2 = a^2\cos^2 B + 2ab\cos A\cos B + b^2\cos^2 A$$

as $4AD\,BD = (AD + BD)^2$ and deduced that $AD = BD$ and so eventually concluded that the triangle is isosceles.

Many candidates gained one mark but then spent a lot of time writing two or more pages of trigonometric equations to little purpose.

Question 3

A surprising number of silly or careless errors were evident in this question. Candidates need to be constantly reminded and encouraged to take care with even the simplest operations. In some cases these errors make the question harder than intended, or, perhaps worse, so much easier that the steps needed to gain the marks for the question are no longer evident in the candidate's solution.

(a) (3 marks)
This part was characterised by many silly arithmetic errors. A surprisingly larger number of candidates made silly errors in verifying the statement was true for $n = 1$. For instance, some candidates wrote '$4 + 14 = 20$ which is divisible by 6.'

A number of candidates had difficulty interpreting the induction hypothesis that the statement is true for $n = k$. Many wrote such things as $4^k + 14 = \frac{6}{p}$ or $4^k + 14 = \frac{p}{6}$. A more subtle error was writing $4^k + 14 = 6k$. Even those who correctly wrote $4^k + 14 = 6p$ often went on to try to establish that $4^{k+1} + 14 = 6p$.

A few tried to apply the idea of a sum, and investigated $18 + 30 + \ldots$

(b) (i) (2 marks)
Candidates who wrote the expanded form for $R\sin(4t + \alpha)$ generally did quite well, although some claimed that $\sin\alpha = \sqrt{3}$ and $\cos\alpha = 1$.

Those who resorted to the remembered formulae $R = \sqrt{a^2 + b^2}$ and $\tan\alpha = \frac{b}{a}$ did not have the same rate of success in finding the correct values for R and α.

(ii) (2 marks)
A significant number did not link this to part (i), and solved $\tan 4t = -\sqrt{3}$.

Many had difficulty solving the equation, giving both correct and incorrect values for t. Arithmetic errors were again common, with many deducing from $4t = \frac{2\pi}{3}$ that $t = \frac{8\pi}{3}$.

A number of candidates did not know what general solutions were and did not attempt this part. Some of these correctly found the general solution of the same equation in the course of answering (c) (iii).

Many others did not know how to find general solutions. A very common error was to find a value for t and then use that value in the formula $n\pi + (-1)^n\alpha$ giving answers such as $t = n\pi - (-1)^n\frac{\pi}{12}$, or even $t = n\pi + (-1)^n \sin^{-1}(4t + \frac{\pi}{3})$.

It appears that candidates need to be encouraged to write a number of consecutive values and try to establish the pattern, rather than learn a formula which may need to be adapted to fit the circumstances of a given question.

(c) Many candidates did not see the link between this part and part (b).

(i) (2 marks)
Many simply assumed that the given function represented simple harmonic motion and then tried to address the idea of centre of motion.

Even amongst those who found the second derivative, many did not know how to link it to the idea of simple harmonic motion.

A most common error was

$$\begin{aligned}\ddot{x} &= -16\sin 4t - 16\sqrt{3}\cos 4t \\ &= -16(\sin 4t - \sqrt{3}\cos 4t) \\ &= -16(x - 1)\,.\end{aligned}$$

Many of those who made the link to (b) appeared to think that $x = 1 + 2\sin(4t + \frac{\pi}{3})$ was the same function as $x = 2\sin(4t + \frac{\pi}{3})$.

This became a problem in (iii) when candidates obtained the correct answer from incorrect reasoning by arguing as follows:

> The maximum speed occurs when $x = 0$. This happens when $2\sin(4t + \frac{\pi}{3}) = 0$. Therefore $t = \frac{\pi}{6}$ from part (b).

(ii) (1 mark)
Many found when the velocity was 0, and found the corresponding x value as the amplitude. Unfortunately, this is only correct for simple harmonic motion about $x = 0$.

(iii) (2 marks)
Having stated that maximum speed was when $x = 1$ or when the acceleration $= 0$, some did not realise that the solution had been found in part (b) (ii).

Others, on the other hand, quoted some other incorrect equation and then quoted the solution from (b) (ii). For example, candidates would write

$$\cos 4t - \sqrt{3}\sin 4t = 0$$
$$\therefore \text{from (b) (ii) } t = \frac{\pi}{6}\,.$$

Many were happy to give a negative value for the first time the particle reaches maximum speed after $t = 0$, or solved this problem by simply changing the sign to positive.

Quite a number realised that the maximum velocity occurred when $\cos(4t + \frac{\pi}{3}) = 1$, but these candidates usually did not appreciate the significance of the word speed in the question, which meant that they actually needed to consider when $\cos(4t + \frac{\pi}{3}) = \pm 1$.

A surprising number stated that maximum speed occurred when $x = 0$, or worse, when $v = 0$.

Others found when the particle stopped (at one of the extremities) and, making the assumption (without actually stating it) that the particle had started at the centre of motion, stated that it would take twice as long to go back to the centre, which would be the time it first reached maximum speed. Others simply thought that the particle would be at the centre of motion at half the period.

Many thought they had been asked to find the maximum speed.

Question 4

This question had three parts. Part (a), worth five marks, involved finding a volume of revolution and using the result to find a related rate of change. Part (b), also worth five marks, was a fairly straight forward question on a function and its inverse, while part (c) was a simple geometry question worth two marks.

(a) (i) (3 marks)
Most candidates scored three marks. Those who did not obtain full

marks frequently tried to integrate before simplifying their expression for x^2 and consequently got lost in the more difficult algebra. Many candidates did not attempt this part, or else tried to use a formula for the volume of a sphere or a hemisphere. It was also common for candidates to integrate correctly without using any limits, and then claim the desired result 'because $y = h$.' Other candidates first substituted $y = h$ and then proceeded to integrate, presumably with respect to h. Relatively few mistakenly rotated the curve around the x-axis.

(ii) (2 marks)
A high proportion scored both marks, and it was encouraging to observe that almost candidates recognised that this part involved a related rate of change.

Many candidates do not know the chain rule. Some misquoted it, using up to five different variables. The most common error was to identify the rate at which the water was rising with $\frac{dV}{dt}$. Those making this error could possibly score one of the two marks. It was also common to use $h = 3\,\text{cm}$ instead of $h = 6\,\text{cm}$. It is not clear whether this was a deliberate choice by candidates because it was a hemisphere or whether the appearance of $3\,\text{cm}^3\,\text{sec}^{-1}$ led to confusion.

It would be reasonable to expect candidates to use t to represent time in a question of this nature. Many did not, leaving examiners to assume, for instance, that $\frac{dh}{dr}$ was the rate at which the height changed with time, even though the variable r was never identified.

It was an unfortunate coincidence that evaluation of $1/(\frac{4}{3}\pi r^2)$ with $r = 6$ gives the same number as that appearing in the correct answer. A number of candidates clearly showed that they had obtained their answer in this way.

(b) (i) (2 marks)
It is very clear that the vast bulk of the candidature does not know which direction is vertical and which is horizontal. In marking the question, examiners ignored the words associated with the two equations and awarded one mark for the appearance of $y = 1$ and the other mark for the appearance of $x = 2$.

Many candidates do not understand the notion of an asymptote, with many confusing it with either the domain or the range. The fact that the question had restricted the domain of the function to $x > 2$ led many candidates to (incorrectly) say that therefore there was no vertical asymptote because $x = 2$ was outside the domain. Provided the candidate made this explicit, this had no effect on the number of marks awarded.

Some candidates took a great deal of time and space to explain why the equations of the asymptotes were as they had given them. The answer $x \neq 2$, $y \neq 1$ was very common and, while incorrect, was awarded both marks.

(ii) (2 marks)
Most candidates realised that an interchange of x and y is required in order to find the equation of the inverse function. Candidates should be advised to do this before attempting to change the subject, as a mark is often awarded for this step. Candidates who stumbled in rearranging usually gave up before interchanging, and so lost the opportunity to gain an easy mark. Examiners were generally impressed with the algebraic skill shown in this part.

(iii) (1 mark)
Predictably, a very high proportion of the candidature did not restrict the domain of the inverse function to correspond to the restricted range of the function. Very often, this was the only mark a candidate failed to gain in Question 4.

(c) (i) (1 mark) and (ii) (1 mark)
Unfortunately many candidates must have misread the question and, instead of providing the missing reasons for a two step proof, embarked on a two page proof of their own. Candidates should be aware of the fact that a two mark question in the middle of a paper is unlikely to require such an amount of work.

By far the most common error was for candidates to mistake the facts which were known for those which needed proof. Thus, many candidates assumed to result to be proved in (ii) and gave the reason 'opposite angles are supplementary' as justification for $EDCB$ being a cyclic quadrilateral in (i). The standard of expression was usually quite poor, and relatively few good answers were given.

It is apparent that many think that supplementary angles must be adjacent. Candidates should also be warned against abbreviating too much. For example 'exterior angle of cyclic quadrilateral' could not be counted as a sufficient reason for (ii). An more explicit answer such as 'ext. $\angle$ of cyclic quad = int. opp. $\angle$' was required.

Question 5

This question consisted of three parts taken from different topics studied in the syllabus. For this reason, it was frequently the case that a candidate would do well on one part and not at all well on another. There was little correlation between candidates' expertise at Newton's method and their capacity to deal with the probability.

(a) (1 mark)
This part was poorly done by many candidates. Common incorrect responses were $\sum_{k=8}^{n} k^3$, $\sum_{k=2}^{x} k^3$ and $\sum_{k=2}^{k} n^3$.

Some candidates gave no upper or lower limits and those who did were often unable to match the use of their chosen pronumeral.

(b) (i) (3 marks)
Most candidates knew that it was necessary to solve $P'(x) = 0$ to find the stationary points. However, many then had problems solving the resulting quadratic equation $12x^2 + 4x = 0$. An alarming number of candidates could not factorise the left hand side correctly. Many who did manage to factorise correctly failed to recognise that $x = 0$ was one of the solutions.

Candidates who managed to solve the quadratic and calculated the coordinates of the stationary points correctly almost invariably managed to sketch the graph. Some candidates did not use calculus to investigate the nature of the stationary points, and this was perfectly acceptable.

On the other hand, one of the marks did require the computation of the y coordinates of the stationary points, and some candidates who did not do this correctly created further difficulties for themselves by obtaining values which made it impossible to sketch the cubic.

There were quite a few candidates who attempted to sketch the curve by plotting points. Generally these candidates did not receive many marks as they were unlikely to find the second stationary point by accident. Some candidates only drew the graph for the domain $-1 < x < 0$, having misread the question.

It was disappointing to see that very few candidates bothered to check obvious mistakes such as finding two minima or obtaining y coordinates which were inconsistent with other information known about the graph.

(ii) (2 marks)
Some candidates did not know the formula for Newton's method. This meant that they could not score any marks. Many of those who did know the formula made mistakes in their calculations. Many demonstrated poor arithmetic skills, with the most common error being $-0.25 + 4.25 = 4.5$.

(iii) (1 mark)
This was quite difficult to mark as many candidates did not express their ideas clearly.

The most efficient way to show why Newton's method did not work was to draw an appropriate diagram showing the intersection of the tangent at $x = \frac{1}{4}$ with the x axis. This was awarded the mark even though examiners were not entirely convinced that such candidates really understood what was happening.

Written explanations ranged from stating the obvious, such as 'the tangent cuts the axis at a point further away than the root' or 'the second approximation is not in the given domain' to the ridiculous, such as 'Newton's method does not work with negatives' or 'Newton's method does not work with fractions' or even 'Newton's method reciprocates the first approximation, i.e from -0.25 to 4.'

Some were very confused, stating such things as 'if $f(x)$ and $f''(x)$ have the same sign then Newton's method will (or won't) work.'

(c) (i) (1 mark)
Many candidates missed the significance of the words 'and placed together' in the question, making the most popular incorrect answer $\left(\frac{5}{6}\right)^4$. There were also quite a number of candidates who did not appear to have realised that four fish were selected and gave $\frac{5}{6}$ as their answer.

(ii) (1 mark)
Those who recognised that

$$P(\text{at least one tagged fish}) = 1 - P(0 \text{ tagged fish})$$

easily got this part right by taking the complement of their answer in part (i).

Those who used

$$P(1 \text{ tagged fish}) + P(2 \text{ tagged fish}) + P(3 \text{ tagged fish}) + P(4 \text{ tagged fish})$$

had more of a struggle, and tended to make mistakes by omitting coefficients.

(iii) (1 mark)
A common mistake was to multiply the answer in part (ii) by 7. Others did not see the connection with either of the previous parts.

(iv) (2 marks)
This was quite well done. Most candidates realised that binomial probability was needed. However, many applied the theory incorrectly. For example, they switched the exponents, or used their answer from part (iii), or did not realise that the p and q in their expression were related by $p + q = 1$.

Question 6

This question consisted of two parts, both concerning motion. In part (a) candidates were asked to derive the equations of motion of a projectile and to apply them to solve a problem about a volley ball being served over a net. In part (b) they were asked to show that a given formula for velocity in terms of x followed from a stated formula for acceleration, and to interpret the motion physically.

Given that much of this question was standard bookwork, candidates handled the calculus aspects fairly well. The same, however, could not be said for the physical interpretation, with the majority clearly unable to visualise the motion of a particle from equations for its velocity and acceleration. There is still a small but substantial group of candidates who arm themselves with only some rote learned formulae, and these candidates are almost always inadequately prepared to cope with a projectile question such as this. It is also worth emphasising that, in questions where the answer is given, it is the responsibility of the candidate to ensure that sufficient lines of working are given to convince the examiners that the result has been shown.

(a) (i) (4 marks)
Candidates were required to use calculus to derive the equations of motion of a projectile launched at an angle of 45°. Generally this was well done, with most candidates showing familiarity with the procedure. The first two marks were awarded for deriving the formulae for y and x in terms of t. To be awarded full marks, candidates needed to deal adequately with the constants of integration, evaluating them using the given initial conditions. Some were somewhat sloppy in this regard. The other two marks were for eliminating t to obtain the equation for y in terms of x. This involved handling the special case $\theta = 45°$. Some of the candidates who failed to make the substitution $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$ at an early stage obtained the equation in terms of $\sin\theta$ and $\cos\theta$ and then had difficulty with the trigonometric manipulations necessary to obtain the given result.

(ii) (2 marks)
From the information that the ball just clears the net, values of x and y could be substituted into the equation found in part (i) to obtain a simple equation from which the initial speed, V, could be found and shown to be 10.3 metres per second (to one decimal place). Most candidates realised the need to use the cartesian equation found in part (i) and were usually successful. Those who chose to ignore the result obtained in part (i), using the equations in terms of t instead, had a much harder task and fewer succeeded.

Applying rote-learned formulae proved to be of no real use at all. Some candidates were under the misapprehension that the place where the ball touched the net occurred at the highest point of the projectile's path.

(iii) (2 marks)
The easiest way to find where the ball lands was to substitute the value $y = 0$ into the cartesian equation. This yields a quadratic equation in x. One mark was awarded for setting up this equation and the other for correctly solving it, enabling the answer to be obtained. Poor arithmetic and calculator work cost marks here. Once again, some candidates chose to ignore the earlier results and reverted to the equations in terms of t. This approach was slightly longer.

There was confusion by some as to whether to substitute $y = 0$ or $y = -1$, suggesting an inability to deal with projectiles launched from places other than at ground level. The use of physics or other memorised formulae was rarely successful, although full marks were awarded to a handful of candidates who were able to obtain the correct answer from what turned out to be a difficult method.

(b) (i) (2 marks)
From the expression for acceleration in terms of x, candidates were required to find v^2 in terms of x. The first mark was awarded for correct use of $\frac{d^2x}{dt^2} = \frac{d}{dx}\frac{1}{2}v^2$. The second mark was for using the initial conditions to evaluate the constant. Most candidates managed this successfully, although quite a few incorrectly used v instead of v^2.

(ii) (2 marks)
The candidates were asked to decide, with justification, whether or not the particle returned to the origin. To score full marks, the candidates had to refer to the expressions for acceleration and velocity and explain that the particle comes to rest and returns towards the origin. The best answers showed that v^2 does not again become zero, but not all successful candidates managed to do this. One mark was awarded to candidates for a meaningful partial explanation.

This part was badly done, with hardly any candidates showing enough understanding of the movement of the particle to score full marks. A small number were able to score one mark. Very few understood the implications of the signs of acceleration and velocity, and confusion between negative acceleration and negative velocity was common. Many candidates thought that this question was about simple harmonic motion, and gave simplistic arguments to support their conclusion based solely on whether or not they thought that the particle executed simple harmonic motion.

It is clear from the responses to this question that candidates are quite happy to deal with motion questions on a formal level, but very few are able to conceptualise what the motion is actually all about.

Question 7

Part (a) of this question related to the binomial theorem, part (b) involved integration by substitution, and part (c) required the solution of an inequality. Most candidates attempted the question. Overall, it was not handled particularly well, and for most candidates the problem did not seem to be a lack of time.

(a) (i) (1 mark)
Candidates were asked to obtain an expansion for $(1+x)^{2n}+(1-x)^{2n}$. The question did not ask for a simplified expansion, although this was required for part (ii). Candidates had problems with the last terms in each of the expansions, sometimes writing $\binom{2n}{n}x^n$, or $\pm x^{2n}$. Most of those who could write expansions for $(1+x)^{2n}$ and $(1-x)^{2n}$ realised that the terms in odd powers of x cancelled, but quite a few forgot to double the terms in even powers. Candidates who wrote their expansions using sigma notation had trouble combining the two.

(ii) (1 mark)
The mark was not awarded to candidates who used a calculator to find the sum. Candidates ignore the use of words such as 'hence' in questions at their peril. Generally, those who had found a correct expansion in (i) realised that the result could be obtained by substituting $n = 10$ and $x = 1$. Some candidates confused the required sum with the sum of the coefficients in the expansion of $(1+x)^n$. Some obtained ridiculous answers, such as 0, 1, or 2, and were apparently not bothered by this.

(b) (i) (3 marks) and (ii) (3 marks)
These two parts required candidates to use two different substitutions to find an indefinite integral. Those attempting these parts generally had a fair idea of how to use substitution, but were often unable to complete the question successfully due to lack of skills in algebraic manipulation.

The final mark in part (i) was not awarded unless the answer was given in terms of x, rather than y, and quite a number of candidates failed to do this. Many had trouble substituting for dx in part (i) and finding the correct substitution for $(1-x)$ in part (ii). Most of those who substituted correctly were able to recognise a standard integral and use the table of integrals correctly.

There was a great deal of very sloppy work in the responses to these two parts. Many left out integral signs, or wrote integrals which had no term in dx, dy or dz. Many square root signs simply disappeared.

(iii) (1 mark)
Very few candidates scored this mark. Most candidates did not

attempt it, and of those who did, only a very small number understood what the question was about. Many thought that they needed to solve $2x - 1 = \sqrt{x}$.

There seems to be virtually no understanding of the role of the arbitrary constant in an indefinite integral. The fact that most candidates did not include arbitrary constants in their answers to (b) (i) and (ii) did not help.

Several candidates wrote $2\sin^{-1}\sqrt{x} = \sin^{-1}(2x-1) + C$, but then made no attempt to find C. Even those who realised that the problem was to find such a C rarely did so successfully.

(c) (3 marks)
Candidates were asked to solve $|4x-1| > 2\sqrt{x(1-x)}$. This is a reasonably difficult inequality, and, predictably, was not handled well. Few candidates realised that since both sides are positive, the inequality is equivalent to $(4x-1)^2 > 4x(1-x)$.

The majority of those attempting this part therefore considered the two cases $|4x-1| = 4x-1$ and $|4x-1| = -(4x-1)$. Sometimes this led them, correctly, to two identical inequalities. More often than not, however, consideration of the second case led to an incorrect inequality such as $(1-4x)^2 < (-2\sqrt{x(1-x)})^2$. Again, as in part (b), algebraic skills were appalling. For example, $(4x-1)^2 = 16x^2 - 1$ was quite common, as was the claim that $4x - 1 > 2\sqrt{x(1-x)}$ implies $8x - 2 > \sqrt{x(1-x)}$.

On the whole, candidates have not learnt good strategies for solving inequalities. Few attempted a graph. Solving $|4x-1| = 2\sqrt{x(1-x)}$ is a good strategy, but candidates do not seem to know that they should then test appropriate regions of the real line in the original inequality.

Only a handful of candidates realised that the right hand side of the inequality restricts the values of x to $0 \leq x \leq 1$, and so very few candidates gained three marks. Of those who did consider the restriction arising from $\sqrt{x(1-x)}$, many appeared to think that $\sqrt{0}$ is undefined.

The fact that $\sqrt{x(1-x)}$ appeared in part (b) unfortunately led quite a few candidates to think that the result of (b) (iii) could somehow be used in the solution to this part.

Mathematics 4 Unit (Additional)

Question 1

This question was generally well done with hardly any non-attempts. A large number of candidates gained full marks.

(a) (2 marks)
This was a standard integral and most candidates earned both marks. There were several answers of 90° instead of $\pi/2$ for the evaluation.

(b) (2 marks)
This was a standard integration by parts. It was noticeable that those who explicitly wrote $u = \ln x$, $du = (1/x)\,dx$ and $dv = x^2 dx$, $v = x^3/3$ were more successful than those who appeared to do the process in their heads.

(c) (3 marks)
This was a standard substitution question. It was badly done with many errors in algebra and basic integration. There were several ways of doing the integral. Candidates were awarded one mark for the correct substitution or use of integration by parts, one mark for having the correct integrand after the first step and one mark for the answer. Candidates who used t substitutions were unable to reach an answer.

(d) (4 marks)
This was a difficult substitution. Candidates were awarded one mark for correctly finding something equivalent to $u\,du = -x\,dx$ as a prelude to the substitution. Many candidates did not earn this mark. There were many algebraic errors such as $\sqrt{u^2} = u^2$ and $u\,du$ becoming du during substitution. Candidates were then awarded one mark for a correct integrand after substitution, one mark for the integration and one mark for the answer. There were many candidates who used trigonometric substitution successfully and several who succeeded by using integration by parts.

(e) (i) (1 mark)
This was poorly done. Many candidates could not find the remainder, often due to algebraic errors. Many used or attempted to use the remainder theorem, even though a simple division of polynomials was all that was required.

(ii) (3 marks)
Many candidates did not recognise the easiest method using partial fractions and proceeded to use logarithms and then complete the square. While this method is feasible, there were many algebraic and sign errors which cost candidates marks.

One mark was awarded for recognising the partial fraction approach or the correct logarithm, one mark for the partial fraction decomposition (or equivalent) and one mark for the answer. Incorrect answers in part (i) that were correctly worked through in part (ii) could earn all three marks.

Question 2

This question involved five unrelated parts, each on the topic of complex numbers.

(a) (1 mark)
This part asked candidates to evaluate i^{1998}. It was relatively well answered, with the most straightforward solution being $i^{1998} = (i^2)^{999} = -1^{999} = -1$.

(b) This part concerned the complex number $z = \dfrac{18+4i}{3-i}$.

(i) (1 mark)
Candidates were required to simplify $(18+4i)\overline{(3-i)}$, which was intended to assist with the following two parts. Most received the mark, which was awarded if the candidate had correctly proceeded at least as far as something equivalent to $54+4i^2+18i+12i$. Candidates who gave $10z$ as an answer were also awarded the mark.

(ii) (2 marks)
For this part, candidates needed to express z in cartesian form. One mark was awarded for working which showed that the candidate understood the procedure of multiplying by $\dfrac{3+i}{3+i}$. The second mark was essentially for correctly evaluating the denominator, as the numerator had been found in part (i).

(iii) (2 marks)
The question involved finding $|z|$ and $\arg z$, with each of these worth one mark. Answers for $\arg z$ such as $\tan^{-1}\frac{3}{5}$, $0.22\ldots$ or $12°32'$ were all acceptable, and occurred in roughly equal proportions.

(c) (2 marks)
Candidates were asked to sketch the region in the complex plane where two inequalities both held. The first inequality corresponded to the interior and boundary of the circle centred at $2-i$ with radius 2, while the second inequality referred to the region on and above the real axis. Candidates could earn one mark by shading some region of the correct circle or by restricting their shading to a region which did not extend below the real axis.

Candidates' responses included circles with centres at any one of the points $\pm 2 \pm i$. Examiners did not pay attention to the fact that the circle should have the imaginary axis as a tangent, nor were marks lost for drawing the boundaries as dotted lines. Few candidates attempted to compute the points of intersection of the circle with the real axis. Such computations were not required, and no marks were lost if candidates made mistakes in this process.

(d) (1 mark)
The diagram in this part showed an isosceles right triangle OPQ in the complex plane, with the right angle at O. The points P and Q corresponded to the complex numbers z and w, and candidates were asked to show that $z^2 + w^2 = 0$. The mark was essentially awarded for noticing that $w = iz$, or equivalently that $z = -iw$. The orientation of the diagram meant that $z = iw$ was incorrect, and candidates who proceeded from this premise did not score the mark.

(e) (i) (2 marks)
Candidates were asked to find the three cube roots of -1 by solving $z^3 + 1 = 0$. Candidates were fairly evenly divided between those who used $z^3 = \text{cis}(\pi + 2k\pi)$ and those who attempted to factorise $z^3 + 1$ as $(z+1)(z^2 - z + 1)$. Finding a cube root of -1 other than -1 itself, either in cartesian form or in mod-arg form, was awarded one mark. All three roots were required to obtain both marks.

Many candidates did not factorise correctly. These candidates could still earn one mark by finding the roots of their quadratic factor. Candidates who made minor errors in the mod-arg approach could also earn one of the two marks.

(ii) (2 marks)
This part required candidates to show that if λ is a non-real cube root of -1, then $\lambda^2 = \lambda - 1$. This was easiest for those candidates who had obtained the two possibilities for λ by solving $z^2 - z + 1 = 0$ in part (i).

Some candidates used the formula for the sum of the geometric series $1 - \lambda + \lambda^2 = \frac{(-\lambda)^3 - 1}{-\lambda - 1}$ as the basis for their argument.

Candidates who worked by substitution of the values for λ needed to provide sufficient evidence to show that they had considered both possibilities in order to earn two marks.

(iii) (2 marks)
This question asked candidates to use the preceding result to simplify $(1 - \lambda)^6$. The obvious way to do this was to use part (ii) to rewrite the expression as $(-\lambda^2)^6 = \lambda^{12} = (\lambda^3)^4 = 1$. Another approach is to rewrite the expression as $(1 - 2\lambda + \lambda^2)^3 = (-\lambda)^3 = 1$.

Mere expansion, without using the result from part (ii), did not receive any marks. However, candidates who had begun this way could still earn the marks. For example, $1-6\lambda+15\lambda^2-20\lambda^3+15\lambda^4-6\lambda^5+\lambda^6$ could be rewritten as $1-6\lambda+15\lambda^2+20-15\lambda+6\lambda^2+1$, making use of the fact that $\lambda^3=-1$. This in turn could be rewritten as $22-21\lambda+21\lambda^2 = 1+21(1-\lambda+\lambda^2) = 1$, with the last step effectively making the required use of the result from part (ii).

Candidates who effectively made use of the identity in part (ii), but failed to find the final result were awarded one mark.

Question 3

(a) This involved three variations of curve sketching based on $f(x) = x - \frac{4}{x}$. The general performance of candidates was extremely good with large clear sketches being one of the main features. Only a very few candidates drew all three graphs on the one number plane. Misreading of the function or its variations was minimal.

(i) (2 marks)
The most common and successful method for graphing $f(x) = x - \frac{4}{x}$ was to find any asymptotes, x intercepts and examine the behaviour of the curve around the asymptote and as $x \to \pm\infty$.

Addition (or subtraction) of the ordinates of $y = x$ and $y = -\frac{4}{x}$ tended to lead to errors, as did the use of calculus to invent turning points which do not exist.

(ii) (2 marks)

Realising that the range of $y = \sqrt{x - \dfrac{4}{x}}$ was $y \geq 0$ was the best starting point. Once again, behaviour around the asymptote and as $x \to \infty$, as well as finding any x intercepts, was the most efficient approach.

Most candidates realised that finding the exact location of the point of inflection in the second quadrant would involve lengthy algebraic manipulations which could not be required for part of a question which was worth a total of six marks. Similarly, most recognised that they would not need to determine the precise nature of the tangents at $(\pm 2, 0)$, although it is not difficult to establish that they are both vertical.

A number of candidates graphed $y^2 = x - \frac{4}{x}$, forgetting to eliminate $y = -\sqrt{x - \dfrac{4}{x}}$.

(iii) (2 marks)
Correct concavity was essential in the first quadrant for $y = e^{x-\frac{4}{x}}$. The examiners were prepared to believe that candidates with the

correct concavity knew that $y \to e^x$ as $x \to \infty$, even if this was not stated.

The fact that the range was $y > 0$ and not $y \geq 0$ was well understood, although the standard notation of an open circle was frequently not used. Instead, many candidates sketched the section $x \to 0^+$ as a fading pencil mark.

(b) (i) (2 marks)
The first step to success was to realise that one should express $\int_1^e (\ln x)^n \, dx$ as $\int_1^e 1 \cdot (\ln x)^n \, dx$ and use integration by parts. The most common error in the step that followed was the omission of $\frac{1}{x}$ when differentiating $(\ln x)^n$ to find the new integrand.

Many candidates initially tried to integrate by parts by writing $\int_1^e (\ln x)^n \, dx$ as $\int_1^e (\ln x)(\ln x)^{n-1} \, dx$, but quickly reverted to the correct method when this approach proved to be fruitless.

The method of induction was successfully used by a few candidates, as was the conversion of $u = \ln x$ to $x = e^u$ with an appropriate adjustment to the limits of integration.

(ii) (2 marks)
The most common errors in this part involved algebraic and arithmetic manipulations, including errors in the use of brackets. Candidates who started at I_0 and progressed upwards to I_4 seemed to find the calculations easier. Successful evaluation of either I_0 or I_1 at the beginning almost invariably led to a correct final result.

For some reason, the statement $n = 1, 2, 3, \ldots$ caused many candidates to stop at $I_2 = e - 2I_1$ and evaluate I_1 by parts. In fact, I_0 is particularly easy since $I_0 = \int_1^e (\ln x)^0 \, dx = \int_1^e 1 \, dx = e - 1$, and the identity with $n = 1$ says that $I_1 = e - I_0 = 1$.

The coefficient of I_{n-1} was sometimes left as n or remained constant at 4. Other common errors were claims that $I_0 = 1$ or 0 or e, and $I_1 = 0$ or e. Candidates usually claimed that these results were clear by inspection.

(c) (i) (1 mark)
Differentiation is a much easier process than integration. This fact was appreciated by candidates who differentiated $P = 1000 + Ae^{-kt}$ and showed that it satisfied the differential equation. The main error made by those who started with $\frac{dP}{dt} = -k(P - 1000)$ and then integrated, was incorrect manipulation involving the constant of integration in order to obtain A.

(ii) (3 marks)
This was the part of the question in which candidates enjoyed the greatest degree of success. The work involving use of a calculator was of a particularly pleasing standard.

Common errors involved using 2500 as the value for A, not realising that the word 'initially' referred to $t = 0$, and stating that 1900-1000=1800. Candidates who found a value for t which was negative usually recognised that this must be incorrect, and rectified their error by backtracking.

(iii) (1 mark)
Some candidates may have been perplexed by the words 'mathematical model.' Most candidates answers involved some terminology implying that the population 'will approach 1000'. Another common response was to produce a sketch of the function showing a horizontal asymptote at $P = 1000$.

Question 4

Candidates' answers to this question were marred by numerous algebraic mistakes and transcription errors, especially in part (b). In part (a), candidates lost marks by making mistakes in elementary differentiation. Most were able to make reasonable attempts at parts (a) (i)–(iii). Only a few candidates gained full marks on part (a) (iv). The probability question, part (c), was not well done by the majority of the candidature.

(a) (i) (2 marks)
There were two crucial steps in this part of the question. Candidates needed to write $f(x)$ in the form $(x - k)^2 g(x)$ and then compute the derivative. Once this has been done, the conclusion is trivial. Accordingly, one mark was awarded for each of these steps.

The most common error occurred in the computation of $f'(x)$. Many candidates claimed that $f'(x) = 2(x - k)g'(x)$.

(ii) (1 mark)
Almost all candidates assumed that the roots of the polynomial were real, although this was not stated in the question. The examiners only recall one candidate who recognised the possibility of complex roots. The question was marked on the assumption that the root of multiplicity two was real. The majority of the candidature were able to gain the mark for this question by giving any one of several features such as the graph has a stationary point or the graph has the x axis as a tangent. A correct graphical answer was also acceptable.

Some of the responses which were not accepted were statements such as the graph is a parabola, the graph has a horizontal point of inflection or the graph has a tangent.

(iii) (2 marks)
One mark was awarded for knowing that $P(1)$=0 and $P'(1) = 0$. The second mark was awarded for correctly setting up these equations in terms of a and b and correctly solving them.

The error $P'(1) = 7a + 6b + 1$, which leads to the values $a = 5$ and $b = -6$, was quite frequent. The step $P(1)$=0 and $P'(1) = 0$ was almost never explicitly stated in the reasoning and so candidates making this error were usually awarded one mark.

A handful of candidates attempted to use long division and then equate coefficients to find a and b. A proportion of these were able to get the correct answer.

(iv) (2 marks)
The important steps are to realise the significance of the derivative, to observe that $E(k) = 0$ and $E'(k) = 0$ if $E(x) = 0$ has a double root at $x = k$, and to show that these two equations imply a contradiction. The first mark was awarded for either giving a correct expression for $E'(x)$ or for the statement $E'(k) = 0$. The second mark was harder to gain and was awarded for a correct argument that there are no double roots. Very few candidates were able to do this. A bald statement that $k \neq 0$ without further justification was not enough to gain the second mark.

(b) (i) (3 marks)
There were three elements which were essential to find the correct answer. The first was the fact that $|\boldsymbol{a}|^2 = \ddot{x}^2 + \ddot{y}^2$. The second was the fact that $\boldsymbol{F} = m\boldsymbol{a}$, while the third involved using the equations for the position at time t to show that $|\boldsymbol{a}|$ is a constant. Each of these elements gained one mark.

Candidates were also awarded one mark for correct expressions for both $\ddot{x}$ and $\ddot{y}$ if they had not gained the mark for $|\boldsymbol{a}|^2 = \ddot{x}^2 + \ddot{y}^2$.

Many were unable to correctly compute the acceleration from $\ddot{x}$ and $\ddot{y}$. They frequently made errors in the exponents of T, π or R occurring in the answer.

Another approach was to use either the result $|\boldsymbol{F}| = m|\boldsymbol{v}|^2/R$ or the result $|\boldsymbol{F}| = mR\omega^2$. However, this implicitly assumes that $\boldsymbol{F}$ is normal to the curve, a fact which was not stated in the question. Candidates using this approach could earn up to two marks for showing from either of these equations and the given information that $\boldsymbol{F}$ is constant.

It should be noted that many candidates used the symbols F, a and v to denote the magnitude of the force, acceleration and velocity, and that this practice did not in any way trouble the examiners as the meaning was always clear from the context.

(ii) (2 marks)
In this part, full marks were awarded to candidates who worked correctly from the answer which they had obtained in part (i). However, no marks could be gained if the incorrect expression for $\boldsymbol{F}$ did not contain T, either implicitly or explicitly.

Some candidates did not use the result from (i), but started with either $m|\boldsymbol{v}|^2/R = GMm/R^2$ or $mR\omega^2 = Gmm/R^2$. Candidates using this approach were given the first mark when they were able to derive a correct expression explicitly containing T, and a further mark for expressing T in terms of G, M and R.

There were two very common errors in this part. Many candidates were unable to correctly transcribe their answer from part (i) for use in part (ii), with the most common error being to transcribe T^2 as T. The other common error, made by about 7% of the candidature, was the following cancellation error.

$$\frac{4\pi^2 \not{R}}{T^2} m = \frac{Gmm}{R^{\not{2}}}$$

(c) (3 marks)
Very few candidates scored just one mark in this part, with most scoring either zero or at least two. It was necessary to set up one equation for each case, solve them correctly and then draw the correct conclusion. One mark was awarded for each equation and a further mark for correctly solving both equations, even if no valid conclusion was reached. It was also acceptable for candidates to solve one equation and then test the solutions in the second, showing that there was only one possibility. The most common errors here were to discard the correct answer $w = 1$ or to state the possibilities for w as 1, 6 or 9.

Question 5

This question contained two parts. One was on coordinate geometry methods and locus, while the other concerned volumes by similarity. The question was reasonably well handled by most candidates.

Candidates were particularly challenged by the degree of algebraic manipulation required, and those candidates who were able to set out such work well were rewarded with a far higher degree of success. Numerical and algebraic errors were very common. The approach used in marking this examination limits the number of marks that can be lost through such errors as the marks are awarded for key steps which are done correctly, and not deducted for mistakes which are made.

Many candidates had problems handling the locus in part (a) (iii) and in showing the height of the trapezium in part (b) (iii), which was often not attempted. It was noticeable that candidates from a few centres did not appear to be experienced in finding volumes by similarity.

(a) (i) (1 mark) and (ii) (2 marks)
These were very well attempted, with candidates displaying fine algebraic skills.

It must be noted that many used the quadratic formula to solve

$$(1 + m^2)x^2 + (2a + 2mb)x = 0$$

or used $x_P + x_R (= 0) = -\frac{b}{a}$. Factorisation would have been far simpler.

(iii) (3 marks)
Higher order algebraic manipulation skills were required here and they were found wanting when determining the midpoint. Eliminating m to find the locus was not well attempted. Candidates were confused by their own messy setting out, perhaps because they were rushing.

There were a variety of methods which were used to eliminate m including squaring x and y then adding, $m = \frac{y-b}{x} = \frac{-b \pm \sqrt{b^2 - x^2}}{x}$ and a few recognised a form of the t-results.

Many first determined expressions for both coordinates of P and Q before attempting to find the midpoint. This was a longer path to the midpoint which added to the problem of handling all the algebraic expressions. There were many transcription errors, minus signs were lost and the vinculum was not well handled.

(b) (i) (1 mark)
This was very well done. A variety of approaches were used including Pythagoras' theorem, trigonometry and the area of a triangle. Some candidates did not simplify the height to $\frac{a\sqrt{3}}{2}$.

(ii) (1 mark)
This part required investigation of similar triangles. It was well done, but some candidates took a long path to arrive at $YZ = \frac{ax}{b}$ by involving right angle triangles and Pythagoras' theorem, providing more opportunities for error. For example, many candidates began by calculating $CR = \sqrt{b^2 + \frac{1}{4}a^2}$ and $CY = \sqrt{x^2 + \frac{1}{4}YZ^2}$.

(iii) (2 marks)
As stated earlier, many did not attempt this part. Most could not see how to do it, with some simply expanding the result given in the question.

However, many used a variety of valid methods including ratios of lengths in similar triangles, trigonometry based on $\tan\theta$ where θ is the angle between the top and the horizontal plane, coordinate geometry, a linear relationship such as $h = mx + h_0$ satisfied by $(x, h) = (0, \frac{a\sqrt{3}}{2})$ and (b, a), and areas of trapezia. Clearly such candidates are well prepared for this topic.

Many candidates wrongly believed that $XY = WZ = a$ or that $\angle XWZ = \angle WXY = 60°$.

(iv) (2 marks)
This was generally well done. However, many ended up expanding both the expression they had obtained and the one supplied in the paper in order to show that they were equivalent. One would have expected candidates to be skilled at the basic manipulation required to rewrite $a + \frac{ax}{b}$ as $\frac{a}{b}(b + x)$.

(v) (3 marks)
This was very well done. There were concerns with some candidates' inability to expand $((2 - \sqrt{3})x + b\sqrt{3})(b + x)$ without error and the number of basic numerical errors such as claiming $\sqrt{3} - \frac{\sqrt{3}}{3} = -\frac{2}{3}\sqrt{3}$. There were also many careless transcription errors in candidates' work. The number 2 was often transcribed as 3 or $\sqrt{3}$ while the letter b was confused with 6 or h.

Question 6

There were three distinct parts. The first part on even and odd functions was often poorly done, even by those candidates who did well on parts (b) and (c). Only a handful of candidates gained all fifteen marks.

(a) (i) (1 mark)
An algebraic argument involving a general polynomial in even powers of x yielding a polynomial in odd powers on differentiation, along with the conclusion that the assertion was true was required for the mark. Some candidates thought that a polynomial was even if it has even degree, which is not correct.

An argument based with a simple polynomial, such as $p(x) = x^2$, from which the candidate concluded that the assertion was true was not acceptable.

Some candidates used the sophisticated argument that $Q(x) = Q(-x)$ and so $Q'(x) = -Q'(-x)$, which is the definition of an odd function.

(ii) (1 mark)
A similar argument to that required above, but with a constant of integration resulting in a polynomial which may not be odd was accepted. The assertion that the result is false with an example also scored the mark. For example $P'(x) = 20x^4$ when $P(x) = 4x^5 + 7$ and $P(x)$ is not odd is a perfectly acceptable justification.

Geometric arguments regarding reflection, rotation and gradient were accepted in both parts provided the explanation given was unambiguous.

(b) Many candidates recognised the binomial theorem but could not bring it to bear in a correct fashion.

(i) (1 mark)
The number of marks earned by candidates in this part was usually equal to the value used for 0! in their calculation.

(ii) (2 marks)
Some candidates used $1 - (0.074^{10} + 10 \times 0.074^9 \times 0.926)$ instead of $1 - (0.926^{10} + 10 \times 0.926^9 \times 0.074)$. Those whose work contained the latter statement were awarded two marks, even if their evaluation was not correct to three decimal places.

A fair amount of leniency was extended for those using other methods, particularly those who attempted to sum nine terms. Deviations from the correct answer of 0.166 were usually due to rounding too early in the course of the computation.

(iii) (3 marks)
Candidates who used a calculator to observe that P_2 had the largest numeric value amongst those they had calculated and deduced that 2 was the most likely number of accidents were given one mark.

An argument such as $\frac{P_{n+1}}{P_n} \geq 1$ leads to $\frac{2.6}{n+1} \geq 1$ and $n \leq 1.6$, so $n = 1$ and P_2 is the maximum was required to earn all three marks. In fact, most candidates who had found $n \leq 1.6$ proceeded by arguing separately that $P_1 < P_2$ and $P_2 > P_3 > \ldots$, and so drew the required conclusion.

(c) The first two parts were generally well done. The third part was completed successfully by only a handful of candidates.

(i) (1 mark)
Most candidates earned this mark by using implicit differentiation to determine the gradient.

(ii) (2 marks)
Many candidates did not use the correct equation for the appropriate asymptote. Some used $y = (\pm bx)/a$ or $y = x$ instead of $y = -x$. A significant number of candidates 'massaged' the signs to arrive at the correct result by incorrect means.

(iii) (4 marks)
One mark was given for the gradient of ST. The second was awarded for a correct substitution into a $\tan(\alpha - \beta)$ expansion approach to a solution. The final two marks were for correctly performing the quite considerable algebraic manipulations required to arrive at the result. Recognition that $e^2 = 2$ was essential and candidates who used this fact along the way were awarded at least one of the final two marks.

Question 7

This question tested two distinct sections of the syllabus. The section on polynomials was examined in the first part of the question. This was generally well done with candidates mainly losing marks through careless algebraic manipulations and an inability to explain why $1/w$ is a root if w is a root.

The second section dealt with a bead sliding along a wire. A preliminary result was first established in part (b). The substantial question in part (c) was designed to test the candidate's ability to work with components of motion. Parts (c) (i) and (c) (ii) tested whether the candidate understood various technical terms and could interpret the equations of motion. This question appeared quite difficult for many candidates, but those who persevered found that four of the six marks were very easy to obtain.

(a) This question required candidates to find all the complex roots of a particular polynomial of degree eight.

(i) (2 marks)
While most candidates successfully gained the mark for showing that iw is a root if w is a root, less than half could carefully explain why $1/w$ was a root to gain the second mark. There were a few instances where the candidates interpreted w as a cube root of unity.

(ii) (2 marks)
This part was very well done with most candidates identifying $2^{1/4}$ as a root. The most common error was to quote an incorrect formula for the roots of a quadratic. A surprisingly large number of candidates claimed that $2^{-1/4}$ was $1/16$.

(iii) (2 marks)
The marks were awarded for using the previous parts to identify the eight roots of the polynomial equation. One mark was awarded if at least four correct roots were given.

(b) (i) (2 marks) and (ii) (1 mark)
The three key steps in this part were differentiating $\sin^{-1}(u) - \sqrt{1-u^2}$, showing that $(u+1)/\sqrt{1-u^2} = [(1+u)/(1-u)]^{1/2}$ and evaluating the integral. One mark was awarded for each of these steps. This part was generally well done, but many lost marks due to careless manipulation of minus signs when differentiating. Many candidates were clearly not aware of the information contained in the table of standard integrals attached to the paper, as they felt they had to develop the derivative of $\sin^{-1}(u)$ from first principles.

(c) Very few candidates attempted the mechanics question. The first four marks in this section were very easy to obtain if the candidate was able to interpret the information given in the description of the physical situation. Many did not understand that the words 'from rest' meant $\dot{x} = 0$ and $\dot{y} = 0$. Several noted that $v = 0$ when $t = 0$, but still did not make any connection between the velocity, v, and the value of its two components $\dot{x}$ and $\dot{y}$.

(i) (2 marks)
One mark was given for noting that $E = 3mg/2$ and the second mark was awarded for successfully replacing $\dot{x}^2$ by $2y\dot{y}^2/3$ in the equation for E.

(ii) (2 marks)
These two marks were awarded for substituting $x = 0$ and $y = 0$ into the equations for $\dot{x}$ and $\dot{y}$. The mark for $\dot{y}(t_1)$ was awarded for either $\sqrt{3g}$ or $-\sqrt{3g}$. The better candidates correctly observed that $\dot{y}(t_1)$ was negative.

(iii) (2 marks)
There were very few attempts at this final part. One mark was awarded for writing

$$t = \frac{1}{\sqrt{3g}} \int_0^{3/8} \sqrt{\frac{3+2y}{3-2y}}\, dy$$

and the final mark was awarded for evaluating this integral. This final mark was awarded to fewer than ten candidates as most attempting this step did not correctly apply the change of variable $u = 2y/3$ to transform the integral into the form given in part (b).

Question 8

This question was found to be very difficult by almost all candidates. Only a handful gained more than ten marks, and only one candidate scored full marks. There were some easy marks to be gained and it was a pity that some candidates were not able to gain these marks because they ran out of time.

(a) This part first required candidates to use calculus to identify the minimum value of a function. They were then required to use this information in a difficult proof by mathematical induction.

(i) (2 marks)
The majority of the candidature were able to find the value of t which minimises $f(t) = \frac{s^p}{p} + \frac{t^q}{q} - st$ where p, q and s are fixed and positive and p and $q > 1$. They did this by equating the derivative to zero. Some weaker candidates had trouble recognising which pronumerals were variables and which were constants.

Few candidates were able to show that $t = s^{\frac{1}{q-1}}$, which is the solution of $f'(t) = 0$, was in fact the location of the minimum value. The few who were successful invariably used the second derivative test. Correct use of this method required mention of the fact that $q > 1$.

(ii) (1 mark)
The candidates were required to show that $\frac{s^p}{p} + \frac{t^q}{q} > st$. Those few who were successful showed that $f(s^{\frac{1}{q-1}}) = 0$. This proved very difficult as candidates had to realise the need to use the condition $p = \frac{q}{q-1}$ somewhere in their algebra.

(iii) (4 marks)
This part asked candidates used mathematical induction to prove the extended arithmetic-geometric mean inequality

$$(x_1 x_2 \ldots x_n)^{\frac{1}{n}} \leq \frac{x_1 + x_2 + \cdots + x_n}{n}.$$

Almost all were familiar with induction and most gained one mark for proving the result true when $n = 1$.

Very few could go any further with this question as the correct use of the induction hypothesis that the statement is true for $n = k$ in the next step proved too difficult an obstacle. This step needed adept use of indices on the left hand side or, more easily, use of the induction hypothesis on the right hand side broken up as

$$\frac{x_1 + x_2 + \cdots + x_k}{k+1} + \frac{x_{k+1}}{k+1}.$$

Of the few who could use the inductive assumption correctly, only three or four candidates went on to complete the proof. To do this, a candidate had to use the inequality in part (ii) and, in particular, realise that $p = \frac{k}{k+1}$. This was quite an achievement for those who did so in the limited time available.

(iv) (1 mark)
This part followed immediately once it was realised that

$$\frac{y_1}{y_2}\frac{y_2}{y_3}\cdots\frac{y_n}{y_1} = 1\,.$$

However, only a small number noticed this.

(b) This part involved proving results about cyclic polygons. Those who attempted this section scored quite well, with a considerable number scoring five or more marks. It was clear that a number of candidates ran short of time.

(i) (1 mark) and (ii) (1 mark)
These parts were very well done with most candidates seeing that it was best to use the fact that opposite angles of a cyclic quadrilateral add to 180°.

(iii) (2 marks)
The result follows from combining the results in parts (i) and (ii) and noting $\angle EDC = \angle EDA + \angle CDA$ and $\angle FAD = \angle FAB + \angle BAD$. A good number of candidates were able to this. However, the process used was often inefficient and the reasoning unclear.

(iv) (2 marks)
Those who were able to provide clear reasoning in part (iii) and drew a diagram indicating how to break the cyclic octagon into cyclic quadrilaterals were usually able to gain the marks for this part.

(v) (1 mark)
In order to gain the mark for this part it was necessary to make a statement which was equivalent to

$$\angle A_1 - \angle A_2 + \angle A_3 - \angle A_4 + \cdots - \angle A_n = 0$$

for even integers n.

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