2003 HSC Notes from the Marking Centre Mathematics Extension 2

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2003 HSC NOTES FROM THE MARKING CENTRE MATHEMATICS EXTENSION 2

Introduction

This document has been produced for the teachers and candidates of the Stage 6 course, Mathematics Extension 2. It is based on comments provided by markers on each of the questions from the Mathematics Extension 2 paper. The comments outline common sources of error and contain advice on examination technique and how best to present answers for certain types of questions.

It is essential for this document to be read in conjunction with the relevant syllabus, the 2003 Higher School Certificate Examination, the Marking Guidelines and other support documents that have been developed by the Board of Studies to assist in the teaching and learning of the Mathematics Extension 2 course.

Question 1

(a) This part was generally well done. Most candidates attempted this by substituting either $u = 1+e^x$ or $u = e^x$, while others used the reverse of the function of a function rule. Where candidates were not awarded full marks, it was generally due to errors in determining limits or in substituting the limits.

(b) Generally well done. Nearly all candidates knew how to integrate by parts. However, many candidates had difficulty as a result of not choosing their u and dv so as to avoid integrating $\ln x$.

(c) Most candidates were able to complete the square and nearly all candidates were able to correctly apply their completion of the square to the correct standard integral.

(d) (i) This part was generally well done.

(ii) Generally well done. The most common errors were leaving out the 3 for $3\ln x$ or giving incorrect constants for $\frac{1}{2}\tan^{-1}\frac{x}{2}$. Some candidates didn't know how to apply part (i) in doing this integration.

(e) Most candidates were able to correctly substitute $x = \sin\theta$ to find the new expression and limits. Some made it difficult to find the limits by confusing the limit $\frac{3}{\sqrt{2}}$ with $\frac{\sqrt{3}}{2}$. Many candidates were not able to recognise that $\int \frac{d\theta}{\cos^2\theta} = \int \sec^2\theta\, d\theta$ and so had difficulty going any further.

Question 2

This question assessed knowledge and skills in complex numbers. Although quite well done overall, there were some parts which were poorly done. At times inefficient methods were used.

(a) This part tested basic arithmetic skills and was well done by almost all candidates.

(b) (i) A common error was the argument in the fourth quadrant rather than in the second quadrant. Those candidates who drew a diagram were less likely to make this mistake.

(ii) This part was generally well done but many candidates did not use part (i), often finding all 4 roots from scratch or by remembering a formula for the roots of unity.

(iii) A common error was to say or imply that since $-1+i$ and $-1-i$ are roots then $(-1+i)(-1+i)$ is also a root. There is obvious confusion between the idea of roots and factors. Errors in expanding the linear factors were common. Many candidates wasted time by fully factorising z^4+4 over the reals instead of merely finding one real factor. Some, having found one quadratic, found the other by long division.

(c) This part was generally well done. However the corner of the wedge was often wrongly positioned at the origin.

(d) Many candidates had clearly not seen this application of de Moivre's theorem to find an expression for $\cos 5\theta$. Also a number of candidates did not know how to expand using the binomial theorem or Pascal's triangle.

(e) This part was poorly done. Those who used geometric methods were more successful. Few candidates were able to provide a correct algebraic method. A small number demonstrated some innovative solutions using trigonometry.

Question 3

This question proved to be a challenge for the average candidate.

(a) (i) Reasonably well done. Candidates should mark axes and indicate clearly where curves cross axes. They should clearly identify the graph to be marked as opposed to the working lines.

(ii) Many candidates did not gain full marks for this part. They correctly drew the graph for the region $-1 < x < 1$ but did not draw the graph outside this region. Many stated that the $f(x)$ and $|f(x)|$ cancelled each other out but did not draw this.

(iii) Reasonably well done. Most candidates gained full marks.

(iv) Reasonably well done. Again, candidates are encouraged to indicate clearly where curves cross axes and to clearly mark asymptotes.

(b) Well done. Candidates who did not gain full marks for this section generally made arithmetic errors (a frequent answer was $\frac{\sqrt{13}}{3}$) or failed to specify that the directrix was $x = \pm\frac{9}{\sqrt{5}}$. A bare answer of $\pm\frac{9}{\sqrt{5}}$ was not awarded a mark. Another common error was $y = \pm\frac{b}{a}x$ and other combinations of *a*, *b* and *e*.

(c) (i) There were several ways to successfully answer the question. Successful responses typically started with $\pi(3-x)^2 - \pi(x-1)^2$ or $\pi(3-x_1)^2 - \pi(3-x_2)^2$, showed that $x = 2 \pm \sqrt{1-y}$ (or equivalent) and argued through to the given conclusion. Others translated the graph to rotate around the *y*-axis. The most common error was to ignore the rotation around $x = 3$ and do the rotation around the *y*-axis.

(ii) This part was reasonably well done, although there were many candidates who integrated incorrectly to obtain $\frac{-8\pi}{3}$. Some candidates successfully used the method of shells but this resulted in a harder integral and was time consuming. They would be advised to use the given information.

Question 4

Question 4 consisted of deriving the inward force when there was uniform circular motion with an application; a theoretical conic question requiring candidates to prove a property of the hyperbola and a counting technique question with a probability application.

(a) (i) Those candidates who attempted this part generally were successful. Some started but did not progress far. Many candidates did not read the question carefully. There were a few approaches to deriving the result, including assuming non-uniform circular motion then stating $\dot{\omega} = 0$.

(ii) Many found this part very challenging.

(b) (i) This part was poorly done overall. It appeared that many candidates were not conversant with deriving the parametric form of the tangent at *P*. There were many forms for the equation of the tangent other than:
$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$ or $bx\sec\theta - ay\tan\theta = ab$.
Many were of a complicated nature. This made it difficult to do part (iii).

(ii) This was reasonably well attempted. Sometimes the candidate's algebraic manipulation skills were compromised and this lead to errors. Most tried to solve using simultaneous equations. Only a few checked that the points satisfied the equations. Some showed the points only lay on the tangent or asymptotes, but not both.

(iii) This part was found to be challenging. There were many different approaches to finding the area. For example,

$\frac{1}{2}OA \times \perp h_{from\ B}$; $\frac{1}{2}OB \times \perp h_{from\ A}$; $\frac{1}{2}AB \times \perp h_{from\ O}$; $\frac{1}{2}OA \times OB \sin \angle AOB$.

(c) (i) Either the candidates knew how to do this or they had no idea.

(ii) Many careless expressions were used to represent the probability. Many answers were not in the range $0 < \mathrm{P(E)} < 1$. Candidates could not properly express, in words, the complementary event required.

Question 5

Most candidates were able to answer one or more parts of the question. One notable feature in all parts was that some candidates labelled their work poorly, often omitting to say which part was being attempted.

(a) (i) An explanation of the value of the sum of the roots was required, answers like $\alpha + \beta + \gamma = \frac{b}{a}$ were not uncommon. The proof that $\alpha^2 + \beta^2 + \gamma^2 = -2p$ was successfully attempted by a large number of candidates, with most relying on the identity $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$. Many candidates also determined the coefficients of the polynomial with zeros $\alpha^2, \beta^2, \gamma^2$. This method was also commonly used in showing that $\alpha^3 + \beta^3 + \gamma^3 = -3q$, by replacing x with its cube root, although many noted and used the fact that $\alpha^3 + p\alpha + q = 0$, making the result in (ii) relatively easy to prove. Since the results were given, candidates had to justify each result clearly in order to gain marks.

(ii) Many candidates appeared to believe that proving the result for $n = 4$ was equivalent to proving it for $n > 3$. A number attempted a proof by induction but were ultimately unsuccessful, although a few did prove the result required within their attempt at induction without realising that they had done so. Those who observed that $\alpha^3 + p\alpha + q = 0$ were generally successful in gaining some marks.

(iii) Most candidates were aware that the result from (ii) was required, but were unable to proceed.

(b) The part on resisted motion proved to be challenging, although many candidates did show a very clear understanding.

(i) The requirement to 'derive' the result meant that showing that $\dot{x} = ue^{-kt}\cos\alpha$ was a solution of $\ddot{x} = -k\dot{x}$ was not sufficient to gain any marks. One notable feature of this part was that many candidates were confused about the meaning of $\dot{x}$, equating it to $u\cos\alpha$ whenever it appeared to be convenient to do so, without realising that it took this value only initially. A minority of candidates successfully adopted the strategy of integrating the original equation $\ddot{x} = -k\dot{x}$ with respect to t to get $\dot{x} = u\cos\alpha - kx$, and then using a second integration together with the initial conditions $t = 0 \Rightarrow x = 0$ to reach the final result.

(ii) Candidates were required to verify a solution of a differential equation and show that it satisfied the initial condition; many candidates successfully chose to integrate $\ddot{y} = -k\dot{y} - g$. Those who simply chose to verify the solution often lost marks either because their verification was not clearly set out, with intricate steps being omitted, or because of their failure to check the initial condition. A significant number of students were careless in their removal of the parentheses.

(iii) The vast majority of candidates stated and attempted to solve the appropriate equation.

(iv) Candidates had to integrate the expression for $\dot{x}$ and then evaluate its limit as $t \to \infty$. Some candidates performed the integration correctly, but some carelessly disregarded the constant.

Question 6

This question contained several proofs involving inequalities, and many of the candidates showed a lack of appreciation for logic and rigour in the setting out of a mathematical proof.
Some candidates set out to prove all the inequalities by showing that $LHS - RHS \geq 0$, which for these questions was a more difficult approach. Students need to appreciate that this is not automatically the only method of proving an inequality.

(a) (i) Generally well done.

(ii) Fairly well done. A small percentage of candidates failed to use part (i) and tried other approaches such as integration by parts, which inevitably proved to be futile.

(b) (i) Generally well done.

(ii) This part caused some difficulty with about half the students not getting any marks. In this question a typical approach was to start with the required result and to square both sides. This method rarely earned full marks. The simplest algebraic method was to write $\left(x + \sqrt{x}\right)^2 = x^2 + x + 2x\sqrt{x} \geq x^2 + x$ (since $x \geq 0$) from which the result follows. A handful of students noticed that x, $\sqrt{x}$, and $\sqrt{x(x+1)}$ form the sides of a right-angled triangle, giving the result for positive values of x (the result is trivially true for $x = 0$).

(iii) This part was poorly done. In this question many candidates did not demonstrate the need to prove two initial cases and to assume the truth of (at least) the two previous cases in the induction hypothesis. Few were able to deal successfully with the algebra required to complete the proof.

(c) (i) The simplest method was to note that $\left(\sqrt{x} - \sqrt{y}\right)^2 = x + y - 2\sqrt{xy} \geq 0$, giving the result.

(ii) This required establishing 3 similar inequalities and summing to give the result. Using part (i) with $x = a^4$ and $y = b^4$ gives $a^4 + b^4 \geq 2a^2b^2$, etc. Not all candidates made use of part (i) but there were several other equally successful approaches.

(iii) This was done in a similar fashion to part (ii) but was found to be more difficult than part (ii).

(iv) This required linking parts (ii) and (iii) and substituting $d = a + b + c$. While many candidates could see how to obtain the result, some of the proofs lacked a logical flow.

Question 7

Many candidates successfully answered parts (a) and (b) of this question. Part (c) was more challenging and was not attempted by a large number of the candidates.

(a) Most candidates gained full marks for this part.

(b) (i) This part was generally well done. Unsuccessful attempts generally gave incorrect reasons for a pair of angles to be equal.

(ii) Generally well done.

(iii) This part was omitted by many candidates. Candidates who proved that the triangles *DSP* and *DTP* were congruent generally went on to gain full marks.

(c) (i) It was important for candidates to clearly indicate that they were using $\cos(\frac{\alpha}{2^n})\sin(\frac{\alpha}{2^n}) = \frac{1}{2}\sin(\frac{\alpha}{2^{n-1}})$, and that they understood what P_{n-1} was.

(ii) Candidates who recognised that $P_{n-1}\sin\left(\frac{\alpha}{2^{n-1}}\right) = \frac{1}{2}P_{n-2}\sin\left(\frac{\alpha}{2^{n-2}}\right)$ were generally awarded the mark for this part.

(iii) Many who rewrote the result in (ii) as $\frac{\sin\alpha}{P_n} = 2^n\sin\left(\frac{\alpha}{2^n}\right)$ were able to go on and obtain the 2 marks for this part. Candidates should be aware that attempting to prove an inequality by beginning with the result to be proved is not an appropriate method.

Question 8

(a) (i) Many candidates were able to show that $\omega^2 + \omega + 1$ was equal to zero. Attempts to use de Moivre's theorem were generally unsuccessful. A number of students put $\omega = 1$ to obtain the value 3.

(ii) Most candidates were able to use the binomial theorem.

(iii) Only a very small number of candidates managed to complete this part, since it relied on recalling the expansion of 2^n in terms of a sum of binomial coefficients, and also understanding from (i) how the sums of powers took the values 0, 0 and 3.

(iv) A number of candidates obtained one mark here by recognising the need to replace $1+\omega$ with $-\omega^2$, and so on. Many failed to clearly apply the fact that *n* was a multiple of 6.

(b) (i) A reasonably large number of candidates successfully completed the first integration by parts. However, many who did the first part, paired the x with the $\sin x$ term which led away from the required result.

(ii) The replacement of x^2 with $\frac{\pi^2}{4}-\left(\frac{\pi^2}{4}-x^2\right)$ had to be carried out in an appropriate manner with π^2 also replaced by $\frac{p^2}{q^2}$.

(iii) To answer this part completely candidates needed to consider both I_n and the first two terms in the sequence and note that they are all integers.

(iv) Very few candidates attempted this part.

(v) A small number of candidates managed to deduce that the earlier parts, predicated on the rationality of π, resulted in a contradiction. However, many tried to rearrange the earlier inequalities and 'solve' for p or q or n.

Mathematics Extension 2

2003 HSC Examination Mapping Grid

Question	Marks	Content	Syllabus outcomes
1 (a)	2	4.1	E8
1 (b)	3	4.1	E8
1 (c)	2	4.1	E8
1 (d) (i)	2	7.6	E8
1 (d) (ii)	2	4.1	E8
1 (e)	4	4.1	HE6
2 (a) (i)	1	2.1	E3
2 (a) (ii)	1	2.1	E3
2 (b) (i)	2	2.2	E3
2 (b) (ii)	1	2.4	E3
2 (b) (iii)	2	2.1	E3
2 (c)	3	2.5	E3, E9
2 (d)	3	2.4	E2, E3, E9
2 (e)	2	2.3	E3, E9
3 (a) (i)	2	1.5	E6
3 (a) (ii)	2	1.2, 1.3	E6
3 (a) (iii)	1	1.6	E6
3 (a) (iv)	2	1.8	E6
3 (b)	3	3.1	E3, E4
3 (c) (i)	3	5.1	E7
3 (c) (ii)	2	5.1	E7
4 (a) (i)	3	6.3.2	E5
4 (a) (ii)	1	6.3.2	E5
4 (b) (i)	2	3.2	E3, E4
4 (b) (ii)	2	3.2	E3, E4
4 (b) (iii)	4	3.2	E3, E4
4 (c) (i)	1	8	PE3, E9
4 (c) (ii)	2	8	PE3, E9, H5
5 (a) (i)	3	7.5	E4
5 (a) (ii)	2	7.5	E4, E9
5 (a) (iii)	2	7.5	E4, E9
5 (b) (i)	2	6.2	E5

Question	Marks	Content	Syllabus outcomes
5 (b) (ii)	2	6.2	E5
5 (b) (iii)	2	6.2	E5
5 (b) (iv)	2	6.2	E5
6 (a) (i)	1	8	H5
6 (a) (ii)	2	8	E8
6 (b) (i)	1	8.2	E2
6 (b) (ii)	2	8.2	E2
6 (b) (iii)	3	8.2	E2, HE2
6 (c) (i)	1	8.3	E2, E9
6 (c) (ii)	2	8.3	E2, E9
6 (c) (iii)	2	8.3	E2, E9
6 (c) (iv)	1	8.3	E2, E9
7 (a)	3	5.1	E7
7 (b) (i)	2	8.1	PE3, E2
7 (b) (ii)	2	8.1	PE3, E2
7 (b) (iii)	3	8.1	PE3, E2
7 (c) (i)	2	8	E2, E9
7 (c) (ii)	1	8	E2, E9
7 (c) (iii)	2	8	E2, E9
8 (a) (i)	2	7.4	E3, E4
8 (a) (ii)	1	7.4, 8	E3, E4
8 (a) (iii)	2	7.4, 8	E3, E4, E9
8 (a) (iv)	2	7.4, 8	E3, E4, E9
8 (b) (i)	3	4.1	E2, E8, E9
8 (b) (ii)	1	4.1	E2, E8, E9
8 (b) (iii)	1	4.1	E2, E8, E9
8 (b) (iv)	2	4.1	E2, E8, E9
8 (b) (v)	1	4.1	E2, E8, E9

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2003 HSC Mathematics Extension 2
Marking Guidelines

Question 1 (a)
Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Appropriate substitution done correctly or correct primitive or equivalent merit	1

Question 1 (b)
Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution (+ C may be omitted)	3
• Obtains $\frac{x^4}{4}\ln x - \int \frac{x^4}{4} \cdot \frac{1}{x}\,dx$, or a single minor error or equivalent	2
• Reasonable attempt to use the method of integration by parts	1

Question 1 (c)
Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct answer (may omit the + C)	2
• Completes square or correct integral following from incorrect completion of square	1

Question 1 (d) (i)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Obtains $a(x^2+4)+(bx-1)(x-1)\equiv 5x^2-3x+13$ and either attempts to equate coeffs of x^2, x and const. or substitute $x = 1$ or equivalent progress	1

Question 1 (d) (ii)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution (+ C may be omitted) a and b must both be non-zero	2
• Correct treatment of at least one of $\int \frac{a\,dx}{x-1}$, $\int \frac{bx\,dx}{x^2+4}$ or $\int \frac{-dx}{x^2+4}$	1

Question 1 (e)

Outcomes assessed: HE6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	4
• Rewrites as $\frac{1}{9}\int_0^{\frac{\pi}{4}} \sec^2\theta\,d\theta$ or equivalent progress	3
• Obtains correct definite integral in terms of q or equivalent progress	2
• Rewrites integral in terms of q correctly or changes limits of integration correctly	1

Question 2 (a) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (a) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (b) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Correct modulus or argument	1

Question 2 (b) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (b) (iii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Recognition that $(z+1\pm i)$ are factors	1

Question 2 (c)

Outcomes assessed: E3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Correct except for wrong boundaries or ‘centre’ for one inequality	2
• Answer shows understanding of where one of the inequalities is satisfied	1

Question 2 (d)

Outcomes assessed: E2, E3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	3
• Applies de Moivre's theorem and uses $\sin^2\theta = 1 - \cos^2\theta$ but does not obtain the correct expression	2
• Uses de Moivre's theorem and equates real parts or equivalent progress	1

Question 2 (e)

Outcomes assessed: E3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct proof	2
• Writes $z = (\cos 2\theta + i\sin 2\theta)$ and attempts to apply double angle formula to 1 + z or equivalent progress	1

Question 3 (a) (i)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct graph	2
• Substantially correct, but missing one of y-intercept, correct behaviours as $x \to \pm 1$, correct behaviour as $x \to \pm\infty$	1

Question 3 (a) (ii)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct graph [open circles at $(\pm 1, 0)$ not required]	2
• Correct for $-1 < x < 1$ or correct except for missing y intercept, or equivalent	1

Question 3 (a) (iii)
Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct shape (y-intercept not required)	1

Question 3 (a) (iv)
Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct graph (y-intercept not required)	2
• Correct behaviour at $x \pm 1$ or correct behaviour as $x \to \pm\infty$	1

Question 3 (b)
Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Correct foci and directrices for incorrect eccentricity, or correct eccentricity and either correct foci or correct directrices or equivalent merit	2
• Correct eccentricity or equivalent merit	1

Question 3 (c) (i)
Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Obtains correct expression for the area of the annulus in terms of x and attempts to use the fact that $y = (x-1)(3-x)$ appropriately or equivalent	2
• Obtains correct expression for the area of the annulus in terms of appropriate x	1

Question 3 (c) (ii)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Obtains $V = 4\pi \int_0^1 \sqrt{1-y}\, dy$ or equivalent merit	1

Question 4 (a) (i)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Obtains value for x and y components of F and makes a reasonable attempt to resolve normally and tangentially or equivalent	2
• Obtains values for x and y components of F	1

Question 4 (a) (ii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 4 (b) (i)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Obtains $\frac{dy}{dx} = \frac{b^2}{a^2}\left(\frac{x}{y}\right)$ or equivalent	1

Question 4 (b) (ii)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Demonstrates knowledge of the fact that the asymptotes are $y = \pm\frac{b}{a}x$ and finds the intersection of the tangent in (i) with one of these lines or shows that one of A or B lies on the tangent and the appropriate asymptote	1

Question 4 (b) (iii)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Correct proof	4
• Obtains a correct expression for area which is independent of q eg $\frac{1}{2}\left(a^2+b^2\right)\sin 2\alpha$	3
• Obtains an expression for the area in terms of q and other constants	2
• Calculates OA or OB or equivalent	1

Question 4 (c) (i)

Outcomes assessed: PE3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 4 (c) (ii)

Outcomes assessed: PE3, E9, H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Recognise that this is the complement of each door being chosen by exactly one person	1

Question 5 (a) (i)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Correct explanation/proof for any 2 of s_1, s_2, s_3	2
• Correct explanation/proof for any of s_1, s_2, s_3	1

Question 5 (a) (ii)

Outcomes assessed: E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Recognition that $\alpha^3 = -p\alpha - q$ in this context or equivalent	1

Question 5 (a) (iii)

Outcomes assessed: E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Application of the result in (ii) with $n = 5$	1

Question 5 (b) (i)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Uses $\frac{dv}{dt} = -kv$, or equivalent	1

Question 5 (b) (ii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Verifies that $\ddot{y} = -k\dot{y} - g$ but does not check initial condition, or equivalent	1

Question 5 (b) (iii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to solve $\left(ku\sin\alpha + g\right)e^{-kt} - g = 0$	1

Question 5 (b) (iv)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Finds $x = \dfrac{u\cos\alpha \; e^{-kt}}{-k}\left(+C\right)$	1

Question 6 (a) (i)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (a) (ii)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct answer (ignore + C)	2
• Rewrites in terms of $\cos 5x$ and $\cos x$	1

Question 6 (b) (i)
Outcomes assessed: E2

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 6 (b) (ii)
Outcomes assessed: E2

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Expands $\left(x+\sqrt{x}\right)^2$ or equivalent progress	1

Question 6 (b) (iii)
Outcomes assessed: E2, HE2

MARKING GUIDELINES

Criteria	Marks
• Correct solution including verification of the initial case	3
• Establishes the induction step using induction as outlined in section 8.2 of the syllabus, but does not verify both n = 1 and n = 2, or equivalent	2
• Verifies at least n = 1 and makes an appropriate induction hypothesis	1

Question 6 (c) (i)
Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 6 (c) (ii)
Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to apply (i) with suitable x and y	1

Question 6 (c) (iii)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to apply (i) with suitable x and y	1

Question 6 (c) (iv)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 7 (a)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Obtains the correct definite integral, or equivalent progress	2
• Attempts to apply the method of cylindrical shells	1

Question 7 (b) (i)

Outcomes assessed: E2, PE3

MARKING GUIDELINES

Criteria	Marks
• Correct proof with reasons	2
• Shows 1 pair of corresponding angles with reasons or correct proof with reasons omitted	1

Question 7 (b) (ii)

Outcomes assessed: PE3, E2

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Uses similarity to show $\frac{SP}{AP} = \frac{BP}{SP}$ or equivalent OR • Deduces that $PT = PS$ given $SP^2 = AP \times BP$ (and $TP^2 = AP \times BP$)	1

Question 7 (b) (iii)

Outcomes assessed: E2, PE3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Proves $\angle PTD = 90°$, or equivalent progress	2
• Proves $\Delta SPD \equiv \Delta TPD$ without full justification or equivalent	1

Question 7 (c) (i)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Uses $\sin\left(\frac{\alpha}{2^{n-1}}\right) = 2\sin\left(\frac{\alpha}{2^n}\right)\cos\left(\frac{\alpha}{2^n}\right)$ or equivalent	1

Question 7 (c) (ii)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 7 (c) (iii)
Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Recognises the need to use $\sin\left(\frac{\alpha}{2^n}\right) < \left(\frac{\alpha}{2^n}\right)$ or equivalent	1

Question 8 (a) (i)
Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Solution shows knowledge that $1+\omega+\omega^2=0$ or that $1+\omega^k+\omega^{2k}=3$ in the case $\omega^k=1$ or equivalent progress	1

Question 8 (a) (ii)
Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution (only one need be correct)	1

Question 8 (a) (iii)
Outcomes assessed: E3, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Solution involves $(1+\omega)^n + \left(1+\omega^2\right)^n + (1+1)^n$ or equivalent	1

Question 8 (a) (iv)

Outcomes assessed: E3, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Applies $1+\omega+\omega^2=0$ in this context, or equivalent	1

Question 8 (b) (i)

Outcomes assessed: E2, E8, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Integrates by parts twice	2
• Integrates by parts correctly once	1

Question 8 (b) (ii)

Outcomes assessed: E2, E8, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 8 (b) (iii)

Outcomes assessed: E2, E8, E9

MARKING GUIDELINES

Criteria	Marks
• Correct explanation	1

Question 8 (b) (iv)

Outcomes assessed: E2, E8, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• A solution which bounds the integrand in the definition of I_n	1

Question 8 (b) (v)

Outcomes assessed: E2, E8, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1