BOARD OF STUDIES
NEW SOUTH WALES

HIGHER SCHOOL CERTIFICATE EXAMINATION

1999

MATHEMATICS

2/3 UNIT (COMMON)

Time allowed—Three hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on page 12.
- Board-approved calculators may be used.
- Answer each question in a SEPARATE Writing Booklet.
- You may ask for extra Writing Booklets if you need them.

QUESTION 1 Use a SEPARATE Writing Booklet. **Marks**

(a) The points A and B have coordinates $(3, -4)$ and $(7, 2)$ respectively. Find the coordinates of the midpoint of AB. **2**

(b) Find the value of e^3, correct to three significant figures. **2**

(c) Solve $3 - 2x \geq 7$. **2**

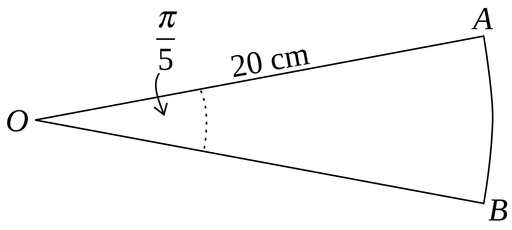
(d) Solve the simultaneous equations **2**

$$x + y = 1$$

$$2x - y = 5.$$

(e) Find integers a and b such that $\left(5 - \sqrt{2}\right)^2 = a + b\sqrt{2}$. **2**

(f) **2**



NOT TO SCALE

In the diagram, AB is an arc of a circle with centre O. The radius OA is 20 cm. The angle AOB is $\frac{\pi}{5}$ radians. Find the area of the sector AOB.

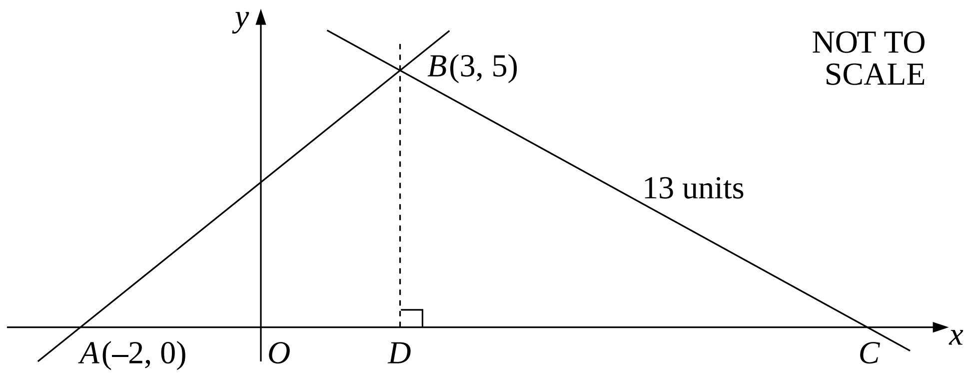
QUESTION 2 Use a SEPARATE Writing Booklet. **Marks**

(a) Find: **4**

(i) $\displaystyle\int\left(\frac{1}{x^2}+\frac{1}{x}\right)dx$

(ii) $\displaystyle\int\cos(2x+1)\,dx$.

(b) **8**



The diagram shows the points $A(-2, 0)$, $B(3, 5)$ and the point C which lies on the x axis. The point D also lies on the x axis such that BD is perpendicular to AC.

(i) Show that the gradient of AB is 1.

(ii) Find the equation of the line AB.

(iii) What is the size of $\angle BAC$?

(iv) The length of BC is 13 units. Find the length of DC.

(v) Calculate the area of ΔABC.

(vi) Calculate the size of $\angle ABC$, to the nearest degree.

QUESTION 3 Use a SEPARATE Writing Booklet. **Marks**

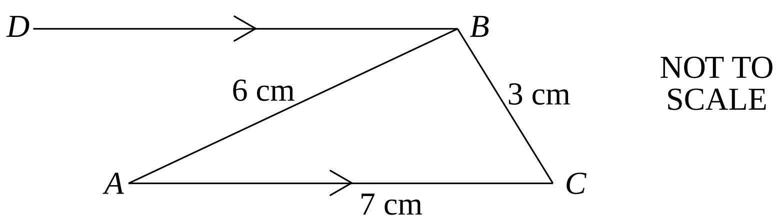
(a) Differentiate the following functions: **4**

(i) $x \tan x$

(ii) $\dfrac{e^x}{1+x}$.

(b) Find the equation of the normal to the curve $y = \sqrt{x+2}$ at the point (7, 3). **4**

(c) **4**



In the diagram, AC is parallel to DB, AB is 6 cm, BC is 3 cm and AC is 7 cm.

(i) Use the cosine rule to find the size of $\angle ACB$, to the nearest degree.

(ii) Hence find the size of $\angle DBC$, giving reasons for your answer.

QUESTION 4 Use a SEPARATE Writing Booklet. **Marks**

(a) An infinite geometric series has a first term of 8 and a limiting sum of 12. Calculate the common ratio. **2**

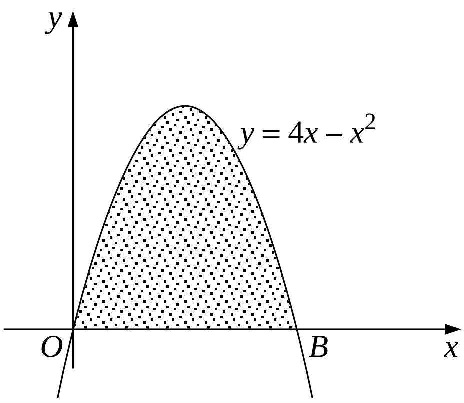
(b) A market gardener plants cabbages in rows. The first row has 35 cabbages. The second row has 39 cabbages. Each succeeding row has 4 more cabbages than the previous row. **5**

(i) Calculate the number of cabbages in the 12th row.

(ii) Which row would be the first to contain more than 200 cabbages?

(iii) The farmer plants only 945 cabbages. How many rows are needed?

(c) **5**



The diagram shows the graph of the function $y = 4x - x^2$.

(i) Find the x coordinate of the point B where the curve crosses the positive x axis.

(ii) Find the area of the shaded region contained by the curve $y = 4x - x^2$ and the x axis.

(iii) Write down a pair of inequalities that specify the shaded region.

QUESTION 5 Use a SEPARATE Writing Booklet. **Marks**

(a) Consider the curve given by $y = x^3 - 6x^2 + 9x + 1$. **8**

(i) Find $\dfrac{dy}{dx}$.

(ii) Find the coordinates of the two stationary points.

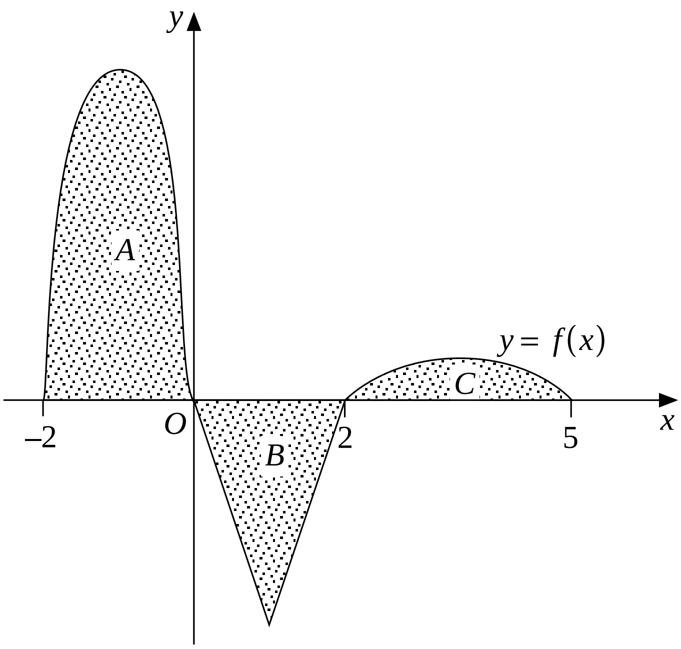
(iii) Determine the nature of the stationary points.

(iv) Sketch the curve for $x \geq 0$.

(b) Let $\log_a 2 = x$ and $\log_a 3 = y$.

Find an expression for $\log_a 12$ in terms of x and y. **2**

(c) **2**



The graph of the function f is shown in the diagram. The shaded areas are bounded by $y = f(x)$ and the x axis. The shaded area A is 8 square units, the shaded area B is 3 square units and the shaded area C is 1 square unit.

Evaluate $\displaystyle\int_{-2}^{5} f(x)\,dx$.

QUESTION 6 Use a SEPARATE Writing Booklet. **Marks**

(a) The mass M kg of a radioactive substance present after t years is given by $M = 10e^{-kt}$, where k is a positive constant. After 100 years the mass has reduced to 5 kg. **7**

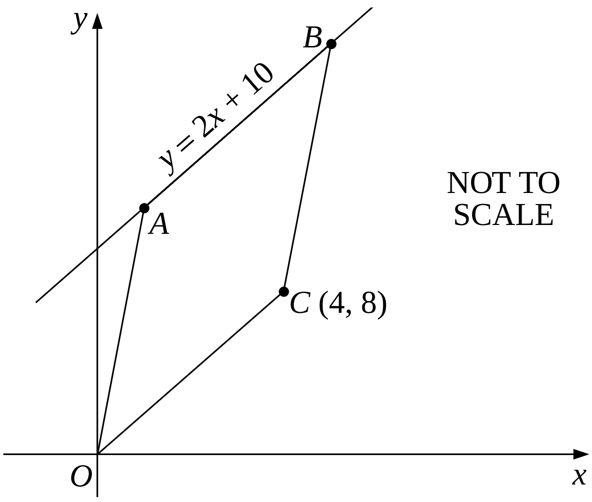
(i) What was the initial mass?

(ii) Find the value of k.

(iii) What amount of the radioactive substance would remain after a period of 1000 years?

(iv) How long would it take for the initial mass to reduce to 8 kg?

(b) **5**



The equation of AB is $y = 2x + 10$. The point C is $(4, 8)$.

Copy or trace the diagram into your Writing Booklet.

(i) Show that OC and AB are parallel.

(ii) State why $\angle ABO = \angle BOC$.

(iii) The line OB divides the quadrilateral $OABC$ into two congruent triangles. Prove that $OABC$ is a parallelogram.

QUESTION 7 Use a SEPARATE Writing Booklet. **Marks**

(a) Isabella invests $\$P$ at 8% per annum compounded annually. She intends to withdraw \$3000 at the end of each of the next six years to cover school fees. **5**

(i) Write down an expression for the amount $\$A_1$ remaining in the account following the withdrawal of the first \$3000.

(ii) Find an expression for the amount $\$A_2$ remaining in the account after the second withdrawal.

(iii) Calculate the amount $\$P$ that Isabella needs to invest if the account balance is to be \$0 at the end of six years.

(b) A particle P is moving along the x axis. Its position at time t seconds is given by **7**

$$x = 2\sin t - t, \qquad t \geq 0.$$

(i) Find an expression for the velocity of the particle.

(ii) In what direction is the particle moving at $t = 0$?

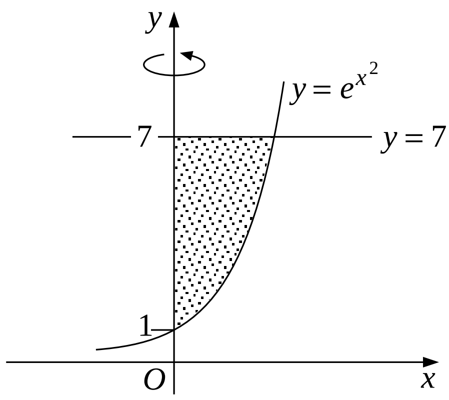
(iii) Determine when the particle first comes to rest.

(iv) When is the acceleration negative for $0 \leq t \leq 2\pi$?

(v) Calculate the total distance travelled by the particle in the first π seconds.

QUESTION 8 Use a SEPARATE Writing Booklet. **Marks**

(a) **5**



The shaded region bounded by $y = e^{x^2}$, $y = 7$ and the y axis is rotated around the y axis to form a solid.

(i) Show that the volume of the solid is given by $V = \pi \int_1^7 \log_e y \, dy$.

(ii) Copy and complete the table. Give your answers correct to 3 decimal places.

y	1	4	7
$\log_e y$			

(iii) Use the trapezoidal rule with 3 function values to approximate the volume, V.

(b) A box contains five cards. Each card is labelled with a number. The numbers on the cards are 0, 3, 3, 5, 5. **5**

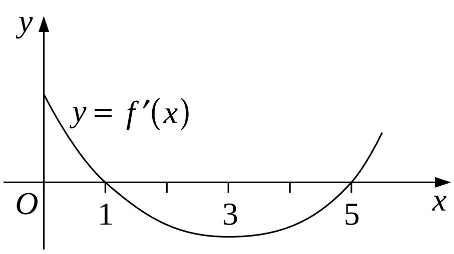
Cameron draws one card at random from the box and then draws a second card at random *without* replacing the first card drawn.

(i) What is the probability that he draws a '5', then a '3'?

(ii) What is the probability that the sum of the two numbers drawn is at least 8?

(iii) What is the probability that the second card drawn is labelled '3'?

(c) **2**

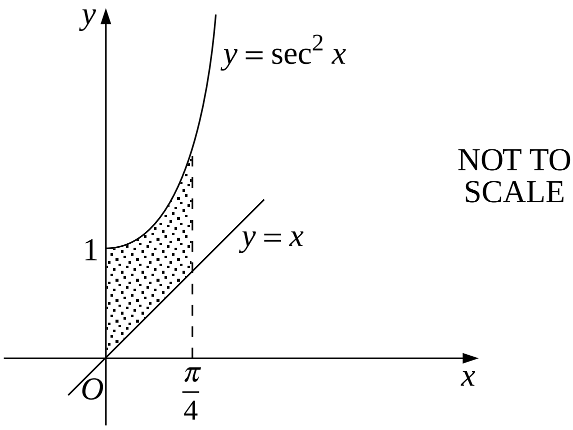


The diagram shows the graph of the gradient function of the curve $y = f(x)$.

For what value of x does $f(x)$ have a local minimum? Justify your answer.

QUESTION 9 Use a SEPARATE Writing Booklet. **Marks**

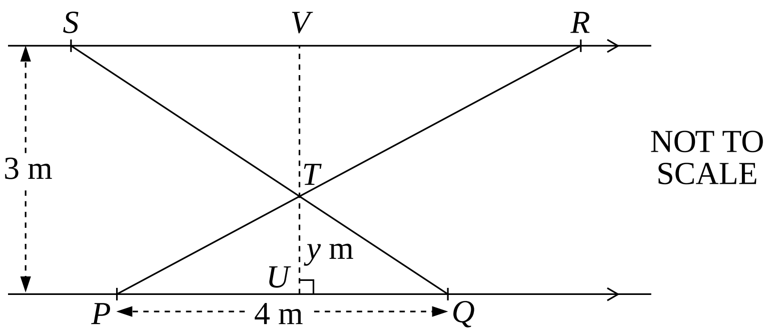
(a) **3**



The diagram shows the graphs of the functions $y = \sec^2 x$ and $y = x$ between $x = 0$ and $x = \dfrac{\pi}{4}$.

Calculate the area of the shaded region.

(b) **9**



In the diagram, PQ and SR are parallel railings which are 3 metres apart. The points P and Q are fixed 4 metres apart on the lower railing. Two crossbars PR and QS intersect at T as shown in the diagram. The line through T perpendicular to PQ intersects PQ at U and SR at V. The length of UT is y metres.

(i) By using similar triangles, or otherwise, show that $\dfrac{SR}{PQ} = \dfrac{VT}{UT}$.

(ii) Show that $SR = \dfrac{12}{y} - 4$.

(iii) Hence show that the total area A of ΔPTQ and ΔRTS is $A = 4y - 12 + \dfrac{18}{y}$.

(iv) Find the value of y that minimises A. Justify your answer.

QUESTION 10 Use a SEPARATE Writing Booklet. **Marks**

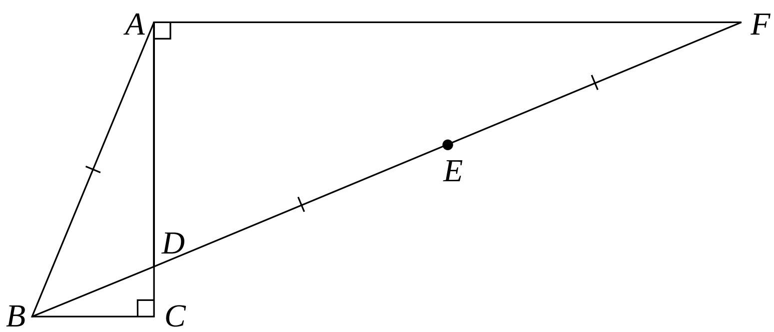
(a) (i) Show that $x = \dfrac{\pi}{3}$ is a solution of $\sin x = \dfrac{1}{2}\tan x$. **6**

(ii) On the same set of axes, sketch the graphs of the functions $y = \sin x$ and $y = \dfrac{1}{2}\tan x$ for $-\pi \le x \le \pi$.

(iii) Hence find all solutions of $\sin x = \dfrac{1}{2}\tan x$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

(iv) Use your graphs to solve $\sin x \le \dfrac{1}{2}\tan x$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

(b) **6**



In the diagram, $AC \perp BC$, $AC \perp AF$ and $AB = DE = EF$.

Copy or trace the diagram into your Writing Booklet.

(i) Show that $\angle DBC = \angle DFA$.

(ii) On your diagram, mark the point G on the line AF such that $EG \parallel AC$.

Show that $\Delta AGE \equiv \Delta FGE$.

(iii) Prove that $\angle ABD = 2\angle DBC$.

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$